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1. Real Numbers

Exercise 1.1

1. Question

If a and b are two odd positive integers such that a > b, then prove that one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Answer

Given: a and b are two odd positive integers such that a>b.

To prove: out of the numbers $\frac{a+b}{2}, \frac{a-b}{2}$ one is odd and other is even.

Proof: a and b both are odd positive integers,Since,

$$\left(\frac{a+b}{2}\right)+\left(\frac{a-b}{2}\right)=\left(\frac{a+b+a-b}{2}\right)=a$$

which is an odd number as "a" is given to be a odd number.Hence one of the number out of $\frac{a+b}{2}, \frac{a-b}{2}$ must be even and other must be odd because adding two even numbers gives an even number and adding two odd numbers gives an even number. Here, $\frac{a+b}{2}$ will give an odd number and $\frac{a-b}{2}$ will give an even number.

EXAMPLE:

Take a = 7 and b = 3 such that a>b

Now, $\frac{a+b}{2}=\frac{7+3}{2}=\frac{10}{2}=5$ which is odd

And $\frac{a-b}{2}=\frac{7-3}{2}=\frac{4}{2}=2$ which is even

Conclusion: Out of out of the numbers $\frac{a+b}{2}, \frac{a-b}{2}$,$\frac{a+b}{2}$ is an odd number and $\frac{a-b}{2}$ is an even number.

2. Question

Prove that the products of two consecutive positive integers is divisible by 2.

Answer

Let the numbers are a and a-1

Product of these number: $a(a - 1) = a^2-a$

Case 1: When a is even:

a=2p

then $(2p)^2 - 2p \Rightarrow 4p^2 - 2p$

2p(2p-1) ……. it is divisible by 2

Case 2: When a is odd:

a = 2p+1

then $(2p+1)^2 - (2p+1) \Rightarrow 4p^2 + 4p + 1 - 2p - 1$

$= 4p^2 + 2p \Rightarrow 2p(2p + 1)$ …….. it is divisible by 2

Hence, we conclude that product of two consecutive integers is always divisible by 2

3. Question

Prove that the product of three consecutive positive integers is divisible by 6.

Answer

Let a be any positive integer and b=6. By division lemma there exists integers q and r such that

a = 6q+r where 0≤r<6a = 6q , or a = 6q+1 or, a=6q+2 or, a=6q+3or a=6q+4 or a=6q+5 Let n is any positive integer.

Since any positive integer is of the form 6q or, 6q + 1 or, 6q + 2 or, 6q + 3 or, 6q + 4 or, 6q + 5.

Case 1:

If n = 6q then

n(n + 1)(n + 2) = 6q (6q + 1)(6q + 2), which is divisible by 6.

Case 2:

If n = 6q + 1, then

n(n + 1)(n + 2) = (6q + 1)(6q + 2) (6q + 3)

= (6q + 1)2(3q + 1)3(2q + 1)

= 6(6q + 1)(3q + 1)(2q + 1), which is divisible by 6.

Case 3:

If n = 6q + 2, then

n(n + 1)(n + 2) = (6q + 2) (6q + 3)(6q + 4)

= 2(3q + 1)3(2q + 1)(6q + 4)

= 6(3q + 1)(2q + 1)(6q + 4), which is divisible by 6.

Case 4:

If n = 6q + 3, then

n(n + 1)(n + 2) = (6q + 3) (6q + 4)(6q + 5)

= 3(2q + 1)2(3q + 2)(6q + 5)

= 6(2q + 1)(3q + 2)(6q + 5), which is divisible by 6.Case 5:

If n = 6q + 4, then

n(n + 1)(n + 2) = (6q + 4) (6q + 5)(6q + 6)

= (6q + 4) (6q + 5)6(q + 1)

= 6(6q + 4) (6q + 5)(q + 1), which is divisible by 6.

Case 6:

If n = 6q + 5, then

n(n + 1)(n + 2) = (6q + 5) (6q + 6)(6q + 7)

= (6q + 5) 6(q + 1)(6q + 7)

= 6(6q + 5) (q + 1)(6q + 7), which is divisible by 6.

Hence, n(n + 1)(n + 2) is divisible by 6

4. Question

For any positive integer, prove that $n^3 - n$ divisible by 6.

Answer

$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$

For a number to be divisible by 6, it should be divisible by 2 and 3 both,

Divisibility by 3:

n – 1, n and n + 1 are three consecutive whole numbers.

By Euclid's division lemma

n + 1 = 3q + r, for some integer k and r < 3

As, r < 3 possible values of 'r' are 0, 1 and 2.

If r = 0

n + 1 = 3q

⇒ n + 1 is divisible by 3

⇒ n(n – 1)(n + 1) is divisible by 3

⇒ (n^3 – n) is divisible by 3

If r = 1

⇒ n + 1 = 3q + 1

⇒ n = 3q

⇒ n is divisible by 3

⇒ n(n – 1)(n + 1) is divisible by 3

⇒ (n^3 – n) is divisible by 3

If r = 2

⇒ n + 1 = 3q + 2

⇒ n + 1 – 2 = 3q

⇒ n – 1 = 3q

⇒ n - 1 is divisible by 3

⇒ n(n – 1)(n + 1) is divisible by 3

⇒ (n^3 – n) is divisible by 3

Divisibility by 2:

If n is even

Clearly, n(n – 1)(n + 1) is divisible by 2

If n is odd

⇒ n + 1 is even

⇒ n + 1 is divisible by 2

⇒ n(n – 1)(n + 1) is divisible by 2

Hence, for any value of n, n^3 - n is divisible by 2 and 3 both, therefore n^3 – n is divisible by 6.

5. Question

Prove that if a positive integer is of the form 6q + 5, then it is of the form 3q + 2 for some integer q, but not conversely.

Answer

let A = 6q + 5, be any number, where q is any positive integer.

Part 1: To show A is in the form of 3q + 2, where q is another integerA= 6q + 5 = 6q + 3 + 2 = 3(2q + 1) + 2 = 3q' + 2As, q is any positive integer, q' = 3q + 2 is also a positive integerand hence 6q + 5, is in form of 3q' + 5

Part 2: To show converse is not true, i.e. if a no is in the form of 3q + 2, then it may or may not be in the form of 6q + 5For example, consider:8 = 3(2) + 2 is in the 3q + 2 form, but it can't be expand in

6q + 5 form.

6. Question

Prove that the square of any positive integer of the form 5q + 1 is of the same form.

Answer

Let N = 5p + 1. Then,

According to the condition:

$N^2 = 25p^2 + 10p + 1 \Rightarrow 5(5p^2 + 2p) + 1 \Rightarrow 5A+1$

Where $A = 5p^2 + 2p$

Therefore N^2 is of the form 5m + 1.

7. Question

Prove that the square of any positive integer is of the form 3m or, 3m + 1 but not of the form 3m + 2.

Answer

we know, that any positive integer N is of the form 3q, 3q + 1 or, 3q + 2.

When N = 3q, then

$N^2 = 9q^2 = 3(3q)^2 = 3m$ where $m = 3q^2$

And, as q is an integer, $m = 3q^2$ is also an integer.

When N = 3q + 1,

then $N^2 = (3q + 1)^2 = 9q^2 + 6q + 1 \Rightarrow 3q(3q + 2) + 1 = 3m + 1$ where $m = q(3q + 2)$And, as q is an integer, m = q(3q + 2) is also an integer.

When N = 3q + 2, then $N^2 = (3q + 2)^2 = 9q^2 + 12q + 4 \Rightarrow 3(3q^2 + 4q + 1) + 1$

$3m + 1$ where, $m = 3q^2 + 4q + 1$

And, as q is an integer, $m = 3q^2 + 4q + 1$ is also an integer.

Therefore,

N^2 is of the form 3m, 3m + 1 but not of the form 3m + 2

8. Question

Prove that the square of any positive integer is of the form 4q or 4q + 1 for some integer q.

Answer

Since any positive integer n is of the form 2p or, 2p + 1

When n = 2p, then $n^2 = 4p^2 = 4a$ where $a = p^2$

When n = 2p + 1, then $n^2 = (2p + 1)^2 = 4p^2 + 4p + 1 \Rightarrow 4p(p + 1) + 1$

$\Rightarrow 4m + 1$ where $m = p(p + 1)$

Therefore square of any positive integer is of the form 4q or 4q + 1 for some integer q

9. Question

Prove that the square of any positive integer is of the form 5q, 5q + 1, 5q + 4 for some integer q.

Answer

Since any positive integer n is of the form 5p or 5p + 1, or 5p + 2 or 5m + 3 or 5p +4.

When n = 5p, then

$n^2 = (5p)^2 = 25p^2 \Rightarrow 5\ (5p^2) = 5a$, where $a = 5p^2$

When n = 5p + 1,

then $n^2 = (5p + 1)^2 = 25p^2 + 10p + 1$

$\Rightarrow 5p(5p + 2) + 1 \Rightarrow 5a + 1$ Where $a = p(5p + 2)$

When n = 5p + 2,

then $n^2 = (5p + 2)^2 \Rightarrow 25p^2 + 20p + 4 \Rightarrow 5p(5p + 4) + 4$

$\Rightarrow 5a + 4$ where $a = p(5p + 4)$

When n = 5p + 3, then $n^2 = 25p^2 + 30p + 9$

$\Rightarrow 5(5p^2 + 6p + 1) + 4 \Rightarrow 5a + 4$ where $a = 5p^2 + 6m + 1$

When n = 5p + 4, then $n^2 = (5p + 4)^2 = 25p^2 + 40p + 16 \Rightarrow 5(5p^2 + 8m + 3) + 1 = 5a + 1$ where $a = 5p^2 + 8m + 3$

Therefore from above results we got that n2 is of the form 5q or, 5q + 1 or, 5q + 4.

10. Question

Show that the square of an odd positive integer is of the form 8q + 1, for some integer q.

Answer

To show: the square of an odd positive integer is of the form 8q + 1, for some integer q.

Solution: Let a be any positive integer and b=4. Applying the Euclid's division lemma with a and b=4 we havea = 4p + r where $0 \le r < 4$ and p is some integer,$\Rightarrow$ r can be 0,1,2,3$\Rightarrow$ a = 4p + 0 , a = 4p + 1 , a = 4p+ 2 , a = 4p + 3 , Since a is odd integer,So a = 4p + 1 or a = 4p + 3 So any odd integer is of the form a = 4p+ 1 or a = 4p+ 3 .Since any odd positive integer n is of the form 4p + 1 or 4p + 3.

When n = 4p + 1,

then $n^2 = (4p + 1)^2$

Apply the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (4p + 1)^2 = 16p^2 + 8p + 1$ ..... (1)

Take 8p common out of $16p^2 + 8p \Rightarrow 8p$

$(2p+ 1) + 1 = 8q + 1$ where $q = p(2p + 1)$

If $n = 4p + 3$, then $n^2 = (4p + 3)^2$

Apply the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (4p + 3)^2 = 16p^2 + 24p + 9$

Now $16p^2 + 24p + 9$ can be written as $16p^2 + 24p + 8 + 1$

$\Rightarrow (4p + 3)^2 = 8(2p^2 + 3p + 1) + 1 = 8q + 1$ where $q = 2p^2 + 3p + 1$

From above results we got that n^2 is of the form $8q + 1$.

Note: To show that the square of an odd positive integer is of the form 8q + 1 We have started the question from taking b=4 initially because whenwe take square of any form of 4 such as 4p+1 we end up having the values which are the multiple of 8 as in (1).While attempting these types of questions always remember to start with the value which would end up giving the value the question demands.and follow the above steps.

11. Question

Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Answer

To prove: any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Solution: Let 'a' be any odd positive integer we need to prove that a is of the form 6q+1, or 6q+3, or 6q+5, where q is some integer.

Since a is an integer consider b = 6 another integer applying Euclid's division lemma there exist integers q and r such that we get,

$a = 6q + r$ for some integer $q \neq 0$, and $r = 0, 1, 2, 3, 4, 5$ since $0 \leq r < 6$.

Therefore according to question:

$a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

However since a is odd so 'a' cannot take the values 6q, 6q+2 and 6q+4

(since all these are even integers, hence divisible by 2)

Therefore $a = 6q + 1$, $a = 6q + 3$, $a = 6q + 5$

Exercise 1.2

1. Question

Define HCF of two positive integers and find the HCF of the following pairs of numbers:

(i) 32 and 54 (ii) 18 and 24

(iii) 70 and 30 (iv) 56 and 88

(v) 475 and 495 (vi) 75 and 243

(vii)240 and 6552(viii)155 and 1385

(ix) 100 and 190 (x) 105 and 120

Answer

Definition of HCF (Highest Common Factor): The largest positive integer which divides two or more integers without any remainder is called Highest Common Factor (HCF) or Greatest Common Divisor or Greatest Common Factor (GCF).

(i) Prime factorization of 32 and 54 are:

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$54 = 2 \times 3 \times 3 \times 3$

From above prime factorization we got that the highest common factor of 32 and 54 is 2

(ii) Prime factorization of 18 and 24 are:

$18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

From above prime factorization we got that the highest common factor of 18 and 24 is $3 \times 2 \Rightarrow 6$

(iii) Prime factorization of 30 and 70 are:

$30 = 2 \times 3 \times 5$

$70 = 2 \times 5 \times 7$

From above prime factorization we got that the highest common factor of 30 and 70 is $2 \times 5 \Rightarrow 10$

(iv) Prime factorization of 56 and 88 are:

$56 = 2 \times 2 \times 2 \times 7$

$88 = 2 \times 2 \times 2 \times 11$

From above prime factorization we got that the highest common factor of 56 and 88 is $2 \times 2 \times 2 \Rightarrow 8$

(v) Prime factorization of 475 and 495 are:

$475 = 5 \times 5 \times 19$

$495 = 3 \times 3 \times 5 \times 11$

From above prime factorization we got that the highest common factor of 475 and 495 is 5

(vi) Prime factorization of 75 and 243 are:

$75 = 3 \times 5 \times 5$

$243 = 3 \times 3 \times 3 \times 3 \times 3$

From above prime factorization we got that the highest common factor of 75 and 243 is 3

(vii) Prime factorization of 75 and 243 are:

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$6552 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 13$

From above prime factorization we got that the highest common factor of 240 and 6552 is $2 \times 2 \times 2 \times 3 \Rightarrow 24$

(viii) Prime factorization of 155 and 1385 are:

$155 = 5 \times 31$

$1385 = 5 \times 277$

From above prime factorization we got that the highest common factor of 155 and 1385 is 5

(ix) Prime factorization of 100 and 190 are:

$100 = 2 \times 2 \times 5 \times 5$

$190 = 2 \times 5 \times 19$

From above prime factorization we got that the highest common factor of 155 and 1385 is $2 \times 5 \Rightarrow 10$

(x) Prime factorization of 105 and 120 are:

$105 = 3 \times 5 \times 7$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

From above prime factorization we got that the highest common factor of 105 and 120 is $3 \times 5 \Rightarrow 15$

2. Question

Use Euclid's division algorithm to find the HCF of

(i) 135 and 225 (ii)196 & 38220

(iii) 867 & 255.

Answer

(i) Concept used : To obtain the HCF of two positive integers, say c and d, with c > d,we followthe steps below:Step 1 : Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.Now, We know that,

= 225>135

Applying Euclid's division algorithm:(Dividend = Divisor × Quotient + Remainder)

$225 = 135 \times 1 + 90$

Here remainder = 90,

So, Again Applying Euclid’s division algorithm

135 = 90×1+45

Here remainder = 45,

So, Again Applying Euclid’s division algorithm

90 = 45×2+0

Remainder = 0,

Hence,

HCF of (135, 225) = 45

(ii) Concept used : To obtain the HCF of two positive integers, say c and d, with c > d,we followthe steps below:Step 1 : Apply Euclid’s division lemma, to c and d. So, we find whole numbers, q and r such that c = dq + r, 0 ≤ r < d.Step 2 : If r = 0, d is the HCF of c and d. If r ≠ 0, apply the division lemma to d and r.Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.Now, We know that,

38220>196

So, Applying Euclid’s division algorithm

38220 = 196×195+0 (Dividend = Divisor × Quotient + Remainder)

Remainder = 0

Hence,

HCF of (196, 38220) = 196

(iii) Concept used : To obtain the HCF of two positive integers, say c and d, with c > d,we followthe steps below:Step 1 : Apply Euclid’s division lemma, to c and d. So, we find whole numbers, q and r such that c = dq + r, 0 ≤ r < d.Step 2 : If r = 0, d is the HCF of c and d. If r ≠ 0, apply the division lemma to d and r.Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.Now, We know that,

867>255

So, Applying Euclid’s division algorithm

867 = 255×3+102 (Dividend = Divisor × Quotient + Remainder)

Remainder = 102

So, Again Applying Euclid’s division algorithm

255 = 102×2+51

Remainder = 51

So, Again Applying Euclid’s division algorithm

102 = 51×2+0

Remainder = 0

Hence,

(HCF 0f 867 and 255) = 51

3. Question

Find the HCF of the following pairs of integers and express it as a linear combination of them

(i) 963 & 657 (ii) 592 & 252

(iii) 506 & 1155 (iv) 1288 & 575

Answer

(i) Using Euclid’s Division Lemma

a = bq + r, (o ≤r<b)

963 = 657×1 + 306

657 = 306×2 + 45

306 = 45×6+36

45 = 36×1+9

36 = 9×4+0

∴ HCF (657, 963) = 9

linear form:657a + 306b = 9The above equation have many solutions, one of them is a = -15, b = 22i.e. 9 = 657(-15) + 306(22)

(ii) Using Euclid’s Division Lemma

a = bq + r, (o ≤r<b)

592 = 252×2+88

252 = 88×2+76

88 = 76×1+12

76 = 12×6+4

12 = 4×3+0

∴ HCF (592, 252) = 4

linear form:592a + 252b = 4The above equation have many solutions, one of them is a = 77, b = -20i.e. 4 = 592(77) + 252(20)

(iii) Using Euclid’s Division Lemma

a = bq + r, (o ≤r<b)

1155 = 506×2+143

506 = 143×3+77

143 = 77×1+66

77 = 66×1+11

66 = 11×6+0

∴ HCF (506, 1155) = 11

linear form:506a + 1155b = 11The above equation have many solutions, one of them is a = 16, b = -7i.e. 11 = 506(16) + 1155(-7)

(iv) Using Euclid's Division Lemma

a = bq + r, (o ≤r<b)

1288 = 575×2+138

575 = 138×4+23

138 = 23×6+0

∴ HCF (1288, 575) = 23

linear form:1288a + 575b = 23

The above equation have many solutions, one of them is a = 4, b = 9

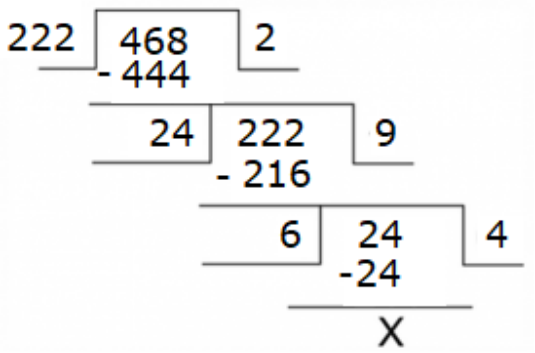
i.e. 23 = 1288(4) + 575(9)

4. Question

Express the HCF of 468 and 222 as 468x + 222y where x, y are integers in two different ways.

Answer

HCF of 468 and 222 is found by division mehtod:



Therefore, HCF (468,222) = 6

Now, we need to express the HCF of 468 and 222 as 468x + 222y where x and y are any two integers.

Now, HCF i.e. 6 can be written as,

HCF = 222 - 216 = 222 - (24 × 9)

Writing 468 = 222 × 2 + 24, we get,

⇒ HCF = 222 - {(468 – 222 x 2) × 9}

⇒ HCF = 222 - {(468 ×9) – (222 × 2 × 9)}

⇒ HCF = 222 - (468 × 9) + (222 × 18)

⇒ HCF = 222 + (222 × 18) - (468 × 9)

Taking 222 common from the first two terms, we get,

⇒ HCF = 222[1 + 18] – 468 × 9

⇒ HCF = 222 × 19 – 468 × 9

⇒ HCF = 468 ×(-9) + 222 ×(19)

Let, say, x = -9 and y =19

Then, HCF = 468 ×(x) + 222 ×(y)

Therefore the HCF of 468 and 222 is written in the form of 468x + 222y where, -9 and 19 are the two integers.

5. Question

If the HCF of 408 and 1032 is expressible in the form 1032 m – 408 × 5, find m.

Answer

Prime factors of 408: 2 × 2 × 2 x 3 × 17

Prime factors of 1032: 2 × 2 ×2 × 3 × 43

HCF of 408 and 1032 = 2 × 2 × 2 × 3 = 24

According to question:

24 = 1032 m – 408 × 5

24 = 1032 m – 2040

2064 = 1032 m

$\therefore m = \frac{2064}{1032} \Rightarrow 2$

Therefore m = 2

6. Question

If the HCF of 657 and 963 is expressible in the form 657 x + 963 × -15, find x.

Answer

Prime factors of 657: 3 × 3 × 73

Prime factors of 963: 3 × 3 × 107

HCF of 657 and 963 = 3 × 3 = 9

According to question: the HCF of 657 and 963 is expressible in the form 657 x + 963 × -15

⇒ 9 = 657x + 963 × -15

⇒ 9 = 657x - 14445

⇒ 9 + 14445 = 657x

∴ x = 14454/657 = 22

Therefore x = 22

7. Question

Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Answer

To find: the largest number which divides 615 and 963 leaving remainder 6 in each case.

Solution: Let the HCF of 615 and 963 be x.

Since it is given remainder is 6 in each case, Therefore for the numbers to be completely divisible, 6 should be subtracted from both the numbers.

Therefore new numbers are:

615-6 = 609

963-6 = 957

Prime factors of 609 = 3 × 3 × 29

Prime factors of 957= 3 × 11 × 29

Therefore HCF of 609 and 957 is: 3 × 29 = 87

Hence the largest number which divides 615 and 963 leaving remainder 6 in each case is 87.

8. Question

Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.

Answer

The new numbers after subtracting remainders are:

285-9 = 276

1249-7 = 1242

Prime factors of 276 = 23 × 3 × 2

Prime factors of 1242 = 2 × 3 × 3 × 3 × 23

Therefore HCF of 276 and 1242 is: 2 × 3 × 23 = 138

Hence the greatest number which divides 285 and 1249 leaving remainder 9 and 7 respectively is138

9. Question

Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.

Answer

The new numbers after subtracting remainders are:

280-4 = 276

1245-3 = 1242

Prime factors of 276 = 23 × 3 × 2

Prime factors of 1242 = 2 × 3 × 3 × 3 × 23

Therefore HCF of 276 and 1242 is: 2 × 3 × 23 = 138

Hence the greatest number which divides 280 and 1245 leaving remainder 4 and 3 respectively is138

10. Question

What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively?

Answer

The new numbers after subtracting remainders are:

626-1 = 625

3127-2 = 3125

15628-3 = 15625

Prime factors of 625 = 5 × 5 × 5 × 5

Prime factors of 3125 = 5 × 5 × 5 × 5 × 5

Prime factors of 15625 = 5 × 5 × 5 × 5 × 5 ×x 5

Therefore HCF of 625, 3125 and 15625 is: 5 × 5 × 5 × 5= 625

Hence the largest number which divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively is 625

11. Question

Find the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively.

Answer

The new numbers after subtracting remainders are:

445-4 = 441

572-5 = 567

699-6 = 693

Prime factors of 441= 3 × 3 × 7 × 7

Prime factors of 567 = 3 × 3 × 3 × 3 × 7

Prime factors of 693 = 3 × 3 × 7 ×11

Therefore HCF of 441, 567 and 693 is: 3 x 3 x7 = 63

Hence the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively is 63

12. Question

Find the greatest number which divides 2011 and 2623 leaving remainder 9 and 5 respectively.

Answer

The new numbers after subtracting remainders are:

2011-9 = 2002

2623-5 = 2618

Prime factors of 2002= 2 × 7 × 11 × 13

Prime factors of 2618 = 2 × 7 × 11 × 17

Therefore HCF of 2002 and 2618 is: 2 × 7 × 11 = 154

Hence the greatest number which divides 2011 and 2623 leaving remainder 9 and 5 respectively is 154

13. Question

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer

To find maximum number of columns we should find HCF of 616 and 32

Using Euclid’s algorithms:

Let a = 616 and b = 32

a = bq + r, (o ≤r<b)

616 = 32×19+8

32 = 8×4+0

∴ HCF of 616 and 32 is 8

Therefore the maximum number of columns in which army contingent to march is 8

14. Question

A merchant has 120 litres of oil of one kind, 180 litres of another kind and 240 litres of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Answer

To find greatest capacity of tin we should find HCF of 120 and 180 and 240

Prime factors of 120= 2 × 2 × 2 × 3 × 5

Prime factors of 180 = 2 × 2 × 3 × 3 × 5

Prime factors of 240 = 2 × 2 × 2 × 2 × 3 × 5

Therefore HCF of 120, 180 and 240 is: 2 × 2 × 3 × 5 = 60

Therefore the greatest capacity of a tin is 60 Liters

15. Question

During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

Answer

For having the full packs of both and the same number of pencils and crayons, we need to find LCM of 24 and 32

Prime factors of 24 = 2 × 2 × 2 × 3

Prime factors of 32 = 2 × 2 × 2 × 2 × 2

Therefore LCM of 24 and 32 is: 2 × 2 × 2 × 2 × 2 × 3 = 96

Number of colour pencil packs = $\frac{96}{24}$ = 4

Number of crayons packs = $\frac{96}{32}$ = 3

Hence 4 packets of colour pencils and 3 packets of crayons would be bought.

16. Question

144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Answer

To find greatest number of cartons each stack would have, we should find HCF of 144 and 90

Prime factors of 144 = 2 × 2 × 2 × 2 × 3 × 3

Prime factors of 90 = 2 × 3 × 3 × 5

Therefore HCF of 144 and 90 is: 2 × 3 × 3 = 18

Therefore the greatest number of cartons each stack would have is: 18

17. Question

Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Answer

Given : Two brands of chocolates are available in packs of 24 and 15 respectively.

To find : the least number of boxes of each kind.

Solution:

To find the least number of boxes of each kind.we need to find LCM of 24 and 15

Prime factors of 24 = 2 × 2 × 2 × 3

Prime factors of 15 = 3 × 5

Therefore LCM of 24 and 15 is: 2 × 2 × 2 × 3 × 5 = 120

Number of boxes for first chocolate kind = $\frac{120}{24}$ = 5

Number of boxes for second chocolate kind = $\frac{120}{15}$ = 8

Hence 5 boxes of first kind and 8 boxes of second kind needed to buy.

18. Question

A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required?

Answer

Given: The size of the bathroom is 10 ft. by 8 ft.

To find: the size in inches of the tile required that has to be cut and how many such tiles are required?

Solution: To find the largest size of tile, we should find HCF of 10 and 8

Prime factors of 10 = 2 × 5

Prime factors of 8 = 2 × 2 × 2

Therefore HCF of 10 and 8 is: 2 ftSince i ft = 12 inches

Therefore the largest size of tile is: 2 × 12 inches = 24 inches

Area of bathroom = length of bathroom × breadth of bathroom

= 10 × 8

= 80 sq. ftArea of 1 tile = length of tile × breadth of tile

= 2 × 2

= 4 sq. ft

Number of tiles = $\frac{area\ of\ bathroom}{area\ of\ one\ tile}$

$= \frac{80}{4}$

= 20 tiles

NOTE: Always find the HCF of the given values to find their maximum.

19. Question

15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuit packets in each. How many biscuit packets and how many pastries will each box contain?

Answer

To find the number of biscuit packets and pastries, we should find HCF of 15 and 12

Prime factors of 15 = 3 × 5

Prime factors of 12 = 2 × 2 × 3

Therefore HCF of 15 and 12 is: 3

Number of pastries = $\frac{15}{3}$ = 5

Number of biscuit packets = $\frac{12}{3}$ = 4

20. Question

105 goats, 140 donkeys and 175 cow have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?

Answer

To find the largest number of animals, we should find HCF of 105, 140 and 175

Prime factors of 105 = 3 × 5 × 7

Prime factors of 140 = 2 × 2 × 5 × 7

Prime factors of 175 = 5 × 5 × 7

Therefore HCF of 105, 140 and 175 is: 5 × 7 = 35

Hence, 35 animals went in each trip

21. Question

The length, breadth and height of a room are 8 m and 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Answer

Given: The length, breadth and height of a room are 8 m and 25 cm, 6 m 75 cm and 4 m 50 cm.

To find: the longest rod which can measure the three dimensions of the room exactly.

Solution: To find the length of largest rod, we should find HCF of 8m and 25 cm, 6 m 75 cm and 4 m 50 cmAs 1 m = 100 cm⇒ 8m and 25 cm = 8 × 100 cm + 25 cm

= 800 cm + 25 cm

= 825 cm⇒ 6 m and 75 cm = 6 × 100 cm + 75 cm

= 600 cm + 75 cm

= 675 cm

⇒ 4 m and 50 cm = 4 × 100 cm + 50 cm

= 400 cm + 50 cm

= 450 cm

Length = 825 cm; Breadth = 675 cm; Height = 450 cm

Prime factors of 825 = 3 × 5 × 5 × 11

Prime factors of 675 = 3 × 3 × 3 × 5 × 5

Prime factors of 450 = 2 × 3 × 3 × 5 × 5

Therefore HCF of 825, 675 and 450 is: 3 × 5 × 5 = 75

Hence, the length of rod is: 75 cm

NOTE: Always find the HCF of the given values to find their maximum.

Exercise 1.3

1. Question

Express each of the following integers as a product of its prime factors:

(i) 420 (ii) 468

(iii) 945 (iv) 7325

Answer

(i) Prime factor of 420 = 2 × 2 × 3 × 5 × 7

(ii) Prime factor of 468 = 2 × 2 × 3 × 3 × 13

(iii) Prime factor of 945 = 3 × 3 × 5 × 7

(v) Prime factor of 7325 = 5 × 5 × 293

2. Question

Determine the prime factorization of each of the following positive integer:

(i) 20570 (ii) 58500

(iii) 45470971

Answer

(i) Prime factors of 20570 = 2 × 5 × 11 × 11 × 17

(ii) Prime factors of 58500 = 2 × 2 × 3 x 3 × 5 x 5 × 13

(iii) Prime factors of 45470971 = 7 × 7 × 13 x 13 × 17 × 17 × 19

3. Question

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer

Numbers are of two types - prime and composite. Prime numbers has only two factors namely 1 and the number itself whereas composite numbers have factors other than 1 and itself.For example: 7 is a prime number as it can be divided by 1 and 7 only whereas 14 is a composite number as it can be divided by 7, 2 and 1 From the question it can be observed that,

7 × 11 × 13 + 13 = 13 × (7 × 11 + 1) = 13 × (77 + 1) [taking 13 common]

= 13 × 78

= 13 × 13 × 6

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 = 5 × (7 × 6 × 4 × 3 × 2 x 1 + 1)

= 5 × (1008 + 1)

= 5 ×1009

In the above we observe that 1009 cannot be factorized further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

4. Question

Check whether 6^n can end with the digit 0 for any natural numbers n.

Answer

If any number ends with the digit 0, it should be divisible by 10 or in other words its prime factorization must include primes 2 and 5 both as 10 = 2 ×5

Prime factorization of 6^n = $(2 \times 3)^n$

In the above equation it is observed that 5 is not in the prime factorization of 6^n

By Fundamental Theorem of Arithmetic Prime factorization of a number is unique. So 5 is not a prime factor of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Exercise 1.4

1. Question

Find the LCM and HCF of the following pairs of integers and verify that LMC × HCF = Product of the integers:

(i) 26 and 91

(ii) 510 and 92

(iii) 336 and 54

Answer

(i) Prime factors of 26 = 2 × 13

Prime factors of 91 = 7 ×13

Hence LCM of 26 and 91 = 2 × 7 × 13 = 182

HCF of 26 and 91 is = 13

LCM × HCF = 182 × 13 = 2366

Product of two numbers = 26 × 91 = 2366

Hence from above result we got LCM × HCF = Product of the numbers

(ii) Prime factors of 510 = 2 × 3 × 5 × 17

Prime factors of 92 = 2 × 2 × 13

Hence LCM of 92 and 510 = 2 × 2 × 3 × 5 × 17 × 23 = 23460

HCF of 92 and 510 is = 2

LCM × HCF = 23460 × 2 = 46920

Product of two numbers = 92×510 = 46920

Hence from above result we got LCM × HCF = Product of the numbers

(iii) Prime factors of 336 = 2 × 2 × 2 × 2 × 3 × 7

Prime factors of 54 = 2 × 3 × 3 × 3

Hence LCM of 336 and 54 = 2 × 2 × 2 × 2 × 3 × 3 × 7 = 3024

HCF of 54 and 336 is = 2 × 3 = 6

LCM × HCF = 3024 × 6 = 18144

Product of two numbers = 336 × 54 = 18144

Hence from above result we got LCM × HCF = Product of the numbers

2. Question

Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

(iv) 40, 36 and 126

(v) 84, 90 and 12

(vi) 24, 15 and 36

Answer

(i) Prime factors of 12 = 2 × 2 × 3

Prime factors of 15 = 3 × 5

Prime factors of 21 = 3 ×7

Hence LCM of 12, 15 and 21 = 2 × 2 × 3 × 5 × 7 = 420

HCF of 12, 15 and 21 = 3

(ii) Prime factors of 17 = 1 × 17

Prime factors of 23 = 1 ×23

Prime factors of 29 = 1 × 29

Hence LCM of 17, 23 and 29 = 11339

HCF of 17, 23 and 29 = 1

(iii) Prime factors of 8 = 1 × 2 × 2 × 2

Prime factors of 9 = 1 × 3 × 3

Prime factors of 25 = 1 × 5 × 5

Hence LCM of 8, 9 and 25 = 1800

HCF of 8, 9 and 25 = 1

(iv) Prime factors of 40 = 2 × 2 × 2 × 5

Prime factors of 36 = 2 × 2 × 3 × 3

Prime factors of 126 = 2 × 3 × 3 × 7

Hence LCM of 40, 36 and 126 = 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

HCF of 40, 36 and 126 = 2

(v) Prime factors of 12 = 2 × 2 × 3

Prime factors of 84 = 2 × 2 × 3 × 7

Prime factors of 90 = 2 × 3 × 3 × 5

Hence LCM of 12, 84 and 90 = 2 × 2 × 3 × 3 × 5 × 7 = 1260

HCF of 12, 84 and 90 = 2 × 3 = 6

(vi) Prime factors of 15 = 3 × 5

Prime factors of 24 = 2 × 2 × 2 × 3

Prime factors of 36 = 2 × 2 × 3 × 3

Hence LCM of 15, 24 and 36 = 2 × 2 × 2 × 3 × 3 × 5 = 360

HCF of 15, 24 and 36 = 3

3. Question

Given that HCF (306, 657) = 9, find LCM (360, 657).

Answer

We know that LCM × HCF = Product of the numbers

Therefore LCM = $\frac{Product\ of\ the\ numbers}{HCF\ of\ the\ numbers} = \frac{306 \times 657}{9}$ = 22338

4. Question

Can two numbers have 16 as their HCF and 380 as their LCM? Give reason.

Answer

NO

LCM should be divisible by HCF since HCF is the common factor of both the numbers. But, in this case 380 is not divisible by 16 therefore, the two numbers are not possible which have 16 as their HCF and 380 as their LCM

5. Question

The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

Answer

We know that LCM × HCF = Product of the numbers

Therefore Other Number = $\frac{LCM \times HCF}{One\ number} = \frac{2175 \times 145}{725}$ = 435

6. Question

The HCF of two numbers is 16 and their product is 3072. Find their LCM.

Answer

We know that LCM × HCF = Product of the numbers

Therefore Other Number = $\frac{Product\ of\ the\ numbers}{HCF\ of\ the\ numbers} = \frac{3072}{16}$ = 192

7. Question

The LCM and HCF of two numbers are 180 and 6 respectively. If one of the numbers is 30, find the other number.

Answer

Given: The LCM and HCF of two numbers are 180 and 6 respectively. One of the numbers is 30.

To find: The other number

Solution: We know that:LCM × HCF = Product of the numbers

LCM × HCF = a × b ..... (1)

Where a and b are numbersOne number i.e a is 30.LCM is 180.HCF is 6.

Substitute the values in eq. (1) to get the other number.

$$\Rightarrow b = \frac{LCM \times HCF}{a}$$

$$\Rightarrow b = \frac{180 \times 6}{30}$$

$$\Rightarrow b = \frac{\cancel{180}^{6} \times 6}{\cancel{30}_{1}}$$

⇒ b = 6 × 6⇒ b = 36

Therefore, the other number is 36.

8. Question

Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

Answer

To find the smallest number we should find LCM of 468 and 520

Prime factors of 468 = 2 × 2 × 3 × 3 × 13

Prime factors of 520 = 2 × 2 × 2 × 5 × 13

Hence LCM of 468 and 520 = 2 × 2 ×2 × 3 × 3 ×5×13 = 4680

Therefore the smallest number which when increased by 17 is exactly divisible by both 520 and 468= LCM − 17 ⇒ 4680 − 17 = 4663

9. Question

Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.

Answer

Greatest number of 6 digits is 999999.

For this number to be divisible by 24, 15 and 36,

Required number must be divisible by the LCM of 24, 15 and 36 i.e., by 360.Now on dividing six digit greatest number by LCM we get 279 as remainder.

Therefore the greatest number of 6 digits exactly divisible by 24, 15 and 36

= Six digit greatest number – remainder

= 999999 – 279

= 999720

10. Question

Determine the number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

Answer

To find : The number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21

Solution : To be exactly divisible by each of 8, 15 and 21,the required number must be divisible by the LCM of 8, 15 and 21 i.e.by 840.

Now on dividing 110000 by 840 we get 800 as remainder.

Therefore the number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21 is 110000 – Remainder ⇒ 110000 – 800 = 109200

Now 109200 gives remainder 0 when divided by 8, 15 and 21 as it is completely divisible by their LCM.

11. Question

Find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

Answer

The smallest number would be the LCM of 28 and 32

LCM of 28 and 32 = 224

Therefore the required smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively would be:

Number = LCM – (sum of the remainders)

⇒ 224 – (12 + 8)

⇒ 224 – 20 = 204

12. Question

What is the smallest number that, when divided by 35, 56 and 91 leaves remainders of 7 in each case?

Answer

Therefore the required smallest number would be the LCM of the numbers

Prime factors of 35 = 5 × 7

Prime factors of 56 = 2 × 2 × 2×7

Prime factors of 91 = 7 × 13

LCM of 35, 56 and 91 ⇒ 2 × 2 × 2 × 5 × 7×13 = 3640

Number = LCM + Remainder

⇒ 3640 + 7 = 3647

13. Question

Find the least number that is divisible by all the numbers between 1 and 10 (both inclusive).

Answer

The least number would be the LCM of the numbers from 1 to 10

We want LCM(1,2,3,4,5,6,7,8,9,10)We can ignore 1, since any counting number is divisible by 1.We prime factor each of the counting numbers from 2 to 10 2 = 23 = 34 = 2×25 = 56 = 2×37 = 78 = 2×2×29 = 3×310 = 2×5 The LCM of all those must have as many factors ofeach prime that appears in any factorization 2 appears at most 3 times as a factor of 83 appears at most 2 times as a factor of 95 appears at most 1 time as a factor if 5 and 107 appears at most 1 time as a factor of 7

So the LCM has 3 factors of 2, 2 factors of 3, and 1 factor each of 5 and 7 LCM = 2×2×2×3×3×5×7 = 2520

Hence, the least number that is divisible by all the numbers between 1 and 10 (both inclusive) is 2520

14. Question

A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.

Answer

Length of the courtyard = 18 m 72 cm= [18(100) + 72] cm [As, 1 m = 100 cm]= 1872 cm

The breadth of the courtyard = 13 m 20 cm= [13(100) + 20] cm = 1320 cm

To find the maximum edge of the tile we need to calculate HCF of length and breadth,

Using Euler’s division lemma: a = pq + r where 0≤r<p

1872 = 1320 × 1 + 552As 'r' is not equals to 0,So apply Euler's division on 1320 and 552,

1320 = 552 × 2 + 216

As 'r' is not equals to 0,So apply Euler's division on 552 and 216,

552 = 216 × 2 + 120

As 'r' is not equals to 0,So apply Euler's division on 216 and 120,

216 = 120×1 + 96

As 'r' is not equals to 0,So apply Euler's division on 120 and 96,

120 = 96 ×1 + 24

As 'r' is not equals to 0,So apply Euler's division on 96 and 24,

96 = 24 × 4 + 0

Therefore HCF of 1872 and 1320 is 24Maximum edge can be 24 cm.

Number of tile = $\frac{Area\ of\ courtyard}{Area\ of\ one\ tile} = \frac{1872 \times 1320}{24 \times 24}$ = 4290 tiles

15. Question

A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60 and 72 km a day, round the field. When will they meet again?

Answer

Given: A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60 and 72 km a day.

To find: When will they meet again

Solution: Circumference of the circular field is 36 km.Use the formula:

$$speed = \frac{Distance}{Time}$$

Time taken by first cyclist = $\frac{circumference\ of\ the\ field}{speed\ of\ first\ cyclist} = \frac{360}{48}$ = 7.5 days As 1 day = 24 hours⇒ 7.5×24 = 180 Hrs

Similarly,

Time taken by second cyclist = $\frac{360}{60}$ = 6 days ⇒ 6×24 = 144 Hrs

Time taken by third cyclist = $\frac{360}{72}$ = 5 days ⇒ 5×24 = 120 we need to find the minimum time at which they will meet.So we need to find LCM of the time they all take.

Now LCM of 180, 144 and 120 is 720 hours,

Therefore the time when all three will meet again = $\frac{720}{24}$ = 30 days

Note: To find the minimum of given values always find their L.C.M.

16. Question

In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Answer

To calculate minimum distance Minimum distance we should calculate LCM

Prime factors of 80 = 2×2×2 × 2 ×5

Prime factors of 85 = 5 ×17

Prime factors of 90 = 2×3×3×5

LCM of 80, 85 and 90 ⇒ 2 ×2 × 2 ×2 × 3×3 ×5 × 17 = 12240Minimum distance is 12240 cm,As 1 m = 100 cmSo 12240 = 120 m 40 cm

Therefore the minimum distance each should walk so that he can cover the distance in complete steps is 122 m 40 cm

Exercise 1.5

1. Question

Show that the following numbers are irrational.

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$

(iii) 6 + $\sqrt{2}$ (iv) 3 - $\sqrt{5}$

Answer

(i) Let assume that $\frac{1}{\sqrt{2}}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $\frac{1}{\sqrt{2}} = \frac{\boldsymbol{p}}{\boldsymbol{q}}$

$\sqrt{2} = \frac{\boldsymbol{q}}{\boldsymbol{p}}$

$\frac{\boldsymbol{q}}{\boldsymbol{p}}$ is a rational number as p and q are integers. Therefore √2 is rational which contradicts the fact that √2 is irrational.

Hence, our assumption is false and we can say that $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let assume that 7√5 is rational therefore it can be written in the form of $\frac{\boldsymbol{p}}{\boldsymbol{q}}$

where p and q are integers and q≠0. Moreover, let p and q have no common divisor > 1.

$7\sqrt{5} = \frac{\boldsymbol{p}}{\boldsymbol{q}}$ for some integers p and q

Therefore $\sqrt{5} = \frac{\boldsymbol{p}}{\boldsymbol{7q}}$

$\frac{\boldsymbol{p}}{\boldsymbol{7q}}$ is rational as p and q are integers, therefore √5 should be rational.

This contradicts the fact that √5 is irrational.

Therefore our assumption that 7√5 is rational is false. Hence 7√5 is irrational.

(iii) Let assume that 6 + √2 is rational therefore it can be written in the form of $\frac{p}{q}$

where p and q are integers and q≠0. Moreover, let p and q have no common divisor > 1.

6 + √2 = $\frac{p}{q}$ for some integers p and q

Therefore √2 = $\frac{p}{q}$ - 6

Since p and q are integers therefore $\frac{p}{q}$ - 6 is rational, hence √2 should be rational.

This contradicts the fact that √2 is irrational. Therefore our assumption is false. Hence, 6 + √2 is irrational.

(iv) Let us assume that 3 - $\sqrt{5}$ is rational

Therefore 3 - $\sqrt{5}$ can be written in the form of $\frac{p}{q}$ where p and q are integers and q≠0

3 - $\sqrt{5}$ = $\frac{p}{q}$ ⇒ $\frac{p}{q}$ -3 = $\sqrt{5}$

⇒ $\frac{p-3q}{q}$ = $\sqrt{5}$

Since p, q and 3 are integers therefore $\frac{p-3q}{q}$ is rational number

But we know $\sqrt{5}$ is irrational number, Therefore it is a contradiction.

Hence 3 - $\sqrt{5}$ is irrational

2. Question

Prove that following numbers are irrationals:

(i) $\frac{2}{\sqrt{7}}$ (ii) $\frac{3}{2\sqrt{5}}$

(iii) $4+\sqrt{2}$ (iv) $5\sqrt{2}$

Answer

(i) Let assume that $\frac{2}{\sqrt{7}}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $\frac{2}{\sqrt{7}} = \frac{p}{q}$

$\sqrt{7} = \frac{2q}{p}$

$\frac{2q}{p}$ is a rational number as p and q are integers.

This contradicts the fact that √7 is irrational, so our assumption is incorrect. Therefore $\frac{2}{\sqrt{7}}$ is irrational

(ii) Let assume that $\frac{3}{2\sqrt{5}}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $\frac{3}{2\sqrt{5}} = \frac{p}{q}$

$\sqrt{5} = \frac{3q}{2p}$

$\frac{3q}{2p}$ is a rational number as p and q are integers.

This contradicts the fact that √5 is irrational, so our assumption is incorrect. Therefore $\frac{3}{2\sqrt{5}}$ is irrational

(iii) Let assume that $4+\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $4+\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - 4$

$\frac{p}{q} - 4$ is a rational number as p and q are integers.

This contradicts the fact that √2 is irrational, so our assumption is incorrect. Therefore $4+\sqrt{2}$ is irrational.

(iv) Let assume that $5\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $5\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{5q}$

$\frac{p}{5q}$ is a rational number as p and q are integers.

This contradicts the fact that √2 is irrational, so our assumption is incorrect. Therefore $5\sqrt{2}$ is irrational.

3. Question

Show that $2-\sqrt{3}$ is an irrational numbers.

Answer

Let assume that $2-\sqrt{3}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $2-\sqrt{3}=\frac{p}{q}$

$\sqrt{3}= 2 - \frac{p}{q}$

$\frac{2q-p}{q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $2-\sqrt{3}$ is irrational.

4. Question

Show that $3+\sqrt{2}$ is an irrational number.

Answer

Let assume that $3+\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $3+\sqrt{2}=\frac{p}{q}$

$\sqrt{2}= \frac{p}{q} - 3$

$\frac{p-3q}{q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $3+\sqrt{2}$ is irrational.

5. Question

Prove that $4-5\sqrt{2}$ is an irrational number.

Answer

Let assume that $4-5\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $4-5\sqrt{2}=\frac{p}{q}$

$5\sqrt{2}= \frac{p}{q} - 4$

$\frac{p-4q}{5q}$ is a rational number as p and q are integers.

This contradicts the fact that √2 is irrational, so our assumption is incorrect. Therefore $4-5\sqrt{2}$ is irrational.

6. Question

Show that $5-2\sqrt{3}$ is an irrational number.

Answer

To prove : $5-2\sqrt{3}$ is an irrational number.**Solution:**Let assume that $5-2\sqrt{3}$ is rational.

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $5-2\sqrt{3}=\frac{p}{q}$

$2\sqrt{3}= 5 - \frac{p}{q} \Rightarrow \sqrt{3} = \frac{5q-p}{2q}$

$\frac{5q-p}{2q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $5-2\sqrt{3}$ is irrational.

Note: Sometimes when something needs to be proved, prove it by contradiction.Where you are asked to prove that a number is irrational prove it by assuming that it is rational numberand then contradict it.

7. Question

Prove that $2\sqrt{3}-1$ is an irrational number.

Answer

Let assume that $2\sqrt{3}-1$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $2\sqrt{3}-1=\frac{p}{q}$

$2\sqrt{3}= \frac{p}{q} + 1$

$$\Rightarrow 2\sqrt{3} = \frac{p+q}{q}$$

$$\Rightarrow \sqrt{3} = \frac{p+q}{2q}$$

$\frac{p+q}{2q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $2\sqrt{3}-1$ is irrational.

8. Question

Prove that $2-3\sqrt{5}$ is an irrational number.

Answer

Given: $2-3\sqrt{5}$

To prove: $2-3\sqrt{5}$ is irrational.

Proof: Let assume that $2-3\sqrt{5}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write $2-3\sqrt{5}=\frac{p}{q}$

$$2-\frac{p}{q}=3\sqrt{5}$$

$\frac{2q-p}{3q}$ = √5

$\frac{2q-p}{3q}$ is a rational number as p and q are integers. This contradicts the fact that √5 is irrational, so our assumption is incorrect. Therefore $2-3\sqrt{5}$ is irrational.

Note: Sometimes when something needs to be proved, prove it by contradiction.Where you are asked to prove that a number is irrational prove it by assuming that it is rational numberand then contradict it.

9. Question

Prove that $\sqrt{5}+\sqrt{3}$ is irrational.

Answer

Let assume that $\sqrt{5}+\sqrt{3}$is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

Therefore we can write √5 = $\frac{p}{q}$ - √3

$(\sqrt{5})^2 = (\frac{p}{q} - \sqrt{3})^2$

$5 = \frac{p2}{q2} - \frac{2p\sqrt{3}}{q} + 3$

$5 - 3 = \frac{p2}{q2} - \frac{2p\sqrt{3}}{q}$

$\frac{p2}{q2}$ - 2 = $\frac{2p\sqrt{3}}{q}$

$\frac{p2-2q2}{qp}$ = √3

$\frac{p2-2q2}{qp}$ is a rational number as p and q are integers. This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $\sqrt{5}+\sqrt{3}$ is irrational.

10. Question

Prove that $\sqrt{3}+\sqrt{4}$ is an irrational number.

Answer

Given: $\sqrt{3}+\sqrt{4}$

To prove: $\sqrt{3}+\sqrt{4}$ is irrational.

Proof: Let assume that $\sqrt{3}+\sqrt{4}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

$$\sqrt{3}+\sqrt{4}=\frac{p}{q}$$

Therefore we can write, $\Rightarrow \sqrt{4}=\frac{p}{q}-\sqrt{3}$

Squaring both sides we get, $\Rightarrow \left(\sqrt{4}\right)^2=\left(\frac{p}{q}-\sqrt{3}\right)^2$

Apply the formula (a - b)^2 = a^2 + b^2 - 2ab in $\left(\frac{p}{q}-\sqrt{3}\right)^2$

$$\Rightarrow 4=\left(\frac{p}{q}\right)^2+\left(\sqrt{3}\right)^2-2\left(\frac{p}{q}\right)\sqrt{3}$$

$$\Rightarrow 4=\frac{p^2}{q^2}+3-\frac{2p\sqrt{3}}{q}$$

$$\Rightarrow 4-3=\frac{p^2}{q^2}-\frac{2p\sqrt{3}}{q}$$

$$\Rightarrow 1=\frac{p^2}{q^2}-\frac{2p\sqrt{3}}{q}$$

$$\Rightarrow \frac{p^2}{q^2} - 1 = \frac{2p\sqrt{3}}{q}$$

$$\Rightarrow \frac{p^2 - q^2}{q^2} = \frac{2p\sqrt{3}}{q}$$

$$\Rightarrow q\left(\frac{p^2 - q^2}{q^2}\right) = 2p\sqrt{3}$$

$$\Rightarrow \frac{p^2 - q^2}{2pq} = \sqrt{3}$$

$\frac{p^2 - q^2}{2pq}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $\sqrt{3} + \sqrt{4}$ is irrational.

Note: Sometimes when something needs to be proved, prove it by contradiction.Where you are asked to prove that a number is irrational prove it by assuming that it is rational numberand then contradict it.

11. Question

Prove that for any prime positive integer p, $\sqrt{p}$ is an irrational number.

Answer

Let assume that √p is rational

Therefore it can be expressed in the form of $\frac{a}{b}$, where a and b are integers and b≠0

Therefore we can write √p = $\frac{a}{b}$

$(\sqrt{p})^2 = (\frac{p}{q})^2$

P = $\frac{a2}{b2}$

$a^2 = pb^2$

Since a^2 is divided by b^2, therefore a is divisible by b.

Let a = kc

$(kc)^2 = pb^2$

$K^2c^2 = pb^2$

Here also b is divided by c, therefore b^2 is divisible by c^2. This contradicts that a and b are co-primes. Hence $\sqrt{p}$ is an irrational number.

12. Question

If p, q are prime positive integers, prove that $\sqrt{p} + \sqrt{q}$ is an irrational number.

Answer

Since it is given that p and q are prime positive integer.Let us assume that $\sqrt{p} + \sqrt{q}$ is a rational number of the form $\frac{a}{b}$,

$$\Rightarrow (\sqrt{q})^2 = \left(\frac{a}{b} - \sqrt{p}\right)^2$$

$$\Rightarrow \sqrt{p} + \sqrt{q} = \frac{a}{b}$$

$$\Rightarrow \sqrt{q} = \frac{a}{b} - \sqrt{p}$$

Squaring both sides we get,

Apply the formula $(a-b)^2 = a^2 + b^2 - 2ab$

$$\Rightarrow (\sqrt{q})^2 = \left(\frac{a}{b}\right)^2 + (\sqrt{p})^2 - 2\left(\frac{a}{b}\right)(\sqrt{p})$$

$$\Rightarrow q = \frac{a^2}{b^2} + p - 2\left(\frac{a}{b}\right)(\sqrt{p})$$

$$2\left(\frac{a}{b}\right)\sqrt{p} = \frac{a^2}{b^2} + p - q$$

$$\Rightarrow 2\left(\frac{a}{b}\right)(\sqrt{p}) = \frac{a^2 - pb^2 - qb^2}{b^2}$$

$$\Rightarrow \sqrt{p} = \frac{b\left(a^2 - pb^2 - qb^2\right)}{ab^2}$$

$$\Rightarrow \sqrt{p} = \frac{a^2 - pb^2 - qb^2}{ab}$$

Since p, q are integers therefore $\frac{a^2 - pb^2 - qb^2}{ab}$ is rational number

But we know $\sqrt{p}$ is irrational number, Therefore it is a contradiction.

Hence $\sqrt{p} + \sqrt{q}$ is irrational

Exercise 1.6

1. Question

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating

(i) $\frac{23}{8}$ (ii) $\frac{125}{441}$

(iii) $\frac{35}{50}$ (iv) $\frac{77}{210}$

(v) $\frac{129}{2^2 \times 5^7 \times 7^{17}}$

Answer

(i) $8 = 2^3$ since the denominator is of the form 2^m therefore the expression $\frac{23}{8}$ is terminating.

(ii) $441 = 3^2 \times 7^2$ since the denominator is not in the form of 2^m and 5^n Therefore the given expression is Non-terminating repeating

(iii) $\frac{7\times5}{10\times5} = \frac{7}{10}$

$10 = 2\times5$ therefore the denominator is of the form $2^m \times 5^n$

Hence, the given expression is Terminating

(iv) $\frac{77}{210} = \frac{7\times11}{30\times7} = \frac{11}{30}$

$30 = 2\times3\times5$

Since the denominator is not of the form $2^m \times 5^n$

Hence, the given expression is Non-terminating repeating Terminating

(v) $\frac{129}{2^2 \times 5^7 \times 7^{17}}$

Since the denominator is not in the form $2^m \times 5^n$

Hence, the given expression is Non-terminating repeating Terminating

2. Question

Write down the decimal expansions of the following rational numbers by writing their denominators in the form $2^m \times 5^n$, where m, n are non-negative integers.

(i) $\frac{3}{8}$ (ii) $\frac{13}{125}$

(iii) $\frac{7}{80}$ (iv) $\frac{14588}{625}$

(v) $\frac{129}{2^2 \times 5^7}$

Answer

(i) $\frac{3}{8} = \frac{3}{2^3} = \frac{3\times5^3}{2^3\times5^3}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{375}{10^3} = 0.375$

(ii) $\frac{13}{125} = \frac{13}{5^3} = \frac{13\times2^3}{2^3\times5^3}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{104}{10^3} = 0.104$

(iii) $\frac{7}{80} = \frac{7}{2^4\times5} = \frac{7\times5^3}{2^4\times5^4}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{875}{10^4} = 0.0875$

(iv) $\frac{14588}{625} = \frac{22\times7\times521}{5^4} = \frac{22\times2^4\times7\times521}{2^4\times5^4}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{22\times2^4\times7\times521}{(2\times5)^4} = 23.3408$

(v) $\frac{129}{2^2 \times 5^7} = \frac{2^5\times129}{2^7\times5^7} = \frac{2^5\times129}{(2\times5)^7}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{2^5\times129}{(2\times5)^7} = 0.0004128$

3. Question

What can you say about the prime factorizations of the denominators of the following rational?

(i) 43.123456789

(ii) $43.\overline{123456789}$

(iii) $27.\overline{142857}$

(iv) 0.120120012000120000…

Answer

(i) $43.123456789 = \frac{43123456789}{100000000} = \frac{43123456789}{2^9 \times 5^9}$

Since Prime factorization of the denominator is in the form $2^m \times 5^n$, where m, n are non-negative integers.

(ii) Since $43.\overline{123456789}$ has non-terminating repeating decimal expression, therefore Prime factorization of the denominator contains factors other than 2 or 5.

(iii) Since $27.\overline{142857}$ has non-terminating repeating decimal expression, therefore Prime factorization of the denominator contains factors other than 2 or 5.

(iv) Since $27.\overline{142857}$ has non-terminating repeating decimal expression, therefore Prime factorization of the denominator contains factors other than 2 or 5.

CCE - Formative Assessment

1. Question

State Euclid's division lemma.

Answer

Euclid's Division Lemma: Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, 0 ≤ r < b. (It is a technique to compute the Highest Common Factor (HCF) of two given positive integers.)

2. Question

State Fundamental Theorem of Arithmetic.

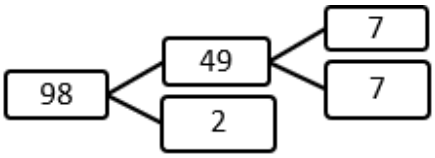
Answer

Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

3. Question

Write 98 as product of its prime factors.

Answer



$98 = 49 \times 2 = 7^2 \times 2$

98 is factorised as a product of primes i.e. $7^2 \times 2$.

4. Question

Write the exponent of 2 in the prime factorization of 144.

Answer

The prime factorization of 144 is as follows:

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow 144 = 2^4 \times 3^2$

We know that the exponent of a number a^m is m.

The exponent of 2 in the prime factorization of 144 is 4.

5. Question

Write the sum of the exponents of prime factors in the prime factorization of 98.

Answer

The prime factorization of 98 is as follows:

$98 = 2 \times 7 \times 7$

$\Rightarrow 98 = 2^1 \times 7^2$

We know that the exponent of a number a^m is m.

$\therefore$The sum of powers = 1 + 2 = 3

The sum of the exponents of prime factors in the prime factorization of 98 is 3.

6. Question

If the prime factorization of a natural number n is $2^3 \times 3^2 \times 5^2 \times 7$, write the number of consecutive zeros in n.

Answer

If any number ends with the digit 0, it should be divisible by 10,

i.e. it will be divisible by 2 and 5.

Prime factorization of n is given as $2^3 \times 3^2 \times 5^2 \times 7$.

It can be observed that there is $(2 \times 5) \times (2 \times 5)$

$\Rightarrow 10 \times 10 = 100$

Thus, there are 2 zeros in n.

The number of consecutive zeros in n is 2.

7. Question

If the product of two numbers is 1080 and their HCF is 30, find their LCM.

Answer

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Here a × b = 1080 and HCF = 30

∴ LCM = (a × b)/ HCF

⇒ LCM = 1080/30

⇒ LCM = 36

The LCM of given product of two numbers is 36.

8. Question

Write the condition to be satisfied by q so that a rational number has a terminating 9 decimal expansion.

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form 2^n 5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

9. Question

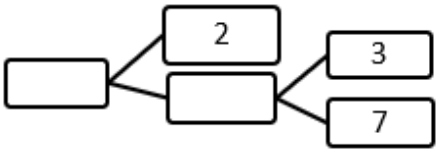
Write the condition to be satisfied by q so that a rational number has a non-terminating terminating decimal expansion.

Answer

Let x = p/q be a rational number, such that the prime factorization of q is not of the form 2^n 5^m, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

10. Question

Complete the missing entries in the following factor tree.



Answer

Let us take the last factors of the tree 3 and 7.

Then 3 × 7 = 21

Now we get the factors in the next level as 21 and 2.

So, 21 × 2 = 42

The missing entries are 42, 21.

11. Question

The decimal expansion of the rational number $\frac{43}{2^4 \times 5^3}$ will terminate after how many places of decimals?

Answer

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then 'x' has a decimal expansion which terminates after n or m places, whichever is maximum

The maximum power of 2 or 5 in the given rational number is 4.

So, it will terminate after 4 places of decimals.

The decimal expansion of the given rational number will terminate after 4 places of decimals.

12. Question

Has the rational number a terminating or a non terminating decimal $\frac{441}{2^2 \times 5^7 \times 7^2}$ representation?

Answer

Let $x = \frac{p}{q}$ be a rational number, then x is terminating if and only if, the denominator q has the form $2^n 5^m$

The denominator is not of the form $2^n 5^m$ as it has 7^2 as its factor.

Therefore, Given rational number has a non-terminating decimal representation.

13. Question

Write whether on simplification $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ gives a rational or an irrational number.

Answer

Given: number $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$

To find: Whether the given number is rational or irrational

Solution: Factorize 45 and 20.

$\Rightarrow \frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{3\times3\times5} + 3\sqrt{2\times2\times5}}{2\sqrt{5}}$

$\Rightarrow \frac{2\sqrt{3^2\times5} + 3\sqrt{2^2\times5}}{2\sqrt{5}}$

$\Rightarrow \frac{2\times3\sqrt{5} + 3\times2\sqrt{5}}{2\sqrt{5}}$

$\Rightarrow \frac{2\sqrt{5}(3+3)}{2\sqrt{5}}$

$\Rightarrow 3 + 3 = 6$

We know that a rational number is defined as the number which can be written in the form of p/q.As 6 can be written as 6/1.

So 6 is a rational number.

The given number after simplification gives a rational number.

14. Question

What is an algorithm?

Answer

An algorithm is a series of well-defined steps which gives a procedure for solving a type of problem. The word algorithm comes from the name of the 9th-century Persian mathematician al-Khwarizmi.

15. Question

What is a lemma?

Answer

A lemma is a proven statement used for proving another statement.

16. Question

If p and q are two prime numbers, then what is their HCF?

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

For any two prime numbers, one of the common prime factors will be 1.

17. Question

If p and q are two prime numbers, then what is their LCM?

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

If p and q are two prime numbers, then $p \times q$ is their LCM.

18. Question

What is the total number of factors of a prime number?

Answer

We know that the factors of a prime number are 1 and the number itself.

The total number of factors of a prime number is 2.

19. Question

What is a composite number?

Answer

Every composite number can be expressed (factorized) as a product of primes.

20. Question

What is the HCF of the smallest composite number and the smallest prime number?

Answer

Smallest composite number = 4

= 2 × 2

Smallest prime number = 2

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

HCF (4, 2) = 2

The HCF of the smallest composite number and the smallest prime number is 2.

21. Question

HCF of two numbers is always a factor of their LCM (True/False).

Answer

True.

LCM of two or more numbers is always divisible by their HCF.

Example: Let us take two numbers 6 and 20.

The prime factorization of

$6 = 2^1 \times 3^1$ and $20 = 2^2 \times 5^1$

HCF = 2

LCM = $2^2 \times 3 \times 5 = 60$

∴ 2 is a factor of 60.

22. Question

π is an irrational number (True/False).

Answer

True

23. Question

The sum of two prime numbers is always a prime number (True/False).

Answer

False

Prime numbers are always odd numbers and the sum of odd numbers is even.

Example: Let two prime numbers be 3 and 5.

Sum of 3 and 5 = 3 + 5 = 8 which is not a prime number.

24. Question

The product of any three consecutive natural numbers is divisible by 6 (True/False).

Answer

True

Since n(n + 1)(n + 2) is divisible by 2 and 3, then it is divisible by 6.

Example: Let us take three consecutive numbers 10, 11, 12.

Product = 10 × 11 × 12 = 1320

∴ 1320 is divisible by 6.

25. Question

Every even integer is of the form 2m, where m is an integer (True/False).

Answer

True

Let a = bq + r. b = 2, q = m

r = 0.

a = 2m

26. Question

Every odd integer is of the form 2m - 1, where m is an integer (True/False).

Answer

True

Let a = bq + r: b = 2, q = m

-2<r<0 i.e., r = -1

a = 2m − 1 for odd integer.

27. Question

The product of two irrational numbers is an irrational number (True/False).

Answer

False

The product of two irrational is sometimes an irrational number.

Example: If we multiply an irrational number with 0 we will get the product as 0 which is a rational number.

28. Question

The sum of two irrational numbers is an irrational number (True/False).

Answer

False

The sum of two irrational numbers is sometimes an irrational number.

Example: If the two irrational numbers have a sum zero, then the sum is a rational number.

29. Question

For what value of n, $2^n \times 5^n$ ends in 5.

Answer

Let us take n = 1.

$2^1 \times 5^1 = 10$

So, for no value of n, $2^n \times 5^n$ ends in 5.

No value of n

30. Question

If a and b are relatively prime numbers, then what is their HCF?

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

For any two relatively prime numbers, one of the common prime factors will be 1.

31. Question

If a and b are relatively prime numbers, then what is their LCM?

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

If a and b are two prime numbers, then a × b is their LCM.

32. Question

Two numbers have 12 as their HCF and 350 as their LCM (True/False).

Answer

False

HCF of two numbers is always a factor of their LCM.

But 12 is not a factor of 350.

1. Question

The exponent of 2 in the prime factorisation of 144, is

A. 4

B. 5

C. 6

D. 3

Answer

The prime factorization of 144 is as follows:

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow 144 = 2^4 \times 3^2$

We know that the exponent of a number a^m is m.

$\therefore$ The exponent of 2 in the prime factorization of 144 is 4.

2. Question

The LCM of two numbers is 1200. Which of the following cannot be their HCF?

A. 600

B. 500

C. 400

D. 200

Answer

We know that LCM of two or more numbers is always divisible by their HCF.

1200 is divisible by 600, 200 and 400 but not by 500.

3. Question

If $n = 2^3 \times 3^4 \times 5^4 \times 7$, then the number of consecutive zeros in n, where n is a natural number, is

A. 2

B. 3

C. 4

D. 7

Answer

If any number ends with the digit 0, it should be divisible by 10,

i.e. it will be divisible by 2 and 5.

Prime factorization of n is given as $2^3 \times 3^4 \times 5^4 \times 7$.

It can be observed that there is (2 × 5) × (2 × 5) × (2 × 5)

⇒ 10 × 10 × 10 = 1000

Thus, there are 3 zeros in n.

4. Question

The sum of the exponents of the prime factors in the prime factorisation of 196, is

A. 1

B. 2

C. 4

D. 6

Answer

The prime factorization of 196 is as follows:

196 = 2 × 2 × 7 × 7

⇒ 98 = $2^2 \times 7^2$

We know that the exponent of a number a^m is m.

∴The sum of powers = 2 + 2 = 4

5. Question

The number of decimal places after which the decimal expansion of the rational number $\frac{23}{2^2 \times 5}$ will terminate, is

A. 1

B. 2

C. 3

D. 4

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n\ 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

The maximum power of 2 or 5 in the given rational number is 2.

So, it will terminate after 2 places of decimals.

6. Question

If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

A. an even number

B. an odd number

C. an odd prime number

D. a prime number

Answer

Let us take $p_1 = 5$ and $p_2 = 3$

Then $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

7. Question

If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3 q$; p, q being prime numbers, then LCM (a, b) is

A. pq

B. $p^3 q^3$

C. $p^3 q^2$

D. $p^2 q^2$

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

So, LCM(a, b) = p^3q^2

8. Question

In Q. No. 7, HCF (a, b) is

A. pq

B. $p^3 q^3$

C. $p^3 q^2$

D. $p^2 q^2$

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

So, HCF(a, b) = pq

9. Question

If two positive integers m and n are expressible in the form $m = pq^3$ and $n = p^3 q^2$ where p, q are prime numbers, then HCF (m, n) =

A. pq

B. pq^2

C. p^3q^3

D. p^2q^3

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

So, HCF(a, b) = pq^2

10. Question

If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =

A. 2

B. 3

C. 4

D. 1

Answer

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Here LCM = 36, HCF = 2 and b = 18

Then, 2 × 36 = a × 18

a = (2 × 36) / 18

a = 4

11. Question

The HCF of 95 and 152, is

A. 57

B. 1

C. 19

D. 38

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

The prime factorization of

95 = 5 × 19

And 152 = 2^3 × 19

∴ HCF = 19

12. Question

If HCF (26, 169) = 13, then LCM (26, 169) =

A. 26

B. 52

C. 338

D. 13

Answer

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Here HCF = 13, a = 26 and b = 169

Then,

13 × LCM = 26 × 169

LCM = (26 × 169) / 13

LCM = 338

13. Question

If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM (a, b, c) = $2^3 \times 3^2 \times 5$, then n =

A. 1

B. 2

C. 3

D. 4

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

Since the power of 3 in LCM is 2,

$C = 3^2 \times 5$

14. Question

The decimal expansion of the rational number 14587 / 1250 will terminate after

A. One decimal place

B. two decimal place

C. three decimal place

D. four decimal place

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

The denominator 1250 can also be written as 2×5^4.

The maximum power of 2 or 5 in 1250 is 4.

So, it will terminate after 4 places of decimals.

15. Question

If p and q are co-prime numbers, then p^2 and q^2 are

A. coprime

B. not coprime

C. even

D. odd

Answer

Given: If p and q are co-prime numbers.

To find: p^2 and q^2 are

Solution: Two numbers are co-prime if their HCF is 1 i.e they have no number common other than 1.

Let us take p = 4 and q = 5.As 4 and 5 has no common factor other than 1, p and q are co-prime.

Now $p^2 = 16$ and $q^2 = 25$,

As 16 and 25 has no common factor other than 1,

So p^2 and q^2 are also co-prime.

16. Question

Which of the following rational numbers have terminating decimal?

(i)16/225 (ii)5/18

(iii)2/21 (iv)7/250

A. (i) and (ii)

B. (ii) and (iii)

C. (i) and (iii)

D. (i) and (iv)

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

(i) Here q = 225

225 can be written as $3^2 \times 5^2$

Since it is in the form of 5^m, it is a terminating decimal.

(ii) Here q = 18

18 can be written as 2×3^2

Since 3 is also there and it is not in the form of 2^n5^m, it is not a terminating decimal.

(iii) Here q = 21

21 can be written as 3 × 7

Since it is not in the form of 2^n5^m, it is not a terminating decimal.

(iv) Here q = 250

250 can be written as 2×5^3

Since it is in the form of 2^n5^m, it is a terminating decimal.

17. Question

If 3 is the least prime factor of number a and 7 is the least prime factor of number b, then the least prime factor of a + b, is

A. 2

B. 3

C. 5

D. 10

Answer

The prime factors of a + b

= 3 + 7 = 10 = 2 × 5

So the least prime factor is 2.

18. Question

$3.\overline{27}$ is

A. an integer

B. a rational number

C. a natural number

D. an irrational number

Answer

Since, the given number is non-terminating recurring,

It is a rational number.

19. Question

The smallest number by which $\sqrt{27}$ should be multiplied so as to get a rational number is

A. $\sqrt{27}$

B. $3\sqrt{3}$

C. $\sqrt{3}$

D. 3

Answer

$\sqrt{27} = \sqrt{3 \times 3 \times 3}$

If we multiply $\sqrt{3}$ we will get a perfect square 9 which is a rational number.

20. Question

The smallest rational number by which 1/3 should be multiplied so that its decimal expansion terminates after one place of decimal, is

A. 3/10

B. 1/10

C. 3

D. 3/100

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form 2^n 5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

So 1/3 should be multiplied by 3/10 so that it is in the form of 2^n5^m.

21. Question

If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by

A. 5

B. 13

C. both 5 and 13

D. None of these

Answer

9 2^n – 4 2^n is of the form a 2^n — b 2^n.

It is divisible by both a - b and a + b.

So, $9\ 2^n - 4\ 2^n$ is divisible by both 9 - 4 = 5 and 9 + 4 = 13.

22. Question

If n is any natural number, then $6^n - 5^n$ always ends with

A. 1

B. 3

C. 5

D. 7

Answer

For any $n \in N$, e and 5^n end with 6 and 5 respectively.

Therefore, $6^n - 5^n$ always ends with 6 - 5 = 1.

23. Question

The LCM and HCF of two rational numbers are equal, then the numbers must be

A. prime

B. co-prime

C. composite

D. equal

Answer

When numbers are equal, LCM and HCF of two rational numbers are equal.

24. Question

If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is

A. 203400

B. 194400

C. 198400

D. 205400

Answer

Given: The sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF.

To find: The product of two numbers.

Solution: Let LCM be x and HCF be y.

Then, by given condition,Sum of LCM and HCFx + y = 1260 (1)

Also, LCM is 900 more than HCF i.e.x = 900 + y

Substituting x = 900 + y in equation (1)

900 + y + y = 1260

900 + 2y = 1260

2y = 360

y = 180

Then, x = 900 + 180 = 108

Thus, LCM = 1080 and HCF = 180

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

1080 × 180 = 194400

25. Question

The remainder when the square of any prime number greater than 3 is divided by 6, is

A. 1

B. 3

C. 2

D. 4

Answer

Any prime number greater than 3 is of the form 6k $\pm$1, where k is a natural number

and (6k $\pm$ 1)2 = 36k^2 $\pm$ 12k + 1 = 6k (6k $\pm$ 2) + 1

Thus, the remainder is 1.

2. Polynomials

Exercise 2.1

1. Question

Find the zeros of each of the following quadratic polynomials and verify the relationship between the zeros and their coefficients:

(i) $f(x) = x^2 - 2x - 8$

(ii) $g(s) = 4s^2 - 4s + 1$

(iii) $h(t) = t^2 - 15$

(iv) $6x^2 - 3 - 7x$

(v) $p(x) = x^2 + 2\sqrt{2x} - 6$

(vi) $q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

(vii) $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

(viii) $g(x) = a(x^2 + 1) - x(a^2 + 1)$

Answer

(i) $f(x) = x^2 - 2x - 8$ factorize the given polynomial by splitting the middle term:

⇒ x^2 - 4x + 2x – 8

⇒ x (x - 4) + 2 (x - 4)

For zeros of f(x), f(x) = 0

⇒(x + 2) (x - 4) = 0x+2=0x=-2x-4=0x=4

⇒x = -2, 4

Therefore zeros of the polynomial are -2 & 4

In a polynomial the relations hold are as follows:sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ product of zeroes is equal to$\frac{Constant\ term}{Coefficient\ of\ x^2}$

For the given polynomial,

Sum of zeros = -2 + 4 = 2

And $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ is -(-2) = 2

Hence the value of $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and sum of zeroes are same.

Product of zeros = -2 × 4 = -8

$\frac{Constant\ term}{Coefficient\ of\ x^2}$ is -8.

Hence the value of $\frac{Constant\ term}{Coefficient\ of\ x^2}$ and product of zeroes are same.

(ii) $g(s) = 4s^2 - 4s + 1$

factorize the given polynomial by splitting the middle term:

⇒ $4s^2$ -2s - 2s + 1

⇒ 2s (2s - 1) -1 (2s - 1)

For zeros of g(s), g(s) = 0

(2s - 1) (2s - 1) = 0

2s - 1=0$s = \frac{1}{2}$

s = $\frac{1}{2}, \frac{1}{2}$

Therefore zeros of the polynomial are $\frac{1}{2}, \frac{1}{2}$

In a polynomial the relations hold are as follows: sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$product of zeroes is equal to$\frac{Constant\ term}{Coefficient\ of\ x^2}$

For the given polynomial,

Sum of zeros = $\frac{1}{2} + \frac{1}{2}$ = 1$\frac{-coefficient\ of\ s}{coefficient\ of\ s^2} = \frac{4}{4} = 1$

Hence the value of $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and sum of zeroes are same.

Product of zeros = $-\frac{1}{2} \times -\frac{1}{2} = \frac{1}{4}$ $\frac{constant\ term}{coefficient\ of\ s^2} = \frac{1}{4}$

Hence the value of $\frac{Constant\ term}{Coefficient\ of\ x^2}$ and product of zeroes are same.

(iii) $h(t) = t^2 - 15$ use the formula $a^2 - b^2 = (a+b)(a-b)$ to solve the above equation,Here a is t and b is $\sqrt{15}$.

Solve the given expression as: $t^2 - \left(\sqrt{15}\right)^2 = \left(t + \sqrt{15}\right)\left(t - \sqrt{15}\right)$

For zeros of h(t), h(t) = 0

$t+\sqrt{15}=0$

$t=-\sqrt{15}$

$t-\sqrt{15}=0$

$t=\sqrt{15}$

Therefore zeros of the given polynomial are t = √15 & -√15

In a polynomial the relations hold are as follows:sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$product of zeroes is equal to$\frac{Constant\ term}{Coefficient\ of\ x^2}$For the given polynomial,

Sum of zeros = √15 + (- √15) = 0

The value of $-\frac{Coefficient\ of\ t}{Coefficient\ of\ t^2}$ is 0.

Hence, the value of $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and sum of zeroes are same.

Product of zeros $=-\sqrt{15}\times\sqrt{15}$

The value of $\frac{Constant\ term}{Coefficient\ of\ t^2}$ is $-\sqrt{15}$.

Hence the value of $\frac{Constant\ term}{Coefficient\ of\ x^2}$ and product of zeroes are same.

(iv) f(x) = $6x^2-3-7x$

Write the equation in the form of ax^2+bx+c as:

$6x^2 - 7x -3$

factorize the given polynomial by splitting the middle term:

⇒ $6x^2 - 9x + 2x - 3$

⇒ 3x(2x - 3) +1(2x - 3)

⇒ (3x + 1) (2x - 3)

For zeros of f(x),f(x) = 0

⇒ (3x + 1) (2x - 3) = 0

x = $\frac{-1}{3},\frac{3}{2}$

Therefore zeros of the polynomial are $\frac{-1}{3},\frac{3}{2}$

In a polynomial the relations hold are as follows:sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$product of zeroes is equal to$\frac{Constant\ term}{Coefficient\ of\ x^2}$

Sum of zeros = $\frac{-1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6} = \frac{7}{6} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = $\frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6} = \frac{-1}{2} = \frac{constant\ term}{coefficient\ of\ x^2}$

(v) $p(x) = x^2 + 2\sqrt{2}x - 6$ P (x) = x^2 + 3√2x - √2x - 6

For zeros of p(x), p(x) = 0

⇒ x (x + 3√2) -√2 (x + 3√2) = 0

⇒ (x - √2) (x + 3√2) = 0

x = √2, -3√2

Therefore zeros of the polynomial are √2 & -3√2

Sum of zeros = √2 -3√2 = -2√2 = -2√2 = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = √2 × -3√2 = -6 = -6 = $\frac{constant\ term}{coefficient\ of\ x^2}$

(vi) q (x) = √3x^2 + 10x + 7√3

⇒ √3x^2 + 10x + 7√3

⇒ √3x^2 + 7x + 3x + 7√3

⇒ √3x (x +$\frac{7}{\sqrt{3}}$) + 3 (x +$\frac{7}{\sqrt{3}}$)

⇒ (√3x + 3) (x +$\frac{7}{\sqrt{3}}$)

For zeros of Q(x), Q(x) = 0

(√3x + 3) (x +$\frac{7}{\sqrt{3}}$) = 0

X = $\frac{-3}{\sqrt{3}}$, $\frac{-7}{\sqrt{3}}$

Therefore zeros of the polynomial are $\frac{-3}{\sqrt{3}}$, $\frac{-7}{\sqrt{3}}$

Sum of zeros = $\frac{-3}{\sqrt{3}} + \frac{-7}{\sqrt{3}} = \frac{-10}{\sqrt{3}} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = = $\frac{-3}{\sqrt{3}} \times \frac{-7}{\sqrt{3}} = 7 = 7 = \frac{Constant\ term}{Coefficient\ of\ x2}$

(vii) $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

$f(x) = x^2 - \sqrt{3}x - x + \sqrt{3}$

$f(x) = x(x - \sqrt{3}) - 1(x - \sqrt{3})$

$f(x) = (x - 1)(x - \sqrt{3})$

For zeros of f(x), f(x) = 0

$(x - 1)(x - \sqrt{3}) = 0$

$X = 1, \sqrt{3}$

Therefore zeros of the polynomial are 1 & $\sqrt{3}$

Sum of zeros = $1 + \sqrt{3} = \sqrt{3} + 1 = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = $1 \times \sqrt{3} = \sqrt{3} = \frac{constant\ term}{coefficient\ of\ x^2}$

(viii) $g(x) = a(x^2 + 13) - x(a^2 + 1)$

$g(x) = ax^2 - a^2x - x + ag(x) = ax^2 - (a^2 + 1)x + a$

$g(x) = ax(x - a) - 1(x - a)$

$g(x) = (ax - 1)(x - a)$

For zeros of g(x), g(x) = 0

$(ax - 1)(x - a) = 0$

$X = \frac{1}{a}, a$

Therefore zeros of the polynomial are $\frac{1}{a}$ & a

Sum of zeros

$$= \frac{1}{a} + a$$

$$= \frac{1 + a^2}{a}$$

$$= \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$

Product of zeros = $\frac{1}{a}$ × a = 1 = 1 = $\frac{constant\ term}{coefficient\ of\ x^2}$

2. Question

If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$

Answer

α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$

Sum of the roots = $\alpha + \beta = \frac{constant\ term}{coefficient\ of\ x^2} = -\frac{(-5)}{4} = \frac{5}{4}$

Product of the roots = $\alpha \times \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-1}{4}$

Now,

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

On substituting values from above, we get

$= \frac{-1}{4} \times \frac{5}{4} = -\frac{5}{16}$

3. Question

If α and β are the zeros of the quadratic polynomial $f(t) = t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{(-4)}{1} = 4$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = \frac{3}{1} = 3$Now, to evaluate

$\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta)$

On substituting values from above, we get

⇒ $3^3 \times 4 = 108$

4. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$, find the value of

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Answer

Given: α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$

To find: the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution:

α and β are the roots of the given eqn.We know,

Sum of the roots = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

⇒ $\alpha + \beta = -\frac{(-5)}{1} = 5$

AndProduct of the root = $\frac{constant\ term}{coefficient\ of\ x^2}$ $= \frac{4}{1} = 4$

⇒ $\alpha \times \beta = \frac{4}{1} = 4$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$,

$$= \frac{\beta + \alpha - 2\alpha^2\beta^2}{\alpha\beta}$$

$$= \frac{\beta + \alpha - 2(\alpha\beta)^2}{\alpha\beta}$$

On substituting values from above, we get $= \frac{5 - 2(4)^2}{4}$

$= \frac{5 - 2(16)}{4}$ $= -\frac{27}{4}$

5. Question

If α and β are the zeros of the quadratic polynomial $p(y) = 5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $= -\frac{(-7)}{5} = \frac{7}{5}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2}$ $= \frac{1}{5}$

On substituting values from above, we get

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{(\beta+\alpha)}{\alpha\beta} = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{\frac{7}{5}}{\frac{1}{5}} = 7$

6. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $= -\frac{(-1)}{1} = 1$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2}$ $= \frac{-4}{1} = -4$

$$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{(\beta+\alpha)}{\alpha\beta} - \alpha\beta$$

On substituting values from above, we get $= \frac{-1}{4} + 4$

$$= \frac{-1+16}{4}$$

$$= \frac{15}{4}$$

7. Question

If α and β are the zeros of the quadratic polynomial $f(x) = 6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{1}{6}$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = \frac{-2}{6} = \frac{-1}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha 2 + \beta 2)}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

Using $(a + b)^2 = a^2 + b^2 + 2ab$

On substituting values from above, we get

$\Rightarrow \frac{\frac{-1 \times -1}{6 \times 6} - \left(\frac{2}{3}\right)}{-\frac{1}{3}} = \frac{-25}{12}$

8. Question

If α and β are the zeros of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = -(-6/3) = 2

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{4}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{(\alpha^2+\beta^2)+2(\alpha+\beta)}{\alpha\beta} + 3\ \alpha\ \beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta + 2(\alpha+\beta)}{\alpha\beta} + 3\ \alpha\ \beta$

Using $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow \frac{(2)^2 - 2\times\frac{4}{3}+2\times 2}{\frac{4}{3}} + \frac{3\times 4}{3}$$

$$\Rightarrow \frac{4 - \frac{8}{3}+4}{\frac{4}{3}} + 4$$

$$= 8$$

9. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 + x - 2$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Answer

Given : α and β are the zeros of the quadratic polynomial $f(x) = x^2 + x - 2$

To find : the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Solution : α and β are the roots of the given eq.

Sum of root: $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-1}{1} = -1$

Product of the roots: $\alpha \times \beta = \frac{constant\ term}{coefficient\ of x^2}$

$$= \frac{-2}{1} = -2$$

Now, $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta - \alpha}{\alpha\beta}$

Use $(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$ to find the value of β-α.
$(\beta+\alpha)^2=\beta^2+\alpha^2+2\beta\alpha$add and subtract $2\beta\alpha$ on right hand side of the above equation.
$(\beta+\alpha)^2=\beta^2+\alpha^2+2\beta\alpha+2\beta\alpha-2\beta\alpha(\beta+\alpha)^2=(\beta-\alpha)^2+4\beta\alpha$

$$(\beta-\alpha) = \sqrt{(\beta+\alpha)^2 - 4\alpha\beta}$$

$$\frac{(\beta-\alpha)}{\alpha\beta} = \frac{\sqrt{(\beta+\alpha)^2 - 4\alpha\beta}}{4\alpha\beta}$$

$$\Rightarrow \frac{\sqrt{(-1)^2-4(-2)}}{-2}=\frac{\sqrt{9}}{-2}=\frac{-3}{2}$$

$$\therefore \frac{1}{\alpha}-\frac{1}{\beta}=\frac{-3}{2}$$

10. Question

If one zero of the quadratic polynomial $f(x)=4x^2-8kx-9$ is negative of the other, find the value of k.

Answer

Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is $-\alpha$

Sum of the roots = $\alpha-\alpha=\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $-\frac{-8k}{4}$ = 2k

$\alpha-\alpha$ = 2k

0 = 2k

therefore,

k = 0

11. Question

If the sum of the zeros of the quadratic polynomial $f(t)=kt^2+2t+3k$ is equal to their product, find the value of k.

Answer

Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

Sum of the roots = $\alpha+\beta=-\frac{Coefficient\ of\ x}{Coefficient\ of\ x2}=-\frac{2}{k}$

Product of the roots = $\alpha\times\beta=\frac{Constant\ term}{Coefficient\ of\ x2}=\frac{3k}{k}$ = 3

According to the question: sum of zeros = product of zeros

$-\frac{2}{k}$ = 3

k = $-\frac{2}{3}$

12. Question

If the squared difference of the zeros of the quadratic polynomial $f(x)=x^2+px+45$ is equal to 144, find the value of p.

Answer

Consider $f(x) = x^2 + px + 45$,Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $-\frac{p}{1}$ = -p

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2}$ = 45

According to the question: squared difference of the zeros = 144

$$(\alpha - \beta)^2 = 144$$

Apply the formula $(x-y)^2 = x^2 + y^2 - 2xy$

$$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 144$$

$$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta + 2\alpha\beta - 2\alpha\beta = 144$$

Apply the formula $(x+y)^2 = x^2 + y^2 + 2xy$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ = 144

$\Rightarrow (-p)^2 - 4\times 45 = 144$

$\Rightarrow p^2 - 4\times 45 = 144$

$\Rightarrow p^2 - 180 = 144$

$\Rightarrow p^2 = 144 + 180$

$\Rightarrow p^2 = 324$

$\Rightarrow p = \pm 18$

13. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$,

prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Answer

Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = -\frac{-p}{1} = p$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = q$

Therefore, we have

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{(\alpha\beta)^2}$$
$$= \frac{(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$
$$= \frac{((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

Putting the values from above, we get

$$= \frac{(p^2 - 2q)^2 - 2q^2}{q^2}$$
$$= \frac{(p^4 - 4p^2q + 4q^2) - 2q^2}{q^2}$$
$$= \frac{p^4 - 4p^2q + 2q^2}{q^2}$$
$$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$$

Hence, Proved.

14. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$, show that $(\alpha+1)(\beta+1) = 1 - c$.

Answer

Given: α and β are the zeros of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$

To show: $(\alpha+1)(\beta+1) = 1 - c$ ..... (1)

solution: one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = x^2 - p(x+1) - cf(x) = x^2 - px - p - c f(x) = x^2 - px - (p + c)

Sum of the roots is: $\alpha + \beta = -\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

$$\Rightarrow \alpha + \beta = -\frac{(-p)}{1} = p$$

Product of coefficient is: $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2}$

$$\alpha \times \beta = \frac{-(p+c)}{1} = -(p+c)$$

Solve LHS of (1) to get,

⇒ $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

On substituting values, we get $(\alpha+1)(\beta+1)$ = -(p+c) + p + 1

⇒$(\alpha+1)(\beta+1) = -p-c+p+1$ ⇒ $(\alpha+1)(\beta+1) = 1-c$Hence proved

15. Question

If α and β are the zeros of the quadratic polynomial such that α + β = 24 and α - β = 8, find a quadratic polynomial having α and β as its zeros.

Answer

A quadratic equation when sum and product of its zeros is given by:

f(x) = k{x^2 - (sum of zeros)x + product of the zeros}, where k is a constant

α + β = 24 ....(1)

α - β = 8 ....(2)

Adding 1 and 2 we get,α + β + α - β = 24 + 8⇒ 2α = 32⇒ α = 16Substitute value in 1 to get16 + β = 24⇒ β = 24-16⇒ β = 8

α = 16 and β = 8

f(x) = k{x^2 - (24)x + 16 × 8}

f(x) = k(x^2 - 24x + 128)

If we will put the different values of k, we will find the different quadratic equations.

16. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeros are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

Answer

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constantConsider the polynomial $f(x) = x^2 - 1$,

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = -\frac{0}{1} = 0$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = -1$

New equation have zeroes as $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$,

⇒ Sum of the zeros of new eq = $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta} = \frac{2(\alpha^2+\beta^2)}{\alpha\beta}$

$= \frac{2\left(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta\right)}{\alpha\beta}$ [Using $(a + b)^2 = a^2 + b^2 + 2ab$]

⇒ $\frac{2(\alpha+\beta)^2 - 2\times 2\alpha\beta}{\alpha\beta} = \frac{2(0)^2 - 4\times -1}{-1} = -4$

Product of the zeros of new eqn = $\frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4$

$f(x) = k\{(x^2 - (-4)x + 4\}$

$f(x) = k(x^2 + 4x + 4)$

17. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeros are $\frac{1}{2\alpha + \beta}$ and $\frac{1}{2\beta + \alpha}$.

Answer

$f(x) = k(x^2 - 6x + 11)$

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{-3}{1} = 3$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = -2$

Sum of the zeros of new eqn = $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha} = \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$

$\Rightarrow \frac{3(\alpha+\beta)}{(2\alpha+\beta)(2\beta+\alpha)}$

$$= \frac{3(\alpha+\beta)}{4\alpha\beta+2\alpha^2+2\beta^2+\alpha\beta}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2\alpha^2+2\beta^2}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha^2+\beta^2)}$$

Now, $= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta)}$

Using $(a + b)^2 = a^2 + b^2 + 2ab$ we get,

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2[(\alpha+\beta)^2-2\alpha\beta]}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha+\beta)^2-4\alpha\beta}$$

$$= \frac{3(\alpha+\beta)}{\alpha\beta+2(\alpha+\beta)^2}$$

$\Rightarrow \frac{3(\alpha+\beta)}{\alpha\beta+2\{(\alpha+\beta)^2\}} = \frac{3\times3}{-2+2\times32} = \frac{9}{16}$

Product of the zeros of new eqn = $\frac{1}{2\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$

$\Rightarrow \frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$

$$= \frac{2}{4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta}$$

$$= \frac{2}{5\alpha\beta + 2(\alpha^2 + \beta^2)}$$

$$= \frac{2}{5\alpha\beta + 2(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta)}$$

$$= \frac{2}{5\alpha\beta + 2[(\alpha + \beta)^2 - 2\alpha\beta]}$$

$$= \frac{2}{5\alpha\beta + 2(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \frac{2}{\alpha\beta + 2(\alpha + \beta)^2}$$

$= \frac{1}{-2+2\times32} = \frac{1}{16}$

$f(x) = k\{(x^2 - \frac{9}{16}x + \frac{1}{16}\}$

$f(x) = k(x^2 - \frac{9}{16}x + \frac{1}{16})$

18. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 + px + q$, from a polynomial whose zeros are $(\alpha + \beta)^2 (\alpha - \beta)^2$.

Answer

$f(x) = k\{x^2 - 2(p^2 - 2q)\,x + p^2(p^2 - 4q)\}$

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{p}{1} = -p$

Product of the roots = $\alpha\times\beta = \frac{Constant\ term}{Coefficient\ of\ x2} = q$

Sum of the zeros of new eqn = $\alpha 2 + \beta 2 + 2\alpha\beta + \alpha 2 + \beta 2 - 2\alpha\beta = 2(\alpha^2 + \beta^2) = 2\{(\alpha + \beta)^2 - 2\alpha\beta\}$

⇒ 2×$p2$-2×$2q$ = 2($p2$- $2q$) [Using (a + b)^2 = a^2 + b^2 + 2ab]

Product of the zeros of new eqn =$(\alpha + \beta)^2(\alpha - \beta)2 = (\alpha + \beta)^2\{(\alpha + \beta)^2 - 4\alpha\beta\}$

⇒ $p2(p2 - 4q)$

$f(x) = k\{(x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q)\}$

19. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 2x + 3$, find a polynomial whose roots are (i) $\alpha + 2, \beta + 2$ (ii) $\frac{\alpha - 1}{\alpha + 1}, \frac{\beta - 1}{\beta + 1}$.

Answer

(i)

$f(x) = k(x^2 - 6x + 11)$

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $-\frac{-2}{1} = 2$

Product of the roots = $\alpha\times\beta = \frac{constant\ term}{coefficient\ of\ x^2}$ = 3

Sum of the zeros of new eqn = $\alpha + \beta + 4 = 2 + 4 = 6$

Product of the zeros of new eqn =$(\alpha + 2)(\beta + 2)$ =$\alpha\beta + 2(\alpha + \beta) + 4 = 3 + 2\times 2 + 4 = 11$

$f(x) = k(x^2 - 6x + 11)$

(ii) A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $-\frac{-2}{1} = 2$

Product of the roots = $\alpha\times\beta = \frac{constant\ term}{coefficient\ of\ x^2}$ = 3

Sum of the zeros of new eqn $= \frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta+1)+(\alpha+1)(\beta-1)}{(\alpha+1)(\beta+1)} =$

$\frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta-\alpha-1}{\alpha\beta+(\alpha+\beta)+1} \quad \frac{2\alpha\beta-2}{\alpha\beta+(\alpha+\beta)+1} = \frac{2\times3-2}{3+2+1} = \frac{2}{3}$

Product of the zeros of new eqn $= \frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)} = \frac{\alpha\beta-(\beta+\alpha)+1}{\alpha\beta+(\alpha+\beta)+1} = \frac{3-2+1}{3+2+1} = \frac{1}{3}$

Therefore eqn is: $f(x) = k\left\{x^2 - \frac{2}{3x} + \frac{1}{3}\right\}$

$\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1} = \frac{\beta-1+\alpha-1}{(\alpha+1)(\beta+1)} = \frac{\alpha+\beta-2}{\alpha\beta+(\alpha+\beta)+1} = \frac{2-2}{\alpha\beta+(\alpha+\beta)+1} = 0$

20. Question

If α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate:

(i) $\alpha - \beta$ (ii) $\frac{1}{\alpha} - \frac{1}{\beta}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ (iv) $\alpha^2\beta + \alpha\beta^2$

(v) $\alpha^4 + \beta^4$ (vi) $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$

(vii) $\frac{\beta}{a\alpha + b} + \frac{\alpha}{a\beta + b}$ (viii) $a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

Answer

(i) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax2 + bx + c$

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha\times\beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

⇒ On substituting values, we get

$\Rightarrow (\alpha - \beta) = \sqrt{\{(\alpha+\beta)^2 - 4\alpha\beta\}} = \sqrt{\left\{\left(\frac{-b}{a}\right)^2 - 4\frac{c}{a}\right\}} = \frac{\sqrt{b^2-4ac}}{a}$

(ii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = ax^2 + bx + c

Sum of the roots = $\alpha+\beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha\times\beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

⇒ On substituting values, we get

$\Rightarrow \frac{1}{\alpha}-\frac{1}{\beta} = \frac{(\beta-\alpha)}{\alpha\beta} = -\frac{\sqrt{\{(\alpha+\beta)^2-4\alpha\beta\}}}{\alpha\beta} = -\frac{\sqrt{\left\{\left(\frac{-b}{a}\right)^2-4\frac{c}{a}\right\}}}{\frac{c}{a}}$

$= \frac{\sqrt{b^2-4ac}}{c}$

(iii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax^2 + bx + c$

Sum of the roots = $\alpha+\beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha\times\beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

⇒ On substituting values, we get

$\Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta = \frac{(\beta+\alpha)-2(\alpha\beta)2}{\alpha\beta} = \frac{(-b/a)-2(c/a)2}{c/a} = -\left(\frac{b}{c}+\frac{2c}{a}\right)$

(iv) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax^2 + bx + c$

Sum of the roots = $\alpha+\beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha\times\beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

⇒ On substituting values, we get

$\Rightarrow \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta) = \frac{c}{a} \times \frac{-b}{a} = \frac{-bc}{a2}$

(v) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax^2 + bx + c$

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

$\Rightarrow \alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)2 = \{(\alpha+\beta)^2 - 2\alpha\beta\}2 - 2(\alpha\beta)2$ [Using $(a + b)^2 = a^2 + b^2 + 2ab$]

On substituting values, we get

$\Rightarrow \alpha^4 + \beta^4 = \left\{\left(-\frac{b}{a}\right)^2 - \left(\frac{2c}{a}\right)\right\}^2 - 2\left(\frac{c}{a}\right)^2$

$\Rightarrow \alpha^4 + \beta^4 = \left(\frac{b2-2ac}{a2}\right)2 - 2\left(\frac{c^2}{a^2}\right)$

$\Rightarrow \alpha^4 + \beta^4 = \frac{(b^2-2ac)^2 - 2a^2c^2}{a^4}$

(vi) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax^2 + bx + c$

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

$\frac{1}{a\alpha+b} + \frac{1}{a\beta+b} = \frac{a\beta+b+a\alpha+b}{\{(a\alpha+b)(a\beta+b)\}} = \frac{a(\alpha+\beta)+2b}{\{(a\alpha+b)(a\beta+b)\}}$

⇒ On substituting values, we get

$\frac{a(\alpha+\beta)+2b}{\{(a\alpha+b)(a\beta+b)\}} = \frac{a(-b/a)+2b}{a2\alpha\beta+ab(\alpha+\beta)+b2}$

$$\Rightarrow \frac{a(-b/a)+2b}{(\frac{a^2c}{a})+ab(-b/a)+b2} = \frac{b}{ac}$$

(vii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax^2 + bx + c$

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{\{(a\alpha+b)(a\beta+b)\}}$$

$$\Rightarrow \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{a2\alpha\beta+ab(\alpha+\beta)+b2} = \frac{a(\alpha+\beta)2-2\alpha\beta+b(\alpha+\beta)}{a2\alpha\beta+ab(\alpha+\beta)+b2} = \frac{a(-b/a)2+b(-\frac{b}{a})}{(\frac{a^2c}{a})+ab(-b/a)+b2}$$

$$\Rightarrow \frac{\frac{b}{a}-(\frac{b^2}{a})}{\frac{a^2c}{a}-b2+b2} = -\frac{2}{a}$$

(viii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x) = ax^2 + bx + c$

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right) =$$

$$\Rightarrow \frac{a(\alpha^3+\beta^3)}{\alpha\beta}+\frac{b(\alpha^2+\beta2)}{\alpha\beta} = \frac{\{a(\alpha^3+\beta^3)+b(\alpha^2+\beta^2)\}}{\alpha\beta} = \frac{\{a(\alpha+\beta)3-3\alpha\beta(\alpha+\beta)\}+\{b(\alpha+\beta)^2-2\alpha\beta\}}{\alpha\beta}$$

On substituting values, we get

$$= \frac{\left\{a(-b/a)3-3c/a(-b/a)\right\}+\left\{b\left(-\frac{b}{a}\right)^2-2c/a\right\}}{c/a}$$

$= \frac{\left\{-\frac{b^3}{a^2}+3bc/a2\right\}-b3/a2-2c/a}{c/a}$ = b

Exercise 2.2

1. Question

Verify that the numbers given along side of the cubic polynomials below are their zeros. Also, verify the relationship between the zeros and coefficients in each case:

(i) $f(x) = 2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2;\ 2, 1, 1$

Answer

(i) $f(x) = 2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$ $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 =$
$\frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{1+1-10+8}{4} = 0$

$f(1) = 2\times13 + 12 - 5\times1 + 2 = 2 + 1 - 5 + 2 = 0$

$f(-2) = 2\times(-2)^3 + (-2)^2 - 5\times(-2) + 2 = -16 + 4 + 10 + 2 = 0$

Let $\alpha = \frac{1}{2}$; β=1; γ=-2

$\alpha + \beta + \gamma = \frac{1}{2} + 1 - 2 = \frac{-1}{2} = -\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}\times1 + 1\times-2 - 2\times\frac{1}{2} = \frac{-5}{2} = \frac{Coefficient\ of\ x}{Coefficient\ of\ x3}$

$\alpha\beta\gamma = \frac{1}{2}\times1\times-2 = -1 = -\frac{Constant\ term}{Coefficient\ of\ x3}$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2;\ 2, 1, 1$ $f(2) = 23 - 4(2)^2 + 5\times2 - 2 = 8 - 16 + 10 - 2 = 0$

$f(1) = 13 - 4(1)^2 + 5\times1 - 2 = 1 - 4 + 5 - 2 = 0$

$f(1) = 13 - 4(1)^2 + 5\times1 - 2 = 1 - 4 + 5 - 2 = 0$

Let α =2; β=1; γ=1

$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 2\times1 + 1\times1 + 1\times2 = 5 = \frac{Coefficient\ of\ x}{Coefficient\ of\ x3}$

$\alpha\beta\gamma = 2\times1\times1 = 2 = -\frac{Constant\ term}{Coefficient\ of\ x3}$

2. Question

Find a cubic polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Answer

A quadratic equation when sum and product of its zeros is given by:

$$f(x) = k\{x^3 - (sum\ of\ the\ zeros)x^2 + (sum of\ zeros\ taken\ two\ at\ a\ time)x^2 - (product\ of\ the\ zeros)\}$$

where k is any non-zero real number.

Here,Sum of zeroes = 3sum of the product of its zeros taken two at a time =-1 product of its zeros = -3

$$\Rightarrow f(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$$

$$\Rightarrow f(x) = k\{x^3 - 3x^2 - x + 3\}$$, where k is any non-zero real number.

3. Question

If the zeros of the polynomial $f(x) = 2x^3 - 15x^2 + 37x - 30$ are in A.P., find them.

Answer

Let $\alpha = a - d, \beta = a\ and\ \gamma = a + d$ are the zeros of the given polynomial.

Sum of the zeros = $-\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$$\Rightarrow \alpha + \beta + \gamma = a - d + a + a + d = -\frac{-15}{2}$$

$$\Rightarrow 3a = \frac{15}{2} = \frac{5}{2}$$

$$\Rightarrow a = \frac{5}{2}$$

$$\alpha\beta\gamma = (a - d)\ a(\ a + d) = -\frac{Constant\ term}{Coefficient\ of\ x3} = \frac{30}{2} = 15$$

$$(a - d)\ a(\ a + d) = 15$$

$$a(\ a^2 - d2) = 15$$

On substituting $a = \frac{5}{2}$

$$\left(\frac{5}{2}\right)\left\{\left(\frac{5}{2}\right)2(-d^2)\right\} = 15$$

$$d2 = 6 - \frac{25}{4} = \frac{1}{4}$$

$$d = \pm\frac{1}{2}$$

$$\alpha = a - d = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$$

$\gamma = a + d = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3$

Therefore zeros of the given equation are: $2, 3, \frac{5}{2}$

4. Question

Find the condition that the zeros of the polynomial $f(x) = x^3 + 3px^2 + 3qx + r$ may be in A.P.

Answer

Let $\alpha = a - d, \beta = a \text{ and } \gamma = a + d$ are the zeros of the given polynomial.

Sum of the zeros = $\frac{-coefficient\ of\ x^3}{coefficient\ of\ x^2}$

$\Rightarrow \alpha + \beta + \gamma = a - d + a + a + d = -3p$

$\Rightarrow 3a = -3p$

$\Rightarrow a = -p$

Since a is the zero of the polynomial, therefore $f(a) = 0$

$\Rightarrow f(a) = a^3 + 3pa^2 + 3qa + r = 0$

$\Rightarrow a^3 + 3pa^2 + 3qa + r = 0$

On substituting $a = -p$, we get

$(-p)^3 + 3p(-p)^2 - 3pq + r = 0$

$\Rightarrow -p^3 + 3p^3 - 3pq + r = 0 \Rightarrow 2p^3 - 3pq + r = 0$

5. Question

If the zeros of the polynomial $f(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., prove that $2b^3 - 3abc + a^2d = 0$.

Answer

Let $\alpha = A - D, \beta = A \text{ and } \gamma = A + D$ are the zeros of the given polynomial.

Sum of the zeros = $-\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\Rightarrow \alpha + \beta + \gamma = A - D + A + A + D = -\frac{3b}{a}$

$\Rightarrow 3A = -\frac{3b}{a}$

$\Rightarrow A = -\frac{b}{a}$

Since A is the zero of the polynomial, therefore $f(A) = 0$

$\Rightarrow f(A) = aA3 + 3bA2 + 3cA + d = 0$

$On\ substituting\ A = -\frac{b}{a}$, we get

$\Rightarrow\ a(-\frac{b}{a})3 + 3b(-\frac{b}{a})2 + 3c(-\frac{b}{a}) + d = 0$

$\Rightarrow\ -\frac{b^3}{a^2} + \frac{3b^3}{a^2} + \frac{3bc}{a} + d = 0$

$\Rightarrow\ \frac{-b3 + 3b3 + 3abc + a2d}{a^2} = 0$

$\Rightarrow -b^3 + 3b^3 + 3abc + a^2d = 0$

$\Rightarrow 2b^3 + 3abc + a^2d = 0$

6. Question

If the zeros of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k.

Answer

Let $\alpha = a - d, \beta = a\ and\ \gamma = a + d$ are the zeros of the given polynomial.

Sum of the zeros = $-\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\Rightarrow \alpha + \beta + \gamma = a - d + a + a + d = 12$

$\Rightarrow 3a = 12$

$\Rightarrow a = 4$

Since a is the zero of the polynomial, therefore $f(a) = 0$

$\Rightarrow f(a) = a3 - 12a2 + 39a + k = 0$

On substituting $a = 4$

$\Rightarrow f(a) = 43 - 12\times42 + 39\times4 + k = 0$

$\Rightarrow 64 - 192 + 156 + k = 0$

$\Rightarrow k = -220 + 192$

$\Rightarrow k = -28$

Exercise 2.3

1. Question

Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x^2 + x + 1$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3, \quad g(x) = 2x^2 + 7x + 1$

(iii) $f(x) = 4x^3 + 8x + 8x^2 + 7, g(x) = 2x^2 - x + 1$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12, \quad g(x) = 2 - 2x + x^2$

Answer

(i) $f(x) = x3 - 6x2 + 11x - 6$ and $g(x) = x2 + x + 1$

Degree of $f(x)\ is\ 3\ and\ degree\ of\ g(x)\ is\ 2$; therefore degree of $q(x)\ is\ 3 - 2 = 1$ and degree of remainder is less than 2,

Let $q(x) = ax + b$ and $r(x) = cx + d$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$$f(x) = q(x) \times g(x) + r(x)$$

On substituting values in the above relation we get,

$$x3 - 6x2 + 11x - 6 = (ax + b) \times (x^2 + x + 1) + (cx + d)$$

$$x3 - 6x2 + 11x - 6 = ax3 + ax2 + ax + bx2 + bx + b + cx + d$$

$$x3 - 6x2 + 11x - 6 = ax3 + x^2(a + b) + x(a + b + c) + (b + d)$$

On comparing coefficients we get,

$$a = 1$$

$$a + b = -6$$

$$a + b + c = 11$$

$$b + d = -6$$

On solving above equations we get,

$$a = 1, b = -7, c = 17, d = 1$$

On substituting these values for $q(x)\ and\ r(x)$

$$q(x) = ax + b = x - 7$$

$$r(x) = cx + d = 17x + 1$$

(ii) $f(x) = 10x^4 + 17x3 - 62x2 + 30x - 3$ and $g(x) = 2x2 + 7x + 1$

Degree of $f(x)\ is\ 4\ and\ degree\ of\ g(x)\ is\ 2$; therefore degree of $q(x)\ is\ 4 - 2 = 2$ and degree of remainder is less than 2.

Let $q(x) = ax2 + bx + c$ and $r(x) = px + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$$f(x) = q(x) \times g(x) + r(x)$$

On substituting values in the above relation we get,

$$10x^4 + 17x3 - 62x2 + 30x - 3 = (ax2 + bx + c) \times (2x2 + 7x + 1) + (px + q)$$

$$10x^4 + 17x3 - 62x2 + 30x - 3 = 2ax^4 + 7ax3 + ax2 + 2bx3 + 7bx2 + bx + 2cx2 + 7cx + c + px + q$$

$$10x^4 + 17x3 - 62x2 + 30x - 3 = 2ax^4 + (7a + 2b)x3 + (a + 7b + 2c)x2 + (b + 7c + p)x + (c + q)$$

On comparing coefficients we get,

$$a = 5$$

$$7a + 2b = 17$$

$$a + 7b + 2c = -62$$

$$b + 7c + p = 30$$

$$c + q = -3$$

On solving above equations we get,

$$a = 1,\ b = -9,\ c = -2,\ p = 53;\ q = -1$$

On substituting these values for $q(x)\ and\ r(x)$

$$q(x) = ax2 + bx + c = 5x2 - 9x - 2$$

$$r(x) = px + q = 53x - 1$$

(iii) $f(x) = 4x3 + 8x2 + 8x + 7$ and $g(x) = 2x2 - x + 1$

Degree of $f(x)\ is\ 3\ and\ degree\ of\ g(x)\ is\ 2$; therefore degree of $q(x)\ is\ 3 - 2 = 1$ and degree of remainder is less than 2,

Let $q(x) = ax + b$ and $r(x) = cx + d$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$$f(x) = q(x) \times g(x) + r(x)$$

On substituting values in the above relation we get,

$$4x3 + 8x2 + 8x + 7 = (ax + b) \times (2x2 - x + 1) + (cx + d)$$

$$4x3 + 8x2 + 8x + 7 = 2ax3 - ax2 + ax + 2bx2 - bx + b + cx + d$$

$$4x3 + 8x2 + 8x + 7 = 2ax3 + x^2(-a + 2b) + x(a - b + c) + (b + d)$$

On comparing coefficients we get,

$$a = 2$$

$$-a + 2b = 8$$

$$a - b + c = 8$$

$$b + d = 7$$

On solving above equations we get,

$$a = 2, b = 5, c = 11, d = 2$$

On substituting these values for $q(x)$ and $r(x)$

$$q(x) = ax + b = 2x + 5$$

$$r(x) = cx + d = 11x + 2$$

(iv) $f(x) = 15x3 - 20x2 + 13x - 12$ and $g(x) = x2 - 2x + 2$

Degree of $f(x)$ is 3 and degree of $g(x)$ is 2; therefore degree of $q(x)$ is $3 - 2 = 1$ and degree of remainder is less than 2,

Let $q(x) = ax + b$ and $r(x) = cx + d$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$$f(x) = q(x) \times g(x) + r(x)$$

On substituting values in the above relation we get,

$$15x3 - 20x2 + 13x - 12 = (ax + b) \times (x2 - 2x + 2) + (cx + d)$$

$$15x3 - 20x2 + 13x - 12 = ax3 - 2ax2 + 2ax + bx2 - 2bx + 2b + cx + d$$

$$15x3 - 20x2 + 13x - 12 = ax3 + x^2(-2a + b) + x(2a - 2b + c) + (2b + d)$$

On comparing coefficients we get,

$$a = 15$$

$$-2a + b = -20$$

$$2a - 2b + c = 13$$

$2b + d = -12$

On solving above equations we get,

$a = 2, b = 10, c = 3, d = -32$

On substituting these values for $q(x)\ and\ r(x)$

$q(x) = x + b = 15x + 10$

$r(x) = cx + d = 3x - 32$

2. Question

Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t) = t^2 - 3, f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $g(x) = x^3 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Answer

(i) $g(t) = t2 - 3$ and $f(t) = 2t^4 + 3t^3 - 2t2 - 9t - 12 = 0$

Degree of $f(t)\ is\ 4\ and\ degree\ of\ g(t)\ is\ 2$; therefore degree of $q(t)\ is\ 4 - 2 = 2$ and degree of remainder is of degree 1 or less,

Let $q(t) = at2 + bt + c$ and $r(t) = pt + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(t) = q(t) \times g(t) + r(t)$

On substituting values in the above relation we get,

$2t^4 + 3t^3 - 2t2 - 9t - 12 = (at^2 + bt + c)(t^2 - 3) + pt + q$

$2t^4 + 3t^3 - 2t2 - 9t - 12 = at^4 - 3at2 + bt3 - 3bt + ct2 - 3c + pt + q$

$2t^4 + 3t^3 - 2t2 - 9t - 12 = at^4 + bt3 - 3at2 + ct2 - 3bt + pt - 3c + q$

$2t^4 + 3t^3 - 2t2 - 9t - 12 = at^4 + bt3 - t^2(3a + c) - t(3b + p) - 3c + q$

On comparing coefficients we get,

$a = 2$

$b = 3$

$3a + c = 2$

$3b + p = 9$

$-3c + q = -12$

On solving above equations we get,

$a = 2, b = 3, c = -4, p = 0, q = 0$

On substituting these values for $q(t)\ and\ r(t)$

$q(t) = at2 + bt + c = 2t2 + 3t - 4$

$r(t) = pt + q = 0t + 0 = 0$ Since remainder is zero, therefore $g(t)\ is\ a\ factor\ of\ f(t)$

(ii) $g(x) = x3 - 3x + 1$ and $f(x) = x^5 - 4x3 + x2 + 3x + 1 = 0$

Degree of $f(x)\ is\ 5\ and\ degree\ of\ g(x)\ is\ 3$; therefore degree of $q(x)\ is\ 5 - 3 = 2$ and degree of remainder is of degree 1 or less,

Let $q(x) = ax2 + bx + c$ and $r(x) = px + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x) \times g(x) + r(x)$

On substituting values in the above relation we get,

$x^5 - 4x3 + x2 + 3x + 1 = (ax2 + bx + c)(x3 - 3x + 1) + px + q$

$x^5 - 4x3 + x2 + 3x + 1$
$= ax^5 - 3ax^3 + ax^2 + bx^4 - 3bx2 + bx + cx3 - 3cx + c + px$
$+ q$

$x^5 - 4x3 + x2 + 3x + 1$
$= ax^5 + bx^4 + x^3(c - 3a) + x^2(a - 3b) + x(b - 3c + p) + q + c$

On comparing coefficients we get,

$a = 1$

$b = 0$

$c - 3a = -4$

$a - 3b = 1$

$b - 3c + p = 3$

$q + c = 1$

On solving above equations we get,

$a = 1, b = 0, c = -1, p = 0, q = 2$

On substituting these values for $q(x)\ and\ r(x)$

$q(t) = ax2 + bx + c \ = x2 - 1$

$r(t) = px + q \ = 0x + 2 = 2$ Since remainder is 2, therefore $g(x)\ is\ not\ a\ factor\ of\ f(x)$

(iii) $g(x) = 2x2 - x + 3$ and $f(x) = 6x^5 - x^4 + 4x3 - 5x2 - x - 15 = 0$

Degree of $f(x)\ is\ 5\ and\ degree\ of\ g(x)\ is\ 2$; therefore degree of $q(x)\ is\ 5 - 2 = 3$ and degree of remainder is of degree 2 or less,

Let $q(x) = ax3 + bx2 + cx + d$ and $r(x) = px + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x) \times g(x) + r(x)$

On substituting values in the above relation we get,

$$6x^5 - x^4 + 4x3 - 5x2 - x - 15 = (ax3 + bx2 + cx + d)(2x2 - x + 3\ ) + px + q$$

$$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 = 2ax^5 - ax^4 + 3ax3 + 2bx^4 - bx3 + 3bx2 + 2cx3 - cx2 + 3cx + 2dx2 - dx + 3d + px + q$$

$$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 = 2ax^5 + (-a + 2b)x^4 + (3a - b + 2c)x^3 + (3b - c + 2d)x^2 + (3c - d + p)x + 3d + q$$

On comparing coefficients we get,

$a = 3$

$-a + 2b = -1$

$3a - b + 2c = 4$

$3b - c + 2d + p = \ -5$

$3c - d + q = \ -1$

$3d + t = \ -15$

On solving above equations we get,

$a = 3, b = 1, c = -2, d = 5\ p = 10, q = -30$

On substituting these values for $q(x)\ and\ r(x)$

$q(x) = ax3 + bx2 + cx + d = 3x3 + x2 - 2x + 5$

$r(x) = px + q \ = 10x - 30$ Since remainder is $10x - 30$, therefore $g(x)\ is\ not\ a\ factor\ of\ f(x)$

3. Question

Obtain all zeros of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are -2 and -1.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -2 and -1 are zeros of $f(x)$. Therefore $(x+2)(x+1) = x2 + 3x + 2$ is a factor of $f(x)$.

Now on dividing $f(x) = 2x^4 + x3 - 14x2 - 19x - 6 \; by \; g(x) = x2 + 3x + 2$ to find other zeros.

$$
\begin{array}{r|l}
x^2+3x+2 & 2x^4+x^3-14x^2-19x-6 \;\big/\; 2x^2-5x-3 \\
\cline{2-2}
 & \underline{2x^4+6x^3+4x^2} \\
 & \quad -5x^3-18x^2-19x-6 \\
 & \quad \underline{-5x^3-15x^2-10x} \\
 & \qquad -3x^2-9x-6 \\
 & \qquad \underline{-3x^2-9x-6} \\
 & \qquad\quad 0
\end{array}
$$

By applying division algorithm, we have:

$2x^4 + x3 - 14x2 - 19x - 6 = (x2 + 3x + 2)(2x2 - 5x - 3)$

$2x^4 + x3 - 14x2 - 19x - 6 = (x + 2)(x + 1)(2x2 - 6x + x - 3)$

$2x^4 + x3 - 14x2 - 19x - 6 = (x + 2)(x + 1)\{(2x + 1)(x - 3)\}$

Hence, the zeros of the given polynomial are: $-\frac{1}{2}, 3, -2, -1$

4. Question

Obtain all zeros of $f(x) = x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -2 is zero of $f(x)$. Therefore $(x+2)$ is a factor of $f(x)$.

Now on divide $f(x) = x^3 + 13x^2 + 32x + 20$, by $(x+2)$ to find other zeros.

$$
\begin{array}{r|l}
x+2 & x^3+13x^2+32x+20 \;\big/\; x^2+11x+10 \\
\cline{2-2}
 & \underline{x^3+2x^2} \\
 & \quad 11x^2+32x+20 \\
 & \quad \underline{11x^2+22x} \\
 & \qquad 10x+20 \\
 & \qquad \underline{10x+20} \\
 & \qquad\quad 0
\end{array}
$$

By applying division algorithm, we have:

$x^3 + 13x^2 + 32x + 20 = (x+2)(x^2+11x+10)$We do factorisation here by splitting the middle term,

$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)(x^2+11x+10) \Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)(x^2+10x+x+10)$

$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)\{x(x+10)+1(x+10)\} \Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)(x+10)(x+1)$

Hence, the zeros of the given polynomial are: $-2, -10, -1$

5. Question

Obtain all zeros of the polynomial $f(x) = x^4 - 3x^3 - x^2 + 9x - 6,$ if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -√3 and √3 are zeros of $f(x).$ Therefore $(x + \sqrt{3})$ and $(x - \sqrt{3})$ are factors of $f(x)$.

Now on dividing $f(x) = x^4 - 3x3 - x2 + 9x - 6 \; by \; g(x) = (x - \sqrt{3})(x + \sqrt{3}) = x2 - 3$ to find other zeros.

$$
\begin{array}{l|l}
x^2 - 3 \;\big)\; x^4 - 3x^3 - x^2 + 9x - 6 \;\big(\; x^2 - 3x + 2 \\
\quad x^4 \quad -3x^2 & \\
\quad - \quad + & \\
\hline
\quad\quad -3x^3 + 2x^2 + 9x - 6 & \\
\quad\quad -3x^3 \quad +9x & \\
\quad\quad + \quad\quad - & \\
\hline
\quad\quad\quad\quad 2x^2 \quad -6 & \\
\quad\quad\quad\quad 2x^2 \quad -6 & \\
\quad\quad\quad\quad - \quad + & \\
\hline
\quad\quad\quad\quad\quad\quad 0 &
\end{array}
$$

By applying division algorithm, we have:

$x^4 - 3x3 - x2 + 9x - 6 = (x - \sqrt{3})(x + \sqrt{3})\,(x2 - 3x + 2)$

$x^4 - 3x3 - x2 + 9x - 6 = (x - \sqrt{3})(x + \sqrt{3})\,(x2 - 2x - x + 2)$

$x^4 - 3x3 - x2 + 9x - 6 = (x - \sqrt{3})(x + \sqrt{3})\,\{(x - 2)(x - 1)\}$

Hence, the zeros of the given polynomial are: $-\sqrt{3}, \sqrt{3}, 1, 2$

6. Question

Find all zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6,$ it its two zeros are $-\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{2}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeros of $f(x).$ Therefore $\left(x + \sqrt{\frac{3}{2}}\right)$ and $\left(x - \sqrt{\frac{3}{2}}\right)$ are factors of $f(x)$.

Now on dividing

$$f(x) = 2x^4 - 2x3 - 7x2 + 3x + 6 \; by \; g(x) = \left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right) = x2 - \frac{3}{2} \Rightarrow \frac{1}{2}(2x^2 - 3)$$

to find other zeros.

$$\begin{array}{l|l} 2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \;\big/\; x^2 - x - 2 \\ & 2x^4 \quad -3x^2 \\ & - \quad + \\ \hline & -2x^3 - 4x^2 + 3x + 6 \\ & -2x^3 \quad +3x \\ & + \quad - \\ \hline & -4x^2 + 6 \\ & -4x^2 + 6 \\ & + \quad - \\ \hline & 0 \end{array}$$

By applying division algorithm, we have:

$$2x^4 - 2x3 - 7x2 + 3x + 6 = \left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)(x2 - x - 2)$$

$$2x^4 - 2x3 - 7x2 + 3x + 6 = \left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)(x2 - 2x + x - 2)$$

$$2x^4 - 2x3 - 7x2 + 3x + 6 = \left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)\{(x - 2)(x + 1)\}$$

Hence, the zeros of the given polynomial are: $-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}, -1, 2$

7. Question

What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

Answer

$x - 2$

By division Algorithm we have,

$$f(x) = g(x) \times q(x) + r(x)$$

$$f(x) - r(x) = g(x) \times q(x)$$

From above relation we got that if we add $-r(x)\; to \; f(x)$ then resulting polynomial is divisible by $g(x)$.

Now find the remainder when $f(x)$ is divisible by $g(x)$.

$$
\begin{array}{r|l}
x^4+2x^3-2x^2+x-1 & x^2+2x-3 \\
x^4+2x^3-3x^2 & \\
\hline
x^2+x-1 & \\
x^2+2x-3 & \\
\hline
-x+2 &
\end{array}
$$

$$r(x) = -x + 2$$

Therefore $-r(x) = x - 2$ has to be added so that resulting polynomial is divisible by $g(x)$

8. Question

What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

Answer

Given : the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$

To find : What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$

Solution : Let g(x) be $x^2 - 4x + 3$ By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

Dividend - Remainder = Quotient× Divisor

$$f(x) = q(x) \times g(x) + r(x)$$

$$f(x) - r(x) = q(x) \times g(x)$$

Now find the remainder when $f(x)$ is divisible by $g(x)$.

$$
\begin{array}{r|l}
x^4+2x^3-13x^2-12x+21 & x^2+6x+8 \\
x^4-4x^3+3x^2 & \\
\hline
6x^3-16x^2-12x+21 & \\
6x^3-24x^2+18x & \\
\hline
8x^2-30x+21 & \\
8x^2-32x+24 & \\
\hline
2x-3 &
\end{array}
$$

$$r(x) = 2x - 3$$

Therefore $r(x) = 2x - 3$ has to be subtracted so that resulting polynomial is divisible by $g(x)$

9. Question

Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and-2.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -2 and 2 are zeros of $f(x).$ Therefore $(x + 2)$ and $(x - 2)$ are factors of $f(x)$.

Now on dividing f(x) by (x - 2)(x + 2) = x^2 - 4 to find other zeros.

$$
\begin{array}{l}
x^2-4 \;\overline{)\; x^4+x^3-34x^2-4x+120 \;)\; x^2+x-30} \\
\quad x^4 \quad -4x^2 \\
\quad - \quad + \\
\hline
\quad\quad x^3 \; -30x^2-4x+120 \\
\quad\quad x^3 \quad\quad -4x \\
\quad\quad - \quad\quad + \\
\hline
\quad\quad\quad -30x^2 \quad +120 \\
\quad\quad\quad -30x^2 \quad +120 \\
\quad\quad\quad + \quad\quad - \\
\hline
\quad\quad\quad\quad 0
\end{array}
$$

By applying division algorithm, we have:

Hence, two other zeroes are -6 and 5

10. Question

Find all zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -√2 and √2 are zeros of $f(x).$ Therefore $\left(x + \sqrt{2}\right)$ and $\left(x - \sqrt{2}\right)$ are factors of $f(x)$.

Now on dividing $f(x) = 2x^4 + 7x3 - 19x2 - 14x + 30 \; by \; g(x) = \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) = x2 - 2$ to find other zeros.

$$
\begin{array}{l}
x^2-2 \;\overline{)\; 2x^4+7x^3-19x^2-14x+30 \;)\; 2x^2+7x-15} \\
\quad 2x^4 \quad -4x^2 \\
\quad - \quad + \\
\hline
\quad\quad 7x^3 \; -15x^2-14x+30 \\
\quad\quad 7x^3 \quad\quad -14x \\
\quad\quad - \quad\quad + \\
\hline
\quad\quad\quad -15x^2 \quad +30 \\
\quad\quad\quad -15x^2 \quad +30 \\
\quad\quad\quad + \quad\quad - \\
\hline
\quad\quad\quad\quad 0
\end{array}
$$

By applying division algorithm, we have:

$2x^4 + 7x3 - 19x2 - 14x + 30 = \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$ $(2x2 + 7x - 15)$

$2x^4 + 7x3 - 19x2 - 14x + 30 = \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$ $(x2 + 6x - 5x - 30)$

$2x^4 + 7x3 - 19x2 - 14x + 30 = \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$ $\{(x - \frac{3}{2})(x + 5)\}$

Hence, the zeros of the given polynomial are: $-\sqrt{2}, \sqrt{2}, -5, \frac{3}{2}$

11. Question

Find all the zeros of the polynomial, $2x^3 + x^2 - 6x - 3$ if two of its zeros are √3 and $-\sqrt{3}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -√3 and √3 are zeros of $f(x).$ Therefore $(x + \sqrt{3})$ and $(x - \sqrt{3})$ are factors of $f(x)$.

Now on dividing $f(x) = 2x3 + x2 - 6x - 3 \; by \; g(x) = (x - \sqrt{3})(x + \sqrt{3}) = x2 - 3$ to find other zeros.

$$
\begin{array}{r|l}
x^2 - 3 \;\overline{\big)\; 2x^3 + x^2 - 6x - 3 \;\big)} & 2x + 1 \\
\quad 2x^3 \quad -6x & \\
\quad - \quad + & \\
\hline
\quad\quad x^2 \quad -3 & \\
\quad\quad x^2 \quad -3 & \\
\quad\quad - \quad + & \\
\hline
\quad\quad\quad 0 &
\end{array}
$$

By applying division algorithm, we have:

$2x3 + x2 - 6x - 3 = (x - \sqrt{3})(x + \sqrt{3})\,(2x + 1)$

Hence, the zeros of the given polynomial are: $-\sqrt{3}, \sqrt{3}, \frac{-1}{2}$

12. Question

Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeros are $-\sqrt{2}$ and $\sqrt{2}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha \; is \; a \; factor \; of \; f(x).$

Since -√2 and √2 are zeros of $f(x).$ Therefore $(x + \sqrt{2})$ and $(x - \sqrt{2})$ are factors of $f(x)$.

Now on dividing $f(x) = x3 + 3x2 - 2x - 6 \; by \; g(x) = (x - \sqrt{2})(x + \sqrt{2}) = x2 - 2$ to find other zeros.

$$
\begin{array}{r|l}
x^2 - 2 \;\overline{\big)\; x^3 + 3x^2 - 2x - 6 \;\big)} & x + 3 \\
\quad x^3 \quad -2x & \\
\quad - \quad + & \\
\hline
\quad\quad 3x^2 \quad -6 & \\
\quad\quad 3x^2 \quad -6 & \\
\quad\quad - \quad + & \\
\hline
\quad\quad\quad 0 &
\end{array}
$$

By applying division algorithm, we have:

$x3 + 3x2 - 2x - 6 = (x - \sqrt{2})(x + \sqrt{2})\,(x + 3)$

Hence, the zeros of the given polynomial are: $-\sqrt{2}, \sqrt{2}, -3$

CCE - Formative Assessment

1. Question

Define a polynomial with real coefficients.

Answer

A polynomial is a mathematical expression containing a sum of powers in one or more variables multiplied by coefficients or we can say an expression of more than two algebraic terms that contain different powers of the same variable.But;

• Not divisible by a variable.

• A variable's exponents can only be 0,1,2,3,... etc.

• It can't have an infinite number of terms.

Example - $5xy^2 – 3x + 5y^3 – 3$

And a polynomial with real coefficients is a product of irreducible polynomials of first and second degrees or in simple words, a polynomial having only real numbers as coefficients is the real coefficient Polynomial.

2. Question

Define degree of a polynomial.

Answer

A degree in a polynomial function is the greatest exponent of that equation, which determines the most number of solutions that a function could have. Or in simple words the degree of a term is the sum of the exponents of the variables that appear in it, and thus is a non-negative integer.

Each equation contains from one to several terms, which are divided by numbers or variables with different exponents.

For Example,

$y = 3x^{13} + 5x^3$

As we can see it has two terms,

$3x^{13}$ and $5x^3$

And the degree of the polynomial is 13, and that's the highest degree of any term in the equation.

3. Question

Write the standard form of a linear polynomial with real coefficients.

Answer

As we know the condition for linear polynomial degree equals to 1

So,

$f(x) = ax^1 + b,$

Where,

$a \neq 0$

4. Question

Write the standard form of a quadratic polynomial with real coefficients.

Answer

To write the standard form of a quadratic polynomial with real coefficients;

Let's take a polynomial,

$f(x) = ax^2 + bx + c$,

As we know that the quadratic polynomial = 2

So,

$a \neq 0$

5. Question

Write the standard form of a cubic polynomial with real coefficients.

Answer

To write the standard form of a cubic polynomial with real coefficients

Let's take,

$f(x) = ax^3 + bx^2 + cx + d$,

Where $a \neq 0$

Because degree of a cubic polynomial is 3

6. Question

Define the value of a polynomial at a point.

Answer

Value of a polynomial at a point is described as the value obtained by the polynomial at that point in time.

For Example:

Polynomial $f(x) = x^3 - 2(1)^2 + 7(1) - 8$

Now the value of the polynomial at $x = 1$

So here we get;

$= 1 - 2 + 7 - 8$

$= -2$

7. Question

Define zero of a polynomial.

Answer

A zero or we can say the root of a polynomial function is a number that, when put in for the variable, makes the function equal to zero. We use the Rational Zero Theorem to find all the zeros of a polynomial function and the possible rational roots of a polynomial equation.

8. Question

The sum and product of the zeros of a quadratic polynomial are $-\frac{1}{2}$ and — 3 respectively. What is the quadratic polynomial?

Answer

Given,

Sum of zeroes (α + β) = −(1/2)

Product of zeroes (αβ) = −3As we know that the quadratic polynomial with zeroes α, β is given by;

$x^2 - (\alpha + \beta)x + (\alpha\beta) = x^2 - [-(1/2)]x + (-3) = x^2 + (1/2)x - 3$

9. Question

Write the family of quadratic polynomials having $-\frac{1}{4}$ and 1 as its zeros.

Answer

Given,

– 1/4 and 1 are the zeros

So,

k[(x + 1/4)(x – 1)]

$k[x^2 - 1x + 1/4x - 1/4]$

$f(x) = k[x^2 - 3/4\ x - 1/4]$

Where k is any non-zero real number.

10. Question

If the product of zeros of the quadratic polynomial $f(x) = x^2 — 4x + k$ is 3, find the value of k.

Answer

Given,

Let the quadratic polynomial be $f(x) = x^2 - 4x + k$Product of the zeroes of the quadratic polynomial = 3Now,

Product of the zeroes = constant term/coefficient of x^2

⇒ 3 = k/1∴ k = 3

11. Question

If the sum of the zeros of the quadratic polynomial f (x) = kx^2 — 3x + 5 is 1, write the value of k.

Answer

Given,

The quadratic polynomial f(x) = kx^2 – 3x + 5Now,

Let two zeroes be a and 1 – a

∵ Sum of zeroes = 1 and also 1 – a > 0 ⇒ a < 1

Therefore;

Sum of the zeroes = 3/k

⇒ a + (1 – a) = 3/k

⇒ 1 = 3/k

⇒ k = 3

12. Question

In Fig. 2.17, the graph of a polynomial p(x) is given. Find the zeros of the polynomial.

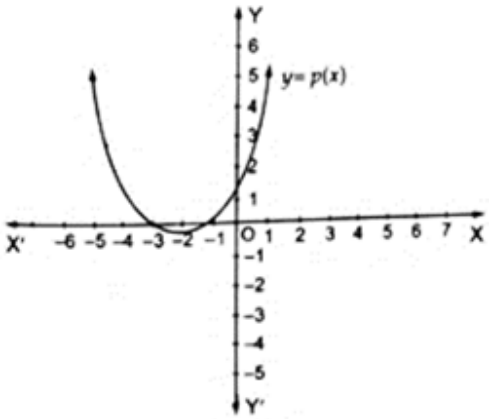


Fig. 2.17

Answer

In figure 2.17, the polynomial has 2 zeroes because the graph cuts x-axis at 2 points i.e. x = – 3 and x = – 1.

13. Question

The graph of a polynomial y = f (x), shown in Fig. 2.18. Find the number of real zeros of f (x).

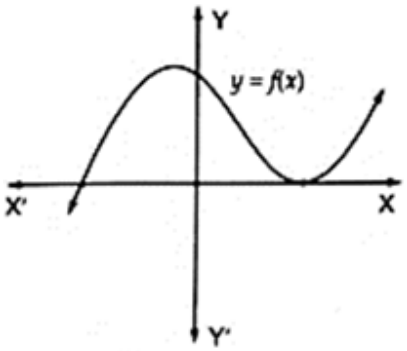


Fig. 2.18

Answer

In figure 2.18, the polynomial has 3 real zeroes because the graph cuts axises at three points.

14. Question

The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown below (Fig. 2.19). Write the signs of 'a' and $b^2 - 4ac$.

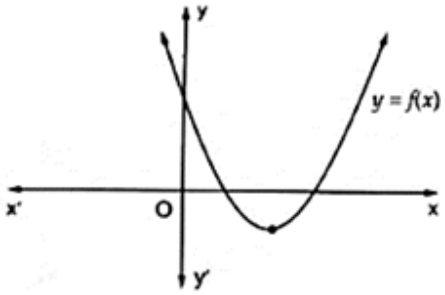


Fig. 2.19

Answer

The signs of a will be positive as $a > 0$ and,

The signs of $b^2 - 4ac$ will be positive as $b^2 - 4ac > 0$

15. Question

The graph of the polynomial $f(x) = ax^2 + box + c$ is as shown in Fig. 2.20. Write the value of $b^2 - 4ac$ and the number of real zeros of f (x).

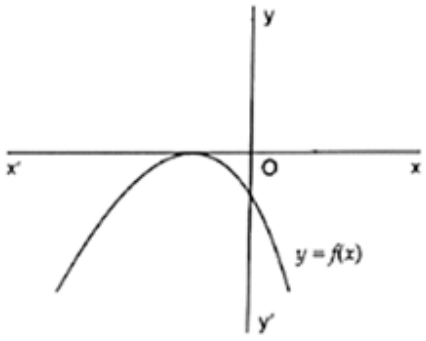


Fig. 2.20

Answer

$b^2 - 4ac = 0$, Two

The given quadratic equation touches the x-axis at only one point.

The root of the quadratic equation is equal and real because if the quadratic equation has two distinct roots, then the graph touches the x-axis at two points.

As we know that the roots are real and equal if the value of discriminant is zero,

So,

$b^2 - 4ac = 0$

16. Question

In Q. No. 14, write the sign of c.

Answer

The y-intercept of the equation is c,

So, the signs of will be positive as $c > 0$

17. Question

In Q. No. 15, write the sign of c.

Answer

The y-intercept of the equation is c,

So, the signs of will be negative as $c < 0$

18. Question

The graph of a polynomial/ (x) is as shown in Fig. 2.21. Write the number of real zeros of f (x).

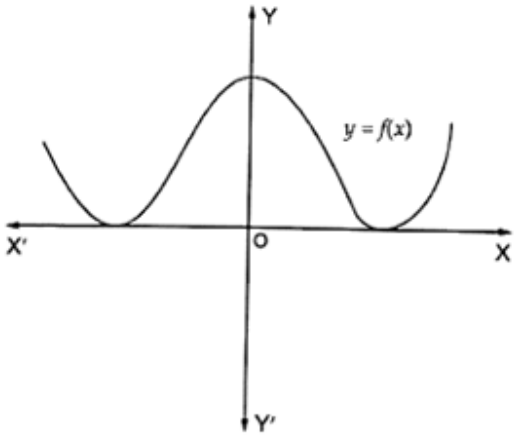


Fig. 2.21

Answer

In the figure 2.21,

The polynomial has 4 real zeroes because the graph cuts x-axis at four points.

19. Question

If x = 1 is a zero of the polynomial f (x) = $x^3 - 2x^2 + 4x + k$, write the value of k.

Answer

Given,

$f(x) = x^3 - 2x^2 + 4x + k$

x = 1 is a zero,

So,

Keeping x = 1 we get,

$f(1) = 1^3 - 2(1)^2 + 4\times1 + k$

f(1) = 1 – 2 + 4 + k

f(1) = 5 – 2 + k

f(1) = 3 + k

– 3 = k

So we get,

k = – 3

20. Question

State division algorithm for polynomials.

Answer

The polynomial long division is an algorithm for dividing a polynomial by another polynomial of the same or lower degree; it is a generalized version of the familiar arithmetic technique called long division. It can be done manually because it separates a complex division problem into smaller ones.

Let's take the Example:

f(x) and g(x) are two polynomials with,

$g(x)\neq0$,

Now we can find the polynomials p(x) and q(x) such that,

$f(x) = p(x) \times g(x) \times q(x)$

Where q(x) = 0 or degree of q(x) < is degree of g(x).

The result is;

Dividend = Quotient × Divisor + Remainder

This is known as the Division Algorithm for polynomials.

21. Question

Give an example of polynomials f(x), g(x), q(x) and r(x) satisfying f(x) = g(x) .q(x)+ r(x), where degree r (x) = 0.

Answer

Let $f(x) = x^3 + x^2 + x + 1$

$g(x) = x + 2$

$q(x) = x^2 - x + 3$

r (x) = -5

Now, f(x) = g(x) .q(x)+ r(x)

Here,

$L.H.S = x^3 + x^2 + x + 1$

And, $R.H.S = [(x + 2) \times (x^2 - x + 3)] + (-5)$

$= x^3 + 2x^2 – x^2 – 2x + 3x + 6 – 5$

$= x^3 + x^2 – 2x + 3x + 6 – 5$

$= x^3 + x^2 + x + 1$

= R.H.S

∴ the given set of polynomials satisfies the equation: f(x) = g(x) .q(x)+ r(x)

22. Question

Write a quadratic polynomial, sum of whose zeros is $2\sqrt{3}$ and their product is 2.

Answer

Given,

Sum of zeros = $2\sqrt{3}$ and

Product = 2

As we know;

$f(x) = x^2 + (– \text{Sum of zeros})x + (\text{Product of zeros})$

So,

$f(x) = x^2 – 2\sqrt{3}x + 2$

23. Question

If fourth degree polynomial is divided by a quadratic polynomial, write the degree of the remainder.

Answer

Degree of reminder is less than the degree of divisor so the degree of reminder could be one or zero depending on the quadratic polynomial.

24. Question

If $f(x) = x^3 + x^2 - ax + b$ is divisible by $x^2 - x$ write the values of a and b.

Answer

a = 2, b = 0

Given,

A polynomial $f(x) = x^3 + x^2 - ax + b$

Which is divisible by $x^2 - x$

Now,

$x^2 - x = (x - 1) = (x - 0)(x - 1)$

(x – 0) and (x – 1) are factors of polynomial f(x)

$\Rightarrow f(0) = 0$

$\Rightarrow 0^3 + 0 - a \times 0 + b = 0$

$\Rightarrow b = 0$

And f(1)

$1^3 + 1 - a \times 1 + b = 0$

$\Rightarrow 2 - a = 0$

$\Rightarrow a = 2$

$\therefore$ a = 2 and b = 0

25. Question

If a – b, a and a + b are zeros of the polynomial $f(x) = 2x^3 - 6x^2 + 5x - 7$, write the value of a.

Answer

Given;

A polynomial $f(x) = 2x^3 - 6x^2 + 5x - 7$

And zeroes are a – b, a and a + b,

Let's take a = α

b = β

c = y

As we know that,

$\alpha + \beta + y = -b/a$

$(a - b) + a + (a + b) = -(-6)/2$

$3a = 3$

$a = 1$

So, the value of a = 1

26. Question

Write the coefficients of the polynomial $p(z) = z^5 - 2z^2 + 4$.

Answer

Given,

Polynomial $p(z) = z^5 - 2z^2 + 4$

Now first break the given equation,

We get,

Coefficient of $z^5 = 1$

Coefficient of $z^4 = 0$

Coefficient of $z^3 = 0$

Coefficient of $z^2 = -2$

Coefficient of z = 0

Constant term = 4

Therefore 1, 0, 0, – 2, 0, 4 are coefficients of the polynomial p(z)

27. Question

Write the zeros of the polynomial $x^2 - x - 6$.

Answer

Given,

Polynomial = $X^2 - x - 6 = 0x^2 - 3x + 2x - 6 = 0x(x - 3) + 2(x - 3) = 0(x + 2)(x - 3) = 0$So – 2 and 3 are the zeros of the given polynomial

28. Question

If (x + a) is a factor of $2x^2 + 2ax + 5x + 10$, find a.

Answer

Given,

$2x^2 + 2ax + 5x + 10$

Factor = (x + a)

Now by using factor theoram we get,

$f(- a) = 2a^2 - 2a^2 - 5a + 10 = 0$

$- 5a + 10 = 0$

$a = 2$

Therefore the value of a = 2.

29. Question

For what value of k, – 4 is a zero of the polynomial $x^2 - x - (2k + 2)$?

Answer

Given,

A polynomial $x^2 - x - (2k + 2)$

– 4 is a zero of the given polynomial.

As – 4 is the zero,so at x = -4, the value of the polynomial $x^2 - x - (2k + 2)$ will be 0.

So we get,

$\Rightarrow x^2 - x - (2k + 2) = 0$

$\Rightarrow (- 4)^2 - (- 4) - (2k + 2) = 0$

$\Rightarrow 16 + 4 - (2k + 2) = 0$

$\Rightarrow 20 - (2k + 2) = 0$

$\Rightarrow - (2k + 2) = - 20$

$\Rightarrow 2k + 2 = 20$

$\Rightarrow 2k = 20 - 2$

$\Rightarrow 2k = 18$

$\Rightarrow k = 18/2 = 9$

30. Question

If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1) x - 1$, then find the value of a.

Answer

Given,

$p(x) = ax^2 - 3(a - 1) x - 1$

Zero is 1

Now,

$p(1) = a(1)^2 - 3(a - 1) \times 1 - 1 = 0$

$p(1) = a - 3a + 3 - 1 = 0$

$p(1) = - 2a + 2 = 0$

$= - 2a = - 2$

$= a = - 2/ - 2 = 1$

So the value of a = 1

31. Question

If α, β are the zeros of a polynomial such that α + β = − 6 and αβ = − 4, then write the polynomial.

Answer

For any polynomial, $f(x) = ax^2 + bx + c$

The sum of zeroes, is given as -b/a

& the product of zeroes is given as c/a

Here, the sum of zeroes is given as -6 = -6/1

$\Rightarrow -\frac{6}{1} = -\frac{b}{a}$

⇒ b = 6 & a = 1

Also, product of zeroes = c/a

$\Rightarrow -4 = \frac{c}{a}$

⇒ c = -4

& a = 1

∴ the polynomial is $f(x) = x^2 + 6x - 4$

32. Question

If α, β are the zeros of the polynomial $2y^2 + 7y + 5$, write the value of α + β + αβ.

Answer

Given,

Polynomial $2y^2 + 7y + 5$

Zeroes = a, 13

Here a = 2

b = 7

c = 5

Lets take α and β are two zero,

So sum of the zeroes will be,

α + β = – b/a = – 7/2

Product of the zeroes = α. β = c/a = 5/2

Now put the values,

α + β + αβ = (α + β) αβ

= – 7/2 + 5/2 = – 2/2 = – 1

33. Question

For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$?

Answer

Given,

Polynomial $2x^2 + x + k$

Zero = 3

By putting x = 3

We get,

$p(x) = 2x^2 + x + k = 0$

$p(3) = 2(3)^2 + 3 + k = 0$

p(3) = 18 + 3 + k = 0

= 21 + k = 0

k = – 21

Hence the answer is – 21

34. Question

For what value of k, is – 3 a zero of the polynomial $x^2 + 11x + k$?

Answer

Given,

Polynomial $x^2 + 11x + k$

Zero of the polynomial = – 3

As we have zero,

$f(x) = x^2 + 11x + k = 0$

By putting x = – 3

$f(-3) = -3^2 + 11\times3 + k = 0$

$f(-3) = 9 + (-33) + k = 0$

$f(-3) = -24 + k = 0$

$f(-3) = k = 24$

So we have the value of k = 24

35. Question

For what value of k, is – 2 a zero of the polynomial $3x^2 + 4x + 2k$?

Answer

Given,

Polynomial $3x^2 + 4x + 2k$

Zero of the polynomial = – 2

As we have zero,

$f(x) = 3x^2 + 4x + 2k = 0$

By putting x = – 2

$f(-2) = 3(-2)^2 + 4(-2) + 2k = 0$

$f(-2) = 12 - 8 + 2k = 0$

$= 4 + 2k = 0$

$= 2k = -4$

$k = -4/2 = -2$

So we have the value of k = – 2

36. Question

If a quadratic polynomial f (x) is factorizable into linear distinct factors, then what is the total number of real and distinct zeros of f(x)?

Answer

Given,

Quadratic polynomial f(x) is factorizable into linear distinct factors;

So,

Let $f(x) = (x - a)(x - b)$, where $a \neq b$

If a, b are the element of R,

Then f(x) must be having two real and distinct zeroes.

37. Question

If a quadratic polynomial f (x) is a square of a linear polynomial, then its two zeroes are coincident. (True/False)

Answer

True

Lets take,

$f(x) = x^2 - 4x + 4$

$= (x - 2)^2$

$= [g(x)]^2$ ……. [g(x) = (x – 2) is a linear polynomial]

Zero of g(x) is 2,

So,

Zeroes of f(x) are 2 and 2.

So we can say that zeroes of f(x) are coincident.

38. Question

If a quadratic polynomial f(x) is not factorizable into linear factors, then it has no real zero. (True/False)

Answer

True

Let a polynomial $f(x) = x^2 + 9$

$f(x) = (x + 3i)(x - 3i)$

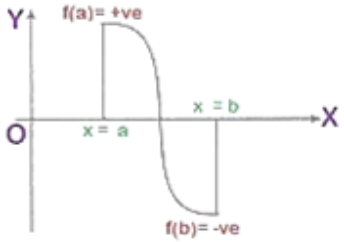
So, the zeroes are always imaginary and not real.

39. Question

If f(x) is a polynomial such that f(a)f(b)< 0, then what is the number of zeros lying between a and b?

Answer

Let's first draw the figure,



As we can see in the figure the number of zeroes can be 1 or 3.

So the least numbers of zeroes lying between a and b is 1.

40. Question

If graph of quadratic polynomial $ax^2 + bx + c$ cuts positive direction of y-axis, then what is the sign of c?

Answer

Positive

If quadratic polynomial $ax^2 + bx + c$ cuts positive direction of y-axis, then it means that the value of y is positive when x = 0 [∵ at y-axis, x = 0]

Now

y = c at x = 0 and y is positive.

So, c is positive.

41. Question

If the graph of quadratic polynomial $ax^2 + bx + c$ cuts negative direction of y-axis, then what is the sign of c?

Answer

Negative

If quadratic polynomial $ax^2 + bx + c$ cuts Negative direction of y-axis, then it means that the value of y is negative when x = 0 [∵ at y-axis, x = 0]

Now,

y = c at x = 0 and y is Negative.

So, c is Negative.

1. Question

If α, β are the zeros of the polynomial $f(x) = x^2 + x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

A. 1

B. – 1

C. 0

D. None of these

Answer

Given,

$f(x) = x^2 + x + 1$

α = a

β = b

c = c

α + β = – b/a

= – 1/1

= – 1 …….. Equ.(i)

αβ = c/a = 1 ….. Equ.(ii)

Dividing (i) from (ii)

We get,

– 1/1 = – 1

So,

1/α + 1/β = – 1

2. Question

If α, β are the zeros of the polynomial $p(x) = 4x^2 + 3x + 7$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

A. $\frac{7}{3}$

B. $-\frac{7}{3}$

C. $\frac{3}{7}$

D. $-\frac{3}{7}$

Answer

Given,

$p(x) = 4x^2 + 3x + 7$

α + β = – b/a

$= – 3x/4x^2 = – 3/4x$ ……. (i)

αβ = c/a

$= 7/4x^2$ ……. (ii)

Dividing (i) from (ii),

We get,

$$\frac{\frac{3}{4x}}{\frac{7}{x^2}} = \frac{-3 \times 4x^2}{4x \times 7}$$

$$= -\frac{3}{7}x$$

So,

1/α + 1/β = – 3/7

3. Question

If one zero of the polynomial f (x) = $(k^2 + 4)\ x^2 + 13x + 4k$ is reciprocal of the other, then k =

A. 2

B. – 2

C. 1

D. – 1

Answer

Given;

f(x) = $(k^2 + 4)\ x^2 + 13x + 4k$,

One zero of the polynomial is reciprocal of the other,

Let a be the one zero,

∴ The other zero will be 1/a

As we know that,

Product of the zeros = c/a = $4k/k^2 + 4$

∴ $4k/k^2 + 4 = 1$

⇒ $4k = k^2 + 4$

⇒ $k^2 + 4 – 4k = 0$

⇒ $(k – 2)^2 = 0$

⇒ k = 2

So the value of k is 2

4. Question

If the sum of the zeros of the polynomial f(x) = $2x^3 – 3kx^2 + 4x – 5$ is 6, then the value of k is

A. 2

B. 4

C. – 2

D. – 4

Answer

Given,

$f(x) = 2x^3 – 3kx^2 + 4x – 5$

Sum of the zeros of the polynomial = 6

Let x, y and z be the zeroes than,

x + y + z = 6……Equation (i)

So,

x + y + z = – b/a

= – (– 3k)/2

From Eq. (i) we get,

3k/2 = 6

k/2 = 6/3

k = 2×2 = 4

5. Question

If α and β are the zeros of the polynomial $f(x) = x^2 + px + q$, then a polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is its zeros is

A. $x^2 + qx + p$

B. $x^2 – px + q$

C. $qx^2 + px + 1$

D. $px^2 + qx + 1$

Answer

Given: If α and β are the zeros of the polynomial $f(x) = x^2 + px + q$,

To find: a polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is its zeros is

Solution:

$f(x) = x^2 + px + q$

We know, $sum\ of\ zeroes = \ -\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

$$product\ of\ zeroes = \frac{constant\ term}{coefficient\ of\ x^2}$$

Since α, β are zeroes of given polynomial,

⇒ α + β = – p

and αβ = q

Let S and P denote respectively the sum and product of zeroes of the required polynomial,So,

$$S = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$\Rightarrow S = \frac{\beta + \alpha}{\alpha\beta} \quad \text{..... (1)}$$

And $P = \frac{1}{\alpha} \times \frac{1}{\beta}$ ..... (2)

Put the values of α + β and αβ in (1) and (2) to get,

$$\Rightarrow S = \frac{-p}{q}$$

And $P = \frac{1}{q}$

We know equation having 2 zeroes is of form,k (x^2 - (sum of zeroes) x + product of zeroes)

For a polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is its zeros the equation becomes,

x^2 + p/q x + 1/q = 0

So here we get,

g(x) = qx^2 + px + 1

6. Question

If α, β are the zeros of polynomial f(x) = x^2 – p (x + 1) – c, then (α + 1) (β + 1) =

A. c – 1

B. 1 – c

C. c

D. 1 + c

Answer

Given,

$f(x) = x^2 - p(x + 1) - c$

α and β are the zeros

So,

$f(x) = x^2 - p(x + 1) - c$

$= x^2 - px - (p + c)$

As

$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

$= - p - c + p + 1$

$= 1 - c$

7. Question

If α, β are the zeros of the polynomial $f(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, then c =

A. 1

B. 0

C. – 1

D. 2

Answer

Given

$f(x) = x^2 - p(x + 1) - c$

α and β are the zeros

Then,

$f(x) = x^2 - p(x + 1) - c$

$= x^2 - px - (p + c)$

As

$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

$= - p - c + p + 1$

$= 1 - c$

So, the value of c,

$c = 1$

8. Question

If $f(x) = ax^2 + bx + c$ has no real zeros and $a + b + c < 0$, then

A. $c = 0$

B. $c > 0$

C. $c < 0$

D. None of these

Answer

Given;

$f(x) = ax^2 + bx + c$ has no real zeroes, and $a + b + c<0$

Suppose $a = -1$,

$b = 1$,

$c = -1$

Then $a + b + c = -1$,

$b^2 - 4ac = -3$

Therefore it is possible that c is less tha zero.

Suppose $c = 0$

Then $b^2 - 4ac = b^2 \geq 0$

So,

f(x) has at least one zero.

Therefore c cannot equal zero.

Suppose $c > 0$.

It must also be true that $b^2 \geq 0$

Then,

$b^2 - 4ac < 0$ only if $a > 0$.

Therefore,

$a + b + c < 0$.

$-b > a + c > 0$

$b^2 > (a + c)^2$

$b^2 > a^2 + 2ac + c^2$

$b^2 - 4ac > (a - c)^2 \geq 0$

As we know that the discriminant can't be both greater than zero and less than zero,

So, C can't be greater than zero.

9. Question

If the diagram in Fig. 2.22 shows the graph of the polynomial $f(x) = ax^2 + bx + c$, then

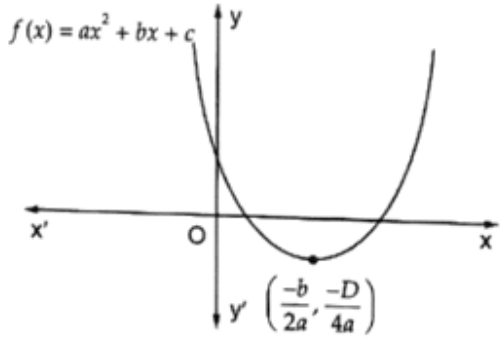


Fig. 2.22

A. $a > 0$, $b<0$ and$c > 0$

B. $a < 0$, $b <0$ and $c< 0$

C. $a < 0$, $b > 0$ and $c > 0$

D. $a < 0$, $b > 0$ and $c < 0$

Answer

As seen from the graph,

The parabola cuts the graph at two points on the positive x-axis.

Hence, both the roots are positive.

Now, for a polynomial, the sum of roots is given as:

$\alpha + \beta = -b/a$

$\therefore$ the sum will be positive as the roots are positive.

Also, the product of roots = c/a has to be positive too.

$\Rightarrow$ a is positive.

Now, since a is positive, therefore for the sum of roots to be negative, b has to be negative.

$\Rightarrow a > 0, b< 0 \& c > 0.$

Therefore, option (a) is correct.

10. Question

Figure 2.23 shows the graph of the polynomial $f(x) = ax^2 + bx + c$ for which

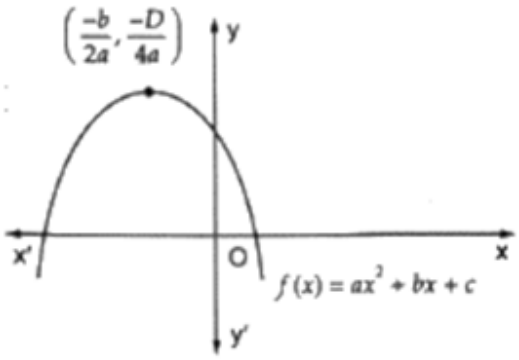


Fig. 2.23

A. $a < 0, b > 0$ and $c > 0$

B. $a < 0, b < 0$ and $c > 0$

C. $a < 0, b < 0$ and $c < 0$

D. $a > 0, b > 0$ and $c < 0$

Answer

As seen from the graph,

The parabola cuts the graph at two points on the x axis.

One root is positive & one root is negative.

Now, for a polynomial, the sum of roots is given as:

$\alpha + \beta = -b/a$

Also, the product of roots = c/a is positive.

Because c is positive,

$\Rightarrow$ a is negative & b is negative.

Therefore, option (b) is correct.

11. Question

If the product of zeros of the polynomial $f(x) = ax^3 - 6x^2 + 11x - 6$ is 4, then a =

A. $\frac{3}{2}$

B. $-\frac{3}{2}$

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

Answer

Given;

Polynomial = $ax^3 - 6x^2 + 11x - 6$

Let a,b,c be the zeroes of $f(x) = ax^3 - 6x^2 + 11x - 6$

Product of the zeroes = – (constant term)/coefficient of x^3

$4 = -(-6)/a$

$4 = 6/a$

$4a = 6$

$a = 6/4$

$a = 3/2$

12. Question

If zeros of the polynomial $f(x) = x^3 - 3px^2 + qx - r$ are in A.P., then

A. $2p^3 = pq - r$

B. $2p^3 = pq + r$

C. $p^3 = pq - r$

D. None of these

Answer

Given;

$f(x) = x^3 - 3px^2 + qx - r$

Let a, b and c be the zeroes of the polynomial $x^3 - 3px^2 + qx - r$

$a + b + c = 3p$

$ab + bc + ac = q$

$abc = r$

$a + b = 2b \Rightarrow b = p \Rightarrow a + c = 2p$

$b(a + c) + ac = q$

$\Rightarrow 2p^2 + ac = q$

$\Rightarrow ac = r/p$

$\therefore 2p^2 + r/p = q$

$2p^3 = pq - r$

13. Question

If the product of two zeros of the polynomial f (x) = $2xZ^3 + 6x^2 - 4x + 9$ is 3, then its third zero is

A. $\frac{3}{2}$

B. $-\frac{2}{3}$

C. $\frac{9}{2}$

D. $-\frac{9}{2}$

Answer

Given;

$f(x) = 2x^3 + 6x^2 - 4x + 9$

Product of the two zeros = 3

a = 2,

b = 6

c = – 4

d = 9

Let the zeros of f(x) be p, q, r

Product of the zeroes = – d/a

p×q×r = – 9/2

3r = – 9/2 … (given p×q = 3)

r = – 3/2

Hence,

Third zero = – 3/2

14. Question

If the polynomial $f(x) = ax^3 + bx - c$ is divisible by the polynomial $g(x) = x^2 + bx + c$, then ab =

A. 1

B. $\frac{1}{c}$

C. –1

D. $-\frac{1}{c}$

Answer

Given;

$f(x) = ax^3 + bx - c$ is divisible by $g(x) = x^2 + bx + c$

Now by division Method,

$$
\begin{array}{r|l}
 & ax - ab \\
\hline
x^2 + bx + c & ax^3 + bx - c \\
 & ax^3 + abx^2 + acx \\
 & - \quad - \quad - \\
\hline
 & -abx^2 + (b - ac)x - c \\
 & -abx^2 - ab^2x - abc \\
 & + \quad + \quad + \\
\hline
 & (b - ac + ab^2)x + abc - c
\end{array}
$$

As f(x) is divisible by g(x), then remainder must be 0,

i.e.

$(ab^2 - ac + b)x + c(ab - 1) = 0$

$\Rightarrow (ab^2 - ac + b)x = 0$ and $c(ab - 1) = 0$

$\Rightarrow ab^2 - ab + b = 0$ ($\because x \neq 0$) and $ab - 1 = 0$ ($\because c \neq 0$)

$\Rightarrow ab - 1 = 0$

$\Rightarrow ab = 1$

15. Question

In Q. No. 14, c =

A. b

B. 2b

C. $2b^2$

D. $-2b$

Answer

Given: $f(x) = ax^3 + bx - c$ is divisible by $g(x) = x^2 + bx + c$

To find: The value of c.

Solution:

Since $f(x) = ax^3 + bx - c$ is divisible by $g(x) = x^2 + bx + c$

Now by division Method first we have to calculate the value of ab,

$$
\begin{array}{r|l}
 & ax - ab \\
\hline
x^2 + bx + c & ax^3 + bx - c \\
 & ax^3 + abx^2 + acx \\
 & - \quad - \quad - \\
\hline
 & -abx^2 + (b - ac)x - c \\
 & -abx^2 - ab^2x - abc \\
 & + \quad + \quad + \\
\hline
 & (b - ac + ab^2)x + abc - c
\end{array}
$$

As f(x) is divisible by g(x), then remainder must be 0,

i.e.

$(ab^2 - ac + b)x + abc - c = 0$

$\Rightarrow (ab^2 - ac + b)x + c(ab - 1) = 0$

$\Rightarrow (ab^2 - ac + b)x = 0$ and $c(ab - 1) = 0$

$\Rightarrow ab^2 - ac + b = 0$ ($\because x \neq 0$) and $ab - 1 = 0$ ($\because c \neq 0$)

$\Rightarrow ab - 1 = 0$

$\Rightarrow ab = 1$

Now take;

$ab^2 - ac + b = 0$

$\Rightarrow (ab)b - ac + b = 0$

Using $ab = 1$ we get;

$\Rightarrow b - ac + b = 0$

$\Rightarrow 2b - ac = 0 \Rightarrow c = 2b/a$ ... (1)Multiply and divide RHS of (1) by b

⇒ c = 2b/a = $2b^2/ab$

As ab = 1

So we get;

$c = 2b^2$

16. Question

If one root of the polynomial f (x) = $5x^2 + 13x + k$ is reciprocal of the other, then the value of k is

A. 0

B. 5

C. $\frac{1}{6}$

D. 6

Answer

Given;

$f(x) = 5x^2 + 13x + k$

Let suppose roots are R and 1/R

Product of the roots = C/R = R×1/R

R×1/R = k/5

1 = k/5

So,

Here k = 5

17. Question

If α, β, γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ =

A. $-\frac{b}{d}$

B. $\frac{c}{d}$

C. $-\frac{c}{d}$

D. $-\frac{c}{a}$

Answer

Given,

$f(x) = ax^3 + bx^2 + cx + d$,

As we know,

$\alpha + \beta + \gamma = b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = - c/a$ …….(i)

$\alpha\beta\gamma = d/a$ …….. (ii)

Now,

Divide (i) by (ii)

We get

$1/\gamma + 1/\alpha + 1/\beta = - c/d$

18. Question

If α, β, γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\alpha^2 + \beta^2 + \gamma^2 =$

A. $\frac{b^2 - ac}{a^2}$

B. $\frac{b^2 - 2ac}{a}$

C. $\frac{b^2 + 2ac}{b^2}$

D. $\frac{b^2 + 2ac}{a^2}$

Answer

$f(x) = ax^3 + bx^2 + cx + d$,

Then the roots are : α , β and γ

Now, sum of roots:

$\alpha + \beta + \gamma = - b/a$ ….(i)

The product of zeroes,

α × β × γ = -d/a….(ii)

And,

αβ + βγ + γα = c/a…(iii)

Now,

$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\beta\gamma + 2\gamma\alpha$

$\Rightarrow \left(-\frac{b}{a}\right)^2 = a^2 + \beta^2 + \gamma^2 + 2\left(\frac{c}{a}\right)$

$\Rightarrow \frac{b^2}{a^2} - \frac{2c}{a} = a^2 + \beta^2 + \gamma^2$

$\Rightarrow a^2 + \beta^2 + \gamma^2 = \frac{(b^2 - 2ac)}{a}$

19. Question

If α, β, γ are the zeros of the polynomial $f(x) = x^3 - px^2 + qx - r$, then $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} =$

A. $\frac{r}{p}$

B. $\frac{p}{r}$

C. $-\frac{p}{r}$

D. $-\frac{r}{p}$

Answer

Given,

$f(x) = x^3 - px^2 + qx - r$

α, β and y are the Zeros,

α + β + y = − p

αβ + βy + yα = − q

αβy = − r

1/αβ + 1/βy + 1/yα = p/r

20. Question

If α, β are the zeros of the polynomial $f(x) = ax^2 + bx + c$, then $\frac{1}{\alpha^2} + \frac{1}{\beta^2} =$

A. $\frac{b^2 - 2ac}{a^2}$

B. $\frac{b^2 - 2ac}{c^2}$

C. $\frac{b^2 + 2ac}{a^2}$

D. $\frac{b^2 + 2ac}{c^2}$

Answer

Given

$f(x) = ax^2 + bx + c$

α, β are the Zeros

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$

$1/\alpha + 1/\beta = \frac{-\frac{b}{a}}{\frac{c}{a}}$

$1/\alpha^2 + 1/\beta^2 = (1/\alpha + 1/\beta)^2 - 2/\alpha\beta$

$= (-b/c)^2 - 2a/c$

$= \frac{b^2 - 2ac}{c^2}$

21. Question

If two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are each equal to zero, then the third zero is

A. $\frac{-d}{a}$

B. $\frac{c}{a}$

C. $\frac{-b}{a}$

D. $\frac{b}{a}$

Answer

Given,

$ax^3 + bx^2 + cx + d$

By Putting x = 0

0 + d = 0

d = 0

$ax^3 + bx^2 + cx + d = 0$

$x(ax^2 + bx + c) = 0$

Put x = 0

c = 0

$ax^2 + bx = 0$

x(ax + b) = 0

Hence,

x = – b/a

22. Question

If two zeros of $x^3 + x^2 – 5x – 5$ are √5 and –√5 , then its third zero is

A. 1

B. – 1

C. 2

D. – 2

Answer

Given,

$x^3 + x^2 – 5x – 5$

Zeros = √5 and – √5

$x^2 – 5$ is a root

$$
\begin{array}{r}
x+1 \\
x^2-5\overline{)x^3-x^2-5x-5} \\
-x^3-5x \\
+ \\
\hline
x^2-5
\end{array}
$$

$(x^2 - 5)(x + 1) = 0$

$x = -1$

So,

The third zero is -1

23. Question

The product of the zeros of $x^3 + 4x^2 + x - 6$ is

A. -4

B. 4

C. 6

D. -6

Answer

Given,

$p(x) = x^3 + 4x^2 + x - 6$

As

$a = 1$

$b = 4$

$c = 1$

$d = -6$

So the product of the zeros will be;

$= -d/a$

$= -(-6)/1$

$= 6$

24. Question

What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial?

A. 1

B. 2

C. 4

D. 5

Answer

Given;

$p(x) = x^2 - 5x + 4$

3 is the zero of the resulting polynomial

So we have;

$p(3) = (3)^2 - 5\times3 + 4$

$p(3) = 9 - 15 + 4$

$p(3) = -2$

As – 2 is the remainder,

So 2 should be added to the given polynomial to get 3 as the zero of the resulting polynomial.

We can also verify the answer;

By adding 2 to the given polynomial we get;

$P(x) = x^2 - 5x + 4 + 2$

$P(x) = x^2 - 5x + 6$

$P(3) = 9 - 15 + 6$

$P(3) = 15 - 15$

$P(3) = 0$

Hence proved that 2 is the right answer to get 3 as the zero of resulting polynomial.

25. Question

What should be subtracted to the polynomial $x^2 - 16x + 30$, so that 15 is the zero of the resulting polynomial?

A. 30

B. 14

C. 15

D. 16

Answer

Given,

$p(x) = x^2 - 16x + 30$

15 is the zero of the resulting polynomial,

So we get;

$p(15) = (15)^2 - 16 \times 15 + 30$

$p(15) = 225 - 240 + 30$

$p(15) = 15$

$\therefore$ 15 should be subtracted from the given polynomial to get 15 as the zero of the resulting polynomial.

We can also verify the answer;

By adding 2 to the given polynomial we get;

$P(x) = x^2 - 16x + 30 - 15$

$P(x) = x^2 - 16x + 15$

$P(15) = 225 - 240 + 15$

$P(15) = 240 - 240$

$P(15) = 0$

Hence proved that 15 is the right answer to get 15 as the zero of resulting polynomial.

26. Question

A quadratic polynomial, the sum of whose zeroes is 0 and one zero is 3, is

A. $x^2 - 9$

B. $x^2 + 9$

C. $x^2 + 3$

D. $x^2 - 3$

Answer

Given,

One of the polynomial is 3,

Sum of the zeros = 0

Now

Let the other zero be x,

$x + 3 = 0$

$x = -3$

So,

Zeros of the polynomial are 3 and – 3

Product of the zeros = 3 × – 3 = – 9

Polynomial with zeros 3 and – 3 = x^2 – (0)x + (– 9) = x^2 – 9

27. Question

If two zeroes of the polynomial x^3 + x^2 – 9x – 9 are 3 and – 3, then its third zero is

A. – 1

B. 1

C. – 9

D. 9

Answer

Given

x^3 + x^2 – 9x – 9

Zeros = 3 and – 3

$$
\begin{array}{r}
x+1 \\
x^2-9\overline{)\,x^3+x^2-9x-9} \\
-x^3-9x \\
+ \\
\hline
x^2-9
\end{array}
$$

x = – 1

So its third zero is – 1

28. Question

If √5 and – √5 are two zeroes of the polynomial x^3 + 3x^2 – 5x – 15, then its third zero is

A. 3

B. – 3

C. 5

D. – 5

Answer

Given,

x^3 + 3x^2 – 5x – 15

Zeros = √5 and – √5

$$
\begin{array}{r}
x+1 \\
x^2-9\overline{)x^3+x^2-9x-9} \\
-x^3-9x \\
+ \\
\hline
x^2-9
\end{array}
$$

So the third zero = – 3

29. Question

If x + 2 is a factor of x^2 + ax + 2b and a + b = 4, then

A. a = 1,b = 3

B. a = 3,b = 1

C. a = – 1,b = 5

D. a = 5,b = – 1

Answer

Given;

A factor = x + 2

So,

x + 2 = 0

x = – 2

$p(x) = x^2 + ax + 2b$

$p(-2) = (-2)^2 + (-2)a + 2b$

0 = 4 – 2a + 2b

– 4 = – 2 (a – b)

2 = a – b……Equation (i)

4 = a + b …….. Equation (ii)

By Adding (i) and (ii) we get,

2 – a + b + 4 – a – b = – 2a + 2

2a = 2

So we get, a = 3 and b = 1

30. Question

The polynomial which when divided by – x^2 + x – 1 gives a quotient x – 2 and remainder 3, is

A. $x^3 - 3x^2 + 3x - 5$

B. $-x^3 - 3x^2 - 3x - 5$

C. $-x^3 + 3x^2 - 3x + 5$

D. $x^3 - 3x^2 - 3x + 5$

Answer

Given,

Divisor = $-x^2 + x - 1$

Quotient = $x - 2$

Remainder = 3

Now we have to find out Dividend…

As we know that,

Dividend = Divisor × Quotient + Remainder

Dividend = $(-x^2 + x - 1)(x - 2) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

So,

The required polynomial is $-x^3 + 3x^2 - 3x + 5$

3. Pair of Linear Equations in Two Variables

Exercise 3.1

1. Question

Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a rig on the items kept in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. Each ride costs Rs 3, and a game of Hoopla costs Rs 4. If she spent Rs 20 in the fair, represent this situation algebraically and graphically.

Answer

Let the number of times Akhila played Hoopla be x and number of times she played Giant wheel be y.

Given, number of times she played Hoopla is half the number of rides she had on the Giant Wheel.

⇒ x = 2y

⇒ x − 2y = 0For above equation, we have following table

x	0	2	4
y	0	1	2

Each ride costs Rs 3, and a game of Hoopla costs Rs 4 and she spent Rs 20 in the fair.

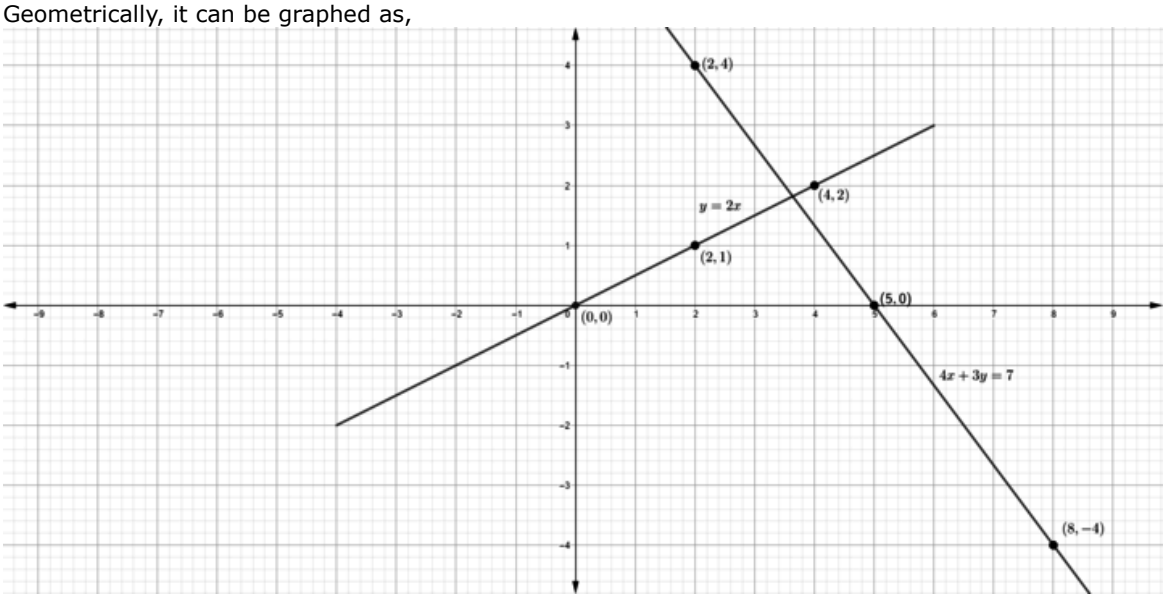
⇒ 4x + 3y = 20

x	5	2	8
y	0	4	-4

Thus, algebraically it is represented by :

x − 2y = 0 and 4x + 3y = 0

Geometrically, it can be graphed as,



2. Question

Aftab tells his daughter, 'Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Is not this interesting? Represent this situation algebraically and graphically.

Answer

Let the present age of father be x and present age of daughter be y.

Aftab tells his daughter, 'Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be."

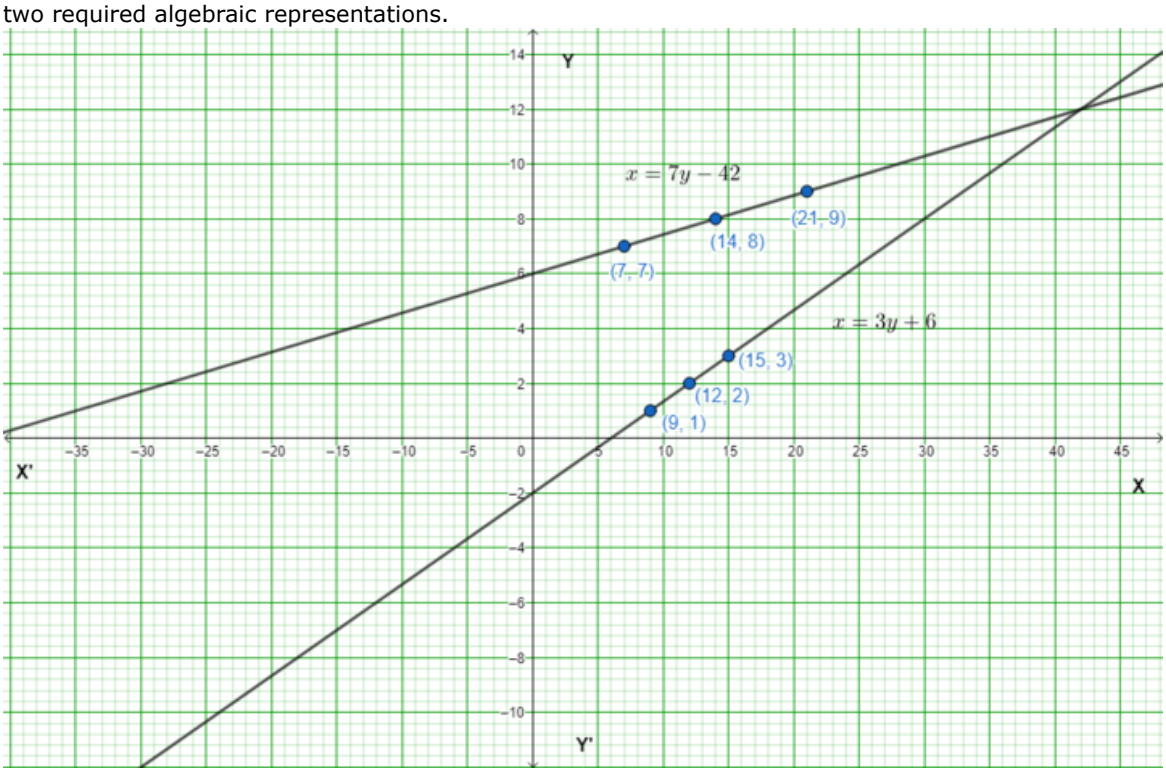
∴ x – 7 = 7(y – 7)⇒ x - 7 = 7y - 49⇒ x – 7y + 42 = 0⇒ x = 7y - 42⇒ x = 7(y - 6) [1]The coordinates satisfying above equation are

x	7	14	21
y	7	8	9

Also, x + 3 = 3(y + 3)⇒ x + 3 = 3y + 9⇒ x – 3y – 6 = 0⇒ x = 3y + 6⇒ x = 3(y + 2) [2]

x	9	12	15
y	1	2	3

The graph of both the equation is the required graphical representation and the equations [1] and [2] are two required algebraic representations.



3. Question

The path of a train A is given by the equation 3x + 4y – 12 = 0 and the path of another train B is given by the equation 6x + 8y – 48 = 0. Represent this situation graphically.

Answer

Given equations are 3x + 4y – 12 = 0 and 6x + 8y – 48 = 0

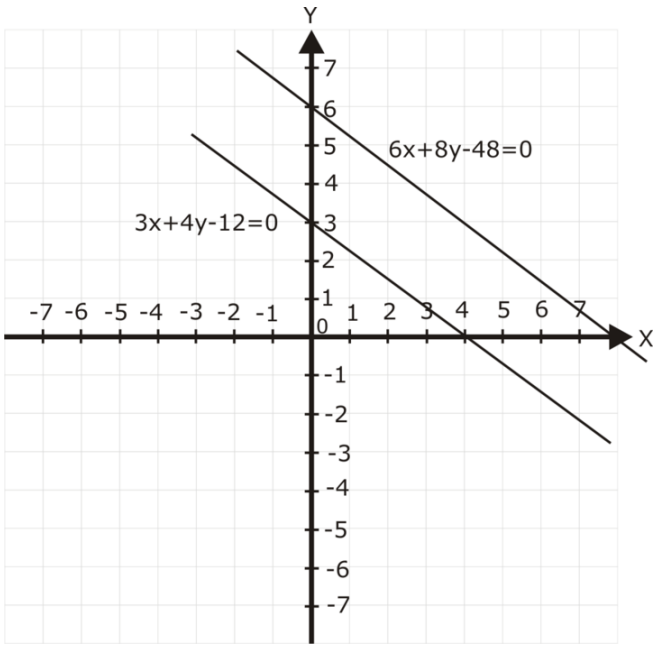
Let us plot the given equations. For this we will need some points to plot for the equation.

3 x + 4 y - 12 = 0

4 y = 12 - 3 xNow let us take random values of x and obtain the corresponding values of y by putting in the equation.at x = 0, 4 y = 12 - 3 x 04 y = 12y = 3Now at y = 0,3 x + 4 x 0 = 123 x = 12x = 4So we have two points for the equation 3 x + 4 y - 12 = 0 and those are (0, 3) and (4, 0)6 x + 8 y - 48 = 0Now let us find points for this equationat x = 0,6 x 0 + 8 y

= 488 y = 48y = 6at y = 0,6 x + 8 x 0 = 486 x = 48x = 8So we have two points for the equation 6 x + 8 y - 48 = 0 and those are (8, 0) and (0, 6)3x + 4y − 12 = 0 passes through (4, 0) and (0, 3)

6x + 8y − 48 = 0 passes through (8, 0) and (0, 6)

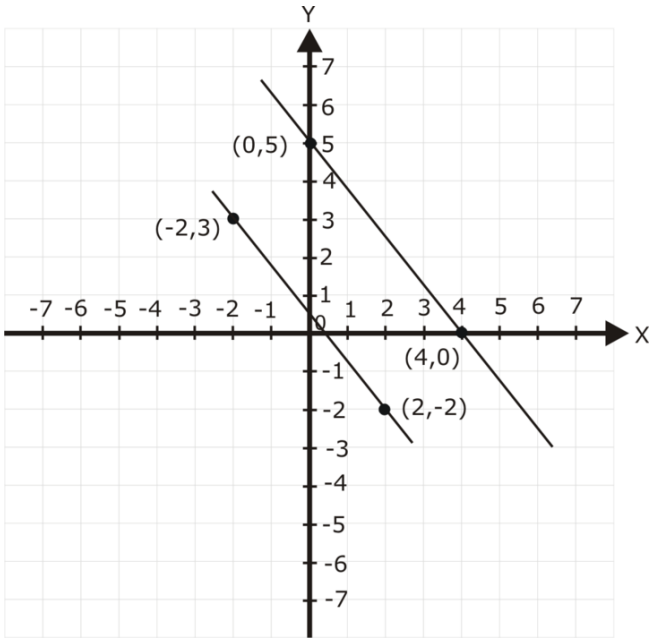


4. Question

Gloria is walking along the path joining (-2, 3) and (2, -2), while Suresh is walking along the path joining (0, 5) and (4, 0). Represent this situation graphically.

Answer

Given, Gloria is walking along the path joining (-2, 3) and (2, -2), while Suresh is walking along the path joining (0, 5) and (4, 0).



5. Question

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Answer

Two lines, a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0

If $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, then the lines coincide

If $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines are parallel

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines intersect

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

$$\Rightarrow \frac{a1}{a2} = \frac{5}{7}, \frac{b1}{b2} = -\frac{4}{6}, \frac{c1}{c2} = \frac{8}{-9}$$

Thus, $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$.

The lines intersect.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

$$\Rightarrow \frac{a1}{a2} = \frac{9}{18} = \frac{1}{2}, \frac{b1}{b2} = \frac{3}{6} = \frac{1}{2}, \frac{c1}{c2} = \frac{12}{24} = \frac{1}{2}$$

Thus, $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

The lines are coincident.

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

$$\Rightarrow \frac{a1}{a2} = \frac{6}{2} = 3, \frac{b1}{b2} = -\frac{3}{-1} = 3, \frac{c1}{c2} = \frac{10}{9}$$

Thus, $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$

The lines are parallel.

6. Question

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines (ii) parallel lines

(iii) coincident lines.

Answer

Two lines, a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0

If $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, then the lines coincide

If $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines are parallel

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines intersect

Given the linear equation $2x + 3y - 8 = 0$.

An intersecting line is x + 2y – 4 =0

A parallel line is 4x + 6y – 12 = 0

A coincident line is 4x + 6y – 16 = 0

7. Question

The cost of 2 kg of apples and l kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer

Let the cost of 1 kg of apple be x and cost of 1 kg of grape be y.

Given, cost of 2 kg of apples and l kg of grapes on a day was found to be Rs 160.

∴ 2x + y = 160

Also, after a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300.

∴ 4x + 2y = 300Now to present these equations graphically plot the points of respective lines,For 2x + y = 160,y = 160 - 2xWhen x = 50, y = 60When x=60, y = 40Table is :

x	50	60
y	60	40

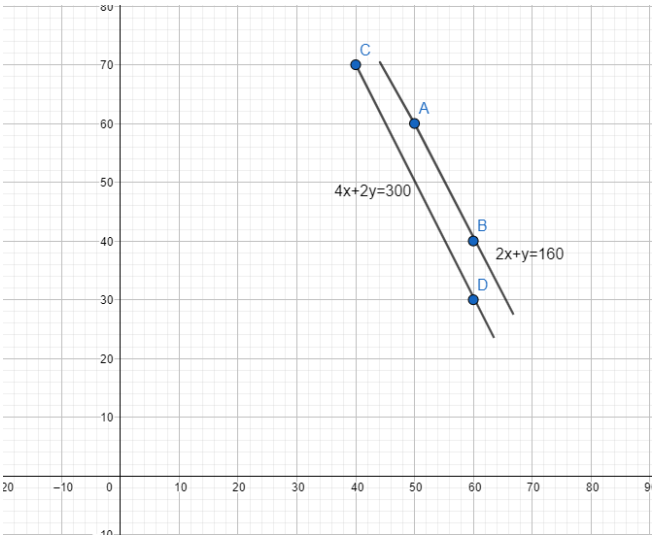
Plot the points A(50,60) and B(60,40)

For 4x + 2y = 300, $y = \frac{300 - 4x}{2}$

⇒ y = 150 - 2xWhen x = 40, y = 70When x = 60, y = 30Table is :

x	40	60
y	70	30

Plot the points C(40,-30) and D(60,30)



Exercise 3.2

1. Question

Solve the following systems of equations graphically:

$x + y = 3$
$2x + 5y = 12$

Answer

For the solutions first we make graph for the equations.

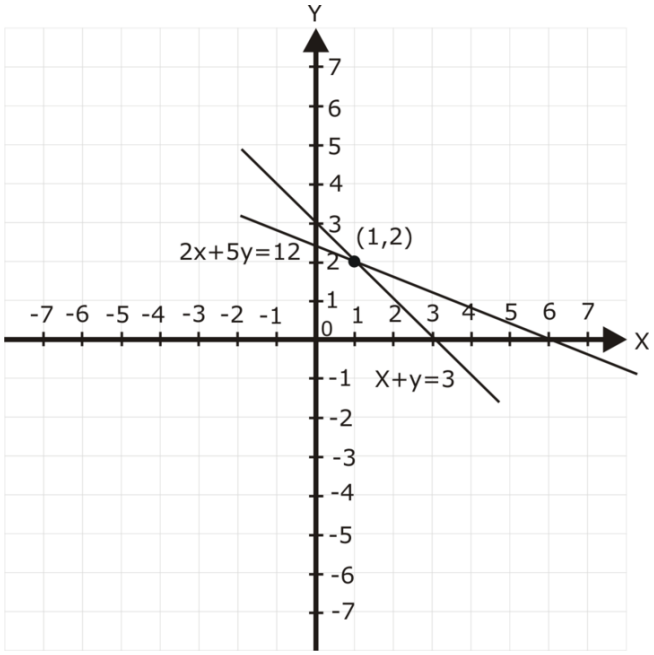
For x+y=3,

x	0	3
y	3	0

x + y = 3 passes through (3,0) and (0,3)For 2x+5y=12,

x	6	0
y	0	2.4

2x + 5y = 12 passes through (6, 0) and (0, 2.4)Now we plot the points and join them to make graph.Wherever, they intersect that is common solution to both equations.



They meet at point x = 1 and y = 2.So (1,2) is solution.

2. Question

Solve the following systems of equations graphically:

$x - 2y = 5$
$2x + 3y = 10$

Answer

Given: The system of equations: $\begin{matrix} x - 2y = 5 \\ 2x + 3y = 10 \end{matrix}$

To find: The solution of above system.

Solution:

For x - 2y = 5,Put x = 5, we get y = 0Put x = 1, we get y = -2Table for x - 2y = 5

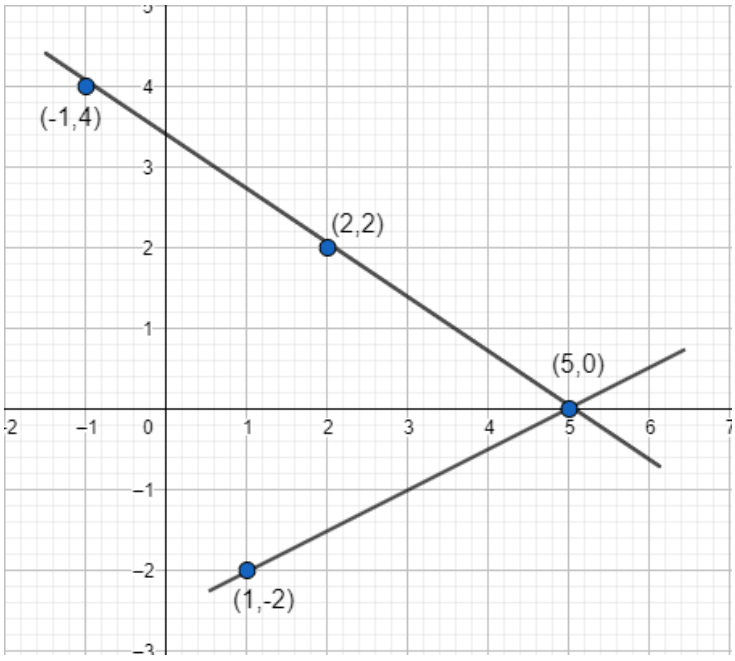
x	5	1
y	0	-2

x - 2y = 5, passes through (0,5) and (1,-2)

For 2x + 3y =10 Put x = -1 , we get y = 4Put x = 2, we get y = 2Table for 2x + 3y =10

x	-1	2
y	4	2

2x + 3y =10 passes through (0,5) and (2,2)Plot the points in the graph,



The lines meet at the point (5,0)So x=5 and y=0.

3. Question

Solve the following systems of equations graphically:

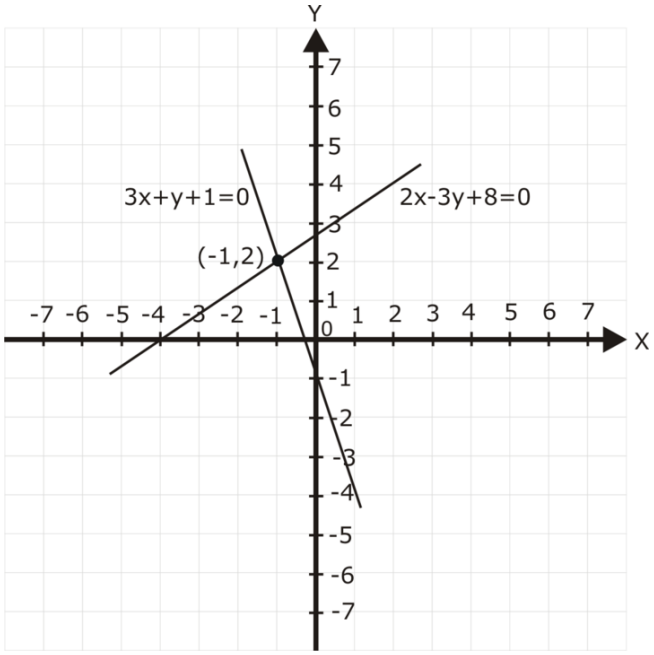
$3x + y + 1 = 0$
$2x - 3y + 8 = 0$

Answer

3x + y + 1= 0 passes through (-1/3, 0) and (0, -1)

2x – 3y + 8 = 0 passes through (-4, 0) and (0, 8/3)

They meet at point x = -1 and y = 2



4. Question

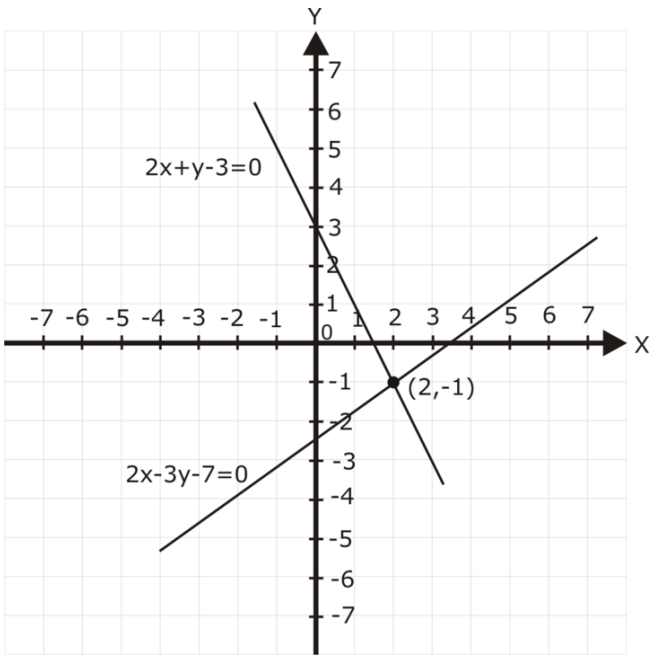
Solve the following systems of equations graphically:

$2x + y - 3 = 0$
$2x - 3y - 7 = 0$

Answer

2x + y − 3 = 0 passes through (3/2, 0) and (0, 3)

2x − 3y − 7 = 0 passes through (7/2, 0) and (0, - 7/3)



They meet at point x = 2, y = -1

5. Question

Solve the following systems of equations graphically:

$x + y = 6$
$x - y = 2$

Answer

Given: $x + y = 6$
$x - y = 2$

To show: The given pair of equations graphically.

Solution: For x + y = 6,Put y = 0, we get x = 6

Put x = 0, we get y = 6

Table for x + y = 6

x	6	0
y	0	6

x + y = 6 passes through (6, 0) and (0, 6)

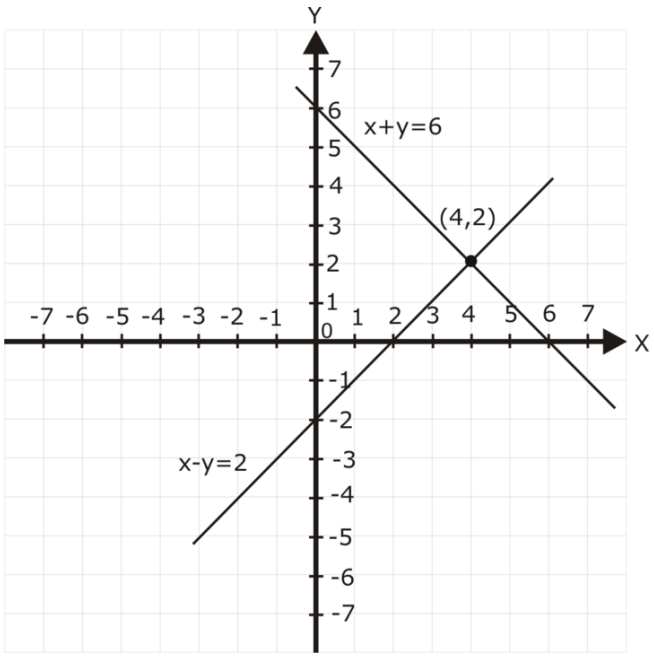
For x – y = 2 Put y = 0, we get x = 2

Put x = 0, we get y = -2

Table for x – y = 2

x	2	0
y	0	-2

x – y = 2 passes through (2, 0) and (0, - 2)Plot the points in the graph,



They meet at point x = 4, y = 2

6. Question

Solve the following systems of equations graphically:

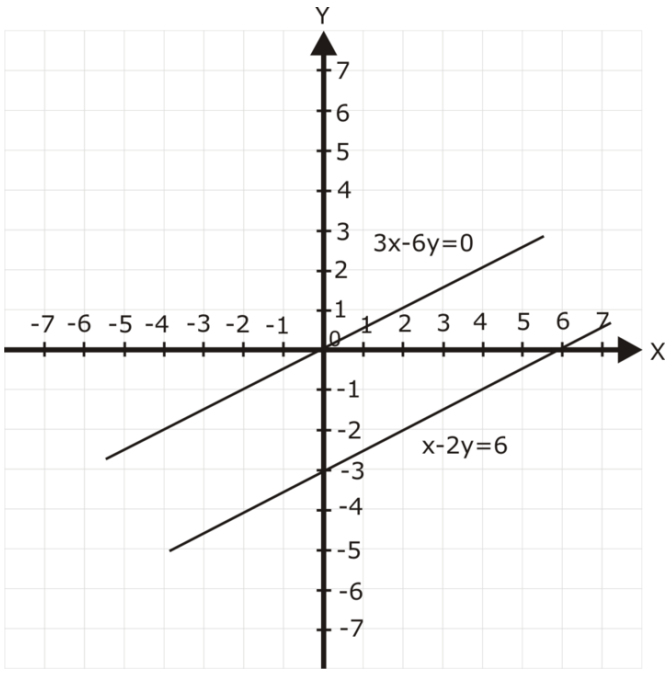
$x - 2y = 6$
$3x - 6y = 0$

Answer

X – 2y = 6 passes through (6, 0) and (0, -3)

3x – 6y = 0 passes through origin

Both lines are parallel, thus it has no solution.



7. Question

Solve the following systems of equations graphically:

$x + y = 4$
$2x - 3y = 3$

Answer

For x + y = 4,Put y = 0, we get x = 4

Put x = 0, we get y = 4

Table for x + y = 4

x	4	0
y	0	4

x + y = 4 passes through (4, 0) and (0, 4)

For 2x – 3y = 3$y = \frac{2x-3}{3}$

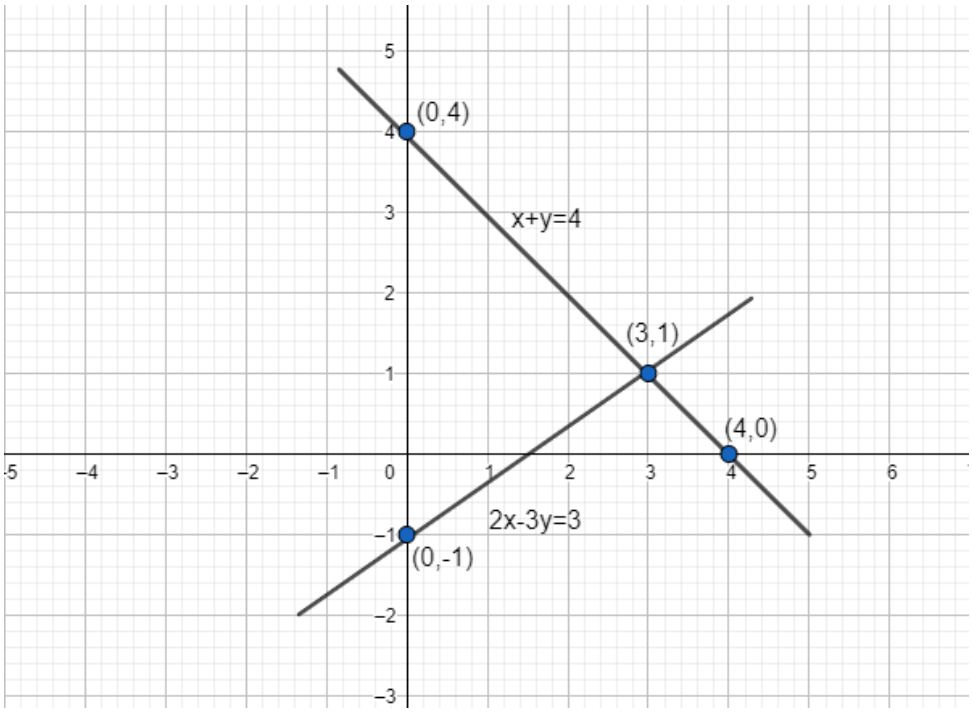
Put y = 1, we get x = 3

Put x = 0, we get y = -1

Table for x – y = 2

x	3	0
y	1	-1

2x – 3y = 3 passes through (3, 1) and (0, -)Plot the points in the graph



They meet at point (3,1)Hence x = 3, y = 1.

8. Question

Solve the following systems of equations graphically:

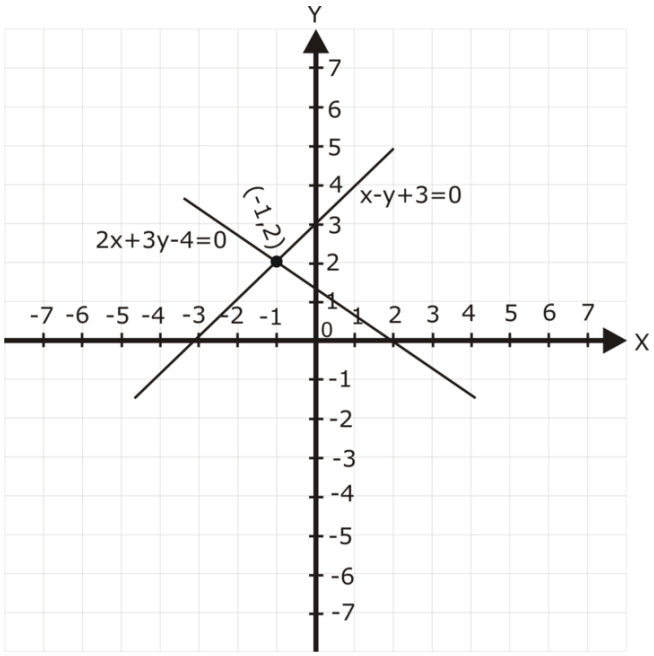
$2x + 3y = 4$
$x - y + 3 = 0$

Answer

2x + 3y = 4 passes through (2, 0) and (0, 4/3)

X – y + 3 = 0 passes through (-3, 0) and (0, 3)

They meet at point x = -1, y = 2



9. Question

Solve the following systems of equations graphically:

$2x - 3y + 13 = 0$
$3x - 2y + 12 = 0$

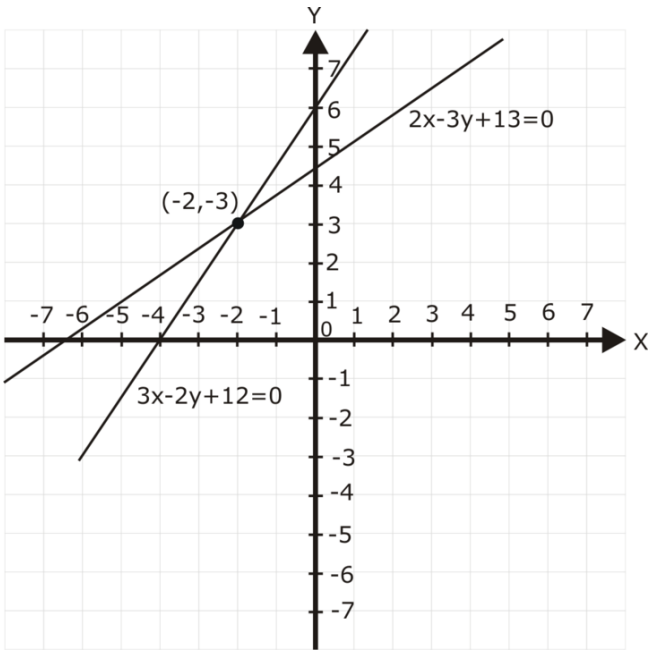
Answer

On solving graphically,

2x – 3y + 13 = 0 passes through (-6.5, 0) and (0, 13/3)

3x – 2y + 12 = 0 passes through (-4, 0) and (0, 6)

Thus the meeting point of the straight lines is, (-2, - 3)



10. Question

Solve the following systems of equations graphically:

$2x + 3y + 5 = 0$
$3x - 2y - 12 = 0$

Answer

For the equation 2x + 3y + 5 = 0, $y = \frac{-5-2x}{3}$

When x = -1 , y =- 1

When x = 5 , y =- 5

Table is :

x	-1	5
y	-1	-5

Plot the points A(-1,-1) B(-5,5) on the graph

For the equation 3x – 2y – 12 = 0 , $y = \frac{3x - 12}{2}$

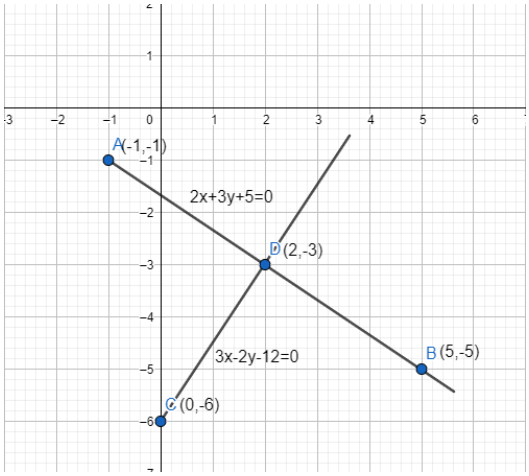
When x = 0, y = -6

When x = 2 , y = -3

Table is :

x	0	2
y	-6	-3

Plot the points C(0,-6) D(2,-3) on the graph



Since the two lines meet at the point (2,-3)Therefore, x = 2 and y = -3

11. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$2x + 3y = 6$
$4x + 6y = 12$

Answer

Given: the system of equations: $2x + 3y = 6$, $4x + 6y = 12$

To show:systems of equations has infinitely many solutions.

Solution:

consider the equation 2x + 3y = 6

To plot its graph,we have $y = \frac{6 - 2x}{3}$

Putting x = 0 we get y = 2

putting y = 0 we get x = 3

The table for points of 2x + 3y = 6 is:

x	0	3
y	2	0

Plot A(0,2) and B(3,0) in the graph

Consider the equation 4x + 6y = 12

To plot its graph,we have $y = \frac{12 - 4x}{6}$

Putting x = 6 we get y = -2

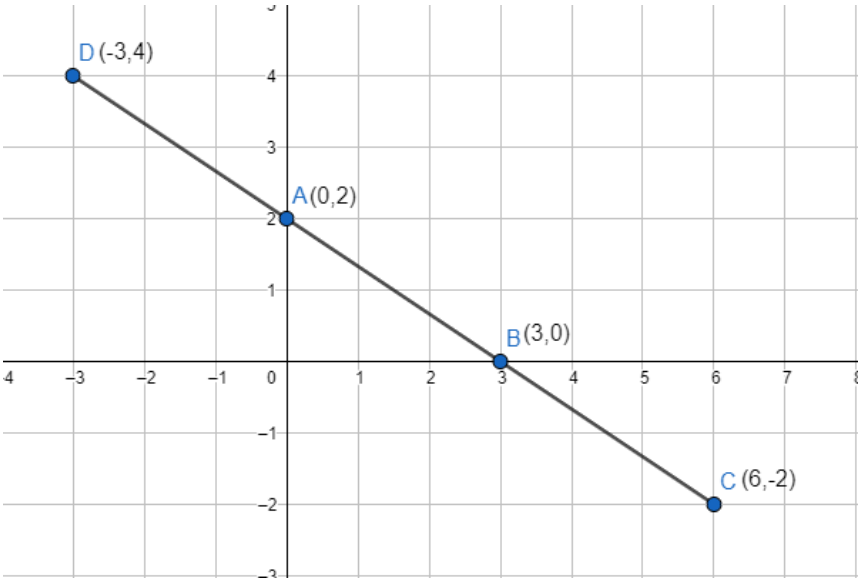
Putting y = 4 we get x = -3

The table for the points 4x + 6y = 12 is:

x	6	-3
y	-2	4

Plot C(6,-2) and D(-3, 4) in the graph

Now plot the graphs for these equations as:



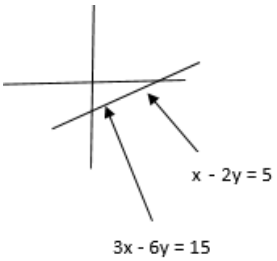
As Both the lines pass through the same points and are coinciding.There can be infinite points lying on both the lines.Hence,systems of equations has infinitely many solutions.

12. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$x - 2y = 5$
$3x - 6y = 15$

Answer



Both pass through the same points and are coinciding.

13. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$3x + y = 8$
$6x + 2y = 16$

Answer

Given: the system of equations: $\begin{matrix} 3x + y = 8 \\ 6x + 2y = 16 \end{matrix}$

To show:systems of equations has infinitely many solutions.

Solution:consider the equation 3x+y=8

To plot its graph,we have y=8-3x

Putting x=0 we get y=8

putting y=2 we get x=2

The table for points of 3x+y=8 is:

x	0	2
y	8	2

Plot A(0,8) and B(2,2) in the graph

Consider the equation 6x+2y=16

To plot its graph,we have $y = \frac{16 - 2x}{6}$

Putting x=2 we get y=2

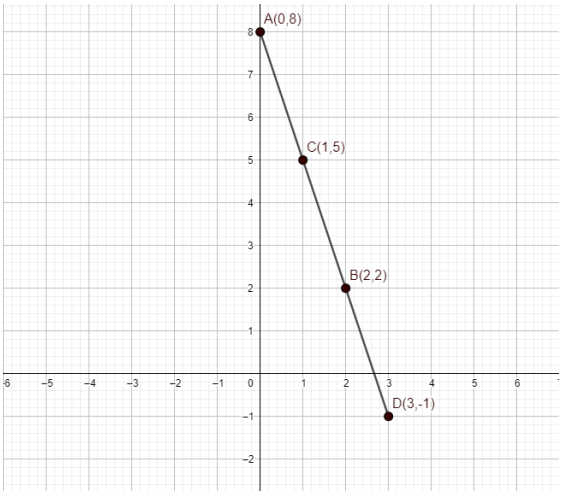
Putting y=0 we get x=8

The table for the points 6x+2y=16 is:

x	1	3
y	5	-1

Plot C(1,5) and D(3,-1) in the graph

Now plot the graphs for these equations as:



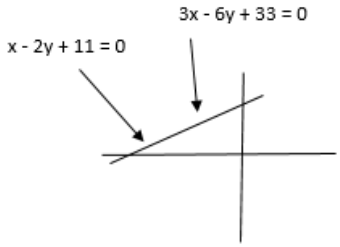
As Both the lines pass through the same points and are coinciding.There can be infinite points lying on both the lines.Hence,systems of equations has infinitely many solutions.

14. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$x - 2y + 11 = 0$
$3x - 6y + 33 = 0$

Answer



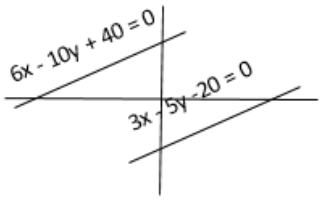
Both pass through the same points and are coinciding.

15. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$3x - 5y = 20$
$6x - 10y = -40$

Answer



3x – 5y = 20 passes through (20/3, 0) and (0, -4)

6x – 10y = - 40 passes through (-20/3, 0) and (0, 4)

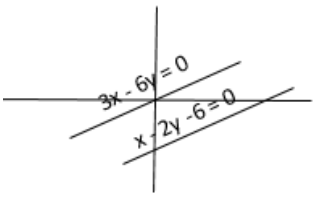
Both are parallel lines, thus system of equations is in-consistent.

16. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$x - 2y = 6$
$3x - 6y = 0$

Answer



X – 2y = 6 passes through (6, 0) and (0, -3)

3x – 6y = 0 passes through origin

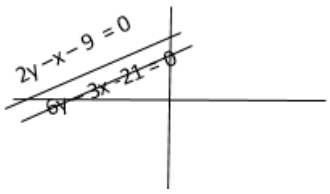
Both are parallel lines, thus system of equations is in-consistent.

17. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$2y - x = 9$
$6y - 3x = 21$

Answer



2y – x = 9 passes through (-9, 0) and (0, 9/2)

6y – 3x = 21 passes through (-7, 0) and (0, 7/2)

Both are parallel lines, thus system of equations is in-consistent.

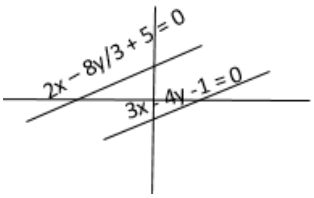
18. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$3x - 4y - 1 = 0$

$2x - \frac{8}{3}y + 5 = 0$

Answer



3x – 4y – 1 = 0 passes through (1/3, 0) and (0, -1/4)

2x – 8y/3 + 5 = 0 passes through (-5/2, 0) and (0, 15/8)

Both are parallel lines, thus system of equations is in-consistent.

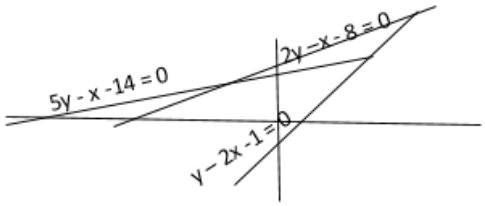
19. Question

Determine graphically the vertices of the triangle, the equations of whose sides are given below:

(i) $2y - x = 8, 5y - x = 14$ *and* $y - 2x = 1$

(ii) $y = x, y = 0, and\ 3x + 3y = 10$

Answer



(i) $2y - x = 8, 5y - x = 14\ and\ y - 2x = 1$

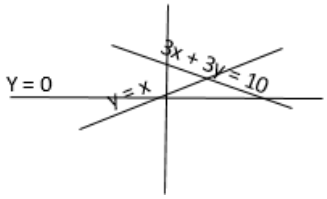
2y – x = 8 passes through (-8, 0) and (0, 4)

5y – x = 14 passes through (-14, 0) and (0, 14/5)

Y – 2x = 1 passes through (0, 1) and (-1/2, 0)

Vertices of the triangle are (-4, 2), (1, 3) and (2, 5)

(ii) $y = x, y = 0, and\ 3x + 3y = 10$



3x + 3y = 10 passes through (10/3, 0) and (0, 10/3)

Vertices of the triangle are (0, 0), (10/3, 0) and (5/3, 5/3)

20. Question

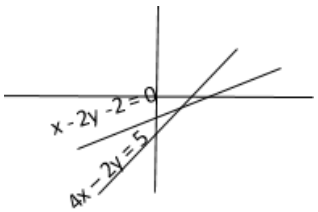
Determine, graphically whether the system of equations is consistent or in-consistent.

$x - 2y = 2, 4x - 2y = 5$

Answer

X – 2y = 2 passes through (2, 0) and (0, -1)

4x – 2y = 5 passes through (5/4, 0) and (0, - 5/2)



Thus the system of equations is consistent and it has a unique solution.

21. Question

Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not:

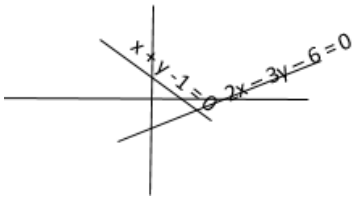
(i) $2x - 3y = 6, x + y = 1$

(ii) $2y = 4x - 6, 2x = y + 3$

Answer

(i) 2x – 3y = 6 passes through (3, 0) and (0, -2)

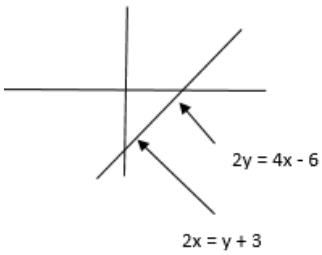
X + y = 1 passes through (0, 1) and (1, 0)



Thus it has unique solution

(ii) 2y = 4x -6 passes through (3/2, 0) and (0, -3)

2x = y + 3 passes through (3/2, 0) and (0, -3)



The lines are coincident, it has infinite solutions

22. Question

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

(i) $2x - 5y + 4 = 0$

$2x + y - 8 = 0$

(ii) $3x + 2y = 12$

$5x - 2y = 4$

(iii) $2x + y - 11 = 0$

$x - y - 1 = 0$

(iv) $x + 2y - 7 = 0$

$2x - y - 4 = 0$

(v) $3x + y - 5 = 0$

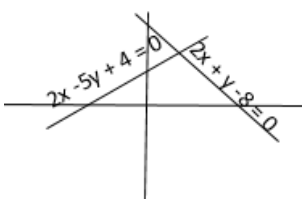
$2x - y - 5 = 0$

(vi) $2x - y - 5 = 0$

$x - y - 3 = 0$

Answer

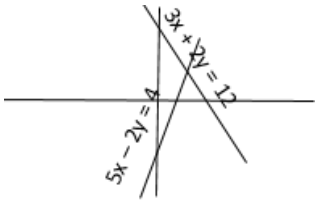
(i)



It has a unique solution at x = 3 and y = 2

Coordinates where lines meet y axis is (0, 4/5) and (0, 8)

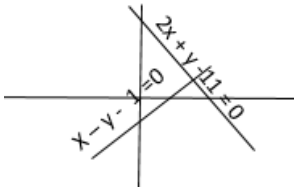
(ii)



Unique solution at x = 2 and y = 3.

Lines meet the y axis at (0, 6) and (0, - 2)

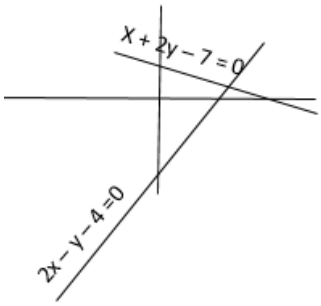
(iii)



Unique solution at x = 4 and y = 3

Lines meet the y axis at (0, 11) and (0, -1)

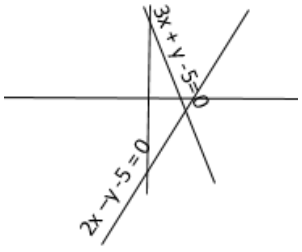
(iv)



Unique solution at x = 3 and y = 2

Lines meet y axis at (0, 7/2) and (0,- 4)

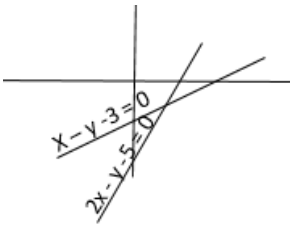
(v)



Unique solution at x = 2 and y = -1

Lines meet the y-axis at (0, 5) and (0, - 5)

(vi)



Unique solution at x = 2 and y = -1

Lines meet y-axis at (0, -5) and (0, -3)

23. Question

Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are:

(i) $y = x, y = 2x$ *and* $y + x = 6$

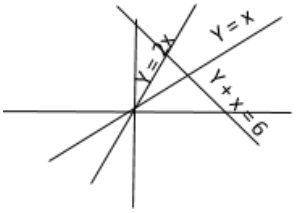
(ii) $y = x, 3y = x, x + y = 8$

Answer

(i) $y = x, y = 2x$ *and* $y + x = 6$

The triangle formed has vertices

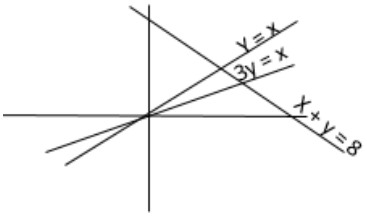
(0, 0), (2, 4) and (3,3)



(ii) $y = x, 3y = x, x + y = 8$

The triangle formed has vertices

(0, 0), (4, 4) and (6, 2)



24. Question

Solve the following system of linear equations graphically and shade the region between the two lines and x-axis:

(i) $2x + 3y = 12,$

$x - y = 1$

(ii) $3x + 2y - 4 = 0,$

$2x - 2y - 7 = 0$

(iii) $3x + 2y - 11 = 0$

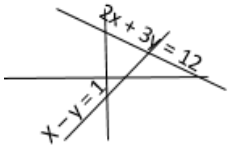
$2x - 3y + 10 = 0$

Answer

i) 2x + 3y = 12 passes through (6, 0) and (0, 4)

x – y = 1 passes through (1, 0) and (0, -1)

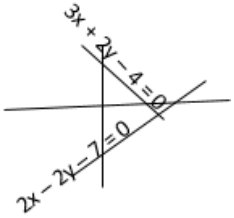
They meet at x = 3, y = 2



ii) 3x + 2y – 4 = 0 passes through, (4/3, 0) and (0, 2)

2x – 2y – 7 = 0 passes through (7/2, 0) and (0, -7/2)

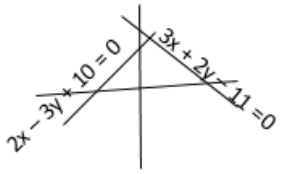
They meet at x = 2, y = -1



iii) 3x + 2y – 11 = 0 passes through (11/3, 0) and (0, 11/2)

2x – 3y + 10 = 0 passes through (-5, 0) and (0, 10/3)

They meet at x = 1, y = 4



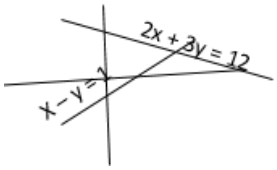
25. Question

Draw the graphs of the following equations on the same graph paper:

$2x + 3y = 12$
$x - y = 1$

Find the coordinates of the vertices of the triangle formed by the two straight lines and the y-axis.

Answer



$2x + 3y = 12$
$x - y = 1$

Lines meet at x = 3, y = 2

Coordinates of the vertices of the triangle formed with y –axis

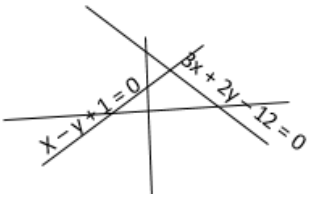
(0, - 1), (0, 4) and (3, 2)

26. Question

Draw the graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and x-axis and shade the triangular area. Calculate the area bounded by these lines and x-axis.

Answer

$x - y + 1 = 0$ and $3x + 2y - 12 = 0$



Coordinates of the vertices of triangle formed with x-axis

(2, 3), (-1, 0) and (4, 0)

Height of the triangle = 3 units

Base of the triangle = 5 units

Area = 0.5 × base × height = 7.5 sq units

27. Question

Solve graphically the system of linear equations:

$4x - 3y + 4 = 0$

$4x + 3y - 20 = 0$

Find the area bounded by these lines and x-axis.

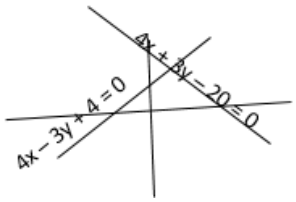
Answer

They meet at x = 2, y = 4

Height of the triangle formed = 4 units

Base of the triangle formed = 5 + 1 = 6

Area of the region bounded by these lines and x-axis



= 0.5 × 4 × 6

= 12 sq. units

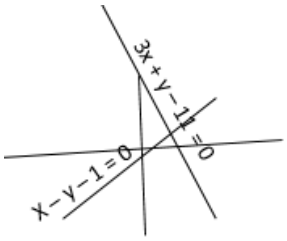
28. Question

Solve the following system of linear equations graphically:

$3x + y - 11 = 0, x - y - 1 = 0$

Shade the region bounded by these lines and y-axis. Also, find the area of the region bounded by the these lines and y-axis.

Answer



$3x + y - 11 = 0, x - y - 1 = 0$

They meet at x = 3, y = 2

Height of the triangle formed with y-axis = 3 units

Base of the triangle formed = 11 + 1 = 12 units

Area of the bounded region = 0.5 × 3 × 36 = 18 sq. units

29. Question

Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

(i) $2x + y = 6$

$x - 2y = -2$

(ii) $2x - y = 2$

$4x - y = 8$

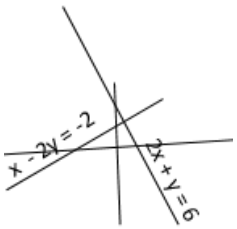
(iii) $x + 2y = 5$

$2x - 3y = -4$

(iv) $2x + 3y = 8$

$x - 2y = -3$

Answer



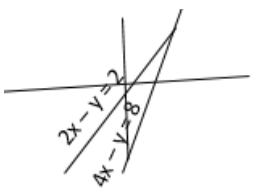
i) $2x + y = 6 \quad x - 2y = -2$

Unique solution at x = 2, y = 2

Lines meet x-axis at (3, 0) and (-2, 0)

ii) $2x - y = 2$

$4x - y = 8$

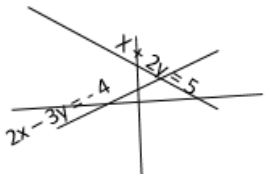


Unique solution at x = 3, y = 3

Lines meet x-axis at (1, 0) and (2, 0)

iii) $x+2y=5$

$2x-3y=-4$

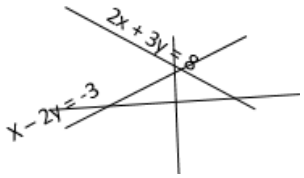


Unique solution at x =1, y = 2

Lines meet x-axis at (5, 0) and (-2, 0)

iv) $2x+3y=8$

$x-2y=-3$



Unique solution at x =1, y = 2

Lines meet x-axis at (4, 0) and (-3, 0)

30. Question

Draw the graphs of the following equations:

$2x-3y+6=0$
$2x+3y-18=0$
$y-2=0$

Find the vertices of the triangle so obtained. Also, find the area of the triangle.

Answer

For equation, 2x - 3y + 6 = 0

Coordinates satisfying graph are,

x	0	-3
y	2	0

For equation, 2x + 3y - 18 = 0

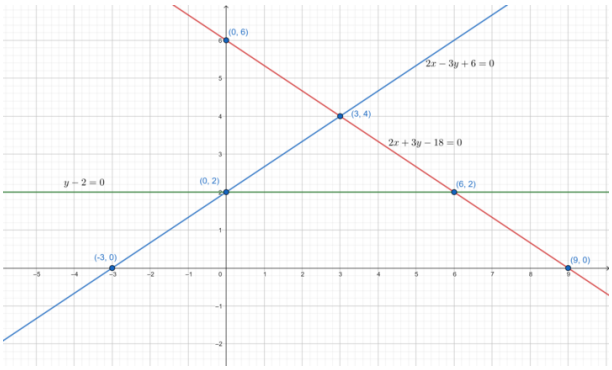
Coordinates satisfying graph are,

x	0	9
y	6	0

and y - 2 = 0

⇒ y = 2, which is a straight line parallel to x-axis intersecting the y-axis at 2

On plotting all the three line on graphs, we get the graph as follows



On observing from graph,

Vertices of the triangle obtained are

(0, 2), (3, 4) and (6, 2)

Height of triangle = 4 – 2 = 2

Base of the triangle = 6

Area of the triangle

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times 6 \times 2$$

= 6 square units

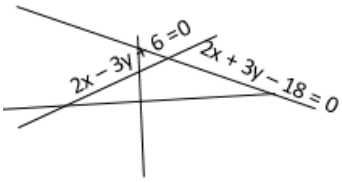
31. Question

Solve the following system of equations graphically:

$2x - 3y + 6 = 0$
$2x + 3y - 18 = 0$

Also, find the area of the region bounded by these two lines and y-axis.

Answer



Unique solution at x = 3 and y = 4

Vertices of triangle formed with y-axis are

(0, 2), (3, 4) and (0, 6)

Height of the triangle obtained = 3 units

Base of the triangle obtained = 6 – 2 = 4 units

Area of the triangle = 0.5 × 3 × 4 = 6 sq. units

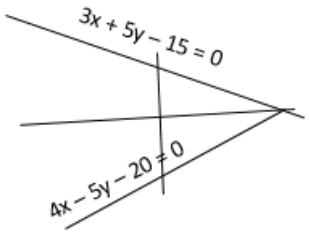
32. Question

Solve the following system of linear equations graphically:

$4x - 5y - 20 = 0$
$3x + 5y - 15 = 0$

Determine the vertices of the triangle formed by the lines representing the above equation and the y-axis.

Answer



$4x - 5y - 20 = 0$
$3x + 5y - 15 = 0$

Unique solution at x = 5 and y = 0

Vertices of the triangle with y axis are

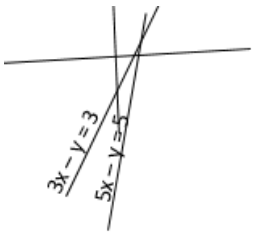
(0, 3), (0, - 4) and (5, 0)

33. Question

Draw the graphs of the equations $5x - y = 5$

and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and y-axis. Calculate the area of the triangle so formed.

Answer



$5x - y = 5$

and $3x - y = 3$

Coordinates of the vertices of the triangle formed by these lines and y-axis

(1, 0), (0, -3), (0, -5)

Height of triangle = 1 unit

Base of triangle = 5 – 3 = 2 unit

Area of triangle = 0.5 × 1 × 2 = 1 sq. units

34. Question

Form the pair of linear equations in the following problems, and find their solution graphically:

(i) 10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and a pen.

(iii) Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of

skirts is four less than four times the number of pants purchased." Help her friends to find how many pants and skirts Champa bought.

Answer

i) Let the number of boys and girls be 'a' and 'b'.

10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys

⇒ a + b = 10 and b = a + 4

Thus, a = 3 and b = 7

ii) Let the cost of 1 pen be a and 1 pencil be b.

5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46

⇒ 7a + 5b = 50 and 5a + 7b = 46

Solving by multiplying 1^{st} by 7 and 2^{nd} by 5 and subtracting

⇒ 24a = 120

⇒ a = Rs. 5

Thus, 35 + 5b = 50

⇒ b = Rs. 3

iii) Let the number of pants and skirts bought be 'a' and 'b'

Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased."

⇒ b = 2a − 2 and b = 4a − 4

⇒ 2a − 2 = 4a − 4

⇒ a = 1

Thus, b = 2 − 2 = 0

35. Question

Solve the following system of equations graphically:

Shade the region between the lines and the y-axis

(i) $3x - 4y = 7$

$5x + 2y = 3$

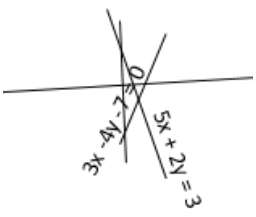
(ii) $4x - y = 4$

$3x + 2y = 14$

Answer

i) $3x - 4y = 7$

$5x + 2y = 3$

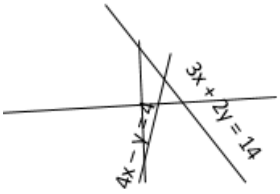


Unique solution at x = 1, y = -1

ii) $4x - y = 4$

$3x + 2y = 14$

Unique solution at x = 2, y = 4



36. Question

Represent the following pair of equations graphically and write the coordinates of points where the lines intersects y-axis

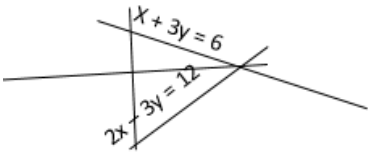
$x + 3y = 6$

$2x - 3y = 12$

Answer

$x + 3y = 6$

$2x - 3y = 12$



Lines meet y-axis at (0, 2) and (0, -4)

37. Question

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is

(i) intersecting lines (ii) Parallel lines

(iii) coincident lines

Answer

Two lines, a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0

If $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, then the lines coincide

If $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines are parallel

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines intersect

Given line $2x + 3y - 8 = 0$

i) intersecting line : 3x + 2y – 6 = 0

ii) parallel line : 4x + 6y = 15

iii) coincident line : 4x + 6y – 16 = 0

Exercise 3.3

1. Question

Solve the following systems of equations:

$11x + 15y + 23 = 0$

$7x - 2y - 20 = 0$

Answer

$11x + 15y + 23 = 0$ [1]

$7x - 2y - 20 = 0$ [2]

Multiplying the 1st equation by 2 and 2nd equation by 15, we get22x + 30y + 46 = 0 [3]105x - 30y - 300 = 0 [4]Adding [3] and [4]

⇒ 22x + 30y + 46 + 105x – 30y – 300 = 0

⇒ 127x = 254

⇒ x = 2

Substituting value of x in eq1.

⇒ 11(2) + 15y + 23 = 0

⇒ 22 + 15y + 23 = 0

⇒ 15y = -45

⇒ y = -3

2. Question

Solve the following systems of equations:

$3x - 7y + 10 = 0$

$y - 2x - 3 = 0$

Answer

$3x - 7y + 10 = 0$ $y - 2x - 3 = 0$

Multiplying eq1 by 2 and eq2 by 3 and adding.

⇒ 6x – 14y + 20 + 3y – 6x – 9 = 0

⇒ -11y = -11

⇒ y = 1

Substituting value of y in eq1, we get x = -1

3. Question

Solve the following systems of equations:

$0.4x + 0.3y = 1.7$

$0.7x - 0.2y = 0.8$

Answer

$0.4x + 0.3y = 1.7$

$0.7x - 0.2y = 0.8$

Multiplying eq1 by 0.2 and eq2 by 0.3 and adding

⇒ 0.08x + 0.06y + 0.21x – 0.06y = 0.34 + 0.24

⇒ 0.29x = 0.58

⇒ x = 2

Substituting value of x in eq1, 0.8x + 0.3y = 1.7

⇒ y = 3

4. Question

Solve the following systems of equations:

$\frac{x}{2} + y = 0.8$

$\frac{7}{x + \frac{y}{2}} = 10$

Answer

$\frac{x}{2} + y = 0.8$

$\frac{7}{x + \frac{y}{2}} = 10$

⇒ x + y/2 = 0.7

Multiplying eq1 by 2 and subtracting eq2 from it.

⇒ x + 2y – x – y/2 = 1.6 – 0.7

⇒ 3y/2 = 0.9

⇒ y = 0.6

Thus, x/2 + 0.6 = 0.8

⇒ x = 0.4

5. Question

Solve the following systems of equations:

$7(y + 3) - 2(x + 2) = 14$

$4(y - 2) + 3(x - 3) = 2$

Answer

$7(y + 3) - 2(x + 2) = 14$ 7y + 21 - 2x - 4 = 147y - 2x = 14 - 21 + 4

⇒ 7y – 2x = -3 ---- (1)

$4(y - 2) + 3(x - 3) = 2$ 4y - 8 + 3x - 9 = 24y + 3x = 2 + 8 + 9

⇒ 4y + 3x = 19 ------ (2)

Multiplying eq1 by 3 and eq2 by 2 and adding them

3(7y – 2x) + 2(4y + 3x) = -3 (3) + 19(2)

⇒ 21y – 6x + 8y + 6x = -9 + 38⇒ 29 y = 29

⇒ y = 1

Put the value of y in (1) to get,

7 – 2x = -3- 2x = -3 - 7-2x = -10

⇒ x = 5

6. Question

Solve the following systems of equations:

$\frac{x}{7}+\frac{y}{3}=5$

$\frac{x}{2}-\frac{y}{9}=6$

Answer

$\frac{x}{7}+\frac{y}{3}=5$

⇒ 3x + 7y = 105

$\frac{x}{2}-\frac{y}{9}=6$

⇒ 9x – 2y = 108

Multiplying eq1 by 3 and subtracting eq2 from it

⇒ 9x + 21y – 9x + 2y = 315 – 108

⇒ 23y = 207

⇒ y = 9

Thus, 9x – 18 = 108

⇒ x = 14

7. Question

Solve the following systems of equations:

$\frac{x}{3}+\frac{y}{4}=11$

$\frac{5x}{6}-\frac{y}{3}=-7$

Answer

$\frac{x}{3}+\frac{y}{4}=11$

⇒ 4x + 3y = 132

$\frac{5x}{6}-\frac{y}{3}=-7$

⇒ 5x – 2y = -42

Multiplying eq1 by 2 and eq2 by 3 and adding them

⇒ 8x + 6y + 15x – 6y = 264 – 126

⇒ 23x = 138

⇒ x = 6

Thus, 24 + 3y = 132

⇒ y = 36

8. Question

Solve the following systems of equations:

$\frac{4}{x}+3y=8$

$\frac{6}{x} - 4y = -5$

Answer

$\frac{4}{x} + 3y = 8$

$\frac{6}{x} - 4y = -5$

Multiplying eq1 by 4 and eq2 by 3 and adding

⇒ 16/x + 12y + 18/x – 12y = 32 – 15⇒ 34/x = 17

⇒ x = 2

Thus, 2 + 3y = 8⇒ 3y = 6

⇒ y = 2

9. Question

Solve the following systems of equations:

$x + \frac{y}{2} = 4$

$\frac{x}{3} + 2y = 5$

Answer

$x + \frac{y}{2} = 4$ ⇒ 2x + y = 8 …(i)

$\frac{x}{3} + 2y = 5$ ⇒ x + 6y = 15 …(ii)

Multiplying eq2 by 2 and subtracting from eq1

⇒ 2x + y - 2x -12y = 8 – 30

⇒ -11y = -22

⇒ y = 2

Thus, 2x + 2 = 8

⇒ x = 3

10. Question

Solve the following systems of equations:

$x + 2y = \frac{3}{2}$

$2x + y = \frac{3}{2}$

Answer

$x + 2y = \frac{3}{2}$ ⇒ 2x + 4y = 3 …(i)

$2x + y = \frac{3}{2}$ ⇒ 4x + 2y = 3 …(ii)

Multiplying eq2 by 2 and subtracting from eq1

2x + 4y – 8x – 4y = 3 – 6

⇒ -6x = - 3

⇒ x = 1/2

Thus, 2 × 1/2 + 4y = 3 ⇒ y = 1/2

11. Question

Solve the following systems of equations:

$\sqrt{2x} - \sqrt{3y} = 0$

$\sqrt{3x} - \sqrt{8y} = 0$

Answer

Given: The following systems of equations:

$\sqrt{2x} - \sqrt{3y} = 0$

$\sqrt{3x} - \sqrt{8y} = 0$

To find : The values of x and y.

Solution: $\sqrt{2x} - \sqrt{3y} = 0$

$\Rightarrow \sqrt{2x} = \sqrt{3y}$

Squaring both sides to get,

2x = 3y

$\Rightarrow x = \frac{3y}{2}$

Now,

$\sqrt{3x} - \sqrt{8y} = 0 \Rightarrow \sqrt{3x} = \sqrt{8y}$

Squaring both sides to get, 3x = 8y ..... (1)

Substitute the value of x in (1),

$$3\left(\frac{3y}{2}\right) = 8y$$

$$\frac{9y}{2} = 8y$$

$$\frac{9y}{2} - 8y = 0$$

$$\frac{9y - 16y}{2} = 0$$

$$\frac{-7y}{2} = 0$$

$$-7y = 0$$

$$y = 0$$

Substitute the value of y in 2x = 3y⇒2x=3(0)⇒2x=0⇒x=0

Solution is x=0,y=0.

12. Question

Solve the following systems of equations:

$3x - \frac{Y+7}{11} + 2 = 10$

$2y + \frac{X+11}{7} = 10$

Answer

Consider $3x - \frac{Y+7}{11} + 2 = 10$,

$$\Rightarrow \frac{11(3x) - y - 7 + 2(11)}{11} = 10$$

$$\Rightarrow \frac{33x - y - 7 + 22}{11} = 10$$

⇒ 33x - y - 7 + 22 = 110⇒ 33x - y - 95 = 0⇒ 33x - 95 = y ..... (1)

Consider $2y + \frac{X+11}{7} = 10$,

$$\Rightarrow \frac{7(2y) + x + 11}{7} = 10$$

$$\Rightarrow \frac{14y + x + 11}{7} = 10$$

⇒ 14y + x + 11 = 70

⇒ 14y + x = 59 .... (2)

Put the value of y from (1) in (2) to get,14(33x-95) + x = 59 462x - 1330 + x = 59463x = 1389x = 3Put the value of x in (1) to get, 33(3) - 95 = y99 - 95 = y4 = y⇒ **x = 3 and y = 4**

13. Question

Solve the following systems of equations:

$2x - \frac{3}{y} = 9$

$3x + \frac{7}{y} = 2,\ y \neq 0$

Answer

$2x - \frac{3}{y} = 9$

$3x + \frac{7}{y} = 2,\ y \neq 0$

Multiplying eq1 by 3 and eq2 by 2 and subtracting eq1 from eq2

⇒ 6x + 21/y – 6x + 6/y = 4 – 27

⇒ 23/y = - 23

⇒ y = -1

Thus, 2x + 3 = 9

⇒ x = 3

14. Question

Solve the following systems of equations:

$0.5x + 0.7y = 0.74$

$0.3x + 0.5y = 0.5$

Answer

$0.5x + 0.7y = 0.74$

$0.3x + 0.5y = 0.5$

Multiplying eq1 by 0.3 and eq2 by 0.5 and subtracting eq1 from eq2

⇒ 0.15x + 0.25y – 0.15x – 0.21y = 0.25 – 0.222

⇒ 0.04y = 0.028

⇒ y = 0.7

Thus, x = 0.5

15. Question

Solve the following systems of equations:

$\frac{1}{7x} + \frac{1}{6y} = 3$

$$\frac{1}{2x} - \frac{1}{3y} = 5$$

Answer

Given:

systems of equations:

$\frac{1}{7x} + \frac{1}{6y} = 3$

$\frac{1}{2x} - \frac{1}{6y} = 5$

To find: The values of x and y.

Solution:

Let's take,

$$\frac{1}{x} = u, \frac{1}{y} = v$$

The equations become:

$$\frac{1}{7}u + \frac{1}{6}v = 3$$

$$\frac{6u + 7v}{42} = 3$$

6u + 7v = 126 ..... (1)

$$\frac{1}{2}u - \frac{1}{3}v = 5$$

$$\frac{3u-2v}{6} = 5$$

3u - 2v = 30 ..... (2)Now multiply eq. (2) with 2 and subtract it from eq. (1)⇒ 6u + 7v - 2(3u - 2v) = 126 - 2(30)⇒ 6u + 7v - 6u +4v = 126 - 60⇒ 11v = 66⇒ v = 6Now put the value of v in eq.

(2) to get,⇒ 3u-2(6)=30⇒ 3u-12=30⇒ 3u=42⇒ u=14

Now

$$u = \frac{1}{x} = 14$$

$$x = \frac{1}{14}$$

And

$$v = \frac{1}{y} = 6$$

$$y = \frac{1}{6}$$

16. Question

Solve the following systems of equations:

$\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Answer

$\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Multiplying eq 1 by 1/2 and eq2 by 1/3 and subtracting

⇒ 1/4x – 1/9x = 1 – 13/18

⇒ 5/36x = 5/18

⇒ x = 1/2

Thus, 1 + 1/3y = 2

⇒ y = 1/3

17. Question

Solve the following systems of equations:

$\frac{x+y}{xy} = 2$

$\frac{x-y}{xy} = 6$

Answer

$\frac{x+y}{xy} = 2$

$\frac{x-y}{xy} = 6$

Adding both equation

⇒ 2x/xy = 8

⇒ y = 1/4

Thus, $\frac{\left(x+\frac{1}{4}\right)}{\frac{x}{4}} = 2$

⇒ x + 1/4 = x/2

⇒ x = -1/2

18. Question

Solve the following systems of equations:

$\frac{15}{u} + \frac{2}{v} = 17$

$\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$

Answer

$\frac{15}{u} + \frac{2}{v} = 17$

$\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$

Multiplying eq2 by 2 and subtracting

13/u = 17 – 72/5

⇒ u = 5

Thus, 3 + 2/v = 17

⇒ v = 1/7

19. Question

Solve the following systems of equations:

$\frac{3}{x} - \frac{1}{y} = -9$

$\frac{2}{x} + \frac{3}{y} = 5$

Answer

$\frac{3}{x} - \frac{1}{y} = -9$

$\frac{2}{x} + \frac{3}{y} = 5$

Multiplying eq1 by 3 and adding to eq1

⇒ 11/x = -27 +5

⇒ x = -1/2

Thus, -4 + 3/y = 5

⇒ y = 1/3

20. Question

Solve the following systems of equations:

$\frac{2}{x}+\frac{5}{y}=1$

$\frac{60}{x}+\frac{40}{y}=19,\ x\neq 0,\ y\neq 0$

Answer

$\frac{2}{x}+\frac{5}{y}=1$

$\frac{60}{x}+\frac{40}{y}=19,\ x\neq 0,\ y\neq 0$

Multiplying eq1 by 8 and subtracting from eq2

⇒ 44/x = 19 – 8

⇒ x = 4

Thus, 2/4 + 5/y = 1

⇒ 5/y = 1/2

⇒ y = 10

21. Question

Solve the following systems of equations:

$\frac{1}{5x}+\frac{1}{6y}=12$

$\frac{1}{3x}-\frac{3}{7y}=8,\quad x\neq 0,\ y\neq 0$

Answer

$\frac{1}{5x}+\frac{1}{6y}=12$

$\frac{1}{3x}-\frac{3}{7y}=8,\quad x\neq 0,\ y\neq 0$

Multiplying eq1 by 1/3 and eq2 by 1/5 and subtracting

⇒ 1/18y + 3/35y = 4 - 8/5

⇒ 89/630y = 12/5

⇒ y = 89/1512

Thus, 1/5x + 1512/534 = 12

⇒ 1/5x = 816/89

⇒ x = 89/4080

22. Question

Solve the following systems of equations:

$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$

$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, *where*, $x \neq 0, y \neq 0$

Answer

$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$

$$\Rightarrow \frac{2y+3x}{xy}=\frac{9}{xy}$$

⇒ 2y + 3x = 9 ----- (1)

$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, *where*, $x \neq 0, y \neq 0$

$$\Rightarrow \frac{4y+9x}{xy}=\frac{21}{xy}$$

⇒ 4y + 9x = 21 ---- (2)

Multiplying eq1 by 2 and subtracting it from eq 2 we get,

⇒ 4y + 9x - 2(2y + 3x) = 21 - 2(9)⇒ 4y + 9x - 4y - 6x = 21 - 18⇒ 3x = 3

⇒ x = 1

Put the value of x in 1 to get,

Thus, 2y + 3(1) = 92y + 3 = 92y = 9 - 32y = 6y = 3

hence x = 1 and y = 3

23. Question

Solve the following systems of equations:

$\frac{6}{x+y}=\frac{7}{x-y}+3$

$$\frac{1}{2(x+y)}=\frac{1}{3(x-y)}$$

where, x + y ≠ 0 and x - y ≠ 0

Answer

Given : the following systems of equations:

$\frac{6}{x+y}=\frac{7}{x-y}+3$ ..... (1)

$$\frac{1}{2(x+y)}=\frac{1}{3(x-y)} \quad \text{..... (2)}$$

To find : The value of x and y.

Solution :

Take $\frac{1}{x+y}=u \ and \ \frac{1}{x-y}=v$

The equations become:6u=7v+36u-7v=3 ..... (3)And, $\frac{1}{2}u = \frac{1}{3}v$

⇒ 3u=2v 3u-2v=0 ..... (4)To find the value of u and v,Multiply eq. 4 with 2 and subtract it from eq. 3⇒ 6u-7v-2(3u-2v)=3-0⇒ 6u-7v-6u+4v=3⇒ -3v=3⇒ v=-1Put the value of v in eq. 4 to get,⇒ 3u-2(-1)=0⇒ 3u+2=0⇒ 3u=-2 $u = \frac{-2}{3}$

Now,

$$\Rightarrow u = \frac{1}{x+y} = \frac{-2}{3}$$

$$\Rightarrow \frac{1}{x+y} = \frac{-2}{3}$$

⇒ 3 = -2(x+y)⇒ 3=-2x-2y ..... (5)

$$\Rightarrow v = \frac{1}{x-y} = -1$$

$$\Rightarrow \frac{1}{x-y} = -1$$

⇒ x-y=-1 .... (6)Multiply eq. 6 with 2 and add to eq. 5⇒ 2(x-y)+(-2x-2y)=-2+3⇒ -2y-2y-2x-2y=1⇒ -4y=1⇒ $y = \frac{-1}{4}$

Now put the value of y in eq. 6 to get $x - \left(\frac{-1}{4}\right) = -1$

$$x + \frac{1}{4} = -1$$

$$x = -1 - \frac{1}{4}$$

$$x = \frac{-4-1}{4}$$

$$\Rightarrow x = \frac{-5}{4}$$

Hence the values are $x = \frac{-5}{4},\ y = \frac{-1}{4}$.

24. Question

Solve the following systems of equations:

$\frac{xy}{x+y} = \frac{6}{5}$

$\frac{xy}{y-x} = 6$ where, x + y ≠ 0 and x - y ≠ 0

Answer

$\frac{xy}{x+y} = \frac{6}{5}$

$\frac{xy}{y-x} = 6$

Dividing the two equations

⇒ (y –x)/(x + y) = 1/5

⇒ 5y – 5x = x + y

⇒ y = 3x/2

Thus, $x \times \frac{\frac{3x}{2}}{x+\frac{3x}{2}} = \frac{6}{5}$

⇒ 3x/5 = 6/5

⇒ x = 2

Thus, y = 3

25. Question

Solve the following systems of equations:

$\frac{22}{x+y} + \frac{15}{x-y} = 5$

$\frac{55}{x+y} + \frac{45}{x-y} = 14$

Answer

$\frac{22}{x+y} + \frac{15}{x-y} = 5$

$\frac{55}{x+y} + \frac{45}{x-y} = 14$

Multiplying eq1 by 3 and subtracting from eq2

⇒ -11/(x + y) = -1

⇒ x + y = 11 ---------- (3)

Multiplying eq1 by 5 and eq2 by 2 and subtracting

⇒ 15/(x –y) = 3

⇒ x – y = 5 ------ (4)

(3) + (4)

⇒ 2x = 16

⇒ x = 8

Thus, y = 3

26. Question

Solve the following systems of equations:

$\frac{5}{x+y} - \frac{2}{x-y} = -1$

$\frac{15}{x+y} + \frac{7}{x-y} = 10$

Answer

$$\frac{5}{x+y}-\frac{2}{x-y}=-1$$

$$\frac{15}{x+y}+\frac{7}{x-y}=10$$

Multiplying eq1 by 3 and subtracting from eq2

⇒ 13/(x – y) = 13

⇒ x – y = 1 ------ (3)

Multiplying eq1 by 7 and eq2 by 2 and adding

⇒ 65/(x + y) = 13

⇒ x + y = 5 ------- (4)

Thus, 2x = 6

⇒ x = 3

Y = x – 1

⇒ y = 2

27. Question

Solve the following systems of equations:

$$\frac{3}{x+y}+\frac{2}{x-y}=2$$

$$\frac{9}{x+y}-\frac{4}{x-y}=1$$

Answer

$$\frac{3}{x+y}+\frac{2}{x-y}=2$$

$$\frac{9}{x+y}-\frac{4}{x-y}=1$$

Multiplying eq1 by 2 and adding to eq2

⇒ 15/(x + y) = 5

⇒ x + y = 3 ------- (3)

Multiplying eq1 by 3 and subtracting eq2 from it

⇒ 10/(x – y) = 5

⇒ x – y = 2 ---- (4)

Adding the two equation

⇒ 2x = 5

⇒ x = 5/2

Thus, y = 1/2

28. Question

Solve the following systems of equations:

$$\frac{1}{2(x+2y)}+\frac{5}{3(3x-2y)}=\frac{-3}{2}$$

$$\frac{5}{4(x+2y)}-\frac{4}{5(3x-2y)}=\frac{73}{60}$$

Answer

$\frac{1}{2(x+2y)}+\frac{5}{3(3x-2y)}=\frac{-3}{2}$

$$\frac{5}{4(x+2y)}-\frac{4}{5(3x-2y)}=\frac{73}{60}$$

Multiplying eq1 by 5/2 and subtracting from eq2

$$\Rightarrow -\frac{4}{5(3x-2y)}-\frac{25}{6(3x-2y)}=\frac{73}{60}+\frac{15}{4}$$

$$\Rightarrow -\frac{149}{30(3x-2y)}=\frac{298}{60}$$

⇒ 3x – 2y = - 1 ------- (3)

Multiplying eq1 by 12/25 and adding to eq2

$$\Rightarrow \frac{6}{25(x+2y)}+\frac{5}{4(x+2y)}=-\frac{18}{25}+\frac{73}{60}$$

$$\Rightarrow \frac{149}{100(x+2y)}=\frac{149}{300}$$

⇒ x + 2y = 3 ------- (4)

Solving (3) and (4)

We get, x = 1/2 and y = 5/4

29. Question

Solve the following systems of equations:

$\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2}$

$\frac{10}{x+1}+\frac{2}{y-1}=\frac{5}{2}$ *where*, $x \neq -1, y \neq 1$

Answer

$$\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2}$$

$$\frac{10}{x+1}+\frac{2}{y-1}=\frac{5}{2}$$

Adding eq1 and eq2

⇒ 15/(x + 1) = 3

⇒ x + 1 = 5

⇒ x = 4

Thus, 5/5 – 2/(y – 1) = 12

⇒ y – 1 = 4

⇒ y = 5

30. Question

Solve the following systems of equations:

$x + y = 5xy$

$x + 2y = 13xy, \quad x \neq 0, y \neq 0$

Answer

$x + y = 5xy$

$x + 2y = 13xy, \quad x \neq 0, y \neq 0$

Subtracting the two eq.

⇒ -y = -8xy

⇒ x = 1/8

Thus, 1/8 + y = 5y/8

⇒ y = 1/3

31. Question

Solve the following systems of equations:

$x + y = 2xy$

$\frac{x - y}{xy} = 6, \quad x \neq 0, y \neq 0$

Answer

x + y = 2xy

x – y = 6xy

Adding the two equation

⇒ 2x = 8xy

⇒ y = 1/4

Thus, x + 1/4 = 2 × x × 1/4

⇒ x/2 = -1/4

⇒ x = -1/2

32. Question

Solve the following systems of equations:

$2(3u - v) = 5uv$

$2(u + 3v) = 5uv$

Answer

$2(3u - v) = 5uv$

$2(u + 3v) = 5uv$

Equating both equations

⇒ 6u – 2v = 2u + 6v

⇒ u = 2v

Substituting value of u

⇒ 2(6v – v) = 5 × 2v × v

⇒ v = 1

Thus, 2(3u – 1) = 5u

⇒ 6u – 2 = 5u

⇒ u = 2

33. Question

Solve the following systems of equations:

$$\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$$

$$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$$

Answer

$$\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$$

$$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$$

Multiplying eq1 by 5 and eq2 by 2 and subtracting

⇒ 13/(3x – 2y) = 13

⇒ 3x – 2y = 1------- (3)

Multiplying eq2 by 3 and subtracting eq1 from it

⇒ 13/(3x +2y) = 13/5

⇒ 3x + 2y = 5 ------ (4)

(3) + (4)

⇒ 6x = 6

⇒ x = 1

Thus, 3 + 2 = 5

⇒ y = 1

34. Question

Solve the following systems of equations:

$$\frac{4}{x}+3y=14$$

$$\frac{3}{x}-4y=23$$

Answer

$$\frac{4}{x}+3y=14$$

$$\frac{3}{x}-4y=23$$

Multiply eq1 by 4 and eq2 by 3 and adding

⇒ 25/x = 125

⇒ x = 1/5

Thus, 20 + 3y = 14

⇒ y =- 2

35. Question

Solve the following systems of equations:

$99x + 101y = 499$

$101x + 99y = 501$

Answer

$99x + 101y = 499$

$101x + 99y = 501$

Adding both equations

⇒ 200x + 200y = 1000

⇒ x + y = 5 ------ (1)

Subtracting both equation

⇒ 2y – 2x = - 2

⇒ x – y = 1 ----- (2)

Adding 1 and 2

⇒ 2x = 6

⇒ x = 3

Thus, 3 + y =5

⇒ y = 2

36. Question

Solve the following systems of equations:

$23x - 29y = 98$

$29x - 23y = 110$

Answer

$23x - 29y = 98$

$29x - 23y = 110$

Adding both equation

⇒ 52x – 52y = 208

⇒ x – y = 4 ----- (1)

Subtracting both equation

⇒ -6x - 6y = -12

⇒ x + y = 12 ----- (2)

(1) + (2)

⇒ 2x = 16

⇒ x = 8

Thus, y = 4

37. Question

Solve the following systems of equations:

$x - y + z = 4$

$x - 2y + 3z = 9$

$2x + y + 3z = 1$

Answer

$x - y + z = 4$

$x - 2y + 3z = 9$

$2x + y + 3z = 1$

Subtracting eq2 and eq3

⇒ x + 3y = 0

⇒ x = -3y

Substituting in eq1

⇒ -3y – y + z = 4

⇒ z = 4 + 4y

Substituting in eq2

⇒ -3y – 2y + 12 + 12y = 1

⇒ 7y = -11

⇒ y = -11/7

Thus, x = 33/7

And z = 4 – 44/7 = - 16/7

38. Question

Solve the following systems of equations:

$x - y + z = 4$

$x + y + z = 2$

$2x + y - 3z = 0$

Answer

$x - y + z = 4$

$x + y + z = 2$

$2x + y - 3z = 0$

Adding eq 1 and eq2

⇒ 2(x + z) = 6

⇒ x = 3 – z

(3) – (2) – (1)

⇒ y – 5z = - 4

⇒ y = -4 + 5z

Substituting in eq2

⇒ 3 – z - 4 + 5z + z = 2

⇒ 5z = 3

⇒ z = 3/5

Thus, x = 3 – z

⇒ x = 12/5

Y = 5z – 4

⇒ y = -1

39. Question

Solve the following systems of equations:

$$\frac{44}{x+y}+\frac{30}{x-y}=10$$

$$\frac{55}{x+y}+\frac{40}{x-y}=13$$

Answer

$$\frac{44}{x+y}+\frac{30}{x-y}=10$$

$$\frac{55}{x+y}+\frac{40}{x-y}=13$$

Multiplying eq1 by 5 and eq2 by 4 and subtracting

⇒ -10/(x – y) = -2

⇒ x – y = 5

Multiply eq1 by 4 and eq2 by 3 and subtracting

⇒ 11/(x + y) = 1

⇒ (x + y) = 11

Thus, 2x = 16

⇒ x = 8

∴ y = 8 – 5 = 3

40. Question

Solve the following systems of equations:

$$\frac{4}{x}+5y=7$$

$$\frac{3}{x}+4y=5$$

Answer

$$\frac{4}{x}+5y=7$$

$$\frac{3}{x}+4y=5$$

Multiply eq1 by 4 and eq2 by 5 and subtracting

⇒ 1/x = 3

⇒ x = 1/3

Thus, 12 + 5y = 7

⇒ y = 1

41. Question

Solve the following systems of equations:

$\frac{2}{x}+\frac{3}{y}=13$

$\frac{5}{x}-\frac{4}{y}=-2$

Answer

$\frac{2}{x}+\frac{3}{y}=13$

$\frac{5}{x}-\frac{4}{y}=-2$

Multiplying eq1 by 4 and eq2 by 3 and adding

⇒ 23/x = 46

⇒ x = 1/2

Thus, 4 + 3/y = 13

⇒ y = 1/3

42. Question

Solve the following systems of equations:

$\frac{5}{x-1}+\frac{1}{y-2}=2$

$$\frac{6}{x-1}-\frac{3}{y-2}=1$$

Answer

Given : The system of equations: $\frac{5}{x-1}+\frac{1}{y-2}=2$

$$\frac{6}{x-1}-\frac{3}{y-2}=1$$

To find: The values of x and y

Solution: Let $\frac{1}{x-y}=u \ and \ \frac{1}{y-2}=v$

Now system of equations become:

5u + v = 2 ..... (1) 6u - 3v = 1 ..... (2)

Now multiply equation (1) by 3 and add to equation (2),3(5u + v)+ 6u - 3v = 3(2) +115u +3v + 6u -3v = 6+121u = 7

$u=\frac{7}{21}$

$u=\frac{1}{3}$

so, $u = \frac{1}{x-1} = \frac{1}{3}$

⇒x - 1 =3⇒x = 3 + 1⇒x = 4Now put $u = \frac{1}{3}$ in the equation (1) to get v,

$$\frac{5}{3} + v = 2$$

$$v = 2 - \frac{5}{3}$$

$$v = \frac{6-5}{3}$$

$$v = \frac{1}{3}$$

So, $v = \frac{1}{y-2} = \frac{1}{3}$

⇒y - 2 = 3⇒y = 3 + 2⇒y=5Hence the values are x= 4 and y=5.

43. Question

Solve the following systems of equations:

$\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{9}{x-y} = -2$

Answer

$\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{9}{x-y} = -2$

Multiplying eq1 by 9 and eq2 by 2 and adding

⇒ 120/(x + y) = 32

⇒ x + y = 15/4 ---- (1)

Multiplying eq1 by 3 and eq2 by 2 and subtracting

⇒ 24/(x – y) = 16

⇒ x – y = 3/2 ---- (2)

Adding (1) and (2)

⇒ 2x = 21/4

⇒ x = 21/8

Thus, 21/8 – y = 3/2

⇒ y = 9/8

44. Question

Solve the following systems of equations:

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$$

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$$

Answer

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$$

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$$

Multiplying eq2 by 2 and adding to eq1

⇒ 2/(3x + y) = 3/4 - 2/8

⇒ 3x + y = 4 ---- (3)

Multiplying eq2 by 2 and subtracting from eq1

⇒ 2/(3x – y) = 1

⇒ 3x – y = 2 ----- (4)

Adding (3) and (4)

⇒ 6x = 6

⇒ x = 1

Thus, y = 3 – 2 = 1

45. Question

Solve the following systems of equations:

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Answer

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Multiplying eq1 by 3 and adding to eq2

⇒ 10/√x = 5

⇒ √x = 2

⇒ x = 4

Thus,

⇒ 2/√4 + 3/√y = 2

⇒ y = 9

46. Question

Solve the following systems of equations:

$$\frac{7x-2y}{xy}=5$$

$\frac{8x+7y}{xy}=15$

Answer

$\frac{7x-2y}{xy}=5$

⇒ 7/y – 2/x = 5 ---- (1)

$\frac{8x+7y}{xy}=15$

⇒ 8/y + 7/x = 15 ---- (2)

Multiplying eq1 by 7 and eq2 by 2 and adding

⇒ 65/y = 65

⇒ y = 1

Thus, 7 – 2/x = 5

⇒ x = 1

47. Question

Solve the following systems of equations:

$152x-378y=-74$

$-378x+158y=-604$

Answer

$152x-378y=-74$

$-378x+158y=-604$

Adding (1) and (2)

⇒ -226x – 226y = -678

⇒ x + y = 3----- (3)

(1) – (2)

⇒ 530x – 530y = 530

⇒ x – y = 1 ------ (4)

Adding (3) and (4)

⇒ 2x = 4

⇒ x = 2

Thus, y = 1

Exercise 3.4

1. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$x+2y+1=0$

$2x-3y-12=0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $x+2y+1=0$

$2x-3y-12=0$

$$\frac{x}{-24+3}=\frac{y}{2+12}=\frac{1}{-3-4}$$

⇒ x = -21/-7 = 3

⇒ y = 14/-7 = -2

2. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$3x+2y+25=0$

$2x+y+10=0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $3x+2y+25=0$

$2x+y+10=0$

$$\therefore \frac{x}{2\times10-1\times25}=\frac{y}{2\times25-3\times10}=\frac{1}{3\times1-2\times2}$$

$$\Rightarrow \frac{x}{-5}=\frac{y}{20}=-1$$

⇒ x= 5 and y = -20

3. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2x+y=35$

$3x+4y=65$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $2x+y=35$

$3x + 4y = 65$

$$\frac{x}{-65+140} = \frac{y}{-105+130} = \frac{1}{8-3}$$

⇒ x = 75/5 = 15

⇒ y = 25/5 = 5

4. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2x - y = 6$

$x - y = 2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $2x - y = 6$

$x - y = 2$

$$\frac{x}{2-6} = \frac{y}{-6+4} = \frac{1}{-2+1}$$

⇒ x = -4/-1 = 4

⇒ y = -2/-1 = 2

5. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{x+y}{xy} = 2,\ \frac{x-y}{xy} = 6$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$\frac{x+y}{xy} = 2,\ \frac{x-y}{xy} = 6$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = 2, -\frac{1}{x} + \frac{1}{y} = 6$$

$$\frac{\frac{1}{x}}{-6+2} = \frac{\frac{1}{y}}{2+6} = \frac{1}{1+1}$$

⇒ 1/x = -2

⇒ x = -1/2

Also, 1/y = 4

⇒ y = 1/4

6. Question

Solve each of the following systems of equations by the method of cross-multiplication:

ax + by = a – b

$bx - ay = a + b$

Answer

Method of cross multiplication

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Given,

ax + by = a – b

$bx - ay = a + b$

This can be written as:

ax+by-(a-b)=0bx-ay-(a+b)=0So,

$$\frac{x}{(b)(-(a+b)) - (-a)(-(a-b))} = \frac{-y}{a(-(a+b)) - b(-(a-b))} = \frac{1}{a(-a) - b(b)}$$

$$\frac{x}{(b)(-a-b) + a(b-a)} = \frac{-y}{a(-a-b) - b(b-a)} = \frac{1}{-a^2 - b^2}$$

$$\frac{x}{-ba - b^2 + ab - a^2} = \frac{-y}{-a^2 - ab - b^2 + ab} = \frac{1}{-a^2 - b^2}$$

$$\frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 - b^2} = \frac{1}{-a^2 - b^2}$$

$$\frac{x}{-b^2 - a^2} = \frac{y}{a^2 + b^2} = \frac{1}{-a^2 - b^2}$$

$$\frac{x}{-b^2 - a^2} = \frac{1}{-a^2 - b^2} \; and \; \frac{y}{a^2 + b^2} = \frac{1}{-a^2 - b^2}$$

$$x = \frac{-b^2 - a^2}{-a^2 - b^2} \; and \; y = \frac{a^2 + b^2}{-a^2 - b^2}$$

$$x = \frac{-b^2 - a^2}{-a^2 - b^2} \; and \; y = \frac{-(-a^2 - b^2)}{-a^2 - b^2}$$

⇒ x = 1 and y = -1

7. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$x + ay = b$

$ax - by = c$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $x + ay = b$

$ax - by = c$

$$\frac{x}{-ac - b^2} = \frac{y}{-ab + c} = \frac{1}{-b - a^2}$$

$$\Rightarrow x = \frac{ac + b^2}{a^2 + b}$$

$$\Rightarrow y = \frac{ab + c}{a^2 + b}$$

8. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$ax + by = a^2$

$bx + ay = b^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $ax + by = a^2$

$bx + ay = b^2$

$$\Rightarrow \frac{x}{-b^3 + a^3} = \frac{y}{-ba^2 + ab^2} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow x = \frac{a^3 - b^3}{a^2 - b^2}$$

$$\Rightarrow x = \frac{(a-b)(a^2+ab+b^2)}{(a-b)(a+b)} = \frac{a^2+ab+b^2}{a+b}$$

$$y = \frac{ab(b-a)}{(a-b)(a+b)} = -\frac{ab}{a+b}$$

9. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{x}{a} + \frac{y}{b} = 2$

$ax - by = a^2 - b^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$\frac{x}{a} + \frac{y}{b} = 2$

$ax - by = a^2 - b^2$

$$\frac{x}{\frac{b^2-a^2}{b} - 2b} = \frac{y}{-2a + \frac{a^2-b^2}{a}} = \frac{1}{-\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \frac{x}{\frac{-a^2-b^2}{b}} = \frac{y}{\frac{-a^2-b^2}{a}} = \frac{1}{\frac{-a^2-b^2}{ab}}$$

⇒ x = a and y = b

10. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2$

$$\frac{x}{-\frac{2}{b}+\frac{a+b}{b^2}}=\frac{y}{-\frac{a+b}{a^2}+\frac{2}{a}}=\frac{1}{\frac{1}{ab^2}-\frac{1}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a-b}{b^2}}=\frac{y}{\frac{a-b}{a^2}}=\frac{1}{\frac{1}{a^2b^2}(a-b)}$$

⇒ x = a^2, y = b^2

11. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{x}{a}=\frac{y}{b}$

$ax+by=a^2+b^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $\frac{x}{a}=\frac{y}{b}$

$ax+by=a^2+b^2$

$$\Rightarrow \frac{x}{\frac{a^2+b^2}{b}}=\frac{y}{\frac{a^2+b^2}{a}}=\frac{1}{\frac{b}{a}+\frac{a}{b}}$$

$$\Rightarrow \frac{x}{\frac{a^2+b^2}{b}}=\frac{y}{\frac{a^2+b^2}{a}}=\frac{ab}{a^2+b^2}$$

⇒ x = a and y = b

12. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{5}{x+y}-\frac{2}{x-y}=-1$

$\frac{15}{x+y}+\frac{7}{x-y}=10$ *where* $x\neq 0$ *and* $y\neq 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given,

$\frac{5}{x+y}-\frac{2}{x-y}=-1$

$\frac{15}{x+y}+\frac{7}{x-y}=10$

Multiplying eq1 by 3 and subtracting from eq2

$$\Rightarrow \frac{13}{x-y}=13$$

⇒ x – y = 1 ------- (3)

Multiplying eq2 by 2 and eq1 by 7 and adding

⇒ 65/(x + y) = 13

⇒ x + y = 5 ---------- (4)

Thus,

$$\frac{x}{5+1}=\frac{y}{-1+5}=\frac{1}{1+1}$$

⇒ x = 6/2 = 3 and y = 4/2 = 2

13. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{2}{x}+\frac{3}{y}=13$

$\frac{5}{x}-\frac{4}{y}=-2$ *where* $x \neq 0$ *and* $y \neq 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $\frac{2}{x}+\frac{3}{y}=13$

$\frac{5}{x}-\frac{4}{y}=-2$

$$\Rightarrow \frac{\frac{1}{x}}{6-52}=\frac{\frac{1}{y}}{-65-4}=\frac{1}{-8-15}$$

⇒ 1/x = 2

⇒ x = 1/2

Also, 1/y = 3

⇒ y = 1/3

14. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$ax+by=\frac{a+b}{2}$

3x + 5y = 4

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given,

$ax+by=\frac{a+b}{2}$

3x + 5y = 8

$$\Rightarrow \frac{x}{-4b+\frac{(5a+5b)}{2}}=\frac{y}{\frac{-3a-3b}{2}+4a}=\frac{1}{5a-3b}$$

$$\Rightarrow \frac{2x}{5a-3b}=\frac{2y}{5a-3b}=\frac{1}{5a-3b}$$

⇒ x = 1/2 , y = 1/2

15. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2ax+3by=a+2b$

$3ax+2by=2a+b$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $2ax+3by=a+2b$

$3ax+2by=2a+b$

$$\Rightarrow \frac{x}{-6ab-3b^2+2ab+4b^2}=\frac{y}{-3a^2-6ab+4a^2+2ab}=\frac{1}{4ab-9ab}$$

$$\Rightarrow \frac{x}{b^2-4ab}=\frac{y}{a^2-4ab}=\frac{1}{-5ab}$$

$$\Rightarrow x=\frac{4a-b}{5a}, y=\frac{4b-a}{5b}$$

16. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$5ax+6by=28$

$3ax+4by=18$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $5ax+6by=28$

$3ax+4by=18$

$$\frac{x}{-108b+112b}=\frac{y}{-84a+90a}=\frac{1}{20ab-18ab}$$

$$\Rightarrow \frac{x}{4b}=\frac{y}{6a}=\frac{1}{2ab}$$

⇒ x = 2/a and y = 3/b

17. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$(a+2b)x+(2a-b)y=2$

(a – 2b)x + (2a + b)y = 3

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

$(a+2b)x+(2a-b)y=2$

(a – 2b)x + (2a + b)y = 3

$$\Rightarrow \frac{x}{3b-6a+4a+2b}=\frac{y}{3a+6b-2a+4b}$$
$$=\frac{1}{(2a+b)(a+2b)-(a-2b)(2a-b)}$$

$$\Rightarrow \frac{x}{5b-2a}=\frac{y}{a+10b}=\frac{1}{2a^2+ab+4ab+2b^2-(2a^2-ab-4ab+2b^2)}$$
$$=\frac{1}{10ab}$$

$$\Rightarrow x=\frac{5b-2a}{10ab}, y=\frac{a+10b}{10ab}$$

18. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a-b}\right)$

$x+y=2a^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

$x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a-b}\right)$

$x+y=2a^2$

$$\Rightarrow \frac{x}{2a^2\times\frac{a^2-b^2-ab}{a-b}}=\frac{y}{2a^2\times\frac{a^2+b^2-ab}{a-b}}=\frac{1}{\left(\frac{a^2+b^2-ab}{a-b}+\frac{a^2-b^2-ab}{a-b}\right)}$$

$$\Rightarrow \frac{x}{2a^2(a^2-b^2-ab)}=\frac{y}{2a^2(a^2+b^2-ab)}=\frac{1}{2a(a-b)}$$

$$\Rightarrow x=\frac{a(a^2-b^2-ab)}{a-b}, y=\frac{a(a^2+b^2-ab)}{a-b}$$

19. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$bx+cy=a+b \quad ax\left(\frac{1}{a-b}-\frac{1}{a+b}\right)+cy\left(\frac{1}{b-a}-\frac{1}{b+a}\right)=\frac{2a}{a+b}$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

$bx+cy=a+b \quad ax\left(\frac{1}{a-b}-\frac{1}{a+b}\right)+cy\left(\frac{1}{b-a}-\frac{1}{b+a}\right)=\frac{2a}{a+b}$

$$\Rightarrow \frac{x}{-\frac{2ac}{a+b}+(a+b)\times c\times\frac{2a}{b^2-a^2}}=\frac{y}{-(a+b)\times a\times\frac{2b}{a^2-b^2}+\frac{2ab}{a+b}}=\frac{1}{-bc\times\frac{2a}{a^2-b^2}-ac\times\frac{2b}{a^2-b^2}}$$

$$\Rightarrow \frac{x}{-\frac{2ac}{a+b}-\frac{2ac}{a-b}}=\frac{y}{\frac{2ab}{a+b}-\frac{2ab}{a-b}}=\frac{1}{-\frac{4abc}{a^2-b^2}}$$

$$\Rightarrow \frac{x}{-2ac\times\frac{2a}{a^2-b^2}}=\frac{y}{2ab\times-\frac{2b}{a^2-b^2}}=\frac{1}{-\frac{4abc}{a^2-b^2}}$$

$$\Rightarrow x=\frac{a}{b}, y=\frac{b}{c}$$

20. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$(a-b)x+(a+b)y=2a^2-2b^2$

$(a+b)(x+y)=4ab$

Answer

Method of cross multiplication:For the system of equations:

$a_1x + b_1y + c_1 = 0a_2x + b_2y + c_2 = 0$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

Here,a_1 = (a - b) , b_1 = (a+b) , c_1 = $2b^2$ - $2a^2$$a_2$ = (a+b), b_2 = (a+b), c_2 = - 4abSo,

$$\Rightarrow \frac{x}{(a+b)(-4ab)-(a+b)(2b^2-2a^2)}=\frac{y}{(2b^2-2a^2)(a+b)-(-4ab)(a-b)}=\frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \frac{x}{(a+b)(-4ab)-2(a+b)(b^2-a^2)}=\frac{y}{2(b^2-a^2)(a+b)+4ab(a-b)}=\frac{1}{(a+b)(a-b-a-b)}$$

$$\Rightarrow \frac{x}{2(a+b)\left[-2ab-(b^2-a^2)\right]}=\frac{y}{2(b-a)(b+a)(a+b)+4ab(a-b)}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-2(a+b)\left[2ab+b^2-a^2\right]}=\frac{y}{2(b-a)\left[(a+b)^2-2ab\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-2(a+b)\left[2ab+b^2-a^2\right]}=\frac{y}{2(b-a)\left[a^2+b^2+2ab-2ab\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-2(a+b)\left[2ab+b^2-a^2\right]}=\frac{y}{2(b-a)\left[a^2+b^2\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow x=\frac{-2(a+b)\left[2ab+b^2-a^2\right]}{(a+b)(-2b)}\ and\ y=\frac{2(b-a)\left[a^2+b^2\right]}{(a+b)(-2b)}$$

$$\Rightarrow x=\frac{\left[2ab+b^2-a^2\right]}{b}\ and\ y=\frac{(a-b)\left[a^2+b^2\right]}{b(a+b)}$$

21. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$a^2x+b^2y=c^2$

$b^2x+a^2y=a^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

$a^2x + b^2y = c^2$

$b^2x + a^2y = a^2$

$$\Rightarrow \frac{x}{-b^2a^2 + a^2c^2} = \frac{y}{-b^2c^2 + a^4} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow x = \frac{a^2c^2 - b^2a^2}{a^4 - b^4}, y = \frac{a^4 - b^2c^2}{a^4 - b^4}$$

22. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{57}{x+y} + \frac{6}{x-y} = 5$

$\frac{38}{x+y} + \frac{21}{x-y} = 9$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

$\frac{57}{x+y} + \frac{6}{x-y} = 5$

$\frac{38}{x+y} + \frac{21}{x-y} = 9$

$$\Rightarrow \frac{\frac{1}{x+y}}{-6\times9 + 5\times21} = \frac{\frac{1}{x-y}}{-38\times5 + 57\times9} = \frac{1}{57\times21 - 6\times38}$$

⇒ x + y = 19 and x – y = 8

Solving the above two equation

⇒ x = 11 and y = 8

23. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2(ax - by) + a + 4b = 0$

$2(bx + ay) + b - 4a = 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$2(ax - by) + a + 4b = 0$

2ax - 2by + a + 4b = 0

$2(bx + ay) + b - 4a = 0$

2bx + 2ay + b - 4a = 0

$$\Rightarrow \frac{x}{(-2b)(b-4a)-2a(a+4b)} = \frac{y}{(2b)(a+4b)-(2a)(b-4a)} = \frac{1}{(2a)(2a)-(-2b)(2b)}$$

$$\Rightarrow \frac{x}{-2b^2+8ab-2a^2-8ab} = \frac{y}{2ab+8b^2-2ab+8a^2} = \frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2b^2-2a^2} = \frac{y}{8b^2+8a^2} = \frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2(b^2+a^2)} = \frac{y}{8(b^2+a^2)} = \frac{1}{4(a^2+b^2)}$$

$$\Rightarrow x = \frac{-2(b^2+a^2)}{4(a^2+b^2)} \; and \; y = \frac{8(a^2+b^2)}{4(a^2+b^2)}$$

$$\Rightarrow x = \frac{-1}{2} \; and \; y = 2$$

24. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$6(ax + by) = 3a + 2b$

$6(bx - ay) = 3b - 2a$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1} = \frac{y}{a2c1-a1c2} = \frac{1}{a1b2-a2b1}$$

$6(ax + by) = 3a + 2b$

$6(bx - ay) = 3b - 2a$

$$\Rightarrow \frac{x}{6b(2a-3b)-6a(3a+2b)} = \frac{y}{-6b(3a+2b)+6a(3b-2a)} = \frac{1}{-36a^2-3b^2}$$

$$\Rightarrow \frac{x}{-18a^2-18b^2} = \frac{y}{-12a^2-12b^2} = \frac{1}{-36a^2-36b^2}$$

⇒ x = 1/2, y = 1/3

25. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{a^2}{x} - \frac{b^2}{y} = 0$

$\frac{a^2b}{x} + \frac{b^2a}{y} = a + b,\ x, y \neq 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$\frac{a^2}{x} - \frac{b^2}{y} = 0$

$\frac{a^2b}{x} + \frac{b^2a}{y} = a + b,\ x, y \neq 0$

$$\Rightarrow \frac{\frac{1}{x}}{b^2(a+b)} = \frac{\frac{1}{y}}{a^2(a+b)} = \frac{1}{a^3b^2 + a^2b^3}$$

$$\Rightarrow \frac{\frac{1}{x}}{b^2(a+b)} = \frac{\frac{1}{y}}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

⇒ x = a^2, y = b^2

26. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$mx - ny = m^2 + n^2$

$x + y = 2m$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $mx - ny = m^2 + n^2$

$x + y = 2m$

$$\Rightarrow \frac{x}{2mn + m^2 + n^2} = \frac{y}{-m^2 - n^2 + 2m^2} = \frac{1}{m+n}$$

$$\Rightarrow \frac{x}{(m+n)^2} = \frac{y}{m^2 - n^2} = \frac{1}{m+n}$$

⇒ x = m + n and y = m − n

27. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{ax}{b} - \frac{by}{a} = a + b$

$ax - by = 2ab$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $\frac{ax}{b} - \frac{by}{a} = a + b$

$ax - by = 2ab$

$$\Rightarrow \frac{x}{2b^2 - ab - b^2} = \frac{y}{-a^2 - ab + 2a^2} = \frac{1}{-a + b}$$

$$\Rightarrow \frac{x}{b(b-a)} = \frac{y}{a(a-b)} = \frac{1}{b-a}$$

$$\Rightarrow x = b, y = -a$$

28. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$$

$x + y = 2ab$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$

$x + y = 2ab$

$$\frac{x}{-2a^2 + a^2 + b^2} = \frac{y}{-a^2 - b^2 + 2b^2} = \frac{1}{\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \frac{x}{b^2 - a^2} = \frac{y}{b^2 - a^2} = \frac{ab}{b^2 - a^2}$$

⇒ x = y = ab

Exercise 3.5

1. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$x - 3y = 3$

$3x - 9y = 2$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, unique solution

X – 3y = 3

3x – 9y = 2

Thus, $\frac{1}{3} = \frac{1}{3} \neq \frac{3}{2}$

The system has no solution

2. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$2x + y = 5$

$4x + 2y = 10$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, unique solution

2x + y = 5

4x + 2y = 10

$Thus, \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Infinitely many solution

3. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$3x - 5y = 20$

$6x - 10y = 40$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, unique solution

3x – 5y = 20

6x – 10y = 40

$Thus, \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Infinitely many solution

4. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$x - 2y = 8$

$5x - 10y = 10$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, unique solution

$x - 2y = 8$

$5x - 10y = 10$

$Thus, \frac{1}{5} = \frac{1}{5} \neq \frac{4}{5}$

No solution.

5. Question

Find the value of k for which the following system of equations has a unique solution:

$kx + 2y = 5$

$3x + y = 1$

Answer

Kx + 2y = 5

3x + y = 1

Multiplying eq2 by 2 and subtracting from eq1

(k – 6)x = 3

⇒ x = 3/(k – 6)

Thus, for k ≠ 6 the system has unique solution

6. Question

Find the value of k for which the following system of equations has a unique solution:

$4x + ky + 8 = 0$

$2x + 2y + 2 = 0$

Answer

$4x + ky + 8 = 0$

$2x + 2y + 2 = 0$

Multiplying eq2 by 2 and subtracting from eq1

⇒ (k – 4)y + 6 = 0

⇒ y = 6/(4 – k)

Thus, for k ≠ 4 it has a unique solution

7. Question

Find the value of k for which the following system of equations has a unique solution:

$4x - 5y = k$

$2x - 3y = 12$

Answer

Multiplying eq2 by 2 and subtracting from eq1

⇒ -5y + 6y = k – 24

⇒ y = k – 24

Thus, for any value of k it has a unique solution

8. Question

Find the value of k for which the following system of equations has a unique solution:

$x + 2y = 3$

$5x + ky + 7 = 0$

Answer

$x + 2y = 3$

$5x + ky + 7 = 0$

Multiplying eq1 by 5 and subtracting from eq2

⇒ (k – 10)y = -22

Thus for all k except 10 it has a unique solution

9. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + 3y - 5 = 0$

$6x + ky - 15 = 0$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$2x+3y-5=0$

$6x+ky-15=0$

$$\Rightarrow \frac{2}{6}=\frac{3}{k}=\frac{5}{15}$$

⇒ k = 3 × 3 = 9

10. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$4x+5y=3$

$kx+15y=9$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

4x + 5y = 3

Kx + 15y = 9

$$\Rightarrow \frac{4}{k}=\frac{5}{15}=\frac{3}{9}$$

⇒ k = 4 × 3 = 12

11. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$kx-2y+6=0$

$4x-3y+9=0$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

kx – 2y + 6 = 0

4x – 3y + 9 = 0

$$\frac{k}{4}=\frac{2}{3}=\frac{6}{9}$$

⇒ k = 4 × 2/3 = 8/3

12. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$8x+5y=9$

$kx+10y=18$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

8x + 5y = 9

Kx + 10y = 18

$$\frac{8}{k}=\frac{5}{10}=\frac{9}{18}$$

⇒ k = 2 × 8 = 16

13. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x-3y=7$

$(k+2)x-(2k+1)y=3(2k-1)$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

2x – 3y = 7

(k + 2)x – (2k + 1)y = 3(2k – 1)

$$\frac{2}{k+2}=\frac{3}{2k+1}=\frac{7}{3(2k-1)}$$

⇒ 4k + 2 = 3k + 6

⇒ k = 4

14. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x+3y=2$

(k+1)x + 9y = k + 1

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

2x + 3y = 2

(k + 1)x + 9y = k + 1

$$\Rightarrow \frac{2}{k+1}=\frac{3}{9}=\frac{2}{k+1}$$

⇒ k + 1 = 6

⇒ k = 5

15. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$x + (k + 1)y = 4$

$(k + 1)x + 9y = 5k + 2$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

x + (k + 1)y = 4

(k + 1)x + 9y = 5k + 2

$$\frac{1}{k+1} = \frac{k+1}{9} = \frac{4}{5k+2}$$

⇒ 5k + 2 = 4k + 4

⇒ k = 2

16. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$kx + 3y = 2k + 1$

$2(k + 1)x + 9y = 7k + 1$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

Kx + 3y = 2k + 1

2(k + 1) + 9y = 7k + 1

$$\frac{k}{2(k+1)} = \frac{3}{9} = \frac{2k+1}{7k+1}$$

⇒ 9k = 6k + 6

⇒ 3k = 6

⇒ k = 2

17. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + (k - 2)y = k$

$6x + (2k - 1)y = 2k + 5$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

2x + (k – 2)y = k

6x + (2k – 1y = 2k + 5

$$\frac{2}{6}=\frac{k-2}{2k-1}=\frac{k}{2k+5}$$

⇒ 4k – 2 = 6k – 12

⇒ 2k = 10

⇒ k = 5

18. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x+3y=7$

$(k+1)x+(2k-1)y=4k+1$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$2x+3y=7$

$(k+1)x+(2k-1)y=4k+1$

$$\frac{2}{k+1}=\frac{3}{2k-1}=\frac{7}{4k+1}$$

⇒ 4k – 2 = 3k + 3

⇒ k = 5

19. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x+3y=k$

$(k-1)x+(k+2)y=3k$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$2x+3y=k$

$(k-1)x+(k+2)y=3k$

$$\frac{2}{k-1}=\frac{3}{2+k}=\frac{k}{3k}$$

⇒ 4 + 2k = 3k – 3

⇒ k = 7

20. Question

Find the value of k for which the following system of equations has no solution:

$kx-5y=2$

$6x+2y=7$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$kx - 5y = 2$

$6x + 2y = 7$

$$\frac{k}{6} = -\frac{5}{2}$$

⇒ k = -15

21. Question

Find the value of k for which the following system of equations has no solution:

$x + 2y = 0$

$2x + ky = 5$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$x + 2y = 0$

$2x + ky = 5$

$$\frac{1}{2} = \frac{2}{k}$$

⇒ k = 4

22. Question

Find the value of k for which the following system of equations has no solution:

$3x - 4y + 7 = 0$

$kx + 3y - 5 = 0$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$3x - 4y + 7 = 0$

$kx + 3y - 5 = 0$

$$\frac{3}{k} = -\frac{4}{3}$$

⇒ k = -9/4

23. Question

Find the value of k for which the following system of equations has no solution:

$2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

$$\frac{2}{3} = -\frac{k}{2}$$

⇒ k = -4/3

24. Question

Find the value of k for which the following system of equations has no solution:

$2x + ky = 11$

$5x - 7y = 5$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$2x + ky = 11$

$5x - 7y = 5$

$$\frac{2}{5} = -\frac{k}{7}$$

⇒ k = -14/5

25. Question

Find the value of k for which the following system of equations has no solution:

$kx + 3y = 3$

$12x + ky = 6$

Answer

No solution will be when

a1/b1 = b1/b2 ≠ C1/C2

$kx + 3y = 3$

$12x + ky = 6$

k/12 = 3/k

⇒ k = ± 6

26. Question

For what value of k the following system of equations will be inconsistent?

$4x + 6y = 11$
$2x + ky = 7$

Answer

System will be inconsistent will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$4x + 6y = 11$
$2x + ky = 7$

$$\frac{4}{2} = \frac{6}{k} \neq \frac{11}{7}$$

$$\frac{4}{2} = \frac{6}{k}$$

⇒ k = 3

27. Question

For what value of a, the system of equations is inconsistent

ax + 3y = a - 3

12x + ay = a

Answer

System will be inconsistent will be when

a1/a2 = b1/b2 ≠ c1/c2

ax + 3y = a - 3

12x + ay = a

a/12 = 3/1 ≠ a-3/a

⇒ a = ± 6

But +6 doesn't satisfy

a/12 = 3/1 ≠ 1-3/a

Thus, a = - 6

28. Question

Find the value of k for which the system

$kx + 2y = 5$
$3x + y = 1$

has (i) a unique solution, and (ii) no solution.

Answer

$kx + 2y = 5$
$3x + y = 1$

i) For unique solution:

$$\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\Rightarrow \frac{k}{3} \neq \frac{2}{1}$$

⇒ k ≠ 6

ii) For no solution

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\Rightarrow \frac{k}{3} = \frac{2}{1}$$

⇒ k = 6

29. Question

Prove that there is a value of c (≠ 0) for which the system

$6x + 3y = c - 3$
$12x + cy = c$

Has infinitely many solutions. Find this value.

Answer

For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$6x + 3y = c - 3$
$12x + cy = c$

$$\frac{6}{12} = \frac{3}{c}$$

⇒ c = 6

30. Question

Find the values of k for which the system

$2x + ky = 1$
$3x - 5y = 7$

Will have (i) a unique solution and (ii) no solution. Is there a value of k for which the system has infinitely many solutions?

Answer

$2x + ky = 1$
$3x - 5y = 7$

i) For unique solution:

$$\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\frac{2}{3} \neq \frac{k}{-5}$$

⇒ k ≠ -10/3

ii) For no solution

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\frac{2}{3} = \frac{k}{-5}$$

⇒ k = -10/3

For infinite solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$$\Rightarrow \frac{2}{3}=\frac{k}{-5}\neq\frac{1}{7}$$

Thus, for no value of k will the system have infinite solution

31. Question

For what value of k, the following system of equations will represent the coincident lines?

$x+2y+7=0$
$2x+ky+14=0$

Answer

For coincident lines

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$x+2y+7=0$
$2x+ky+14=0$

$$\frac{1}{2}=\frac{2}{k}=\frac{7}{14}$$

⇒ k = 4

32. Question

Obtain the condition for the following system of linear equations to have a unique solution

$$ax+by=c$$
$$lx+my=n$$

Answer

For unique solution:

$$\frac{a1}{a2}\neq\frac{b1}{b2}\neq\frac{c1}{c2}$$**Given**

$$ax+by=c$$
$$lx+my=n$$

$$\frac{a}{l}\neq\frac{b}{m}$$

⇒ am ≠ lbHence the system of given linear equations will have unique solution when am ≠ lb.

33. Question

Determine the values of a and b so that the following system of linear equations have infinitely many solutions:

$(2a-1)x+3y-5=0$

3x + (b – 1)y – 2 = 0

Answer

For infinitely many solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$(2a-1)x+3y-5=0$

3x + (b – 1)y – 2 = 0

$$\frac{2a-1}{3}=\frac{3}{b-1}=\frac{5}{2}$$

⇒ 2a – 1 = 15/2

⇒ a = 17/4

6 = 5b – 5

⇒ b = 11/5

34. Question

Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$2x-3y=7$

$(a+b)x-(a+b-3)y=4a+b$

Answer

For infinitely many solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$2x-3y=7$

$(a+b)x-(a+b-3)y=4a+b$

$$\frac{2}{a+b}=\frac{3}{a+b-3}=\frac{7}{4a+b}$$

⇒ 2a + 2b – 6 = 3a + 3b and 12a + 3b = 7a + 7b – 21

⇒ a + b = -6 ----- (1) and 5a – 4b = - 21 ------- (2)

Multiplying eq1 by 4 and adding to eq2

⇒ 9a = -45

⇒ a = -5

Thus, b = -1

35. Question

Find the values of p and q for which the following system of linear equations has infinite number of solutions:

$2x+3y=9$

$(p+q)x-(2p-q)y=3(p+q+1)$

Answer

For infinitely many solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$2x+3y=9$

$(p+q)x-(2p-q)y=3(p+q+1)$

$$\frac{2}{p+q}=\frac{3}{-(2p-q)}=\frac{9}{p+q+1}$$

⇒ -4p + 2q = 3p + 3q and 2p + 2q + 2 = 9p + 9q

⇒ q = -7p and 7p + 7q = 2

⇒ 7q – q = 2

⇒ q = 1/3

Thus, p = -1/21

36. Question

Find the values of a and b for which the following system of equations has infinitely many solutions:

(i) $(2a-1)x-3y=5$

$3x+(b-2)y=3$

(ii) $2x-(2a+5y)y=5$

$(2b+1)x-9y=15$

(iii) $(a-1)x+3y=2$

$6x+(1-2b)y=6$

(iv) $3x+4y=12$

$(a+b)x+2(a-b)y=5a-1$

(v) $2x+3y=7$

$(a-b)x+(a+b)y=3a+b-2$

(vi) $2x+3y-7=0$

$(a-1)x+(a+1)y=(3a-1)$

(vii) $2x+3y=7$

$(a-1)x+(a+2)y=3a$

Answer

For infinitely many solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$$\frac{2a-1}{3}=-\frac{3}{b-2}=\frac{5}{3}$$

⇒ 6a – 3 = 15 and -9 = 5b – 10

⇒ a = 3 and b = 1/5

(ii) For infinitely many solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$$\frac{2}{2b+1}=-\frac{2a+5}{-9}=\frac{5}{15}$$

$$\frac{2}{2b+1}=\frac{2a+5}{9}=\frac{1}{3}$$

⇒ 6 = 2b + 1 and 6a + 15 = 9

⇒ b = 5/2 and a = -1

(iii) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{a-1}{6} = \frac{3}{1-2b} = \frac{2}{6}$$

$$\Rightarrow \frac{a-1}{6} = \frac{3}{1-2b} = \frac{1}{3}$$

⇒ 3a – 3 = 6 and 9 = 1 – 2b

⇒ a = 3 and b = -4

(iv) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{5a-1}$$

⇒ 15a – 3 = 12a + 12b and 20a – 4 = 24a – 24b

⇒ 3a – 12b = 3 ------ (1) and 6b – a = 1 ------ (2)

Multiplying eq2 by 3 and adding to eq1

⇒ 6b = 6

⇒ b = 1

Thus, 3a – 12 = 3

⇒ a = 5

(v) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

⇒ 2a + 2b = 3a – 3b and 6a + 2b – 4 = 7a – 7b

⇒ a = 5b and 9b – a = 4

Thus, 9b – 5b = 4

⇒ b = 1

⇒ a = 5

(vi) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x + 3y - 7 = 0$

$(a-1)x + (a+1)y = (3a-1)$

$$\frac{2}{a-1}=\frac{3}{a+1}=-\frac{7}{-(3a-1)}$$

⇒ 2a + 2 = 3a – 3

⇒ a = 5

(vii) For infinitely many solution

$$\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$$

$$\Rightarrow \frac{2}{a-1}=\frac{3}{a+2}=\frac{7}{3a}$$

⇒ 2a + 4 = 3a – 3

⇒ a = 7

Exercise 3.6

1. Question

5 pens and 6 pencils together cost Rs 9 and 3 pens and 2 pencils cost Rs 5. Find the cost of 1 pen and 1 pencil.

Answer

Let the cost of 1 pen be ‘a’ and the cost of 1 pencil be ‘b’.

Given, 5 pens and 6 pencils together cost Rs 9.

5a + 6b = 9 --------- (1)

Given, 3 pens and 2 pencils cost Rs 5.

3a + 2b = 5 -------- (2)

Multiplying eq2 by 3 and subtracting eq1 from it.3(3a + 2b) - (5a + 6b) = 5(3) - 9⇒ 9a + 6b – 5a – 6b = 15 – 9

⇒ 4a = 6

⇒ a = 3/2

Substituting value of 'a' in (1)

⇒ 15/2 + 6b = 9

⇒ 6b = 3/2

⇒ b = 1/4

2. Question

7 audio cassettes and 3 video cassettes cost Rs 1110, while 5 audio cassettes and 4 video cassettes cost Rs 1350. Find the cost of an audio cassette and a video cassette.

Answer

Let the cost of an audio cassette be ‘a’ and a video cassette be ‘b’.

Given, 7 audio cassettes and 3 video cassettes cost Rs 1110.

⇒ 7a + 3b = 1110 -------- (1)

Also, 5 audio cassettes and 4 video cassettes cost Rs 1350.

⇒ 5a + 4b = 1350 -------- (2)

Multiplying eq1 by 4 and eq2 by 3 and subtracting eq2 from eq1.

⇒ 28a + 12b – 15a – 12b = 4440 – 4050

⇒ 13a = 390

⇒ a = Rs. 30

Substituting value of a in eq1

⇒ 7 × 30 + 3b = 1110

⇒ 3b = 900

⇒ b = Rs. 300

3. Question

Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Answer

Let the number of pens be ‘a’ and number of pencils be ‘b’.

Given, Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens.

⇒ a + b = 40 ------ (1)

Also, b + 5 = 4(a – 5)

⇒ b = 4a – 25 ------- (2)

Subtracting eq2 from eq1

⇒ a + b – b = 40 – 4a + 25

⇒ 5a = 65

⇒ a = 13

Substituting the value of 'a' in equation (1), we get⇒ 13 + b = 40⇒ b = 27

Hence, Reena has 13 pens and 27 pencils.

4. Question

4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Answer

Let the cost of 1 table be ‘a’ and cost of 1 chair be ‘b’.

Given, 4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost Rs 1950.

⇒ 4a + 3b = 2250 -------- (1)

Also, 3a + 4b = 1950 --------- (2)

Multiplying eq1 by 3 and eq2 by 4 and subtracting eq2 from eq1.

⇒ 12a + 9b – 12a – 16b = 6750 – 7800

⇒ 7b = 1050

⇒ b = 150

Substituting ‘b’ in eq1

⇒ 4a + 450 = 2250

⇒ a = Rs. 450

Cost of 2 chair and 1 table = 2b + a = Rs. 750

5. Question

3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost Rs 324. Find the total cost of 1 bag and 10 pens.

Answer

Let the cost of 1 bag be ‘a’ and 1 pen be ‘b’.

Given, 3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost Rs 324.

⇒ 3a + 4b = 257 ---------- (1) and 4a + 3b = 324 ---------- (2)

Multiplying eq1 by 4 and eq2 by 3 and subtracting eq2 from eq1.

⇒ 12a + 16b – 12a – 9b = 1028 – 972

⇒ 7b = 56

⇒ b = Rs. 8

Substituting in eq1

⇒ 3a + 32 = 257

⇒ a = Rs. 75

Cost of 1 bag and 10 pens = 75 + 10 × 8 = Rs. 155

6. Question

5 books and 7 pens together cost Rs 79 whereas 7 books and 5 pens together cost Rs 77. Find the total cost of 1 book and 2 pens.

Answer

Let the cost of 1 book be ‘a’ and cost of 1 pen be ‘b’.

Given, 5 books and 7 pens together cost Rs 79 whereas 7 books and 5 pens together cost Rs 77.

⇒ 5a + 7b = 79 ------ (1) and 7a + 5b = 77

Multiplying eq1 by 7 and eq2 by 5 and subtracting eq2 from eq1.

⇒ 35a + 49b – 35a – 25b = 553 – 385

⇒ 24b = 168

⇒ b = Rs. 7

Substituting value of b in eq1, we get

⇒ 5a + 49 = 79

⇒ a = Rs. 6

Cost of 1 book and 2 pens = 6 + 2 × 7 = Rs. 20

7. Question

A and B each have a certain number of mangoes. A says to B, “if you give 30 of your mangoes, I will have twice as many as left with you.” B replies, “if you give me 10, I will have thrice as many as left with you.” How many mangoes does each have?

Answer

Let the number of mangoes A has be ‘a’ and number of mangoes B has be ‘b’.

Given, . A says to B, “if you give 30 of your mangoes, I will have twice as many as left with you.” B replies, “if you give me 10, I will have thrice as many as left with you”

⇒ a + 30 = 2(b – 30)

⇒ a = 2b – 90 --------- (1)

Also, b + 10 = 3(a – 10)

⇒ b = 3a – 40 ---------- (2)

Substituting value of a from eq1 in eq2

⇒ b = 6b – 270 – 40

⇒ 5b = 310

⇒ b = Rs. 62

Substituting value of b in eq1

⇒ a = 124 – 90

⇒ a = Rs. 34

8. Question

On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains Rs 2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss. He gains Rs 1500 on the transaction. Find the actual prices of T.V. and fridge.

Answer

Let the actual price of Tv be 'a' and actual price of fridge be 'b'.

Given, on selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains Rs 2000.

∴ 5% of a + 10% of b = 2000

$$\Rightarrow \frac{5}{100}a + \frac{10}{100}b = 2000$$

⇒ 5a + 10b = 200000 ---------- (1)

Also, if he sells the T.V. at 10% gain and the fridge at 5% loss.

∴ 10% of a – 5% of b = 1500

$$\Rightarrow \frac{10}{100}a - \frac{5}{100}b = 1500$$

⇒ 10a – 5b = 150000 -------- (2)

Multiplying eq2 by 2 and adding eq1 to it

⇒ 5a + 10b + 20a – 10b = 200000 + 300000

⇒ 25a = 500000

⇒ a = Rs. 20000

Thus, 5 × 20000 + 10b = 200000

⇒ b = Rs. 10000

9. Question

The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Answer

Let the cost of each bat be 'a' and each ball be 'b'.

Given, coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750

⇒ 7a + 6b = 3800 ---- (1) and

3a + 5b = 1750 ---- (2)

1. Question

The sum of two numbers is 8. If their sum is four times their difference, find the number.

Answer

Let the numbers be ‘a’ and ‘b’.

Given, sum of two numbers is 8. If their sum is four times their difference, find the number.

⇒ a + b = 8 ------ (1)

Also,

a + b = 4(a – b)

⇒ a + b = 4a – 4b

⇒ a = 5b/3

Substituting value of a in eq1

⇒ 5b/3 + b = 8

⇒ 8b/3 = 8

⇒ b = 3

Thus, a = 5

2. Question

The sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

Answer

Let the one’s digit be ‘a’ and ten’s digit be ‘b’

Given, sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45.

⇒ a + b = 13 ----- (1) and,

10a + b – (10b + a) = 45

⇒ 9a – 9b = 45

⇒ a – b = 5 ---------- (2)

Adding (1) and (2)

⇒ 2a = 18

⇒ a = 9

Thus, b = 4

Number is 10b + a

⇒ Number = 49

3. Question

A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.

Answer

Let the one’s digit be ‘a’ and ten’s digit be ‘b’

Given, number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine.

Multiplying eq1 by 3 and eq2 by 7 and subtracting eq2 from eq1.

⇒ 3(7a + 6b) - 7(3a + 5b) = 3(3800) - 7(1750)

⇒ 21a + 18b – 21a – 35b = 11400 – 12250

⇒ -17b = -850

⇒ 17b = 850

⇒ b = 50Putting this in eq 1, we get7a + 6(50) = 3800⇒ 7a + 300 = 3800⇒ 7a = 3500⇒ a = 500Hence, Each bat cost a = 500 Rupees and Each ball costs b = 50 Rupees.

10. Question

One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you".Tell me what is the amount of their respective capital?

Answer

Given: One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you".

To find: the amount of their respective capital.

Solution: Let the capital of two friends be 'a'and 'b' respectively.

Given, one says, "Give me a hundred, friend! I shall then become twice as rich as you.It means if one is giving Rs. 100 other is losing the Rs. 100.Since a is gaining 100 and b is losing 100.⇒ a + 100 = 2(b – 100)

⇒ a + 100 = 2b – 200

⇒ a - 2b = -200 - 100⇒ a - 2b = -300 ..... (1)In another condition 2nd friend replies, "Give me a ten, I shall be six times as rich as you".It means if one person is gaining 10 other person is losing 10.

Here b is gaining Rs 10 and a is losing Rs 10.⇒ b + 10 = 6(a – 10)⇒ b + 10 = 6a – 60⇒ 6a - b = -60-10⇒ 6a - b = -70 ..... (2)Now solve equations (1) and (2) to get the amount a and b.Multiply eq. (1) with 6 and subtract we. (2) from it.⇒6(a - 2b)-(6a - b) = 6 (-300) -70⇒6a - 12b-6a+b = -1800 -70⇒ -11b= -1870⇒ b = 170put the value of b in the eq.(1) to get value of a,⇒a - 2(170) = -300⇒a - 340 = -300 ⇒ a = -300 + 340⇒ a= 40

Hence the amount of capital of two friends is Rs 40 and Rs 170.

11. Question

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer

Let the fixed charge and the charge for each extra day be 'a'and 'b' respectively.

Given, a lending library has a fixed charge for the first three days and an additional charge for each day thereafter

Saritha paid Rs 27 for a book kept for seven days⇒ a + 4b = 27 -------- (1)

Susy paid Rs 21 for the book she kept for five days⇒ a + 2b = 21 --------- (2)

Subtracting eq2 from eq1

⇒ 2b = 6

⇒ b = 3

Putting this value in eq(1), we get⇒ a + 4(3) = 27⇒ a = 27 - 12⇒ a = 15 Therefore, fixed charge, a = 15 Rupees and charge thereafter, b = 3 Rupees per day

⇒ a = Rs. 15

Exercise 3.7

⇒ a+ b = 5 ----- (1) and

10a + b – (10b + a) = 9

⇒ a – b = 1 ------ (2)

Adding (1) and (2)

Thus, 2a = 6

⇒ a = 3

∴ b = 2

Number is 23.

4. Question

The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Answer

Let the unit's digit be a and tens digit be b.

Number = 10b + a

Reverse of the number = 10a + b

Given, sum of digits of a two digit number is 15.

⇒ a + b = 15 ------ (1)

Also, the number obtained by reversing the order of digits of the given number exceeds the given number by 9.

⇒ 10a + b – 10b – a = 9

⇒ a – b = 1 ------- (2)

Solving (1) and (2), we get

⇒ a = 8 and b = 7

The number is 78.

5. Question

The sum of two-digit number and the number formed by reversing the order of digits is 66. If the two digits differ by 2, find the number. How many such numbers are there?

Answer

Given : The sum of two-digit number and the number formed by reversing the order of digits is 66 and the two digits differ by 2.

To find: the number. How many such numbers are there.

Solution: Let the one's digit be 'a' and ten's digit be 'b'.The 2 digit number is formed as (10×number on tens' digit + number on one's digit)So the number is 10 b + a,If we reverse the digits one's digit will be 'b' and tens' digit will be 'a'.So number after reversing will become 10 a + b.

As it is given that sum of two-digit number and the number formed by reversing the order of digits is 66.

⇒ 10 b + a + 10 a + b = 66

⇒ 11 b + 11 a = 66

⇒ a + b = 6 .... (1)

Also, digits differ by 2.

⇒ a – b = 2 ..... (2)or b – a = 2 ..... (3)

Adding eq 1 and 2 we geta + b + a - b = 6 + 2⇒ 2a = 8⇒ a = 4Putting value of a in 1 we get,4 + b = 6 ⇒ b = 6 - 4⇒ b = 2Adding eq 1 and 3 we geta + b + b - a = 6 + 2⇒ 2b = 8

⇒ b = 4Putting value of a in 1 we get,a + 4 = 6 ⇒ a = 6 - 4⇒ a = 2

Thus, a = 4 and b = 2 or a = 2 and b = 4Now 2 digit number is 10 b + aFor a = 4 and b = 2 The 2 digit number is 10 (2) + 4 = 20 + 4 = 24For a = 2 and b = 4The 2 digit number is 10 (4) + 2 = 40 + 2 = 42

Hence Numbers can be 24 or 42

6. Question

The sum of two numbers is 1000 and the difference between their squares is 256000. Find the numbers.

Answer

Let the numbers be 'a' and 'b'

Given, of two numbers is 1000 and the difference between their squares is 256000

⇒ a + b = 1000 ---- (1) and

⇒ $a^2 - b^2 = 25600$

⇒ (a + b)(a – b) = 25600 [From 1]

⇒ 1000(a - b) = 256000

⇒ a – b = 256 ----- (2)

Adding (1) and (2)

⇒ 2a = 1256

⇒ a = 628

From (1) 628 + b = 1000⇒ b = 1000 - 628 = 372

7. Question

The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, sum of two-digit number and the number formed by reversing the order of digits is 99.

⇒ 10a + b + 10b + a = 99

⇒ a + b = 9

Also, digits differ by 3.

⇒ a – b = 3 or b – a = 3

Adding both equation

2a = 12 or 2b = 12

⇒ a = 6 and b = 3 or a = 3 and b = 6

Number can be 36 or 63

8. Question

A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, number is 4 times the sum of its digits.

$10b + a = 4(a + b)$

$\Rightarrow a = 2b$ -----(1)

Also, if 18 is added to the number, the digits are reversed.

$\Rightarrow 10b + a + 18 = 10a + b$

$\Rightarrow a - b = 2$ ----- (2)

Substituting a from eq1 in eq2

$\Rightarrow b = 2$

Thus, $a = 4$

Number is 24

9. Question

A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Then, the number is $= 10b + a$

Given, the number is 3 more than 4 times the sum of its digits.

$\Rightarrow 10b + a = 4(a + b) + 3$

$\Rightarrow 10b + a = 4a + 4b + 3$

$\Rightarrow 3a = 6b - 3$

$\Rightarrow a = 2b - 1$ -----(1)

Also, if 18 is added to the number, the digits are reversed.

$\Rightarrow 10b + a + 18 = 10a + b$

$\Rightarrow 9a - 9b = 18$

$\Rightarrow a - b = 2$using (1)

$\Rightarrow 2b - 1 - b = 2 \Rightarrow b = 3$

Thus, again from (1)$a = 6 - 1 = 5$

Hence, Number is 35.

10. Question

A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, two-digit number is 4 more than 6 times the sum of its digits.

$\Rightarrow 10b + a = 6(a + b) + 4$

$\Rightarrow 4b = 5a + 4$ ------- (1)

Also, if 18 is subtracted from the number, the digits are reversed.

$\Rightarrow 10b + a - 18 = 10a + b$

⇒ b – a = 2 ------- (2)

Multiplying eq2by 4 and subtracting from eq1

⇒ 4b – 5a – 4b + 4a = 4 – 8

⇒ a = 4

Thus, b = 6

Number is 64

11. Question

A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'.

Given, two digit number is 4 times the sum of its digits and twice the product of the digits

⇒ 10b + a = 4(a + b)

⇒ a = 2b

Also, 10b + a = 2ab

Substituting value of a.

⇒ 10b + 2b = 2 × 2b × b

⇒ b = 3

Thus, a = 6

Number is 36

12. Question

A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'.

Given, two-digit number is such that the product of its digits is 20.

⇒ ab = 20 ----- (1)

Also, if 9 is added to the number, the digits interchange their places.

⇒ 10b + a + 9 = 10a + b

⇒ a – b = 1 ----- (2)

Substituting value of a from eq1 in to eq2

⇒ 20/b – b = 1

⇒ b^2 + b – 20 = 0

⇒ (b + 5)(b – 4) = 0

Thus, b = 4 and a = b + 1 = 5

Number is 45.

13. Question

The difference between two numbers is 26 and one number is three times the other. Find them.

Answer

Let the numbers be a and b.

Given, difference between two numbers is 26 and one number is three times the other.

⇒ a – b = 26 [1] and a = 3b [2]From [1] and [2], we get

⇒ 3b - b = 26

⇒ 2b = 26

⇒ b = 13

Thus, a = 3b = 3(13) = 39

14. Question

The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'.therefore no is = 10b + aReversed no = 10a + b

Given, Sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits.

⇒ a + b = 9 ----- (1)

And 9(10b + a) = 2(10a + b)

⇒ 90b + 9a = 20a + 2b

⇒ 88b = 11a

⇒ a = 8b

Substituting value of a in eq1

⇒ 8b + b = 9

⇒ 9b = 9

⇒ b = 1

Thus, a = 8(1) = 9

Hence, no is 10(1) + 8 = 18

15. Question

Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.

Answer

Given: Seven times a two-digit number is equal to four times the number obtained by reversing the digits and the difference between the digits is 3.

To find: the number.

Solution: Let the one's digit be 'a' and ten's digit be 'b'.The the number will be 10b+aIf the digits are reversed the number will be 10a+b

Given, seven times a two-digit number is equal to four times the number obtained by reversing the digits.

⇒ 7(10b + a) = 4(10a + b)⇒ 7(10b) + 7a = 4(10a)+ 4b⇒ 70b + 7a = 40a + 4b⇒ 70b - 4b = 40a - 7a

⇒ 66b = 33a

⇒ a = 2b ----- (1)

Also, difference between the digits is 3

⇒ a – b = 3

Substitute value of a from (1)

⇒ 2b – b = 3

Thus, b = 3

⇒ a = 2 × 3 = 6

The number is 10b+a = 10(3)+6 = 36

∴ Number is 36.

Exercise 3.8

1. Question

The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Answer

Let the numerator be a and denominator be b.

Given, numerator of a fraction is 4 less than the denominator.

⇒ a = b – 4 --------- (1)

Also, if the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator.

⇒ b + 1 = 8(a – 2)

⇒ b = 8a – 17 -------- (2)

Substituting value of b from (2) in (1).

⇒ a = 8a – 17 – 4

⇒ a = 3

Thus, b = 3 + 4 = 7

Fraction is 3/7.

2. Question

A fraction becomes 9/11 if 2 is added to both numerator and the denominator, it becomes 5/7 if 2 is subtracted from both numerator and the denominator. Find the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’

Given, fraction becomes 9/11 if 2 is added to both numerator and the denominator, it becomes 5/7 if 2 is subtracted from both numerator and the denominator

$$\Rightarrow \frac{(a+2)}{b+2} = \frac{9}{11}$$

⇒ 11a + 22 = 9b + 18

⇒ 11a = 9b – 4 ------ (1)

Also, $\frac{a-2}{b-2} = \frac{5}{7}$

⇒ 7a – 14 = 5b – 10

⇒ 7a – 5b = 4 ------- (2)

Multiplying eq1 by 7 and eq2 by 11 and subtracting eq2 from eq1

⇒ 77a – 63b – 77a + 55b = -28 – 44

⇒ 8b = 72

⇒ b = 9

Thus, 7a – 45 = 4

⇒ a = 7

Fraction is 7/9.

3. Question

A fraction becomes 1/3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/2. Find the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’

Given, fraction becomes 1/3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/2.

$$\Rightarrow \frac{a-1}{b-1} = \frac{1}{3}$$

⇒ 3a – 3 = b – 1

⇒ b = 3a – 2 ------ (1)

Also, if 1 is added to both the numerator and denominator, it becomes 1/2.

$$\Rightarrow \frac{a+1}{b+1} = \frac{1}{2}$$

⇒ b = 2a + 1 ---------- (2)

Equating (1) and (2)

⇒ 3a – 2 = 2a + 1

⇒ a = 3

Thus, b = 7

Fraction is 3/7

4. Question

If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction?

Answer

Let the numerator be ‘a’ and denominator be ‘b’

Given, if add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1.

$$\Rightarrow \frac{a+1}{b-1} = 1$$

⇒ a = b – 2 -------- (1)

Also, it becomes 1/2 if we only add 1 to the denominator.

$$\Rightarrow \frac{a}{b+1} = \frac{1}{2}$$

⇒ 2a = b + 1 --------- (2)

Subtracting eq 1 from eq2

⇒ 2a – a = b + 1 – b + 2

⇒ a = 3

Thus, b = 5

Fraction is 3/5

5. Question

If the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes 6/5. And, if the denominator is doubled and the numerator is increased by 8, the fraction becomes 2/5. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, if the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes 6/5

$$\Rightarrow \frac{2a}{b-5} = \frac{6}{5}$$

⇒ 10a = 6b – 30

⇒ 5a = 3b – 15 ---------- (1)

Also, if the denominator is doubled and the numerator is increased by 8, the fraction becomes 2/5

$$\Rightarrow \frac{a+8}{2b} = \frac{2}{5}$$

⇒ 5a + 40 = 4b -------- (2)

Subtracting eq2 from eq1

⇒ 5a – 5a – 40 = 3b – 15 – 4b

⇒ b = 25

Thus, 5a = 75 – 15

⇒ a = 12

Fraction is 12/25

6. Question

When 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes 1/4. And, when 6 is added to numerator and the denominator is multiplied by 3, it becomes 2/3. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes 1/4.

$$\Rightarrow \frac{a-2}{b+3} = \frac{1}{4}$$

⇒ 4a – 8 = b + 3

⇒ 4a – b = 11 ------ (1)

Also, when 6 is added to numerator and the denominator is multiplied by 3, it becomes 2/3.

$$\Rightarrow \frac{a+6}{3b} = \frac{2}{3}$$

⇒ 3a + 18 = 2b

⇒ a + 6 = 2b -------- (2)

Multiplying eq2 by 4 and subtracting from eq1

⇒ 4a – b – 4a – 24 = 11 – 8b

⇒ 7b = 35

⇒ b = 5

Thus, a = 4

Fraction is 4/5

7. Question

The sum of a numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to 1/3. Find the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’

Given, sum of a numerator and denominator of a fraction is 18.

⇒ a + b = 18 ----- (1)

Also, if the denominator is increased by 2, the fraction reduces to 1/3.

⇒ a/(b + 2) = 1/3

⇒ 3a - b = 2 ------ (2)

Adding (1) and (2)

⇒ 4a = 20

⇒ a = 5

Thus, b = 13

Fraction is 5/13

8. Question

If 2 is added to the numerator of a fraction, it reduces to 1/2 and if 1 is subtracted from the denominator, it reduces to 1/3. Find the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’.

Given, if 2 is added to the numerator of a fraction, it reduces to 1/2 and if 1 is subtracted from the denominator, it reduces to 1/3

⇒ (a + 2)/b = 1/2

⇒ 2a + 4 = b ------ (1)

Also, a/(b – 1) = 1/3

⇒ 3a + 1 = b ------- (2)

Subtracting eq2 from eq1

⇒ 2a + 4 – 3a – 1 = b – b

⇒ a = 3

Thus, b = 2a + 4 = 10

Fraction is 3/10

9. Question

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'.

Given, sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3.

⇒ a + b = 2a + 4

⇒ b – a = 4 -------- (1)

Also, $\frac{a+3}{b+3} = \frac{2}{3}$

⇒ 3a + 9 = 2b + 6

⇒ 3a – 2b = - 3 ------(2)

Multiplying eq1 by 2 and adding to eq1

⇒ 2b – 2a + 3a – 2b = 8 - 3

⇒ a = 5

Thus b = 9

Fraction is 5/9

10. Question

The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'.

Given, sum of the numerator and denominator of a fraction is 3 less than twice the denominator.

⇒ a + b = 2b – 3

⇒ a = b – 3 ------ (1)

Also, if the numerator and denominator are decreased by 1, the numerator becomes half the denominator.

⇒ a – 1 = (b – 1)/2

⇒ b = 2a - 1 ------ (2)

Substituting value of b in eq1.

⇒ a = 2a – 1 – 3

⇒ a = 4.

Thus, b = 2 × 4 – 1 = 7

11. Question

The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 1/2. Find the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’.

Given, sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 1/2.

⇒ a + b = 12 ------ (1)

Also, a/(b + 3) = 1/2

⇒ 2a – b = 3 ------- (2)

Adding eq1 and eq2

⇒ a + b + 2a – b = 15

⇒ 3a = 15

⇒ a = 5

Thus, b = 7

Fraction is 5/7.

Exercise 3.9

1. Question

A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Answer

Given: A father is three times as old as his son. After twelve years, his age will be twice as that of his son.**To find:** The present age of father and his son.**Solution:** Let the present age of father be ‘a’ and present age of son be b.

Given, father is three times as old as his son.

⇒ a = 3b ---- (1)

Also, after twelve years, his age will be twice as that of his son then.12 years later age of father will be a+12 and son will be b+12.

⇒ a + 12 = 2(b + 12)⇒ a+12 = 2b + 24

⇒ a = 2b + 12

Put the value of a from eq. (1)

⇒ 3b = 2b + 12

⇒ 3b - 2b = 12

⇒ b = 12

Put the value of a in eq. 1 to get,a=3ba=3(12)a=36

Thus, present age of father is 36 years and present age of son is 12 years.

2. Question

Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. What are the present ages of A and B?

Answer

Let the present ages of A and B be ‘a’ and ‘b’ respectively.

Given, ten years later, A will be twice as old as B and five years ago, A was three times as old as B

⇒ a + 10 = 2(b + 10)

⇒ a = 2b + 10 ------- (1)

Also, a – 5 = 3(b – 5)

⇒ a = 3b – 10 -------- (2)

Equating eq2 and eq1

⇒ 2b + 10 = 3b – 10

⇒ b = 20

Thus, a = 2b + 10 = 50

3. Question

A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old as his sister S. If the ages of the father and sister differ by 40 years, find the age of A.

Answer

Let the present ages of A, B, F and S be 'a', 'b', 'c' and 'd' respectively.

Given, A is elder to B by 2 years

⇒ a = b + 2 -------- (1)

A's father F is twice as old as A and B is twice as old as his sister S

⇒ c = 2a and b = 2d ------- (2)

Also, the ages of the father and sister differ by 40 years.

⇒ c – d = 40 ------- (3)

From [2], we have

⇒ 2a – b/2 = 40

$$2a - \frac{(a-2)}{2} = 40$$

$$\Rightarrow \frac{4a - a + 2}{2} = 40$$

$$\Rightarrow 3a + 2 = 80$$

$$\Rightarrow 3a = 78$$

$$\Rightarrow a = 26$$

Hence, Age of 'A' is 26 years.

4. Question

Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages.

Answer

Let the present age of man be 'a' and present age of son be b.

Given, Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son

⇒ a + 6 = 3(b + 6)

⇒ a = 3b + 12 ------ (1)

Also, three years ago he was nine times as old as his son

⇒ a – 3 = 9(b – 3)

⇒ a = 9b – 24 ----- (2)

Equating eq1 and eq2

⇒ 3b + 12 = 9b – 24

⇒ b = 6

Thus, a = 3b + 12 = 30

Hence age of man is 30 yrs and age of son is 6 yrs.

5. Question

Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then. Find their present ages.

Answer

Let the present age of father be 'a' and present age of son be b.

Given, ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then.

⇒ a – 10 = 12(b – 10)

⇒ a = 12b + 110 ------ (1)

Also, a + 10 = 2(b + 10)

⇒ a = 2b + 10 ------- (2)

Equating eq1 and eq2.

⇒ 12b + 110 = 2b + 10

⇒ b = 10

Thus, a = 30

6. Question

The present age of a father is three years more than three times the age of the son. Three years hence father's age will be 10 years more than twice the age of the son. Determine their present ages.

Answer

Let the father's age be 'a' and son's age be 'b'

Given, present age of a father is three years more than three times the age of the son.

⇒ a = 3b + 3 ---------- (1)

Also, three years hence father's age will be 10 years more than twice the age of the son.

⇒ a + 3 = 2(b + 3) + 10

⇒ a = 2b + 13 --------- (2)

Equating (1) and (2), we get

3b + 3 = 2b + 13

⇒ b = 10 years

Father's age (a) = 3 × 10 + 3 = 33

7. Question

A father is three times as old as his son. In 12 years time, he will be twice as old as hi son. Find the present ages of father and the son.

Answer

Let the present age of father be ‘a’ and present age of son be b.

Given, father is three times as old as his son. In 12 years time, he will be twice as old as hi son.

⇒ a = 3b and

a + 12 = 2(b + 12)

⇒ 3b + 12 = 2b + 24

⇒ b = 12

Thus, a = 36

8. Question

Father’s age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Answer

Given: Father’s age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children.

To find: the age of father.

Solution: Let the present age of father be ‘a’ and present sum of age of both sons be b.

Given, Father’s age is three times the sum of ages of his two children.

⇒ a = 3b

After 5 years his age will be twice the sum of ages of two children.So age of father will be a + 5 and as there are two sons, age of both sons combined after 5 years will be b + 10.

⇒ a + 5 = 2(b + 10)

Put the value of a in above equation,

⇒ 3b + 5 = 2b + 20

⇒ 3b - 2b = 20 - 15

⇒ b = 15

Thus, a = 3b = 3 (15) = 45Hence father's present age is 45 years.

9. Question

Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

Answer

Let the present age of father be ‘a’ and present age of son be b.

Given, two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son.

⇒ a – 2 = 5(b – 2)

⇒ a = 5b – 8 ------ (1)

Also, a + 2 = 3(b + 2) + 8

⇒ a = 3b + 12 ----- (2)

Equating (1) and (2)

⇒ 5b – 8 = 3b + 12

⇒ b = 10

From [1], we havea = 5(10) - 8Thus father’s age, a = 50 – 8 = 42 Years

10. Question

Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Answer

Given: Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu.

To find: The age of Nuri and Sonu.

Solution: Let the present age of Nuri and his son be ‘a’ and ‘b’ respectively.

Given, Five years ago, Nuri was thrice as old as Sonu.

This implies age of Nuri and Sonu five years ago was a - 5 and b - 5.

⇒ a – 5 = 3(b – 5)

⇒ a - 5 = 3b – 15⇒ a = 3b – 15 + 5

⇒ a = 3b – 10 ------- (1)

Ten years later, Nuri will be twice as old as Sonu.This implies after ten years the age of Nuri and Sonu will be a + 10 and b + 10.

⇒ a + 10 = 2(b + 10)

⇒ a + 10 = 2b + 20⇒ a = 2b + 20 - 10

⇒ a = 2b + 10 -------- (2)

Equating (1) and (2), we get

3b – 10 = 2b + 10

⇒ 3b - 2b = 10 + 10⇒ b = 20

Thus, a = 60 – 10 = 50Hence Nuri's present age is 50 years and Sonu's age is 20 years.

11. Question

The ages of two friends Ani and Biju differ by 3 years. Ani’s father Dharam is twice as old as Ani and Biju as twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Answer

Let the ages of Ani, Biju, Dharam and Cathy be a, b, c and d respectively.

Given, ages of two friends Ani and Biju differ by 3 years. Ani’s father Dharam is twice as old as Ani and Biju as twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years.

a – b = 3

c = 2a and b = 2d

c – d = 30

Solving the above four equations

⇒ 2a – b/2 = 30

⇒ 2a – a/2 + 3/2 = 30

⇒ a = 19

Thus, b = 16 years

Exercise 3.10

1. Question

Points A and B are 70 km a part on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

Answer

Let the speed of car from A be 'a' and of car from B be 'b'

Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a − b

Given, if they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. A and B are 70 km a part on a highway

⇒ a + b = 70 ------ (1)

Also, a − b = 70/7 = 10 ------ (2)

Adding (1) and (2)

⇒ 2a = 80

⇒ a = 40 km/hr

Thus, b = 40 − 10 = 30 km/hr

2. Question

A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

Answer

Speed = distance/time

Let the speed of sailor in still water be 'a' and speed of the current be 'b'.

The relative speed of sailor going upstream = a − b

The relative speed of sailor going downstream = a + b

Given, sailor goes 8 km downstream in 40 minutes and returns in 1 hour.

$$\Rightarrow a + b = \frac{8}{\frac{40}{60}}$$

⇒ a + b = 12 ----- (1)

Also,

a − b = 8/1 = 8 ------ (2)

Adding (1) and (2).

⇒ 2a = 20

⇒ a = 10km/hr

Thus, b = 12 − 10 = 2 km/hr

So, speed of sailor is 10 km/h and current is 2 km/h.

3. Question

The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Answer

Given: The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream.

To find: the speed of stream and that of the boat in still water.

Solution: Speed = distance/time

Let the speed of boat be 'a' and speed of stream be 'b'

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstreamand we know, $time = \frac{speed}{distance}$

$\Rightarrow \frac{30}{a-b} + \frac{44}{a+b} = 10$ ------- (1)

Also, $\frac{40}{a-b} + \frac{55}{a+b} = 13$ ------ (2)

Now take, $\frac{1}{a-b} = u, \frac{1}{a+b} = v$

Eq 1 and 2 becomes⇒ 30 u + 44 v = 10Take 2 common out of above equation⇒ 15 u + 22 v - 5 = 0 ..... (3)and40 u + 55 v - 13 = 0 ..... (4)Solve the equations by cross multiplication method

$$\frac{u}{22\times(-13)-55\times(-5)} = \frac{-v}{15\times(-13)-40\times(-5)} = \frac{1}{15\times(55)-40\times(22)}$$

$$\frac{u}{-286+275} = \frac{-v}{-195+200} = \frac{1}{825-880}$$

$$\frac{u}{-11} = \frac{-v}{5} = \frac{1}{-55}$$

$$u = \frac{-11}{-55}\ ,\ v = \frac{-5}{-55}$$

$$u = \frac{1}{5}\ ,\ v = \frac{1}{11}$$

⇒ a- b = 5 .... (4) a + b = 11 .... (5)Add eq 4 and 5 to geta - b + a + b = 5 + 11⇒ 2a = 16⇒ a = 8 Substitute value of a in eq 4 we get,8 - b = 5⇒ b = 8 - 5⇒ b = 3Speed of boat = 8 km/hr and speed of stream = 3 km/hr

4. Question

A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Answer

Speed = distance/time

Let the speed of boat be 'a' and speed of stream be 'b'

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs

$\Rightarrow \frac{24}{a-b}+\frac{28}{a+b}=6$ ------- (1)

Also, $\frac{30}{a-b}+\frac{21}{a+b}=\frac{13}{2}$ ------ (2)

Multiplying eq1 by 3 and eq2 by 4 and (1) – (2)

$$\Rightarrow \frac{72}{a-b}-\frac{120}{a-b}=-8$$

⇒ a - b = 6 ------- (3)

Substituting value of (a - b) into eq1

$$\Rightarrow \frac{24}{6}+\frac{28}{a+b}=6$$

⇒ a + b = 14 ------ (4)

From (1) and (2), we get

a = 10 km/hr and b = 4 km/hr

5. Question

While covering distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking.

Answer

Speed = distance/time

Let the speed of Ajeet be ‘a’ and speed of Amit be ‘b’.

Given, while covering distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit.

$$\Rightarrow \frac{30}{a}-\frac{30}{b}=2$$

Also, $\frac{30}{b}-\frac{30}{2a}=1$

Adding the two equations

⇒ 30/a - 15/a = 3

⇒ a = 5 km/hr

Thus, 30/5 – 30/b = 2

⇒ b = 7.5 km/hr

6. Question

A man walks a certain distance with certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Answer

Given: A man walks a certain distance with certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. '

To find: the distance covered by the man and his original rate of walking.

Solution: Let the original speed, original time taken and distance be 'a', 't' and 'd'.As we know,distance = speed × time

⇒ d = at ------- (1)

Given, if he walks 1/2 km = 0.5 km an hour faster, he takes 1 hour less,

⇒ d = (a + 0.5)(t − 1)⇒ d = at + 0.5t − a − 0.5 ⇒ at = at + 0.5t - a - 0.5 [From 1]⇒ 0.5t - a = 0.5Multiply the above equation by 10 to get,⇒ 5t - 10a = 5⇒ 10a = 5t - 5 -------- (2)

Also if he walks 1 km an hour slower, her takes 3 more hours., ⇒d = (a − 1)(t + 3) ⇒ d = at + 3a - t - 3⇒ at = at + 3a - t - 3 [From 1]⇒ 3a - t = 3⇒ t = 3a - 3 ------- (3)Put the value of t in eq (2)⇒ 10a = 5(3a - 3) - 5⇒ 10a = 15a - 15 - 5⇒ 10a - 15a = - 15 - 5⇒ - 5a = - 20⇒ a = 4Putting back in (3), we get⇒ t = 3(4) - 3 = 9 Therefore, d = at = 4(9) = 36Hence, speed of man = a = 4 km/hrdistance covered by man = d = 36 km

7. Question

Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car. Find the speed of the train and car respectively.

Answer

Let the speed of train be 'a' and speed of car be 'b'.

Speed = distance/time

Given, Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car.

$\Rightarrow \frac{160}{a} + \frac{600}{b} = 8$------- (1)

Also, $\frac{240}{a} + \frac{520}{b} = 8 + \frac{12}{60}$

⇒ 240/a + 520/b = 41/5 ----- (2)

Multiplying eq1 by 3 and eq2 by 2 and subtracting eq2 from eq1

$\Rightarrow \frac{1800}{b} - \frac{1040}{b} = 24 - \frac{82}{5}$

$\Rightarrow \frac{760}{b} = \frac{38}{5}$

⇒ b = 100 km/hr

Thus, 160/a + 6 = 8

⇒ a = 80 km/hr

8. Question

A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

Answer

Given: A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer.

To find: the speed of the train and that of the car.

Solution: Let the speed of train be 'a' and speed of car be 'b'.

Speed = distance/time

Given, man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes.

$\Rightarrow \frac{400}{a} + \frac{200}{b} = 6.5$------- (1)

But, if he travels 200 km by train and the rest by car, he takes half an hour longer.

$\frac{200}{a} + \frac{400}{b} = 7$------- (2)

Multiplying eq1 by 2 and subtracting from eq2

$$\left(\frac{200}{a} + \frac{400}{b}\right) - 2\left(\frac{400}{a} + \frac{200}{b}\right) = 7 - 2(6.5)$$

$$\frac{200}{a} + \frac{400}{b} - \frac{800}{a} - \frac{400}{b} = 7 - 13$$

$$\Rightarrow \frac{200}{a} - \frac{800}{a} = 7 - 13 \Rightarrow \frac{-600}{a} = -6$$

⇒ -600 = -6a

⇒ a = 100 km/hr

Thus, 4 + 200/b = 6.5200/b = 6.5-4

⇒ 200/b = 2.5 ⇒ 2.5b = 200

⇒ b = 80 km/hr

9. Question

Places A and B are 80 km apart form each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes. Find the speeds of the cars.

Answer

Given: places A and B are 80 km apart form each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes.

To find: the speeds of the cars.

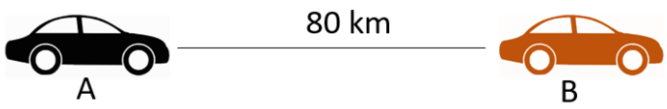
Solution: Let the speed of car from A be 'a' and of car from B be 'b'.

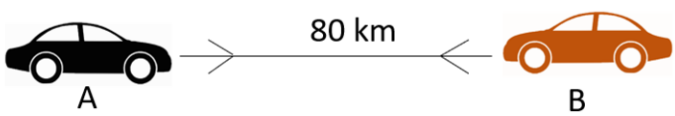
Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a – b

Given, places A and B are 80 km apart form each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes.





⇒ a – b = 80/8 = 10 ------ (1)

$$Also, a + b = \frac{80}{1+\frac{20}{60}} = 60 \text{ ------ } (2)$$

Adding (1) and (2)⇒ a - b + a + b = 10 + 60

⇒ 2a = 70

⇒ a = 35 km/hr

Put the value of a in (1).

Thus, b = 35 – 10 = 25 km/hr

Hence, speed of two cars are 35km/h and 25 km/h

10. Question

A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the oat in still water and the speed of the stream.

Answer

Speed = distance/time

Let the speed of boat be 'a' and speed of stream be 'b'

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time

$\Rightarrow \frac{12}{a-b} + \frac{40}{a+b} = 8$ ------- (1)

Also, $\frac{16}{a-b} + \frac{32}{a+b} = 8$ ------ (2)

Equating (1) and (2), we get

⇒ 8/(a + b) = 4/(a – b)

⇒ a = 3b

Substitute value of a in eq1

⇒ 12/2b + 40/4b = 8

⇒ 16/b = 8

⇒ b = 2 km/hr

Thus, a = 6 km/hr.

11. Question

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus respectively.

Answer

Let the speed of train be 'a' and speed of bus be 'b'.

Speed = distance/time

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer

$\Rightarrow \frac{60}{a} + \frac{240}{b} = 4$------- (1)

Also, $\frac{100}{a}+\frac{200}{b}=4+\frac{10}{60}$

$\frac{100}{a}+\frac{200}{b}=\frac{25}{6}$ ------- (2)

Multiplying eq1 by 5 and eq2 by 3 and subtracting eq2 from eq1

$$\Rightarrow \frac{1200}{b}-\frac{600}{b}=20-\frac{25}{2}$$

⇒ b = 80 km/hr

Thus, 60/a + 3 = 4

⇒ a = 60 km/hrHence speed of train is 60 km/h and bus is 80 km/h.

12. Question

Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Answer

Let the speed of her rowing in still water be 'a' and speed of current be 'b.'We know, $\text{speed}=\frac{\text{distance}}{\text{time}}$

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours

⇒ a + b = 20/2 = 10 ---- (1) and a – b = 4/2 = 2 ------ (2)

Adding eq1 and eq2

⇒ 2a = 12

⇒ a = 6 km/hr

Also, b = a – 2 = 4 km/hr

Therefore, her speed of rowing in still water is 6 km/h and the speed of the current is 4 km/h

13. Question

A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed) he is ahead of B by $1\frac{1}{2}$ hours. Find the speeds of A and B.

Answer

Let the speed of A be 'a' and speed of B be 'b'.

Given, A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed) he is ahead of B by $1\frac{1}{2}$ hours

$\Rightarrow \frac{30}{a}-\frac{30}{b}=3$ -------- (1)

$Also, \frac{30}{b}-\frac{30}{2a}=1.5$ ------ (2)

Adding (1) and (2)

⇒ 15/a = 4.5

⇒ a = 150/45 = 10/3 km/hr

Thus,

$$\frac{30}{\frac{10}{3}} - \frac{30}{b} = 3$$

⇒ b = 5 km/hr

14. Question

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi.

Answer

Let the speed of train be 'a' and speed of taxi be 'b'.

Speed = distance/time

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer.

$\Rightarrow \frac{300}{a} + \frac{200}{b} = 5.5$------- (1)

Also, $\frac{260}{a} + \frac{240}{b} = 5.5 + \frac{6}{60}$

$\frac{260}{a} + \frac{240}{b} = 5.6$ ------- (2)

Multiplying eq1 by 6 and eq2 by 5 and subtracting eq2 from eq1

$$\Rightarrow \frac{1800}{a} - \frac{1300}{a} = 33 - 28$$

⇒ a = 100 km/hr

Thus, 3 + 200/b = 5.5

⇒ b = 80 m/hr

15. Question

A train covered a certain distance at a uniform speed. If the train could have been 10 km/hr. faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hours more than the scheduled time. Find the distance covered by train.

Answer

Let the speed of train be 's', scheduled time be 't' and distance be 'd'.

Speed = distance/time

Given, covered a certain distance at a uniform speed. If the train could have been 10 km/hr. faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hours more than the scheduled time.

⇒ d = st ------- (1)

d = (s + 10)(t – 2) ------ (2)

d = (s – 10)(t + 3) ------ (3)

Solving the above three equation we get, d = 600km, s = 50 km/hr and t = 12 hours

16. Question

Places A and B are 100 km apart on a highway. One car starts form A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of two cars?

Answer

Let the speed of car from A be ‘a’ and of car from B be ‘b’

Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a − b

Given, places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour

⇒ a − b = 100/5 = 20 (1)

Also, a + b = 100/1 = 100 (2)

Adding (1) and (2)

a - b + a + b = 20 + 100

⇒ 2a = 120

⇒ a = 60 km/hr

Putting value of a in (1) we get,

Thus, b = 60 − 20 = 40 km/hrSpeed of two cars are 60 km/h and 40 km/h

Exercise 3.11

1. Question

If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.

Answer

Area of a rectangle = l × b

Given, if in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units.

⇒ (l + 2)(b − 2) = lb − 28

⇒ 2b − 2l = - 24 ------ (1)

Also, (l − 1)(b + 2) = lb + 33

⇒ 2l − b = 35 ------- (2)

Adding (1) and (2), we get

⇒ b = 11

Thus, 2l = 46, l = 23

Area of the rectangle = lb = 253 square units.

2. Question

The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. If the length is decreased by 7 metres and breadth is increased by 4 metres, the area is decreased by 21 sq. metres. Find the dimensions of the rectangle.

Answer

Area of a rectangle = l × b

Given, area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. If the length is decreased by 7 metres and breadth is increased by 4 metres, the area is decreased by 21 sq.

metres.

⇒ (l + 7)(b – 3) = lblb - 3l + 7b - 21 = lb

⇒ 7b – 3l = 21 ------ (1)

Also, (l – 7)(b + 4) = lb - 21

lb + 4l - 7b - 28 = lb - 21

⇒ 4l – 7b = 7 ------- (2)

Adding (1) and (2), we get7b - 3l + 4l - 7b = 21 + 7

⇒ l = 28 m

put the value of l in 1 to get7b - 3(28) = 21

Thus, 7b – 84 = 21

7b = 21 + 847b = 105

⇒ b = 15 mHence the length is 28 m and breadth is 15 m.

3. Question

In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimensions of the rectangle.

Answer

Area of a rectangle = l × b

Given, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres.

⇒ (l + 3)(b – 4) = lb – 67

⇒ 3b – 4l = -55 ------ (1)

Also, (l – 1)(b + 4) = lb + 89

⇒ 4l – b = 93 ------- (2)

Adding (1) and (2), we get

⇒2 b = 38

⇒ b = 19

Thus, 4l – 19 = 93

⇒ l = 28m

4. Question

The incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16. If each saves Rs 1250, find their incomes.

Answer

Given, incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16.

Let the incomes of X and Y be 8a and 7a

Let the expenditures of X and Y be 19b and 16b

Each saves Rs. 1250

⇒ 8a – 19b = 1250 -------- (1) and 7a – 16b = 1250 ------- (2)

Equating both equation

⇒ 8a – 19b = 7a – 16b

⇒ a = 3b

Substituting value of a in eq1

⇒ 24b – 19b = 1250

⇒ b = 250

Thus, a = 750

X's income = 8a = Rs. 6000

Y's income = 7a = Rs. 5250

5. Question

A and B each has some money. If A gives Rs 30 to B, then B will have twice the money left with A. But, if B gives Rs 10 to A, then a will have thrice as much as is left with B. How much money does each have?

Answer

Let the amount of money which A has be 'a' and which B has be 'b'.

Given, if A gives Rs 30 to B, then B will have twice the money left with A.

⇒ b + 30 = 2(a – 30)⇒ b + 30 = 2a – 60⇒ b = 2a-60-30⇒ b = 2a – 90 --------- (1)But, if B gives Rs 10 to A, then A will have thrice as much as is left with B

a + 10 = 3(b - 10)a+10=3b-30a=3b-30-10

⇒ a = 3b - 40 --------- (2)

Substituting 'a' from eq2 in eq1b = 2(3b-40) – 90

⇒ b = 6b - 80 – 90

⇒ b - 6b =- 80 – 90⇒ -5b = -170⇒ b = -170/-5

⇒ b = Rs. 34

Substitute value of b in eq. 2,a=3(34)-40

Thus, a = 102 – 40 = Rs. 62Hence, A has Rs 62 and B has Rs 34.

6. Question

There are two examination rooms A and B. If 10 candidates are sent from A to B, the number of students in each room is same. If 20 candidates are sent from B to A, the number of students in A is double the number of students in B. Find the number of students in each room.

Answer

Let the number of candidates in rooms A and B be 'a' and 'b' respectively.

Given, if 10 candidates are sent from A to B, the number of students in each room is same.

⇒ a – 10 = b + 10

⇒ a – b = 20 ----- (1)

Also, if 20 candidates are sent from B to A, the number of students in A is double the number of students in B.

⇒ a + 20 = 2(b – 20)

⇒ a – 2b = -60 ------- (2)

Subtracting eq2 from eq1

⇒ a – b – a + 2b = 20 + 60

⇒ b = 80

Thus, a = 80 + 20 = 100

7. Question

2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?

Answer

Given:2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. **To find:** How long would it take one man and one boy to do it.**Solution:**Let the number of days in which 1 man and 1 boy can do the work be 'a' and 'b' respectively.

In 1 day,In a days 1 man can work⇒ In 1 day 1/a man can do work⇒ 2 men can do work in 2/a daysSimilarlyIn b days 1 boy can work⇒ In 1 day 1/b boy can do work⇒ 7 boys can do work in 7/b daysGiven, 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys.

$\Rightarrow \frac{2}{a}+\frac{7}{b}=\frac{1}{4}$ -------- (1)

$\Rightarrow \frac{4}{a}+\frac{4}{b}=\frac{1}{3}$ -------- (2)

Multiplying eq1 by 2 and subtracting eq2 from eq1

$$\Rightarrow 2\left(\frac{2}{a}\right)+2\left(\frac{7}{b}\right)-\frac{4}{a}-\frac{4}{b}=2\left(\frac{1}{4}\right)-\frac{1}{3}$$

$$\Rightarrow \frac{4}{a}+\frac{14}{b}-\frac{4}{a}-\frac{4}{b}=\frac{2}{4}-\frac{1}{3}$$

$$\Rightarrow \frac{14}{b}-\frac{4}{b}=\frac{1}{2}-\frac{1}{3}$$

$$\Rightarrow \frac{14-4}{b}=\frac{3-2}{6}$$

⇒ 10/b = 1/6

⇒ b = 60 days

Substitute the value of b in eq.2 $\Rightarrow \frac{4}{a}+\frac{4}{60}=\frac{1}{3}$

$$\Rightarrow \frac{4}{a}+\frac{\cancel{4}^{1}}{\cancel{60}_{15}}=\frac{1}{3}$$

Thus, 4/a + 1/15 = 1/3

$$\Rightarrow \frac{4}{a}=\frac{1}{3}-\frac{1}{15}$$

$$\Rightarrow \frac{4}{a}=\frac{5-1}{15}$$

$$\Rightarrow \frac{4}{a}=\frac{4}{15}$$

⇒ 4/a = 4/15

⇒ a = 15 days

thus 1 man can do work in 15 days and 1 boy can do work in 60 days.

8. Question

In a ΔABC, $\angle A = x°, \angle B = (3x - 2)°,$

$\angle C = y°$. Also, $\angle C - \angle B = 9°$. Find the three angles.

Answer

Sum of angles of a triangle = 180°

Given, $\angle A = x°, \angle B = (3x - 2)°,$

$\angle C = y°$.

⇒ x + 3x – 2 + y = 180

⇒ 4x + y = 182 ------ (1)

Also, $\angle C - \angle B = 9°$.

⇒ y – 3x + 2 = 9

⇒ y – 3x = 7 ------- (2)

(1) – (2)

⇒ 4x + y – y + 3x = 182 – 7

⇒ 7x = 175

⇒ x = 25°

Thus, y = 75 + 7 = 82°

∠A = 25°

∠B = 3X – 2 = 73°

∠C = 82°

9. Question

In a cyclic quadrilateral ABCD $\angle A = (2x + 4)°, \angle B = (y + 3)°, \angle C = (2y + 10)°, \angle D = (4x - 5)°$. Find the four angles.

Answer

Opposite angles of a cyclic quadrilateral are supplementary

∠A + ∠C = 180°

∠B + ∠D = 180°

Given, $\angle A = (2x + 4)°, \angle B = (y + 3)°, \angle C = (2y + 10)°, \angle D = (4x - 5)°$

2x + 4 + 2y + 10 = 180

⇒ x + y = 83 ------- (1)

y + 3 + 4x – 5 = 180

⇒ y + 4x = 182 -------- (2)

(1) – (2)

⇒ x – 4x = 83 – 182

⇒ x = 33

Thus, y = 50

$\angle A = 2x + 4 = 70°$

$\angle B = y + 3 = 53°$

$\angle C = 2y + 10 = 110°$

$\angle D = 4x - 5 = 127°$

10. Question

A railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmedabad costs Rs 216 and one full and one half reserved first class tickets cost Rs 327. What is the basic first class full fare and what is the reservation charge?

Answer

Let the basic full fare be 'a', half far be 'b' and reservation charges be 'r'.

Given, railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket.

a = 2b ----- (1)

Also, reserved first class ticket from Mumbai to Ahmedabad costs Rs 216 and one full and one half reserved first class tickets cost Rs 327.

$\Rightarrow$ a + r = 216 ------ (2)

Also, a + b + 2r = 327 ----- (3)

Substituting value of a in eq2 and eq3

$\Rightarrow$ 2b + r = 216 and 3b + 2r = 327

Solving the above equations, we get

r = Rs. 6 and b = Rs. 105

Thus a = Rs. 210

11. Question

In a $\Delta ABC, \angle A = x°, \angle B = 3x°, \angle C = y°$. If 3y - 5x = 30, prove that the triangle is right angled.

Answer

Sum of angles of a triangle = 180°

Given, $\angle A = x^0, \angle B = 3x^0, \angle C = y^0$

$\therefore$ x + 3x + y = 180

$\Rightarrow$ y + 4x = 180 ------- (1)

Also, 3y – 5x = 30 ------- (2)

Multiplying eq1 by 5 and eq2 by 4 and adding them

$\Rightarrow$ 5y + 12y = 900 + 120

$\Rightarrow$ y = 60

Thus, 60 + 4x = 180

$\Rightarrow$ x = 30

$\angle A = 30°$, $\angle B = 90°$ and $\angle C = 60°$

Triangle is right angled at B.

12. Question

The car hire charges in a city computerise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is Rs 89 and for a journey of 20 km, the charge paid is Rs 145. What will a person have

to pay for travelling a distance of 30 km?

Answer

Let the fixed charge be ‘a’ and the charge per km travelled be ‘b’

Given, for a journey of 12 km, the charge paid is Rs 89 and for a journey of 20 km, the charge paid is Rs 145

⇒ a + 12b = 89 ----- (1)

and a + 20b = 145 ----- (2)

(1) – (2)

⇒ -8b = -56

⇒ b = 7

Thus, a + 84 = 89

⇒ a = 5

For a distance of 30km, amount paid = distance travelled x charge per kilometre + fixed charge

= 30 × 7 + 5 = Rs. 215

13. Question

A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student a takes food for 20 days, he has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charge and the cost of food per day**.**

Answer

Let the fixed charges be ‘a’ and the cost of food per day be ‘b’

Given, when a student a takes food for 20 days, he has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges.

⇒ a + 20b = 1000

a + 26b = 1180

Subtracting one from another

⇒ 6b = 180

⇒ b = Rs. 30

Thus, a + 600 = 1000

⇒ a = Rs. 400

14. Question

Half the perimeter of a garden, whose length is 4 more than its width is 36 m. Find the dimensions of the garden.

Answer

Given, half the perimeter of a garden, whose length is 4 more than its width is 36 m.

⇒ l = b + 4 and l + b = 36

Solving these two equations, we get

Length = 20 m, Width = 16 m

15. Question

The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Answer

Given: The larger of two supplementary angles exceeds the smaller by 18 degrees.

To find: The measure of both angles.

solution: Let the larger angle be 'a' and the smaller angle be 'b'.

Given, larger of two supplementary angles exceeds the smaller by 18 degrees.

⇒ a = b + 18 ⇒ a - b = 18 ..... (1)The sum of supplementary angles is 180°.⇒a + b = 180.....(2)

Adding the equations 1 and 2 we get,

a-b+a+b=18+180

⇒ 2a = 198

⇒ a = 99°Put the value of b in eq. 1 to get, 99-b=18⇒ -b=18-99⇒ -b=-81⇒ b=81

Thus, b = 81°Hence the measure of smaller angle is 81° and larger angle is 99°.

16. Question

2 Women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the embroidery, and that taken by 1 man alone.

Answer

Let the number of days in which 1 woman and 1 man can do the work be 'a' and 'b' days respectively.

In 1 day, 1 woman completes 1/a part while 1 man does 1/b part

Given, 2 Women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days.

$\Rightarrow \frac{2}{a}+\frac{5}{b}=\frac{1}{4}$ ------ (1)

$\frac{3}{a}+\frac{6}{b}=\frac{1}{3}$ -------(2)

Multiplying eq1 by 3 and eq2 by 2

$$\frac{6}{a}+\frac{15}{b}=\frac{3}{4} \quad (3)$$

$$\frac{6}{a}+\frac{12}{b}=\frac{2}{3} \quad (4)$$

subtracting eq3 from eq4

$$\Rightarrow \frac{15}{b}-\frac{12}{b}=\frac{3}{4}-\frac{2}{3}$$

⇒ 3/b = 1/12

⇒ b = 36 days

Thus, 3/a + 1/6 = 1/3

⇒ 3/a = 1/6

⇒ a = 18 daysHence, A women can finish the embroidery in 18 days and A single man can finish the embroidery in 36 days

17. Question

A wizard having powers of mystic in candations and magical medicines seeing a cock, fight going on, spoke privately to both the owners of cocks. To one he said; if your bird wins, than you give me your stake-money, but if you do not win, I shall give you two third of that. Going to the other, he promised in the same way to give three fourths. From both of them his gain would be only 12 gold coins. Find the stake of money each of the cock-owners have.

Answer

Let the stake money be 'a' and 'b' respectively.

Given, to one he said; if your bird wins, than you give me your stake-money, but if you do not win, I shall give you two third of that. Going to the other, he promised in the same way to give three fourths.

From both of them his gain would be only 12 gold coins

If the 1^{st} one wins

⇒ a - 3b/4 = 12 ----- (1)

If the 2^{nd} one wins

⇒ b – 2a/3 = 12 ----- (2)

Equating 1 and 2

⇒ 5a/3 = 7b/4

⇒ a = 21b/20

Thus, 21b/20 – 3b/4 = 12

⇒ 6b/20 = 12

⇒ b = 40

Thus, a = 21b/20 = Rs. 42

18. Question

Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many note Rs 50 and Rs 100 she received.

Answer

Given: Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all.

To find: how many notes of Rs 50 and Rs 100 she received.

Solution: Let the number of Rs. 50 and Rs. 100 notes be 'a' and 'b'.

Given, Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only.

⇒ 50a + 100b = 2000

⇒ a + 2b = 40 ------- (1)

Also Meena got 25 notes in all.

⇒ a + b = 25 ----- (2)

(1) – (2)a+2b-(a+b)=40-25

⇒ a+2b-a-b=40-25

⇒ b = 15

Put the value of b in eq. 1 to get,a+15=25⇒ a=25-15

Thus, a = 10Therefore, Meena received 10 notes of denomination 50 and 15 notes of denomination 100.

19. Question

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Answer

Given: Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks.

To find: How many questions were there in the test?

Solution: Let the number of right answers be 'a' and number of wrong answers be 'b'

Given, Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Deducted marks are represented by "-" sign.

⇒ 3a – b = 40 ------ (1)Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks

⇒ 4a – 2b = 50 ---- (2)

Multiplying eq1 by 2 and subtracting eq2 from it⇒ 4a - 2b - 2(3a - b) = 50 - 2(40)⇒ 4a - 2b - 6a +2b = 50 - 80⇒ 4a - 6a = 50 - 80

⇒ 6a – 4a = 80 – 50

⇒ 2a = 30

⇒ a = 15Put the value of a in eq. 1 to get,3(15) - b=40⇒ 45 - b = 40⇒ -b = 40 - 45⇒ -b = -5⇒ b = 5

Total number of questions in the test = right answers + wrong answers= a + b= 15 + 5 = 20

20. Question

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more. Find the number of students in the class.

Answer

Let the number of students in a row be 'a' and number of rows be 'b'.

Given, 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more.

Number of students remain constant.

⇒ ab = (a + 3)(b − 1) ab = (a − 3)(b + 2)

So,ab = ab - a + 3b -3a - 3b = -3 ..... (1) andab = ab + 2a - 3b - 62a - 3b = 6 ..... (2)Subtract 1 from 2 to get,2a - 3b - (a - 3b)= 6 - (-3)2a - 3b - a + 3b = 6 + 3a = 9Put the value of a in 1 to get,9 - 3b = -3-3b = -3 -9-3b = -12b = 4

Solving the above equations we get , a = 9 and b = 4

Thus, number of students = ab = (9)(4) = 36

21. Question

One says, "give me hundred, friend! I shall then become twice as rich as you" The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital?

Answer

Given: One says, "give me hundred, friend! I shall then become twice as rich as you" The other replies, "If you give me ten, I shall be six times as rich as you."

To find: the amount of their respective capital.

Solution: Let the capitals be 'a' and 'b'.

Given, one says, "give me hundred, friend! I shall then become twice as rich as you" .Lets assume "b" gives hundred to "a".According to given condition

a + 100 = 2(b − 100)⇒ a + 100 = 2b - 200

⇒ a = 2b - 200 - 100

⇒ a = 2b − 300⇒ a - 2b = -300 ------ (1)Now The other replies, "If you give me ten, I shall be six times as rich as you."

Which means "a" gives 10 to "b".

So, b + 10 = 6(a – 10)⇒ b + 10 = 6a – 60⇒ b = 6a – 60 - 10

⇒ b = 6a – 70⇒ 6a - b = 70 ----- (2)

Multiplying eq1 by 6 and subtract from eq2

⇒ 6a - b - 6 (a - 2b) = 70 - 6 (- 300)⇒ 6a - b - 6 a + 12b = 70 + 1800⇒ 11b = 1870⇒ b = 170substitute the value of b in eq 1 to get, a - 2(170) = -300⇒ a - 340 = -300⇒ a = -300 + 340⇒ a = 40The amount of their respective capital is Rs 40 and Rs 170.

22. Question

In a cyclic quadrilateral ABCD $\angle A = (2x+4)^\circ, \angle B = (y+3)^\circ, \angle C = (2y+10)^\circ, \angle D = (4x-5)^\circ$. Find the four angles.

Answer

In a cyclic quadrilateral sum of opposite angles is 180°.

Given, in a cyclic quadrilateral ABCD $\angle A = (2x+4)^\circ, \angle B = (y+3)^\circ, \angle C = (2y+10)^\circ, \angle D = (4x-5)^\circ$

∴ ∠A + ∠C = 180° and ∠B + ∠D = 180°

⇒ 2x + 4 + 2y + 10 = 180 and y + 3 + 4x – 5 = 180

⇒ x + y = 83 --------- (1) and y + 4x = 182 ---------- (2)

Subtracting eq1 from eq2.

⇒ y + 4x – x – y = 182 – 83

⇒ 3x = 99

⇒ x = 33

Substituting in eq1.

⇒ y = 50

∠A = 2 × 33 + 4 = 70°

∠B = 50 + 3 = 53°

∠C = 2 × 50 + 10 = 110°

∠D = 4 × 33 – 5 = 127°

CCE - Formative Assessment

1. Question

Write the value of k for which the system of equations x + y – 4 = 0 and 2x + ky – 3 = 0 has no solution.

Answer

Given:

Equation 1: x + y = 4

Equation 2: 2x + ky = 3

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have no solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ .....(i)

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = 1$

$b_2 = k$

$c_1 = 4$

$c_2 = 3$

Putting the above values in equation (i) we get:

$$\frac{1}{2} = \frac{1}{k}$$

$$\Rightarrow k = 2$$

Also we find $\frac{1}{2} \neq \frac{4}{3}$

The value of k for which the system of equations has no solution is k = 2

2. Question

Write the value of k for which the system of equations

2x – y = 5

6x + ky = 15

has infinitely many solutions.

Answer

Given:

Equation 1: 2x – y = 5

Equation 2: 6x + ky = 15

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .....(i)

According to the problem:

$a_1 = 2$

$a_2 = 6$

$b_1 = -1$

$b_2 = k$

$c_1 = 5$

$c_2 = 15$

Putting the above values in equation (i) we get:

$$\frac{2}{6} = \frac{-1}{k}$$

$$\Rightarrow 2k = -6 \Rightarrow k = -3$$

Also we find $\frac{2}{6} = \frac{-1}{-3} = \frac{5}{15}$

The value of k for which the system of equations has infinitely many solution is k = – 3

3. Question

Write the value of k for which the system of equations 3x – 2y = 0 and kx + 5y = 0 has infinitely many solutions.

Answer

Given:

Equation 1: 3x – 2y = 0

Equation 2: kx + 5y = 0

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .....(i)

According to the problem:

$a_1 = 3$

$a_2 = k$

$b_1 = -2$

$b_2 = 5$

$c_1 = 0$

$c_2 = 0$

In this problem since c_1 & $c_2 = 0$ so $\frac{c_1}{c_2} = \frac{0}{0}$ which is undefined.

So for this problem the system of linear equations will have infinite solutions if

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ .....(ii)

Putting the above values in equation (ii) we get:

$$\frac{3}{k} = \frac{-2}{5}$$

$$\Rightarrow -2k = 15$$

$$\Rightarrow k = -\frac{15}{2}$$

The value of k for which the system of equations has infinitely many solution is $k = -\frac{15}{2}$

4. Question

Write the values of k for which the system of equations x + ky = 0, 2x – y = 0 has unique solution.

Answer

Given:

Equation 1: x + ky = 0

Equation 2: 2x – y = 0

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = k$

$b_2 = -1$

$c_1 = 0$

$c_2 = 0$

So for this problem the system of linear equations will have unique solution if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ .....(i)

Putting the above values in equation (i) we get:

$$\frac{1}{2} \neq \frac{k}{-1}$$

$$\Rightarrow k \neq -\frac{1}{2}$$

The value of k for which the system of equations has unique solution is $k \neq -\frac{1}{2}$

5. Question

Write the set of values of a and b for which the following system of equations has infinitely many solutions.

2x + 3y = 7

2ax + (a + b) y = 28

Answer

Given:

Equation 1: 2x + 3y = 7

Equation 2: 2ax + (a + b)y = 28

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .....(i)

According to the problem:

$a_1 = 2$

$a_2 = 2a$

$b_1 = 3$

$b_2 = (a + b)$

$c_1 = 7$

$c_2 = 28$

Putting the above values in equation (i) we get:

$\mathbf{\frac{2}{2a} = \frac{3}{a+b} = \frac{7}{28}}$ **...(ii)**

To obtain the value of a & b we need to solve the above equality. First we solve the extreme left and extreme right of the equality to obtain the value of a.

$\mathbf{\Rightarrow \frac{2}{2a} = \frac{7}{28} \Rightarrow 2a*7 = 2*28 \Rightarrow 14a = 56 \Rightarrow a = 4}$

After obtaining the value of a we again solve the extreme left and middle portion of the equality (ii)

$\mathbf{\frac{2}{2*4} = \frac{3}{4+b} \Rightarrow 2*(4 + b) = 3*2*4 \Rightarrow b + 4 = 12 \Rightarrow b = 8}$

The value of a & b for which the system of equations has infinitely many solution is a = 4 & b = 8

6. Question

For what value of k, the following pair of linear equations has infinitely many solutions?

10x + 5y – (k – 5) = 0

20x + 10y – k = 0

Answer

Given:

Equation 1: 10x + 5y = (k – 5) Equation 2: 20x + 10y = k

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \text{ .....(i)}$$

According to the problem:

$a_1 = 10$

$a_2 = 20$

$b_1 = 5$

$b_2 = 10$

$c_1 = k - 5$

$c_2 = k$

Putting the above values in equation (i) we get:

$$\frac{10}{20} = \frac{5}{10} = \frac{k-5}{k} \text{ ...(ii)}$$

On solving the equality (ii) we get

$5k = 10(k - 5) \Rightarrow 5k = 10k - 50 \Rightarrow 5k = 50 \Rightarrow k = 10$

The value of k for which the system of equations has infinitely many solution is k = 10

7. Question

Write the number of solutions of the following pair of linear equations:

x + 2y– 8 = 0

2x + 4y = 16

Answer

Given:

Equation 1: x + 2y = 8 Equation 2: 2x + 4y = 16

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

The system of linear equations needs to be analyzed by checking the nature of ratios of each coefficients in the above two equations.

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = 2$

$b_2 = 4$

$c_1 = 8$

$c_2 = 16$

Comparing the ratios of the coefficients we see:

$\Rightarrow \frac{a_1}{a_2} = \frac{1}{2}$ **...(i)**

$\Rightarrow \frac{b_1}{b_2} = \frac{2}{4}$

$\Rightarrow \frac{b_1}{b_2} = \frac{1}{2}$ **...(ii)**

$\Rightarrow \frac{c_1}{c_2} = \frac{8}{16}$

$\Rightarrow \frac{c_1}{c_2} = \frac{1}{2}$ ...(iii)

On seeing equation (i), (ii) and (iii) we find

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Conclusion: The system of linear equations have infinite number of solution.

The given system of linear equations will have infinite number of solutions for all values of x and y

8. Question

Write the number of solutions of the following pair of linear equations:

x + 3y– 4 = 0

2x + 6y = 7

Answer

Given:

Equation 1: x + 3y = 4 Equation 2: 2x + 6y = 7

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = 3$

$b_2 = 6$

$c_1 = 4$

$c_2 = 7$

Comparing the ratios of the coefficients we see:

$\Rightarrow \frac{a_1}{a_2} = \frac{1}{2}$ **...(i)**

$\Rightarrow \frac{b_1}{b_2} = \frac{3}{6} \Rightarrow \frac{b_1}{b_2} = \frac{1}{2}$ **...(ii)**

$\Rightarrow \frac{c_1}{c_2} = \frac{4}{7}$ ...(iii)

On seeing equation (i), (ii) and (iii) we find

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Conclusion: The system of linear equations has no solutions.

The given system of linear equations will have no solution for all values of x and y

1. Question

The value of k for which the system of equations

kx – y = 2

6x – 2y = 3

has a unique solution, is

A. = 3

B. ≠ 3

C. ≠ 0

D. = 0

Answer

Given:

Equation 1: kx – y = 2

Equation 2: 6x – 2y = 3

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have unique solution we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ .....(i)

According to the problem:

$a_1 = k$

$a_2 = 6$

$b_1 = -1$

$b_2 = -2$

$c_1 = 2$

$c_2 = 3$

Putting the above values in equation (i) we get

$\frac{k}{6} \neq \frac{-1}{-2} \Rightarrow k \neq \frac{-6}{-2} \Rightarrow k \neq 3$

The value of k for which the system of equations has unique solution is $k \neq 3$

2. Question

The value of k for which the system of equations

$2x + 3y = 5$

$4x + ky = 10$

has infinite number of solutions, is

A. 1

B. 3

C. 6

D. 0

Answer

Given:

Equation 1: $2x + 3y = 5$

Equation 2: $4x + ky = 10$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .....(i)

According to the problem:

$a_1 = 2$

$a_2 = 4$

$b_1 = 3$

$b_2 = k$

$c_1 = 5$

$c_2 = 10$

Putting the above values in equation (i) and solving the extreme left and middle portion of the equality we get the value of k

$$\frac{2}{4} = \frac{3}{k}$$

$\Rightarrow 2k = 12 \Rightarrow k = 6$

Also we find $\frac{2}{4} = \frac{3}{6} = \frac{5}{10}$

The value of k for which the system of equations has infinitely many solution is k = 6

3. Question

The value of k for which the system of equations x + 2y – 3 = 0 and 5x + ky + 7 = 0 has no solution, is

A. 10

B. 6

C. 3

D. 1

Answer

Given:

Equation 1: x + 2y = 3

Equation 2: 5x + ky = – 7

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have no solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ .....(i)

According to the problem:

$a_1 = 1$

$a_2 = 5$

$b_1 = 2$

$b_2 = k$

$c_1 = 3$

$c_2 = -7$

Putting the above values in equation (i) and solving we get:

$$\frac{1}{5} = \frac{2}{k}$$

$\Rightarrow k = 10$

Also we find $\frac{1}{5} = \frac{2}{10} \neq \frac{3}{-7}$

The value of k for which the system of equations has no solution is k = 10

4. Question

The value of k for which the system of equations 3x + 5y = 0 and kx + 10y = 0 has a non-zero solution, is

A. 0

B. 2

C. 6

D. 8

Answer

Given:

Equation 1: 3x + 5y = 0

Equation 2: kx + 10y = 0

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = 3$

$a_2 = k$

$b_1 = 5$

$b_2 = 10$

$c_1 = 0$

$c_2 = 0$

The equation will have a non zero solution only when it will satisfy a non trivial solution i.e. the equations should satisfy with values other than x = 0 & y = 0 .For the given system of equations the equation will have a non zero solution if

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ .....(i)

Putting the above values in equation (i) we get:

$$\frac{3}{k} = \frac{5}{10}$$

$\Rightarrow 5k = 30 \Rightarrow k = 6$

The value of k for which the system of equations has non zero solution is k = 6

5. Question

If the system of equations

2x + 3y = 7

8. Quadratic Equations

Exercise 8.1

1. Question

Which of the following are quadratic equations?

(i) $x^2 + 6x - 4 = 0$

(ii) $\sqrt{3}x^2 - 2x + \frac{1}{2} = 0$

(iii) $x^2 + \frac{1}{x^2} = 5$

(iv) $x - \frac{3}{x} = x^2$

(v) $2x^2 - \sqrt{3x} + 9 = 0$

(vi) $x^2 - 2x - \sqrt{x} - 5 = 0$

(vii) $3x^2 - 5x + 9 = x^2 - 7x + 3$

(viii) $x + \frac{1}{x} = 1$

(ix) $x^2 - 3x = 0$

(x) $\left(x + \frac{1}{x}\right)^2 = 3\left(x + \frac{1}{x}\right) + 4$

(xi) $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$

(xii) $x + \frac{1}{x} = x^2, x \neq 0$

(xiii) $16x^2 - 3 = (2x + 5)(5x - 3)$

(xiv) $(x + 2)^3 = x^3 - 4$

(xv) $x(x + 1) + 8 = (x + 2)(x - 2)$

Answer

A polynomial equation is a quadratic equation, if it is of the form $ax^2 + bx + c = 0$ such that $a \neq 0$

(i) $x^2 + 6x - 4 = 0$

It is a quadratic equation.

(ii) $\sqrt{3}x^2 - 2x + \frac{1}{2} = 0$

It is a quadratic equation.

(iii) $x^2 + \frac{1}{x^2} = 5$

$\Rightarrow x^4 - 5x^2 + 1 = 0$

It is not a quadratic equation as the highest power of x is '4'.

(iv) $x-\frac{3}{x}=x^2$

$\Rightarrow x^2 - 3 = x^3$

It is not a quadratic equation.

(v) $2x^2-\sqrt{3x}+9=0$

It is not a quadratic equation as √x is present instead of 'x'.

(vi) $x^2-2x-\sqrt{x}-5=0$

It is not a quadratic equation as an additional √x term is present.

(vii) $3x^2-5x+9=x^2-7x+3$

$\Rightarrow 2x^2 + 2x + 6 = 0$

It is a quadratic equation.

(viii) $x+\frac{1}{x}=1$

$\Rightarrow x^2 + 1 - x = 0$

It is a quadratic equation.

(ix) $x^2-3x=0$

It is a quadratic equation.

(x) $\left(x+\frac{1}{x}\right)^2=3\left(x+\frac{1}{x}\right)+4$

$$\Rightarrow x^2+\frac{1}{x^2}+2=3x+\frac{3}{x}+4$$

$\Rightarrow x^4 + 1 + 2x^2 = 3x^3 + 3x + 4x^2$

It is not a quadratic equation.

(xi) $(2x+1)(3x+2)=6(x-1)(x-2)$

$\Rightarrow 6x^2 + 7x + 2 = 6x^2 - 18x + 12$

$\Rightarrow 25x = 10$

It is not a quadratic equation.

(xii) $x+\frac{1}{x}=x^2, x\neq 0$

$\Rightarrow x^2 + 1 = x^3$

It is not a quadratic equation.

(xiii) $16x^2-3=(2x+5)(5x-3)$

$\Rightarrow 16x^2 - 3 = 10x^2 + 19x - 15$

$\Rightarrow 6x^2 - 19x + 12 = 0$

It is a quadratic equation.

(xiv) $(x+2)^3 = x^3 - 4$

$\Rightarrow x^3 + 8 + 6x^2 + 12x = x^3 - 4$

$\Rightarrow 6x^2 + 12x + 12 = 0$

It is a quadratic equation.

(xv) $x(x+1)+8 = (x+2)(x-2)$

$\Rightarrow x^2 + x + 8 = x^2 - 4$

$\Rightarrow x = -12$

It is not a quadratic equation.

2. Question

In each of the following, determine whether the given values are solutions of the given equation or not:

(i) $x^2 - 3x + 2 = 0, x = 2, x = 1$

(ii) $x^2 + x + 1 = 0, x = 0, x = 1$

(iii) $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -2\sqrt{3}$

(iv) $x + \frac{1}{x} = \frac{13}{6}, x = \frac{5}{6}, x = \frac{4}{3}$

(v) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2, x = 3$

(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, x = -2\sqrt{2}$

(vii) $a^2x^2 - 3abx + 2b^2 = 0, x = a / b, x = b / a$

Answer

We will have to check for each value and see whether it satisfies the equation.

(i) $x^2 - 3x + 2 = 0, x = 2, x = 1$

For x = 2,

$2^2 - 3 \times 2 + 2 = 0$

$\Rightarrow 0 = 0$

Thus, x = 2 is a solution.

For, x = 1

$1^2 - 3 \times 1 + 2 = 0$

$\Rightarrow 0 = 0$

Thus, x = 1 is a solution.

(ii) $x^2 + x + 1 = 0, x = 0, x = 1$

For x = 0,

$\Rightarrow 0 + 0 + 1 = 0$

$\Rightarrow 1 = 0$ which is not true thus x = 0 is not a solution

For x = 1,

$\Rightarrow 1 + 1 + 1 = 0$

⇒ 3 = 0 which is not true thus x = 1 is not a solution

(iii) $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -2\sqrt{3}$

For x= √3

⇒ 3 – 3√3 × √3 + 6 = 0

⇒ 3 – 9 + 6 = 0

⇒ 0 = 0

Thus, x = √3 is a solution

For x = -2√3

⇒ $(-2\sqrt{3})^2$ – 3√3 × -2√3 + 6 = 0

⇒ 4 × 3 + 18 + 6 = 0

⇒ 36 = 0 which is not true, thus x = -2√3 is not a solution

(iv) $x + \frac{1}{x} = \frac{13}{6}, x = \frac{5}{6}, x = \frac{4}{3}$

For x = 5/6

$$\Rightarrow \frac{5}{6} + \frac{6}{5} = \frac{13}{6}$$

$$\Rightarrow \frac{61}{30} = \frac{13}{6}$$

⇒ 61 = 65 which is not true, thus x = 5/6 is not a solution

For x = 4/3

$$\Rightarrow \frac{4}{3} + \frac{3}{4} = \frac{13}{6}$$

⇒ 25/12 = 13/6

⇒ 25 = 26 which is not true, thus x = 4/3 is not a solution

(v) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2, x = 3$

For x = 2,

⇒ 2 × 4 – 2 + 9 = 4 + 4 × 2 + 3

⇒ 15 = 15, thus x = 2 is a solution.

For x = 3

⇒ 2 × 9 – 3 + 9 = 9 + 4 × 3 + 3

⇒ 24 = 24, thus x = 3 is also a solution

(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, x = -2\sqrt{2}$

For x = -√2,

⇒ 2 - √2 × -√2 – 4 = 0

⇒ 2 + 2 – 4 = 0

⇒ 0 = 0

Thus, x = -√2 is a solution

For x = -2√2

⇒ 4 × 2 - √2 × -2√2 – 4 = 0

⇒ 8 + 8 – 4 = 0

⇒ 12 = 0 which is not true, thus x = -2√2 is not a solution

(vii) $a^2x^2 - 3abx + 2b^2 = 0, x = a/b, x = b/a$

For, x = a/b

$$\Rightarrow a^2 \times \frac{a^2}{b^2} - 3ab \times \frac{a}{b} + 2 \times b^2 = 0$$

⇒ $a^4/b^2 - 3a^2 + 2b^2 = 0$ which is not true, thus x = a/b is not a solution

For x = b/a

$$\Rightarrow a^2 \times \frac{b^2}{a^2} - 3ab \times \frac{b}{a} + 2b^2 = 0$$

⇒ $b^2 - 3b^2 + 2b^2 = 0$

⇒ 0 = 0 , thus x = b/a is a solution

3. Question

In each of the following, find the value of k for which the given value is a solution of the given equation:

(i) $7x^2 + kx - 3 = 0, x = 2/3$

(ii) $x^2 - x(a+b) + k = 0, x = a$

(iii) $kx^2 + \sqrt{2}x - 4 = 0, x = \sqrt{2}$

(iv) $x^2 + 3ax + k = 0, x = -a$

Answer

For the given value to be a solution, it should satisfy the quadratic equation

(i) $7x^2 + kx - 3 = 0, x = 2/3$

⇒ 7 × 4/9 + k × 2/3 – 3 = 0

⇒ 2k/3 = 3 – 28/9 = - 1/9

⇒ k = -1/6

(ii) $x^2 - x(a+b) + k = 0, x = a$

⇒ a^2 – a(a + b) + k = 0

⇒ $a^2 - a^2$ – ab + k = 0

⇒ k = ab

(iii) $kx^2 + \sqrt{2}x - 4 = 0, x = \sqrt{2}$

⇒ k × 2 + √2 × √2 – 4 = 0

⇒ 2k = 2

⇒ k = 1

(iv) $x^2 + 3ax + k = 0, x = -a$

⇒ $a^2 - 3a \times a + k = 0$

⇒ $k = 2a^2$

4. Question

If $x = 2/3$ *and* $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

Answer

Given: If $x = 2/3$ *and* $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$

To find: the values of a and b.

Solution: Quadratic equation in roots form:

(x – a)(x – b) = 0, where a and b are the roots

Given, $x = 2/3$ *and* $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$

Quadratic equation is,

(x – 2/3)(x + 3) = 0

⇒ x^2 -2x/3 + 3x – 2 = 0

$$\Rightarrow \frac{3x^2 - 2x + 9x - 6}{3} = 0$$

⇒ $3x^2$ +7x – 6 = 0

On comparing with $ax^2 + 7x + b = 0$

We get, a = 3 and b = -6

5. Question

Determine, if 3 is a root of the equation given below:

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$

Answer

For the given value to be a root, it should satisfy given equation

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$

$$\Rightarrow \sqrt{3^2 - 4\times3 + 3} + \sqrt{3^2 - 9} = \sqrt{4\times3^2 - 14\times3 + 16}$$

⇒ 0 + 0 = √10

Thus x = 3 does not satisfy the given equation and it is not a root of the equation.

Exercise 8.2

1. Question

The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denoted the smaller integer.

Answer

Let the consecutive numbers be ‘a’ and ‘a + 1’ respectively.

Given, product of two consecutive positive integers is 306

a × (a + 1) = 306

$\Rightarrow a^2 + a - 306 = 0$

2. Question

John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they had to start with, if john had x marbles.

Answer

Number of marbles John has is x.

Given, John and Jivanti together have 45 marbles.

Number of marbles which Jivanti has = 45 – x

Now, both of them lost 5 marbles each, and the product of the number of marbles they now have is 128.

So John will have x - 5 marbles and Jivanti will have 45 - x - 5 = 40 - x marbles.

$\Rightarrow (x - 5)(40 - x) = 128$

$\Rightarrow 40x - 200 + 5x - x^2 = 128$

$\Rightarrow 40x - 200 + 5x - x^2 - 128 = 0$

$\Rightarrow 45x - 328 - x^2 = 0$

$\Rightarrow x^2 - 45x + 328 = 0$

3. Question

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day. On a particular day, the total cost of production was Rs.750. If x denotes the number of toys produced that day, form the quadratic equation to find x.

Answer

Number of toys produced that day is 'x'.

Cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day.

$\therefore$ Cost of production of each toy = 55 – x

Given, total cost of production = Rs. 750

$\Rightarrow x \times (55 - x) = 750$

$\Rightarrow -x^2 + 55x - 750 = 0$

$\Rightarrow x^2 - 55x + 750 = 0$

4. Question

The height of a right TRIANGLE IS 7 CM LESS THAN ITS BASE. If the hypotenuse is 13 cm, form the quadratic equation to find the base of the triangle.

Answer

By Pythagoras theorem :

$\text{Hypotenuse}^2 = \text{perpendicular}^2 + \text{base}^2$

Given, height of a right TRIANGLE IS 7 CM LESS THAN ITS BASE and the hypotenuse is 13 cm.

Let the base be 'x'

$\Rightarrow 13^2 = (x - 7)^2 + x^2$

⇒ 169 = x^2 – 14x + 49 + x^2

⇒ x^2 – 7x – 60 = 0

5. Question

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr more than that of the passenger train, form the quadratic equation to find the average speed of express train.

Answer

Given: An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr more than that of the passenger train.

To find: the quadratic equation to find the average speed of express train.

Solution: Let the average speed of passenger train be 'x' km/hr and the time taken by passenger train be 't' hr.

Since an express train takes 1 hour less than a passenger train and the average speed of the express train is 11 km/hr more than that of the passenger train.

So, For the express train average speed = x + 11, time taken = t - 1

Since,Distance = speed × time

Given, total distance traveled = 132 kmFor passenger train:

⇒ x × t = 132

⇒ t = 132/x ..... (1)

Also for express train (x + 11) × (t - 1) = 132

Substitute the value of t from (1),

$$(x+11)\left(\frac{132}{x}-1\right)=132$$

⇒ (x + 11)(132-x) = 132x

⇒ 132x - x^2 +1452 -11x = 132x

⇒ - x^2 -11x +1452 = 0

⇒ x^2 + 11x – 1452 = 0Required quadratic equation

6. Question

A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 4 hour less for the same journey. Form the quadratic equation to find the speed of the train.

Answer

Let the speed of the train be 'a' km/hr and the actual time taken be 't'

Given, train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 4 hour less for the same journey.

Distance = speed × time

⇒ 360 = a × t

⇒ t = 360/a

Also, 360 = (a + 5)(t – 4)

$\Rightarrow 360 = (a+5)\left(\frac{360}{a} - 4\right)$

⇒ 360a = (a + 5)(360 – 4a)

⇒ 360a = 360a + 1800 – $4a^2$ – 20a

⇒ a^2 + 5a – 450 = 0

Upon solving we will get a to be 18.86 and -23.86 but the speed can't be negative so the speed of the train is 18.86 km/hr.

Exercise 8.3

1. Question

Solve the following quadratic equations by factorization:

$(x-4)(x+2)=0$

Answer

$(x-4)(x+2)=0$ is already factorized

⇒ x = 4, -2

2. Question

Solve the following quadratic equations by factorization:

$(2x+3)(3x-7)=0$

Answer

$(2x+3)(3x-7)=0$ is already factorized

⇒ x = -3/2, 7/3

3. Question

Solve the following quadratic equations by factorization:

$x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

⇒ x^2 –x -√3x + √3 = 0

⇒ x(x – 1) - √3(x – 1) = 0

⇒ (x - √3)(x – 1) = 0

⇒ x = √3, 1

4. Question

Solve the following quadratic equations by factorization:

$9x^2-3x-2=0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$9x^2 - 3x - 2 = 0$

⇒ $9x^2 - 6x + 3x - 2 = 0$

⇒ $3x(3x - 2) + (3x - 2) = 0$

⇒ $(3x + 1)(3x - 2) = 0$

⇒ $x = -1/3, 2/3$

5. Question

Solve the following quadratic equations by factorization:

$3\sqrt{5}x^2 + 25x - 10\sqrt{5} = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3\sqrt{5}x^2 + 25x - 10\sqrt{5} = 0$

⇒ $3\sqrt{5}x^2 + 30x - 5x - 10\sqrt{5} = 0$

⇒ $3\sqrt{5}x(x + 2\sqrt{5}) - 5(x + 2\sqrt{5}) = 0$

⇒ $(x + 2\sqrt{5})(3\sqrt{5}x - 5) = 0$

⇒ $x = -2\sqrt{5}, \sqrt{5}/3$

6. Question

Solve the following quadratic equations by factorization:

$6x^2 + 11x + 3 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$6x^2 + 11x + 3 = 0$

⇒ $6x^2 + 9x + 2x + 3 = 0$

⇒ $3x(2x + 3) + (2x + 3) = 0$

⇒ $(3x + 1)(2x + 3) = 0$

⇒ $x = -1/3, - 3/2$

7. Question

Solve the following quadratic equations by factorization:

$5x^2 - 3x - 2 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$5x^2 - 3x - 2 = 0$

⇒ $5x^2 - 5x + 2x - 2 = 0$

⇒ 5x(x – 1) + 2(x – 1) = 0

⇒ (5x + 2)(x – 1) = 0

⇒ x = -2/5, 1

8. Question

Solve the following quadratic equations by factorization:

$48x^2 - 13x - 1 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$48x^2 - 13x - 1 = 0$

⇒ $48x^2 - 16x + 3x - 1 = 0$

⇒ 16x(3x – 1) + (3x – 1) = 0

⇒ (16x + 1)(3x – 1) = 0

⇒ x = -1/16, 1/3

9. Question

Solve the following quadratic equations by factorization:

$3x^2 = -11x - 10$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3x^2 = -11x - 10$

⇒ $3x^2 + 11x + 10 = 0$

⇒ $3x^2 + 6x + 5x + 10 = 0$

⇒ 3x(x + 2) + 5(x + 2) = 0

⇒ (3x + 5)(x + 2) = 0

⇒ x = -5/3, - 2

10. Question

Solve the following quadratic equations by factorization:

$25x(x+1) = -4$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$25x(x+1) = -4$

⇒ $25x^2 + 25x + 4 = 0$

⇒ $25x^2 + 20x + 5x + 4 = 0$

⇒ 5x(5x + 4) + (5x + 4) = 0

⇒ (5x + 1)(5x + 4) = 0

⇒ x = -1/5, -4/5

11. Question

Solve the following quadratic equations by factorization:

$$16x - \frac{10}{x} = 27$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$16x - \frac{10}{x} = 27$$

⇒ $16x^2 - 27x - 10 = 0$

⇒ $16x^2 - 32x + 5x - 10 = 0$

⇒ 16x(x – 2) + 5(x – 2) = 0

⇒ (16x + 5)(x – 2) = 0

⇒ x = -5/16, 2

12. Question

Solve the following quadratic equations by factorization:

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

⇒ $\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

⇒ $\sqrt{3}x^2 - \sqrt{3}\times\sqrt{3}\times\sqrt{2}x + \sqrt{2}x - 2\times\sqrt{3}\times\sqrt{3} = 0$⇒ $\sqrt{3}x^2 - \sqrt{3}\times\sqrt{6}x + \sqrt{2}x - \sqrt{6}\times\sqrt{3} = 0$

⇒ $\sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$

⇒ $(\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$

⇒ $x = \sqrt{6}, -\sqrt{(2/3)}$

13. Question

Solve the following quadratic equations by factorization:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

⇒ $4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

⇒ $4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

⇒ (4x - √3)(√3x + 2) = 0

⇒ x = √3/4, - 2/√3

14. Question

Solve the following quadratic equations by factorization:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

⇒ $\sqrt{2}x^2 - 4x + x - 2\sqrt{2} = 0$

⇒ $\sqrt{2}x(x - 2\sqrt{2}) + (x - 2\sqrt{2}) = 0$

⇒ $(\sqrt{2}x + 1)(x - 2\sqrt{2}) = 0$

⇒ x = -1/√2, 2√2

15. Question

Solve the following quadratic equations by factorization:

$a^2x^2 - 3abx + 2b^2 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$a^2x^2 - 3abx + 2b^2 = 0$

⇒ $a^2x^2 - 2abx - abx + 2b^2 = 0$

⇒ ax(ax – 2b) – b(ax – 2b) = 0

⇒ (ax – b)(ax – 2b) = 0

⇒ x = b/a, 2b/a

16. Question

Solve the following quadratic equations by factorization:

$x^2 - \left(\sqrt{2} + 1\right)x + \sqrt{2} = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - \left(\sqrt{2} + 1\right)x + \sqrt{2} = 0$

⇒ x^2 - √2x – x + √2 = 0

⇒ x(x - √2) - (x - √2) = 0

⇒ (x – 1)(x - √2) = 0

⇒ x = 1, √2

17. Question

Solve the following quadratic equations by factorization:

$9x^2 - 6b^2x - (a^4 - b^4) = 0$

Answer

Given: $9x^2 - 6b^2x - (a^4 - b^4) = 0$

To find: The value of above equation.

Solution: In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

We know:a^2-b^2=(a+b)(a-b)

Consider,

$9x^2 - 6b^2x - (a^4 - b^4) = 0$

Here,

$$a^4 - b^4 = (a^2)^2 - (b^2)^2$$

apply the above formula and solve,

⇒ $9x^2 - 3(a^2 + b^2)x + 3(a^2 - b^2)x - (a^2 + b^2)(a^2 - b^2) = 0$

⇒ $3x(3x - (a^2 + b^2)) + (a^2 - b^2)(3x - (a^2 + b^2)) = 0$

⇒ $(3x + a^2 - b^2)(3x - (a^2 + b^2) = 0$

⇒ $x = \frac{b^2 - a^2}{3}, \frac{a^2 + b^2}{3}$

18. Question

Solve the following quadratic equations by factorization:

$4x^2 + 4bx - (a^2 - b^2) = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

⇒ $4x^2 + 2(a + b)x - 2(a - b)x - (a - b)(a + b) = 0$

⇒ $2x(2x + a + b) - (a - b)(2x - (a + b)) = 0$

⇒ $(2x - (a - b))(2x + a + b) = 0$

$$\Rightarrow x = \frac{a - b}{2}, -\frac{a + b}{2}$$

19. Question

Solve the following quadratic equations by factorization:

$ax^2 + (4a^2 - 3b)x - 12ab = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$ax^2 + (4a^2 - 3b)x - 12ab = 0$

$\Rightarrow ax^2 + 4a^2x - 3bx - 12ab = 0$

$\Rightarrow ax(x + 4a) -3b(x + 4a) = 0$

$\Rightarrow (ax - 3b)(x + 4a) = 0$

$\Rightarrow x = 3b/a, -4a$

20. Question

Solve the following quadratic equations by factorization:

$2x^2 + ax - a^2 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$2x^2 + ax - a^2 = 0$

$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x + a) - a(x + a) = 0$

$\Rightarrow (2x - a)(x + a) = 0$

$\Rightarrow x = a/2, -a$

21. Question

Solve the following quadratic equations by factorization:

$x^2 - 4\sqrt{2x} + 6 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - 4\sqrt{2x} + 6 = 0$

$\Rightarrow x^2 - 3\sqrt{2}x - \sqrt{2}x + 6 = 0$

$\Rightarrow x(x - 3\sqrt{2}) - \sqrt{2}(x - 3\sqrt{2}) = 0$

$\Rightarrow (x - \sqrt{2})(x - 3\sqrt{2}) = 0$

$\Rightarrow x = \sqrt{2}, 3\sqrt{2}$

22. Question

Solve the following quadratic equations by factorization:

$\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

$\Rightarrow 2x^2 - 9 + 3x = 3x^2 - 14 - x$

$\Rightarrow x^2 - 4x - 5 = 0$

⇒ $x^2 - 5x + x - 5 = 0$

⇒ $(x - 5)(x + 1) = 0$

⇒ $x = -1, 5$

23. Question

Solve the following quadratic equations by factorization:

$$\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$$

$$\Rightarrow 3\times(2x^2 - 6x + 2x^2 + 20 - 13x) = 25(x-4)(x-3)$$

⇒ $12x^2 - 57x + 60 = 25x^2 + 300 - 175x$

⇒ $13x^2 - 118x + 240 = 0$

⇒ $13x^2 - 78x - 40x + 240 = 0$

⇒ $13x(x - 6) - 40(x - 6) = 0$

⇒ $(13x - 40)(x - 6) = 0$

⇒ $x = 6, 40/13$

24. Question

Solve the following quadratic equations by factorization:

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

⇒ $4(x^2 + 3x + (x - 1)(x - 2)) = 17(x^2 - 2x)$

⇒ $4(x^2 + 3x + x^2 - 3x + 2) = 17(x^2 - 2x)$

⇒ $8x^2 + 8 = 17x^2 - 34x$

⇒ $9x^2 - 34x - 8 = 0$

⇒ $9x^2 - 36x + 2x - 8 = 0$

⇒ $9x(x - 4) + 2(x - 4) = 0$

⇒ $(9x + 2)(x - 4) = 0$

⇒ $x = -2/9, 4$

25. Question

Solve the following quadratic equations by factorization:

$\frac{x-3}{x+3}-\frac{x+3}{x-3}=\frac{48}{7}, x \neq 3, x \neq -3$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-3}{x+3}-\frac{x+3}{x-3}=\frac{48}{7}, x \neq 3, x \neq -3$

Taking L.C.M

$$\frac{(x\ -\ 3)\,(x\ -\ 3)\ +\ (x\ +\ 3)\,(x+\ 3)}{(x\ -\ 3)\,(x\ +\ 3)}\ =\ \frac{48}{7}$$

$$\frac{x^2\ +\ 9\ -\ 6\ x\ +\ x^2\ +\ 9\ +\ 6\ x}{x^2\ -\ 9}\ =\ \frac{48}{7}$$

$$\frac{2\ x^2\ +\ 18}{x^2\ -\ 9}\ =\ \frac{48}{7}$$

Cross Multiplying we get,

$\Rightarrow 7(2\ x^2 + 18) = 48(x^2 - 9)$

$\Rightarrow -84x = 48x^2 - 432$

$\Rightarrow 4x^2 + 7x - 36 = 0$

$\Rightarrow 4x^2 + 16x - 9x - 36 = 0$

$\Rightarrow 4x(x + 4) -9(x + 4) = 0$

$\Rightarrow (4x - 9)(x + 4) = 0$

$\Rightarrow x = 9/4, -4$

26. Question

Solve the following quadratic equations by factorization:

$\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}, x \neq 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}, x \neq 0$

$\Rightarrow x(x - 1 + 2x - 4) = 6(x^2 -3x + 2)$

$\Rightarrow 3x^2 - 5x = 6x^2 - 18x + 12$

$\Rightarrow 3x^2 - 13x + 12 = 0$

$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$

$\Rightarrow 3x(x - 3) -4(x - 3) = 0$

$\Rightarrow (3x - 4)(x - 3) = 0$

⇒ x = 4/3, 3

27. Question

Solve the following quadratic equations by factorization:

$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}, x \neq 1,-1$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}, x \neq 1,-1$

⇒ $6((x + 1)^2 - (x - 1)^2) = 5(x^2 - 1)$

⇒ $6 \times 4x = 5x^2 - 5$

⇒ $5x^2 - 24x - 5 = 0$

⇒ $5x^2 - 25x + x - 5 = 0$

⇒ $5x(x - 5) + 1(x-5) = 0$

⇒ $(5x + 1)(x - 5) = 0$

⇒ x = 5, -1/5

28. Question

Solve the following quadratic equations by factorization:

$\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=\frac{5}{2}, x \neq -\frac{1}{2}, 1$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=\frac{5}{2}, x \neq -\frac{1}{2}, 1$

⇒ $2(x^2 - 2x + 1 + 4x^2 + 4x + 1) = 5(2x^2 - x - 1)$

⇒ $10x^2 + 4x + 4 = 10x^2 - 5x - 5$

⇒ 9x = -9

⇒ x = -1

29. Question

Solve the following quadratic equations by factorization:

$3x^2-14x-5=0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3x^2-14x-5=0$

⇒ $3x^2 - 15x + x - 5 = 0$

⇒ 3x(x – 5) + 1(x – 5) = 0

⇒ (3x + 1)(x – 5) = 0

⇒ x = 5, -1/3

30. Question

Solve the following quadratic equations by factorization:

$\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

⇒ $\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

⇒ $m^2x^2 + n^2 = mn - 2mnx$

⇒ $m^2x^2 + 2mnx - mn + n^2 = 0$

⇒ $(mx + n)^2 = mn$

⇒ mx + n = ±√mn

$$\Rightarrow x = \frac{-n \pm \sqrt{mn}}{m}$$

31. Question

Solve the following quadratic equations by factorization:

$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$

Let (x – a)/(x – b) = y and a/b = c ..... (1)

So (x – b)/(x – a) = 1/ y and b/a = 1/c .... (2)

$$\Rightarrow y + \frac{1}{y} = c + \frac{1}{c}$$

$$\Rightarrow \frac{y^2 + 1}{y} = \frac{c^2 + 1}{c}$$

⇒ $c(y^2 + 1) = y(c^2+1)$

⇒ $cy^2 + c = c^2y + y$

⇒ $cy^2 - c^2y - y + c = 0$

⇒ cy(y – c) – 1(y – c) = 0

⇒ (cy – 1)(y – c) = 0

⇒ y = 1/c, c

From (1)⇒ (x – a)/(x – b) = c and a/b = c

$$\Rightarrow (x-a)(x-b)=\frac{a}{b}\ and\ (x-b)(x-a)=\frac{b}{a}$$

⇒ x^2 - bx - ax + ab = a/b

⇒ b(x^2 - bx - ax + ab) = a

⇒ bx^2 - b^2x - abx + ab^2 = a

⇒ bx (x - b) - ab (x - b) = a

⇒ (bx - ab) (x - b) = a⇒ (bx - ab) = a and (x - b) = aSo,bx - ab = a ⇒ bx = a + ab⇒ x =(a + ab) / bAnd x - b = a⇒ x = a + bHence x =(a + ab) / b , (a+b)

32. Question

Solve the following quadratic equations by factorization:

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$$

$$\Rightarrow (x-3)(x-4)+(x-1)(x-4)+(x-1)(x-2)=\frac{(x-1)(x-2)(x-3)(x-4)}{6}$$

$$\Rightarrow \left(x^2-4x-3x+12\right)+\left(x^2-4x-x+4\right)+\left(x^2-2x-x+1\right)=\frac{\left(x^2-2x-x+2\right)\left(x^2-4x-3x+12\right)}{6}$$

⇒ 6(x^2 + 12 – 7x + x^2 + 4 – 5x + x^2 – 3x + 2) = (x^2 – 3x + 2)(x^2 + 12 – 7x)

⇒ 6(3x^2 + 18 – 15x) = x^4 + 12x^2 – 7x^3 – 3x^3 – 36x + 21x^2 + 2x^2 + 24 – 14x

⇒ 18x^2 – 90x + 108 = x^4 + 12x^2 – 7x^3 – 3x^3 – 36x + 21x^2 + 2x^2 + 24 – 14x

⇒ x^4 – 10x^3 + 17x^2 + 40x + 84 = 0

Let P(x) = x^4 – 10x^3 + 17x^2 + 40x + 84

At x = -2, $(-2)^4$ - 10$(-2)^3$ + 17$(-2)^2$ + 40(-2) + 84 = 16 + 80 + 68 - 80 + 84

P(x) = 0therefore, x + 2 is a factor of P(x).On dividing P(x) by (x + 2), we get x^3 - 12x^2 + 41x - 42

Let g(x) = x^3 - 12x^2 + 41x - 42, P(x) = (x - 2)g(x)

at x =-2g(x) =0therefore, x + 2 is a factor of g(x).On dividing g(x) by (x + 2), we get x^2 - 14x + 49

Therefore, P(x) = (x - 2)(x - 2)(x^2 - 14x + 49)Using, $(a - b)^2$ =a^2 + b^2 - 2ab, we haveP(x) = $(x + 2)^2(x – 7)^2$

⇒ $(x + 2)^2(x – 7)^2$ = 0

Therefore, possible value of 'x' are -2, -2, and -7, -7

33. Question

Solve the following quadratic equations by factorization:

$(x-5)(x-6)=\frac{25}{(24)^2}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$(x-5)(x-6)=\frac{25}{(24)^2}$

$\Rightarrow x^2 - 11x + 30 = 25/(24)^2$

$\Rightarrow x^2 - 11x + 121/4 - 121/4 + 30 = 25/(24)^2$

$\Rightarrow x^2 - 11x + 121/4 = 25/(24)^2 + 1/4$

$\Rightarrow x^2 - 11x + 121/4 = 676/(4 \times 24^2)$

$\Rightarrow (x - 11/2)^2 = (13/24)^2$

$$\Rightarrow x-\frac{11}{2}=\pm\frac{13}{24}$$

$$\Rightarrow x=\frac{11}{2}+\frac{13}{24}\ or\ x=\frac{11}{2}-\frac{13}{24}$$

$$\Rightarrow x=\frac{145}{24}=6\frac{1}{24}, x=\frac{119}{24}=4\frac{23}{24}$$

34. Question

Solve the following quadratic equations by factorization:

$7x+\frac{3}{x}=35\frac{3}{5}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$7x+\frac{3}{x}=35\frac{3}{5}$

$\Rightarrow 7x^2 + 3 = 178x/5$

$\Rightarrow 35x^2 - 178x + 15 = 0$

$\Rightarrow 35x^2 - 175x - 3x + 15 = 0$

$\Rightarrow 35x(x - 5) -3(x - 5) = 0$

$\Rightarrow (35x - 3)(x - 5) = 0$

$\Rightarrow x = 5, 3/35$

35. Question

Solve the following quadratic equations by factorization:

$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

Answer

Given: $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

To find: The value of x.

Solution: In factorization, we write the coefficient of middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the product of these two factors will be equal to the product of the coefficient of x^2 and the constant term.

$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$ ..... (1)

Take the LCM of the denominators.LCM is (x - a) (x - b) (x - c)

Now solve for (1),

⇒ a (x - b) (x - c) + b (x - a) (x - c) = 2c (x - a) (x - b) ⇒(a (x - b) + b (x - a))(x - c) = 2c (x - a) (x - b)

⇒ (ax – ab + bx – ab)(x – c) = 2c (x^2 – bx - ax +ab)

⇒ (ax – ab + bx – ab)(x – c) = 2cx^2 – 2cbx - 2cax +2cab

⇒ ((a + b)x – 2ab)(x – c) = 2cx^2 – 2c(a + b)x + 2abc

⇒ (a + b)x^2 – (a + b)cx – 2abx + 2abc = 2cx^2 – 2(a + b)cx +2abc

⇒ (a + b – 2c)x^2 + ((a + b)c – 2ab)x = 0

⇒x[(a + b – 2c)x + ((a + b)c – 2ab)] = 0⇒ x = oand (a + b – 2c)x + ((a + b)c – 2ab)=0⇒(a + b – 2c)x = - [(a + b)c – 2ab]⇒(a + b – 2c)x = - (ac + bc – 2ab)

⇒ $x=\frac{-(ac+bc-2ab)}{a+b-2c}$

⇒ $x=\frac{2ab-ac-bc}{a+b-2c}$

Hence x = 0, $\frac{2ab-ac-bc}{a+b-2c}$

36. Question

Solve the following quadratic equations by factorization:

$x^2+2ab=(2a+b)x$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2+2ab=(2a+b)x$

⇒ x^2 – (2a + b)x + 2ab = 0

⇒ x^2 – 2ax – bx + 2ab = 0

⇒ x(x – 2a) – b(x – 2a) = 0

⇒ (x – b)(x – 2a) = 0

⇒ x = b, 2a

37. Question

Solve the following quadratic equations by factorization:

$(a+b)^2 x^2 - 4abx - (a-b)^2 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$(a+b)^2 x^2 - 4abx - (a-b)^2 = 0$

$\Rightarrow (a + b)^2x^2 - (a^2 + b^2 + 2ab - a^2 - b^2 + 2ab)x - (a - b)^2 = 0$

$\Rightarrow (a + b)^2x^2 - (a + b)^2x + (a - b)^2x - (a - b)^2 = 0$

$\Rightarrow (a + b)^2x(x - 1) + (a - b)^2(x - 1) = 0$

$\Rightarrow ((a + b)^2x + (a - b)^2)(x - 1) = 0$

$\Rightarrow x = 1, -\frac{(a-b)^2}{(a+b)^2}$

38. Question

Solve the following quadratic equations by factorization:

$a(x^2+1) - x(a^2+1) = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$a(x^2+1) - x(a^2+1) = 0$

$\Rightarrow ax^2 + a - a^2x - x = 0$

$\Rightarrow ax^2 - x(a^2 + 1) + a = 0$

$\Rightarrow ax(x - a) - 1(x - a) = 0$

$\Rightarrow (ax - 1)(x - a) = 0$

$\Rightarrow x = 1/a, a$

39. Question

Solve the following quadratic equations by factorization:

$x^2 - x - a(a+1) = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - x - a(a+1) = 0$

$\Rightarrow x^2 - a^2 - x - a = 0$

$\Rightarrow (x + a)(x - a) - 1(x + a) = 0$

$\Rightarrow (x + a)(x - a - 1) = 0$

$\Rightarrow x = -a, a + 1$

40. Question

Solve the following quadratic equations by factorization:

$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

$\Rightarrow ax^2 + a^2x + x + a = 0$

$\Rightarrow ax(x + a) + 1(x + a) = 0$

$\Rightarrow (ax + 1)(x + a) = 0$

$\Rightarrow x = -a, -1/a$

41. Question

Solve the following quadratic equations by factorization:

$abx^2 + (b^2 - ac)x - bc = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow abx^2 - acx + b^2x - bc = 0$

$\Rightarrow ax(bx - c) + b(bx - c) = 0$

$\Rightarrow (ax + b)(bx - c) = 0$

$\Rightarrow x = -b/a, c/b$

42. Question

Solve the following quadratic equations by factorization:

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$\Rightarrow b^2x(a^2x + 1) - (a^2x + 1) = 0$

$\Rightarrow (b^2x - 1)(a^2x + 1) = 0$

$\Rightarrow x = -1/a^2, 1/b^2$

43. Question

Solve the following quadratic equations by factorization:

Solve for x: $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2, 4$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x \neq 2,4$

⇒ 3(x– 1)(x – 4) + 3(x – 2)(x – 3) = 10(x – 2)(x – 4)

⇒ $3x^2$ + 12 – 15x + $3x^2$ + 18 – 15x = $10x^2$ – 60x + 80

⇒ $4x^2$ – 30x + 50 = 0

⇒ $2x^2$ – 15x + 25 = 0

⇒ $2x^2$ – 10x – 5x + 25 = 0

⇒ 2x(x – 5) – 5(x – 5) = 0

⇒ (2x – 5)(x – 5) = 0

Thus, x = 5/2, 5

44. Question

Solve the following quadratic equations by factorization:

$3x^2-2\sqrt{6}x+2=0$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3x^2-2\sqrt{6}x+2=0$

⇒ $(\sqrt{3}x)^2 - 2\sqrt{6}x + (\sqrt{2})^2 = 0$

⇒ $(\sqrt{3}x - \sqrt{2})^2 = 0$

⇒ x = $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

45. Question

Solve the following quadratic equations by factorization:

$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x \neq 1,-5$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x \neq 1,-5$

⇒ 7(x + 5 – x + 1) = 6(x – 1)(x + 5)

⇒ 42 = $6x^2$ + 24x – 30

⇒ x^2 + 4x – 12 = 0

⇒ x^2 + 6x – 2x – 12 = 0

⇒ x(x + 6) – 2(x + 6) = 0

⇒ (x – 2)(x + 6) = 0

⇒ x = 2, -6

46. Question

Solve the following quadratic equations by factorization:

$\frac{1}{x}-\frac{1}{x-2}=3, x \neq 0,2$

Answer

Given: $\frac{1}{x}-\frac{1}{x-2}=3, x \neq 0,2$

To find: Solve the given quadratic equations by factorization.

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{x}-\frac{1}{x-2}=3, x \neq 0,2$

⇒ x-2 - x = 3 x (x - 2)⇒ x – 2 – x = 3(x^2 – 2x)

⇒ x – 2 – x = $3x^2$ – 6x

⇒ $3x^2$ – 6x + 2 = 0

Now convert the terms in above equation involving 3,⇒ $3x^2$ – (3+3) x + (3-1) = 0

Now add and subtract √3 in the coefficient of x,⇒ $3x^2$ – (3+√3+3-√3) x + (3-1) = 0

Now 3 = $(\sqrt{3})^2$ and 1 = 1^2

⇒ $(\sqrt{3}x)^2$ – [(3+√3) + (3-√3)]x + (3-1) = 0

⇒ $(\sqrt{3}x)^2$ – (3+√3) x - (3-√3) x + ($\sqrt{3}^2$ - 1^2)= 0

Apply the formula a^2 - b^2 = (a+b) (a-b) in ($\sqrt{3}^2$ - 1^2)

⇒ $(\sqrt{3}x)^2$ – (3+√3) x - (3-√3) x +[(√3-1) (√3+1)]= 0

⇒ $(\sqrt{3}x)^2$ – √3(√3+1) x - √3(√3-1) x +[(√3-1) (√3+1)]= 0

⇒ $(\sqrt{3}x)^2$ – √3(√3+1) x - √3(√3-1) x +[(√3-1) (√3+1)]= 0

⇒ √3x [√3x-(√3+1)] - (√3-1) [√3x-(√3+1)] = 0

⇒[√3x- (√3-1)] [√3x-(√3+1)] = 0

⇒ [√3x- (√3-1)] =0 and [√3x-(√3+1)] = 0

⇒ √3x = (√3-1) and √3x = (√3+1)

⇒ x = (√3-1)/√3 and x = (√3+1)/√3

Rationalizing both values we get,]

$$\Rightarrow x=\frac{\sqrt{3}-1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\ and\ x=\frac{\sqrt{3}+1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{\sqrt{3}(\sqrt{3}-1)}{\left(\sqrt{3}\right)^2} \text{ and } x = \frac{\sqrt{3}(\sqrt{3}+1)}{\left(\sqrt{3}\right)^2}$$

$$\Rightarrow x = \frac{(3-\sqrt{3})}{3} \text{ and } x = \frac{(3+\sqrt{3})}{3}$$

47. Question

Solve the following quadratic equations by factorization:

$$x - \frac{1}{x-3} = 3,\ x \neq 0$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$x - \frac{1}{x-3} = 3, x \neq 0$$

$\Rightarrow x^2 - 3x - 1 = 3x - 9$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow x^2 - 4x - 2x + 8 = 0$

$\Rightarrow x(x - 4) - 2(x - 4) = 0$

$\Rightarrow (x - 2)(x - 4) = 0$

$\Rightarrow x = 2, 4$

48. Question

Solve the following quadratic equations by factorization:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq 4,7$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq 4,7$$

$\Rightarrow 30(x - 7 - x - 4) = 11(x + 4)(x - 7)$

$\Rightarrow -330 = 11x^2 - 308 - 33x$

$\Rightarrow 11x^2 - 33x + 22 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2) - (x - 2) = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1, 2$

49. Question

Solve the following quadratic equations by factorization:

$\frac{1}{x-3}+\frac{2}{x-2}=\frac{8}{x}; x \neq 0,2,3$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{x-3}+\frac{2}{x-2}=\frac{8}{x}; x \neq 0,2,3$

⇒ x((x – 2) + 2(x – 3)) = 8(x – 3)(x – 2)

⇒ $3x^2$ – 8x = $8x^2$ – 40x + 48

⇒ $5x^2$ - 32x + 48 = 0

⇒ $5x^2$ – 20x – 12x + 48 = 0

⇒ 5x(x – 4) – 12(x – 4) = 0

⇒ (5x – 12)(x – 4) = 0

⇒ x = 12/5, 4

50. Question

Solve the following quadratic equations by factorization:

$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$

⇒ 2ab(2x – 2a – b – 2x) = (2a + b)2x(2a + b + 2x)

⇒ 2ab(-2a – b)= 2(2a+ b)(2ax + bx + $2x^2$)

⇒ -ab = 2ax + bx + $2x^2$

⇒ $2x^2$ + 2ax + bx + ab = 0

⇒ 2x(x + a) + b(x + a) = 0

⇒ (2x + b)(x + a) = 0

⇒ x = -a, -b/2

51. Question

Solve the following quadratic equations by factorization:

$\frac{4}{x}-3=\frac{5}{2x+3}, x \neq 0,-\frac{3}{2}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{4}{x}-3=\frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

$\Rightarrow (4 - 3x)(2x + 3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - (x + 2) = 0$

$\Rightarrow (x - 1)(x + 2) = 0$

$\Rightarrow x = 1, - 2$

52. Question

Solve the following quadratic equations by factorization:

$\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}; x \neq 5,7$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}; x \neq 5,7$

$\Rightarrow 3(x^2 - 11x + 28) + 3(x^2 - 11x + 30) = 10(x^2 - 12x + 35)$

$\Rightarrow 4x^2 - 54x + 176 = 0$

$\Rightarrow 2x^2 - 27x + 88 = 0$

$\Rightarrow 2x^2 - 16x - 11x + 88 = 0$

$\Rightarrow 2x(x - 8) - 11(x - 8) = 0$

$\Rightarrow (2x - 11)(x - 8) = 0$

$\Rightarrow x = 11/2, 8$

53. Question

Solve the following quadratic equations by factorization:

$\frac{16}{x}-1=\frac{15}{x+1}; x \neq 0, -1$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{16}{x}-1=\frac{15}{x+1}; x \neq 0, -1$

$\Rightarrow (16 - x)(x + 1) = 15x$

$\Rightarrow -x^2 - x + 16x + 16 = 15x$

⇒ $x^2 = 16$

⇒ x = ±4

54. Question

Solve the following quadratic equations by factorization:

$\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}; x \neq 3,5$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}; x \neq 3,5$

⇒ $3(x^2 - 7x + 10) + 3(x^2 - 7x + 12) = 10(x^2 - 8x + 15)$

⇒ $4x^2 - 38x + 84 = 0$

⇒ $2x^2 - 19x + 42 = 0$

⇒ $2x^2 - 12x - 7x + 42 = 0$

⇒ $2x(x - 6) - 7(x - 6) = 0$

⇒ $(x - 6)(2x - 7) = 0$

⇒ x = 7/2, 6

55. Question

Solve the following quadratic equations by factorization:

$\frac{5+x}{5-x}-\frac{5-x}{5+x}=3\frac{3}{4}; x \neq 5,-5$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{5+x}{5-x}-\frac{5-x}{5+x}=3\frac{3}{4}; x \neq 5,-5$

⇒ $4(25 + x^2 + 10x) - 4(25 + x^2 - 10x) = 15(25 - x^2)$

⇒ $15x^2 + 80x - 375 = 0$

⇒ $3x^2 + 16x - 75 = 0$

⇒ $3x^2 + 25x - 9x - 75 = 0$

⇒ $x(3x + 25) - 3(3x + 25) = 0$

⇒ $(x - 3)(3x + 25) = 0$

⇒ x = 3, - 25/3

56. Question

Solve the following quadratic equations by factorization:

$\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1},\ x \neq -1, \frac{1}{3}$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1}, x \neq -1, \frac{1}{3}$$

⇒ (5 - x)(3x – 1) = 4x + 4

⇒ $-3x^2$ + 16x – 5 = 4x + 4

⇒ $3x^2$ - 12x + 9 = 0

⇒ $3x^2$ – 3x – 9x + 9 = 0

⇒ 3x(x – 1) -9(x – 1) =0

⇒ (3x – 9)(x – 1) = 0

⇒ x = 1, 3

57. Question

Solve the following quadratic equations by factorization:

$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$

Answer

Given: $3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$

to find: Solution of the above quadratic equation.

Solution: In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$

$$\frac{3(3x-1)^2-2\,(2x+3)^2}{(2x+3)(3x-1)}=5$$

$$\Rightarrow \frac{3\left(9x^2+1-6x\right)\ -2\left(4x^2+9+12x\right)}{\left(6x^2-2x+9x-3\right)}=5$$

$$\Rightarrow \frac{3\left(9x^2+1-6x\right)\ -2\left(4x^2+9+12x\right)}{\left(6x^2+7x-3\right)}=5$$

⇒ 3($9x^2$ + 1 – 6x) – 2($4x^2$ + 9 + 12x) = 5($6x^2$ – 3 + 7x)

⇒ 27x^2 + 3 – 18x - 8x^2 - 18 – 24x = 30x^2 - 15 + 35x⇒ 19x^2 - 42x – 15 = 30x^2 - 15 + 35x ⇒19x^2 - 30x^2 - 42x – 35x - 15 + 15 = 0

⇒ -11x^2 - 77x = 0⇒ 11x^2 + 77x = 0

⇒ 11x(x + 7) = 0

⇒ x=0 and (x+7)=0⇒ x=0 and x=-7

⇒ x = 0, - 7

58. Question

Solve the following quadratic equations by factorization:

$$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11; x\neq\frac{3}{5},-\frac{1}{7}$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11; x\neq\frac{3}{5},-\frac{1}{7}$$

⇒ $3(7x + 1)^2 – 4(5x – 3)^2 = 11(5x – 3)(7x + 1)$

⇒ $3(49x^2 + 1 + 14x) – 4(25x^2 + 9 – 30x) = 11(35x^2 – 3 – 16x)$

⇒ $338x^2 – 338x = 0$

⇒ $x(x – 1) = 0$

⇒ x = 0, 1

59. Question

Solve the following quadratic equations by factorization:

$$\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}; x\neq 1,-1,\frac{1}{4}$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}; x\neq 1,-1,\frac{1}{4}$$

⇒ $(3x – 3 + 4x + 4)(4x – 1) = 29(x^2 – 1)$

⇒ $(7x + 1)(4x – 1) = 29x^2 – 29$

⇒ $28x^2 – 3x – 1 = 29x^2 – 29$

⇒ $x^2 + 3x – 28 = 0$

⇒ $x^2 + 7x – 4x – 28 = 0$

⇒ $x(x + 7) – 4(x + 7) = 0$

⇒ $(x – 4)(x + 7) = 0$

⇒ x = 4, - 7

60. Question

Solve the following quadratic equations by factorization:

$$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}; x\neq 0,-1,2$$

Answer

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized

$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}; x \neq 0,-1,2$

⇒ 5x(4x – 8 + 3x + 3) = 46(x + 1)(x – 2)

⇒ $35x^2$ – 25x = $46x^2$ – 92 – 46x

⇒ $11x^2$ - 19x - 92 = 0

⇒ $11x^2$ - 44x + 23x – 92 = 0

⇒ 11x(x – 4) + 23(x – 4) = 0

⇒ (11x + 23)(x – 4) = 0

⇒ x = 4, - 23/11

Exercise 8.4

1. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$x^2-4\sqrt{2x}+6=0$

Answer

Given: $x^2-4\sqrt{2x}+6=0$

To find: the roots of the following quadratic (if they exist) by the method of completing the square.

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.Step 1: Make the coefficient of x^2 unity.In the equation $x^2-4\sqrt{2x}+6=0$,The coefficient of x^2 is 1.Step 2: Shift the constant term on RHS,

$$\Rightarrow x^2-4\sqrt{2x}=-6$$

Step 3: Add square of half of coefficient of x on both the sides.

$$\Rightarrow x^2-2\times 2\sqrt{2x}+\left(2\sqrt{2}\right)^2=-6+\left(2\sqrt{2}\right)^2$$

Step 4: Apply the formula, $(a - b)^2 = a^2 - 2ab + b^2$ on LHS and solve RHS,Here a=x and $b=2\sqrt{2}$

$$\Rightarrow \left(x-2\sqrt{2}\right)^2=-6+\left(2\sqrt{2}\right)^2$$

$$\Rightarrow \left(x-2\sqrt{2}\right)^2=-6+(2)^2\left(\sqrt{2}\right)^2$$

$$\Rightarrow \left(x-2\sqrt{2}\right)^2=-6+(4\times 2)$$

$$\Rightarrow \left(x-2\sqrt{2}\right)^2=-6+8$$

$$\Rightarrow \left(x-2\sqrt{2}\right)^2=2$$

As RHS is positive, the roots exist.Now,take square root on both sides,

$$\Rightarrow \left(x-2\sqrt{2}\right) = \pm\sqrt{2}$$

$$\Rightarrow x = \sqrt{2}+2\sqrt{2} \ and \ x = -\sqrt{2}+2\sqrt{2}$$

$$\Rightarrow x = 3\sqrt{2} \ and \ x = \sqrt{2}$$

2. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$2x^2 - 7x + 3 = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$2x^2 - 7x + 3 = 0$ Divide the equation by 2 to get,

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Now add and subtract the square of half of coefficient of x to get,

$$\Rightarrow x^2 - 2\times\frac{7}{4}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

Use the formula **$(a + b)^2 = a^2 + 2ab + b^2$**

$$\Rightarrow \left(x-\frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left(x-\frac{7}{4}\right)^2 - \frac{25}{16} = 0$$

$\Rightarrow (x - 7/4)^2 = 25/16$

As RHS is positive the roots exist.

$\Rightarrow x - 7/4 = \pm 5/4$

$$\Rightarrow x-\frac{7}{4} = \frac{5}{4} \ and \ x-\frac{7}{4} = -\frac{5}{4}$$

$$\Rightarrow x = \frac{7}{4}+\frac{5}{4} \ and \ x = \frac{7}{4}-\frac{5}{4}$$

$$\Rightarrow x = \frac{12}{4} \ and \ x = \frac{2}{4}$$

$\Rightarrow x = 3, 1/2$

3. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$3x^2 + 11x + 10 = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$3x^2 + 11x + 10 = 0$

$$\Rightarrow x^2 + 2 \times \frac{11}{6}x + \left(\frac{11}{6}\right)^2 - \left(\frac{11}{6}\right)^2 + \frac{10}{3} = 0$$

$$\Rightarrow \left(x + \frac{11}{6}\right)^2 + \frac{10}{3} - \frac{121}{36} = 0$$

$\Rightarrow (x + 11/6)^2 = 1/36$

$\Rightarrow x + 11/6 = \pm 1/6$

$\Rightarrow x = -2, -5/3$

4. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$2x^2 + x - 4 = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$2x^2 + x - 4 = 0$

$\Rightarrow x^2 + x/2 - 2 = 0$

$\Rightarrow x^2 + 2 \times 1/4 \times x + (1/4)^2 - (1/4)^2 - 2 = 0$

$\Rightarrow (x + 1/4)^2 = 33/16$

$\Rightarrow x + 1/4 = \pm \sqrt{33}/4$

$$\Rightarrow x = \frac{\sqrt{33} - 1}{4}, \frac{-\sqrt{33} - 1}{4}$$

5. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$2x^2 + x + 4 = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$2x^2 + x + 4 = 0$

$\Rightarrow x^2 + x/2 + 2 = 0$

$\Rightarrow x^2 + 2 \times 1/4 \times x + (1/4)^2 - (1/4)^2 + 2 = 0$

$\Rightarrow (x + 1/4)^2 = -31/16$

Roots are not real.

6. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$4x^2 + 4\sqrt{3}x + 3 = 0$

Answer

Given: The quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$

To find: the roots of the following quadratic (if they exist) by the method of completing the square.

Solution: We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.Step 1: Make the coefficient of x^2 unity.In the equation $4x^2 + 4\sqrt{3}x + 3 = 0$,The coefficient of x^2 is 4.So to make the coffecient of x^2 equals to 1.divide the whole equation by 4.

The quadratic equation now becomes:

$$\frac{4x^2}{4} + \frac{4\sqrt{3}\,x}{4} + \frac{3}{4} = \frac{0}{4}$$

$$\Rightarrow x^2 + \sqrt{3}\,x + \frac{3}{4} = 0$$

Step 2: Shift the constant term on RHS,

$$\Rightarrow x^2 + \sqrt{3}\,x = -\frac{3}{4}$$

Step 3: Add square of half of coefficient of x on both the sides.

$$\Rightarrow x^2 + \sqrt{3}\,x + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$$

Step 4: Apply the formula,$(a + b)^2 = a^2 + 2ab + b^2$ on LHS and solve RHS,Here a=x and $b = \frac{\sqrt{3}}{2}$

$$\Rightarrow \left(x\ + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{\left(\sqrt{3}\right)^2}{(2)^2}$$

$$\Rightarrow \left(x\ + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{3}{4}$$

$$\Rightarrow \left(x\ + \frac{\sqrt{3}}{2}\right)^2 = 0$$

Step 5: As the RHS is zero, the roots exist.Since the quadratic equations have 2 roots, in this case both roots will be same.

$$\Rightarrow x\ = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

7. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$$\Rightarrow x^2 - 2 \times \left(\frac{3}{2\sqrt{2}}\right) \times x + \left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = 2 + \frac{9}{8} = \frac{25}{8}$$

⇒ x – 3/2√2 = ±5/2√2

⇒ x = 2√2, -1/√2

8. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Answer

Given: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

To find: the roots of the following quadratic (if they exist) by the method of completing the square.

Solution: We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

Step 1: Make the coefficient of x^2 unity.In the equation

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$,

The coefficient of x^2 is $\sqrt{3}$.

So to make the coffecient of x^2 equals to 1.

divide the whole equation by $\sqrt{3}$.

The quadratic equation now becomes:

$$\frac{\sqrt{3}x^2}{\sqrt{3}} + \frac{10x}{\sqrt{3}} + \frac{7\sqrt{3}}{\sqrt{3}} = \frac{0}{\sqrt{3}}$$

$$\Rightarrow x^2 + \frac{10x}{\sqrt{3}} + 7 = 0$$

Step 2: Shift the constant term on RHS,

$$x^2 + \frac{10}{\sqrt{3}} = -7$$

Step 3: Add square of half of coefficient of x on both the sides.

$$\Rightarrow x^2 + \frac{10x}{\sqrt{3}} + \left(\frac{5}{\sqrt{3}}\right)^2 = -7 + \left(\frac{5}{\sqrt{3}}\right)^2$$

Step 4: Apply the formula, $(a + b)^2 = a^2 + 2ab + b^2$ on LHS and solve RHS,Here a=x and $b = \frac{5}{\sqrt{3}}$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}} \right)^2 = -7 + \frac{(5)^2}{(\sqrt{3})^2}$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}} \right)^2 = -7 + \frac{25}{3}$$

$$\Rightarrow \left(x \ + \frac{5}{\sqrt{3}} \right)^2 = \frac{-21 + 25}{3}$$

$$\Rightarrow \left(x \ + \frac{5}{\sqrt{3}} \right)^2 = \frac{4}{3}$$

As RHS is positive, the roots exist.

Step 5: take square root on both sides,

$$\Rightarrow \left(x \ + \frac{5}{\sqrt{3}} \right) = \pm \frac{\sqrt{4}}{\sqrt{3}}$$

$$\Rightarrow \left(x \ + \frac{5}{\sqrt{3}} \right) = \pm \frac{2}{\sqrt{3}}$$

$$\Rightarrow x \ + \frac{5}{\sqrt{3}} = \frac{2}{\sqrt{3}} \ and \ x \ + \frac{5}{\sqrt{3}} = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow x = \frac{2}{\sqrt{3}} - \frac{5}{\sqrt{3}} \ and \ \ x = -\frac{2}{\sqrt{3}} - \frac{5}{\sqrt{3}}$$

$$\Rightarrow x = \frac{-3}{\sqrt{3}} \ and \ \ x = \frac{-7}{\sqrt{3}}$$

$$\Rightarrow x = -\sqrt{3} \ and \ \ x = -\frac{7}{\sqrt{3}}$$

9. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

$\Rightarrow x^2 - (\sqrt{2} + 1)x + ((\sqrt{2} + 1)/2)^2 - ((\sqrt{2} + 1)/2)^2 + \sqrt{2} = 0$

$\Rightarrow (x - (\sqrt{2} + 1)/2)^2 = (2 + 1 + 2\sqrt{2})/4 - \sqrt{2}$

$\Rightarrow (x - (\sqrt{2} + 1)/2)^2 = (2 + 1 - 2\sqrt{2})/4 = ((\sqrt{2} - 1)/2)^2$

$\Rightarrow x - (\sqrt{2} + 1)/2 = \pm(\sqrt{2} - 1)/2$

$\Rightarrow x = \sqrt{2}, 1$

10. Question

Find the roots of the following quadratic (if they exist) by the method of completing the square.

$x^2 - 4ax + 4a^2 - b^2 = 0$

Answer

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$x^2 - 4ax + 4a^2 - b^2 = 0$

$\Rightarrow x^2 - 2 \times 2ax + 4a^2 = b^2$

$\Rightarrow (x - 2a)^2 = b^2$

$\Rightarrow x - 2a = \pm b$

$\Rightarrow x = 2a + b, 2a - b$

Exercise 8.5

1. Question

Write the discriminant of the following quadratic equations:

(i) $2x^2 - 5x + 3 = 0$

(ii) $x^2 + 2x + 4 = 0$

(iii) $(x - 1)(2x - 1) = 0$

(iv) $x^2 - 2x + k = 0, k \in R$

(v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

(vi) $x^2 - x + 1 = 0$

Answer

(i) $2x^2 - 5x + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

$2x^2 - 5x + 3 = 0$

$\Rightarrow D = 25 - 4 \times 2 \times 3 = 1$

(ii) $x^2 + 2x + 4 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

Given, $x^2 + 2x + 4 = 0$

⇒ D = 4 − 4 × 4 × 1 = - 12

(iii) $(x-1)(2x-1) = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

Given, $(x-1)(2x-1) = 0$

⇒ $2x^2 - 3x + 1 = 0$

⇒ D = 9 − 4 × 2 × 1 = 1

(iv) $x^2 - 2x + k = 0, k \in R$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

$x^2 - 2x + k = 0, k \in R$

⇒ D = 4 − 4 × 1 × k = 4 − 4k

(v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

$\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

⇒ D = 8 − 4 × √3 × -2√3 = 32

(vi) $x^2 - x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

Given, $x^2 - x + 1 = 0$

⇒ D = 1 − 4 × 1 = -3

2. Question

In the following determine whether the given quadratic equations have real roots and if so, find the roots:

(i) $16x^2 = 24x + 1$

(ii) $x^2 + x + 2 = 0$

(iii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

(iv) $3x^2 - 2x + 2 = 0$

(v) $2x^2 - 2\sqrt{6}x + 3 = 0$

(vi) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

(vii) $3x^2 + 2\sqrt{5}x - 5 = 0$

(viii) $x^2 - 2x + 1 = 0$

(ix) $2x^2 + 5\sqrt{3}x + 6 = 0$

(x) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(xi) $2x^2 - 2\sqrt{2}x + 1 = 0$

(xii) $3x^2 - 5x + 2 = 0$

Answer

(i) $16x^2 = 24x + 1$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$16x^2 - 24x - 1 = 0$

$\Rightarrow D = 24 \times 24 + 4 \times 16 \times 1 = 640$

Roots are real.

$$x = \frac{24 \pm \sqrt{576 + 4 \times 16}}{32}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{10}}{4}$$

(ii) $x^2 + x + 2 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$x^2 + x + 2 = 0$

$\Rightarrow D = 1 - 4 \times 2 = -7$

Roots are not real

(iii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

$\Rightarrow D = 100 + 4 \times 8\sqrt{3} \times \sqrt{3} = 196$

Roots are real

$$x = \frac{-10 \pm \sqrt{100 + 4 \times 8\sqrt{3} \times \sqrt{3}}}{2\sqrt{3}}$$

⇒ x = (-10 ± 14)/2√3

⇒ x = -4√3, 2/√3

(iv) $3x^2 - 2x + 2 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$3x^2 - 2x + 2 = 0$

⇒ D = 4 − 4 × 2 × 3 = - 20

Roots are not real

(v) $2x^2 - 2\sqrt{6}x + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$2x^2 - 2\sqrt{6}x + 3 = 0$

⇒ D = 4 × 6 − 4 × 3 × 2 = 0

Roots are equal

$$x = \frac{2\sqrt{6} \pm \sqrt{24 - 4 \times 3 \times 2}}{4} = \sqrt{\frac{3}{2}}$$

(vi) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

⇒ $D = 64a^2b^2 - 4 \times 3a^2 \times 4b^2 = 16a^2b^2$

$$X = \frac{-8ab \pm \sqrt{64a^2b^2 - 48a^2b^2}}{6a^2}$$

$$\Rightarrow x = \frac{-8ab \pm 4ab}{6a^2} = -\frac{2b}{a}, -\frac{2b}{3a}$$

(vii) $3x^2 + 2\sqrt{5}x - 5 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$3x^2 + 2\sqrt{5}x - 5 = 0$

$D = 20 + 4 \times 5 \times 3 = 80$

$$\Rightarrow x = \frac{-2\sqrt{5} \pm \sqrt{20 - 4\times3\times-5}}{6}$$

$$\Rightarrow x = \frac{-2\sqrt{5} \pm 4\sqrt{5}}{6}$$

$\Rightarrow x = -\sqrt{5}, \sqrt{5}/3$

(viii) $x^2 - 2x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$x^2 - 2x + 1 = 0$

$\Rightarrow D = 4 - 4 \times 1 \times 1 = 0$

Roots are equal

$x = (2 \pm \sqrt{(4 - 4)})/2 = 1$

(ix) $2x^2 + 5\sqrt{3}x + 6 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$2x^2 + 5\sqrt{3}x + 6 = 0$

$\Rightarrow D = 75 - 4 \times 2 \times 6 = 27$

$$x = \frac{-5\sqrt{3} \pm \sqrt{27}}{4}$$

$$\Rightarrow x = \frac{-5\sqrt{3} \pm 3\sqrt{3}}{4}$$

⇒ x = -2√3, -√3/2

(x) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

⇒ D = 49 – 4 × 5√2 × √2 = 9

$$x = \frac{-7 \pm \sqrt{9}}{2\sqrt{2}} = \frac{-7 \pm 3}{2\sqrt{2}}$$

⇒ x = -5/√2 , -√2

(xi) $2x^2 - 2\sqrt{2}x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$2x^2 - 2\sqrt{2}x + 1 = 0$

$D = (2\sqrt{2})^2 - 4 \times 2 \times 1$

⇒ D = 8 – 8 = 0

Roots are equal

x = (2√2 ±0)/4

⇒ x = 1/√2, 1/√2

(xii) $3x^2 - 5x + 2 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$3x^2 - 5x + 2 = 0$

⇒ D = 25 – 4 × 3 × 2 = 1

x = (5 ± √1)/6

⇒ x = (5 ± 1)/6

⇒ x = 1, 2/3

3. Question

Solve for x:

(i) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3};x\neq 2,4$

(ii) $\frac{1}{x}-\frac{1}{x-2}=3,x\neq 0,2$

(iii) $x+\frac{1}{x}=3,x\neq 0$

(iv) $\frac{16}{x}-1=\frac{15}{x+1},x\neq 0,-1$

Answer

(i) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3};x\neq 2,4$

$$\frac{x-1}{x-2}+\frac{x-3}{x-4}=\frac{10}{3}$$

$$3\left(\frac{x-1}{x-2}+\frac{x-3}{x-4}\right)=10$$

$$3\left(\frac{x-1}{x-2}\right)+3\left(\frac{x-3}{x-4}\right)=10$$

LCM of the denominator is (x-2)(x-4)Now further solve it,

⇒ 3(x– 1)(x – 4) + 3(x – 2)(x – 3) = 10(x – 2)(x – 4)

⇒ $3(x^2-4x-x+4)+3(x^2-3x-2x+6)=10(x^2-4x-2x+8)$⇒ $3(x^2-5x+4)+3(x^2-6x+6)=10(x^2-6x+8)$

⇒ $3x^2+12-15x+3x^2+18-18x=10x^2-60x+80$

⇒ $6x^2-30x+30=10x^2-60x+80$⇒ $6x^2-10x^2-30x+60x+30-80=0$⇒ $-4x^2+30x-50=0$

⇒ $4x^2-30x+50=0$

⇒ $2x^2-15x+25=0$

Factorize it by splitting the middle term.

⇒ $2x^2-10x-5x+25=0$

⇒ 2x(x – 5) – 5(x – 5) = 0

⇒ (2x – 5)(x – 5) = 0⇒2x – 5 = 0 $x=\frac{5}{2}$

⇒ x – 5 = 0 x=5

Thus, $x=5,\frac{5}{2}$

(ii) $\frac{1}{x}-\frac{1}{x-2}=3,x\neq 0,2$

LCM of the denominator is x (x-2)

$$\Rightarrow \frac{x-2-x}{x(x-2)}=3$$

⇒ x – 2 – x = 3x(x-2)

⇒ x – 2 – x = $3x^2$ – 6x

⇒ $3x^2$ – 6x + 2 = 0

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a=3, b=-6, c=2

$$\Rightarrow x=\frac{6\pm\sqrt{36-24}}{6}=\frac{6\pm\sqrt{12}}{6}$$

$$\Rightarrow x=\frac{3\pm\sqrt{3}}{3}$$

(iii) $x+\frac{1}{x}=3, x\neq 0$

LCM of denominators is x.

$$\Rightarrow\frac{x^2+1}{x}=3$$

⇒ x^2 + 1 = 3x

⇒ x^2 – 3x + 1 = 0

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{3\pm\sqrt{3^2-4}}{2}$$

$$x=\frac{3\pm\sqrt{9-4}}{2}$$

$$x=\frac{3\pm\sqrt{5}}{2}$$

(iv) $\frac{16}{x}-1=\frac{15}{x+1}, x\neq 0,-1$

$$\frac{16-x}{x}=\frac{15}{x+1}$$

Use cross multiplication to get, (16 – x)(x + 1) = 15x

⇒ 16x – x^2 + 16 – x = 15x

⇒ 16x –15x - x^2 + 16 – x = 0⇒ - x^2 + 16 = 0

⇒ x^2 = 16

⇒ x = ±4

Exercise 8.6

1. Question

Determine the nature of the roots of the following quadratic equations:

(i) $2x^2 - 3x + 5 = 0$

(ii) $2x^2 - 6x + 3 = 0$

(iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$

(vi) $(x - 2a)(x - 2b) = 4ab$

(vii) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$, $a \neq 0, b \neq 0$

(viii) $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

(ix) $(b + c)x^2 - (a + b + c)x + a = 0$

Answer

(i) $2x^2 - 3x + 5 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$2x^2 - 3x + 5 = 0$

⇒ D = 9 − 4 × 5 × 2 = -31

Roots are not real.

(ii) $2x^2 - 6x + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$2x^2 - 6x + 3 = 0$

⇒ D = 36 − 4 × 2 × 3 = 12

Roots are real and distinct.

(iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

$\Rightarrow D = 4/9 - 4 \times 3/5 \times 1 = -88/45$

Roots are not real.

(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$3x^2 - 4\sqrt{3}x + 4 = 0$

$\Rightarrow D = 48 - 4 \times 3 \times 4 = 0$

Roots are real and equal

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow D = 24 - 4 \times 3 \times 2 = 0$

Roots are real and equal.

(vi) $(x - 2a)(x - 2b) = 4ab$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D < 0, roots are not real

If D > 0, roots are real and unequal

If D = 0, roots are real and equal

$(x - 2a)(x - 2b) = 4ab$

$\Rightarrow x^2 - (2a + 2b)x + 4ab = 4ab$

$\Rightarrow x^2 - (2a + 2b)x = 0$

$D = (2a + 2b)^2 - 0 = (2a + 2b)^2$

Roots are real and distinct

(vii) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$, $a \neq 0, b \neq 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$

$\Rightarrow D = 576a^2b^2c^2d^2 - 4 \times 16 \times 9 \times a^2b^2c^2d^2 = 0$

Roots are real and equal

(viii) $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

$\Rightarrow D = 4(a + b)^2 - 4 \times 2 \times (a^2 + b^2)$

$\Rightarrow D = -4(a^2 + b^2) + 2ab = -(a - b)^2 - 3(a^2 + b^2)$

Roots are not real

(ix) $(b + c)x^2 - (a + b + c)x + a = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$(b + c)x^2 - (a + b + c)x + a = 0$

$\Rightarrow D = (a + b + c)^2 - 4a(b + c)$

$\Rightarrow D = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$

$\Rightarrow D = (a - b - c)^2$

Thus, roots are real and unequal

2. Question

Find the values of k for which the roots are real and equal in each of the following equations:

(i) $kx^2 + 4x + 1 = 0$

(ii) $kx^2 - 2\sqrt{5}x + 4 = 0$

(iii) $3x^2 - 5x + 2k = 0$

(iv) $4x^2 + kx + 9 = 0$

(v) $2kx^2 - 40x + 25 = 0$

(vi) $9x^2 - 24x + k = 0$

(vii) $4x^2 - 3kx + 1 = 0$

(viii) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

(ix) $(3k + 1)x^2 + 2(k + 1)x + k = 0$

(x) $kx^2 + kx + 1 = -4x^2 - x$

(xi) $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$

(xii) $x^2 - 2kx + 7x + 1/4 = 0$

(xiii) $(k + 1)x^2 - 2(3k + 1)x + 8k + 1 = 0$

(xiv) $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$

(xv) $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$

(xvi) $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$

(xvii) $4x^2 - 2(k + 1)x + (k + 4) = 0$

(xviii) $x^2 - 2(k + 1)x + k^2 = 0$

(xix) $k^2x^2 - 2(k - 1)x + 4 = 0$

(xx) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

(xxi) $2x^2 + kx + 3 = 0$

(xxii) $kx(x - 2) + 6 = 0$

(xxiii) $x^2 - 4kx + k = 0$

(xxiv) $kx(x - 2\sqrt{5}) + 10 = 0$

(xxv) $px(x - 3) + 9 = 0$

(xxvi) $4x^2 + px + 3 = 0$

Answer

(i) $kx^2 + 4x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$kx^2 + 4x + 1 = 0$

$\Rightarrow D = 16 - 4k = 0$

⇒ k = 4

(ii) $kx^2 - 2\sqrt{5}x + 4 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$kx^2 - 2\sqrt{5}x + 4 = 0$

⇒ D = 4 × 5 − 4 × 4k = 0

⇒ k = 5/4

(iii) $3x^2 - 5x + 2k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$3x^2 - 5x + 2k = 0$

⇒ D = 25 − 4 × 3 × 2k = 0

⇒ k = 25/24

(iv) $4x^2 + kx + 9 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$4x^2 + kx + 9 = 0$

⇒ D = k^2 − 4 × 4 × 9 = 0

⇒ k^2 − 144 = 0

⇒ k = ±12

(v) $2kx^2 - 40x + 25 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$2kx^2 - 40x + 25 = 0$

⇒ 1600 − 4 × 2k × 25 = 0

⇒ k = 8

(vi) $9x^2 - 24x + k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$9x^2 - 24x + k = 0$

⇒ D = 576 – 4 × 9 × k = 0

⇒ k = 576/36 = 16

(vii) $4x^2 - 3kx + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$4x^2 - 3kx + 1 = 0$

⇒ $D = 9k^2 - 4 \times 4 \times 1 = 0$

⇒ $9k^2 = 16$

⇒ k = ±4/3

(viii) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

⇒ $D = 4(5 + 2k)^2 - 4 \times 3(7 + 10k) = 0$

⇒ $100 + 16k^2 + 80k - 84 - 120k = 0$

⇒ $16k^2 - 40k + 16 = 0$

⇒ $2k^2 - 5k + 2 = 0$

⇒ $2k^2 - 4k - k + 2 = 0$

⇒ $2k(k - 2) - (k - 2) = 0$

⇒ $(2k - 1)(k - 2) = 0$

⇒ k = 2, 1/2

(ix) $(3k + 1)x^2 + 2(k + 1)x + k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$(3k + 1)x^2 + 2(k + 1)x + k = 0$

⇒ $D = 4(k + 1)^2 - 4k(3k + 1) = 0$

⇒ $4k^2 + 8k + 4 - 12k^2 - 4k = 0$

⇒ $2k^2 - k - 1 = 0$

⇒ $2k^2 - 2k + k - 1 = 0$

⇒ $2k(k - 1) + (k - 1) = 0$

⇒ (2k + 1)(k – 1) = 0

⇒ k = 1, -1/2

(x) $kx^2 + kx + 1 = -4x^2 - x$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$kx^2 + kx + 1 = -4x^2 - x$

⇒ $(k + 4)x^2 + (k + 1)x + 1 = 0$

$D = (k + 1)^2 - 4(k + 4) = 0$

⇒ $k^2 + 2k + 1 - 4k - 16 = 0$

⇒ $k^2 - 2k - 15 = 0$

⇒ $k^2 - 5k + 3k - 15 = 0$

⇒ k(k – 5) + 3(k – 5) = 0

⇒ (k + 3)(k – 5) = 0

⇒ k = 5, -3

(xi) $(k+1)x^2 + 2(k+3)x + (k+8) = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$(k+1)x^2 + 2(k+3)x + (k+8) = 0$

⇒ $D = 4(k + 3)^2 - 4(k + 1)(k + 8) = 0$

⇒ $4k^2 + 36 + 24k - 4k^2 - 32 - 36k = 0$

⇒ 12k = 4

⇒ k = 1/3

(xii) $x^2 - 2kx + 7x + 1/4 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$x^2 - 2kx + 7x + 1/4 = 0$

⇒ $D = (7 - 2k)^2 - 4 \times 1/4 = 0$

⇒ $49 + 4k^2 - 28k - 1 = 0$

⇒ $k^2 - 7k + 12 = 0$

⇒ $k^2 - 4k - 3k + 12 = 0$

⇒ k(k – 4) – 3(k – 4) = 0

⇒ (k – 3)(k – 4) = 0

⇒ k = 3, 4

(xiii) $(k+1)x^2-2(3k+1)x+8k+1=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 – 4ac$

If D = 0, roots are real and equal

$(k+1)x^2-2(3k+1)x+8k+1=0$

⇒ $D = 4(3k + 1)^2 – 4(k + 1)(8k + 1) = 0$

⇒ $4 \times (9k^2 + 6k + 1) – 32k^2 – 4 – 36k = 0$

⇒ $36k^2 + 24k + 4 – 32k^2 – 4 – 36k = 0$

⇒ 4k(k – 3) = 0

⇒ k = 0, 3

(xiv) $5x^2-4x+2+k\left(4x^2-2x-1\right)=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 – 4ac$

If D = 0, roots are real and equal

$5x^2-4x+2+k\left(4x^2-2x-1\right)=0$

⇒ $(5 + 4k)x^2 – (4 + 2k)x + 2 – k = 0$

⇒ $D = (4 + 2k)^2 – 4 \times (5 + 4k)(2 – k) = 0$

⇒ $16 + 4k^2 + 16k + 16k^2 – 12k – 40 = 0$

⇒ $20k^2 – 4k – 24 = 0$

⇒ $5k^2 - k - 6 = 0$

⇒ $5k^2 – 6k + 5k – 6 = 0$

⇒ k(5k – 6) + (5k – 6) = 0

⇒ (k + 1)(5k – 6) = 0

⇒ k = -1, 6/5

(xv) $(4-k)x^2+(2k+4)x+(8k+1)=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 – 4ac$

If D = 0, roots are real and equal

$(4-k)x^2+(2k+4)x+(8k+1)=0$

⇒ $D = (2k + 4)^2 – 4 \times (4 – k)(8k + 1) = 0$

⇒ $4k^2 + 16 + 16k + 32k^2 – 16 – 124k = 0$

⇒ $36k^2 - 108k = 0$

⇒ $36k(k - 3) = 0$

⇒ $k = 0, 3$

(xvi) $(2k+1)x^2+2(k+3)x+(k+5)=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$(2k+1)x^2+2(k+3)x+(k+5)=0$

⇒ $D = 4(k + 3)^2 - 4 \times (2k + 1)(k + 5) = 0$

⇒ $4k^2 + 36 + 24k - 8k^2 - 20 - 44k = 0$

⇒ $-4k^2 - 20k + 16 = 0$

⇒ $k^2 + 5k - 4 = 0$

$$\Rightarrow k = \frac{-5 \pm \sqrt{25 + 4\times4}}{2} = \frac{-5 \pm \sqrt{41}}{2}$$

(xvii) $4x^2-2(k+1)x+(k+4)=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$4x^2-2(k+1)x+(k+4)=0$

⇒ $D = 4(k + 1)^2 - 4 \times 4(k + 4) = 0$

⇒ $4k^2 + 8k + 4 - 16k - 64 = 0$

⇒ $k^2 - 2k - 15 = 0$

⇒ $k^2 - 5k + 3k - 15 = 0$

⇒ $(k - 5)(k + 3) = 0$

⇒ $k = -3, 5$

(xviii) $x^2-2(k+1)x+k^2=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$x^2-2(k+1)x+k^2=0$

⇒ $D = 4(k + 1)^2 - 4k^2 = 0$

⇒ $4k^2 + 8k + 4 - 4k^2 = 0$

⇒ $k = -1/2$

(xix) $k^2x^2-2(k-1)x+4=0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$k^2x^2 - 2(k-1)x + 4 = 0$

$\Rightarrow D = 4(k-1)^2 - 4 \times 4k^2 = 0$

$\Rightarrow 4k^2 - 8k + 4 - 16k^2 = 0$

$\Rightarrow 12k^2 + 8k - 4 = 0$

$\Rightarrow 3k^2 + 2k - 1 = 0$

$\Rightarrow 3k^2 + 3k - k - 1 = 0$

$\Rightarrow 3k(k+1) - (k+1) = 0$

$\Rightarrow (3k-1)(k+1) = 0$

$\Rightarrow k = 1/3, -1$

(xx) $(k+1)x^2 - 2(k-1)x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$(k+1)x^2 - 2(k-1)x + 1 = 0$

$\Rightarrow D = 4(k-1)^2 - 4 \times (k+1) = 0$

$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 0$

$\Rightarrow 4k(k-3) = 0$

$\Rightarrow k = 0, 3$

(xxi) $2x^2 + kx + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$2x^2 + kx + 3 = 0$

$\Rightarrow D = k^2 - 4 \times 2 \times 3 = 0$

$\Rightarrow k^2 = = 24$

$\Rightarrow k = \pm 2\sqrt{6}$

(xxii) $kx(x-2) + 6 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$kx(x-2) + 6 = 0$

⇒ $kx^2 - 2kx + 6 = 0$

⇒ $D = 4k^2 - 4 \times 6 \times k = 0$

⇒ $4k(k - 6) = 0$

⇒ k = 0, 6 but k can't be 0 a it is the coefficient of x^2, thus k = 6

(xxiii) $x^2 - 4kx + k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$x^2 - 4kx + k = 0$

⇒ $D = 16k^2 - 4k = 0$

⇒ $4k(4k - 1) = 0$

⇒ k = 0, 1/4

(xxiv) $kx\left(x - 2\sqrt{5}\right) + 10 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$kx\left(x - 2\sqrt{5}\right) + 10 = 0$

⇒ $kx^2 - 2\sqrt{5}kx + 10 = 0$

⇒ $D = 4 \times 5k^2 - 4 \times k \times 10 = 0$

⇒ $k^2 = 2k$

⇒ k = 2

(xxv) $px(x - 3) + 9 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$px(x - 3) + 9 = 0$

⇒ $px^2 - 3px + 9 = 0$

⇒ $D = 9p^2 - 4 \times 9 \times p = 0$

⇒ p = 4

(xxvi) $4x^2 + px + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are real and equal

$4x^2 + px + 3 = 0$

$\Rightarrow D = p^2 - 4 \times 4 \times 3 = 0$

$\Rightarrow p^2 = 48$

$\Rightarrow p = \pm 4\sqrt{3}$

3. Question

In the following, determine the set of values of k for which the given quadratic equation has real roots:

(i) $2x^2 + 3x + k = 0$

(ii) $2x^2 + kx + 3 = 0$

(iii) $2x^2 - 5x - k = 0$

(iv) $kx^2 + 6x + 1 = 0$

(v) $x^2 - kx + 9 = 0$

(vi) $2x^2 + kx + 2 = 0$

(vii) $3x^2 + 2x + k = 0$

(viii) $4x^2 - 3kx + 1 = 0$

(ix) $2x^2 + kx - 4 = 0$

Answer

(i) $2x^2 + 3x + k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$2x^2 + 3x + k = 0$

$\Rightarrow D = 9 - 4 \times 2 \times k$

$\Rightarrow 9 - 8k \geq 0$

$\Rightarrow k \leq 9/8$

(ii) $2x^2 + kx + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$2x^2 + kx + 3 = 0$

$\Rightarrow D = k^2 - 4 \times 2 \times 3$

$D \geq 0$

$\Rightarrow k^2 - 24 \geq 0$

$\Rightarrow (k + 2\sqrt{6})(k - 2\sqrt{6}) \geq 0$

Thus, $k \leq -2\sqrt{6}$ or $k \geq 2\sqrt{6}$

(iii) $2x^2 - 5x - k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$2x^2 - 5x - k = 0$

$\Rightarrow D = 25 - 8k$

$D \geq 0$

$\Rightarrow 25 - 8k \geq 0$

$\Rightarrow k \leq 25/8$

(iv) $kx^2 + 6x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$kx^2 + 6x + 1 = 0$

$\Rightarrow D = 36 - 4k$

$\Rightarrow 36 - 4k \geq 0$

$\Rightarrow k \leq 9$

(v) $x^2 - kx + 9 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$x^2 - kx + 9 = 0$

$\Rightarrow D = k^2 - 36$

$\Rightarrow k^2 - 36 \geq 0$

$\Rightarrow (k - 6)(k + 6) \geq 0$

$\Rightarrow k \geq 6$ or $k \leq -6$

(vi) $2x^2 + kx + 2 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$2x^2 + kx + 2 = 0$

$\Rightarrow D = k^2 - 4 \times 4$

$\Rightarrow k^2 - 16 \geq 0$

$\Rightarrow (k + 4)(k - 4) \geq 0$

$\Rightarrow k \geq 4$ or $k \leq -4$

(vii) $3x^2 + 2x + k = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

D = b^2 – 4ac

If D ≥ 0, roots are real

$3x^2 + 2x + k = 0$

⇒ D = 4 – 12k

⇒ 4 – 12k ≥ 0

⇒ k ≤ 1/3

(viii) $4x^2 - 3kx + 1 = 0$

For a quadratic equation, ax^2 + bx + c = 0,

D = b^2 – 4ac

If D ≥ 0, roots are real

$4x^2 - 3kx + 1 = 0$

⇒ D = $9k^2$ – 16

⇒ $9k^2$ – 16 ≥ 0

⇒ (3k – 4)(3k + 4) ≥ 0

⇒ k ≤ -4/3 or k ≥ (4/3)

(ix) $2x^2 + kx - 4 = 0$

For a quadratic equation, ax^2 + bx + c = 0,

D = b^2 – 4ac

If D ≥ 0, roots are real

$2x^2 + kx - 4 = 0$

⇒ D = k^2 + 4 × 2 × 4 = k^2 + 32

Thus, D is always greater than 0 for all values of k.

4. Question

For what value of k, $(4-k)x^2 + (2k+4)x + (8k+1) = 0$, is a perfect square.

Answer

$(4-k)x^2 + (2k+4)x + (8k+1) = 0$

For the above expression to be a perfect square, D = b^2 – 4ac = 0

⇒ $(2k + 4)^2$ – 4 × (4 – k)(8k + 1) = 0

⇒ $4k^2$ + 16k + 16 + $32k^2$ – 124k – 16 = 0

⇒ $36k^2$ – 108k = 0

⇒ 36k(k – 3) = 0

⇒ k = 0, 3

5. Question

Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$x^2 + kx + 4 = 0$

$\Rightarrow D = k^2 - 16$

Thus, $k^2 - 16 \geq 0$

$\Rightarrow k \geq 4$ or $k \leq -4$

Thus, least positive value of k is 4.

6. Question

Find the values of k for which the given quadratic equation has real and distinct roots:

(i) $kx^2 + 2x + 1 = 0$

(ii) $kx^2 + 6x + 1 = 0$

(iii) $x^2 - kx + 9 = 0$

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and distinct

$kx^2 + 2x + 1 = 0$

$\Rightarrow D = 4 - 4k$

$\Rightarrow 4 - 4k > 0$

$\Rightarrow k < 1$

(ii) $kx^2 + 6x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and distinct

$kx^2 + 6x + 1 = 0$

$\Rightarrow D = 36 - 4k$

$\Rightarrow 36 - 4k > 0$

$\Rightarrow k < 9$

(iii) $x^2 - kx + 9 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and distinct

$x^2 - kx + 9 = 0$

⇒ $D = k^2 - 36$

⇒ $k^2 - 36 > 0$

⇒ $(k + 6)(k - 6) > 0$

⇒ $k < -6$ or $k > 6$

7. Question

If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then prove that 2b = a + c.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal

$(b-c)x^2 + (c-a)x + (a-b) = 0$

⇒ $(c - a)^2 - 4(b - c)(a - b) = 0$

⇒ $c^2 + a^2 - 2ac + 4b^2 - 4ab - 4cb + 4ac = 0$

⇒ $a^2 + 4b^2 + c^2 + 2ac - 4ab - 4bc = 0$

⇒ $(a - 2b + c)^2 = 0$

⇒ 2b = a + c

8. Question

If the roots of the equation $(a^2 + b^2)x^2 - 2(ab + cd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal. For the given equation D would be -

$(a^2 + b^2) - 2(ab + cd) + (c^2 + d^2) = 0$

⇒ $4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

⇒ $a^2c^2 + b^2d^2 + 2acbd - a^2d^2 - a^2c^2 - b^2d^2 - b^2c^2 = 0$

⇒ $a^2d^2 + b^2c^2 - 2abcd = 0$

⇒ $(ad - bc) = 0$

⇒ a/b = c/d

9. Question

If the roots of the equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real, then prove that $b^2 = ac$.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D ≥ 0, roots are real

$ax^2 + 2bx + c = 0$

⇒ $4b^2 - 4ac \geq 0$

⇒ $b^2 \geq ac$ ------ (1)

$bx^2 - 2\sqrt{ac}x + b = 0$

⇒ $4ac - 4b^2 \geq 0$

⇒ $b^2 \leq ac$ ----- (2)

For both (1) and (2) to be true

⇒ $b^2 = ac$

10. Question

If p, q are real and p ≠ q, then show that the roots of the equation $(p-q)x^2 + 5(p+q)x - 2(p-q) = 0$ are real and unequal

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D > 0, roots are real and unequal.

$(p-q)x^2 + 5(p+q)x - 2(p-q) = 0$

⇒ $D = 25(p + q)^2 + 8(p - q)^2$

Thus D > 0 for all p and q as sum of two squares is always positive.

11. Question

If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal, prove that either a = 0 or $a^3 + b^3 + c^3 = 3abc$.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal

Given, roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal.

∴ D = 0

⇒ $[2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

⇒ $4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

⇒ $a^4 + b^2c^2 - 2a^2bc - b^2c^2 - a^2bc + ab^3 + ac^3 = 0$

⇒ $a(a^3 + b^3 + c^3 - 3abc) = 0$

⇒ a = 0 or $a^3 + b^3 + c^3 = 3abc$

12. Question

Show that the equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ has no real roots, when $a \neq b$.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real.

$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

$\Rightarrow D = 4(a + b)^2 - 8(a^2 + b^2)$

$\Rightarrow D = 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$\Rightarrow D = -4(a^2 + b^2 - 2ab) = -4(a - b)^2$

Thus, $D < 0$ for all values of a and b.

$\therefore$ Roots are not real.

13. Question

Prove that both the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are real but they are equal only when $a = b = c$.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real.

$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow x^2 - (a + b)x + ab + x^2 - (b + c)x + bc + x^2 - (a + c)x + ac = 0$

$\Rightarrow 3x^2 - 2(a + b + c)x + ab + bc + ac = 0$

$\Rightarrow D = 4(a + b + c)^2 - 12(ab + bc + ac)$

$\Rightarrow D = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc - 3ab - 3bc - 3ac$

$\Rightarrow D = 1/2 \times (2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc)$

$\Rightarrow D = 1/2 \times ((a - b)^2 + (b - c)^2 + (c - a)^2)$

Thus, D is always greater than 0, and the roots are real

Now, when $a = b = c$,

$D = 0$, thus the roots are equal when $a = b = c$.

14. Question

If a, b, c are real numbers such that $ac \neq 0$, then show that at least one of the equations $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ has real roots.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real.

$ax^2 + bx + c = 0$

⇒ D = b^2 – 4ac

If D > 0 then b^2 > 4ac -------- (1)

$-ax^2 + bx + c = 0$

⇒ D = b^2 + 4ac

If D > 0, b^2 > - 4ac --------- (2)

If (1) is true then (2) is false and vice versa

Thus, one of the equation has real roots.

15. Question

If the equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$.

Answer

For a quadratic equation, ax^2 + bx + c = 0,

D = b^2 – 4ac

If D = 0, roots are equal

$(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

⇒ D = $4m^2c^2 - 4(c^2 - a^2)(1 + m^2) = 0$

⇒ $m^2c^2 - c^2 + a^2 - c^2m^2 + a^2m^2 = 0$

⇒ $c^2 = a^2(1 + m^2)$

16. Question

Find the values of k for which the quadratic equation $(3k+1)x^2 + 2(k+1)x + 1 = 0$ has equal roots. Also, find these roots.

Answer

For a quadratic equation, ax^2 + bx + c = 0,

D = b^2 – 4ac

If D = 0, roots are equal

$(3k+1)x^2 + 2(k+1)x + 1 = 0$

⇒ D = $4(k + 1)^2 - 4(3k + 1) = 0$

⇒ $k^2 + 2k + 1 - 3k - 1 = 0$

⇒ k(k – 1) = 0

⇒ k = 0, 1

When k = 0,

Eq. – x^2 + 2x + 1 = 0

⇒ $(x + 1)^2 = 0$

⇒ x = -1

When k = 1,

Eq. – $4x^2$ + 4x + 1 = 0

⇒ $(2x + 1)^2 = 0$

⇒ x = -1/2

17. Question

Find the values of p for which the quadratic equation $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$ has equal roots. Also, find these roots.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal

$(2p+1)x^2 - (7p+2)x + (7p-3) = 0$

⇒ $D = (7p + 2)^2 - 4(7p - 3)(2p + 1) = 0$

⇒ $49p^2 + 28p + 4 - 56p^2 + 12 - 4p = 0$

⇒ $7p^2 - 24p - 16 = 0$

⇒ $7p^2 - 28p + 4p - 16 = 0$

⇒ 7p(p − 4) + 4(p − 4) = 0

⇒ (7p + 4)(p − 4) = 0

⇒ p = -4/7, 4

18. Question

If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal

Given, -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$

⇒ 2 × 25 − 5p − 15 = 0

⇒ 35 = 5p

⇒ p = 7

Now, the quadratic equation $p(x^2 + x) + k = 0$ has equal roots

⇒ $7x^2 + 7x + k = 0$ has equal roots

⇒ D = 49 − 28k = 0

⇒ k = 49/28 = 7/4

19. Question

If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal

Given, 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$

$\Rightarrow 3 \times 4 + 2p - 8 = 0$

$\Rightarrow 2p = -4$

$\Rightarrow p = -2$

Now, the quadratic equation $4x^2 - 2px + k = 0$ has equal roots

$\Rightarrow 4x^2 + 4x + k = 0$ has equal roots

$\Rightarrow D = 16 - 16k = 0$

$\Rightarrow k = 1$

20. Question

If 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$ and the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots, find the value of b.

Answer

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If D = 0, roots are equal

Given, 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$

$\Rightarrow 3 + a - 2 = 0$

$\Rightarrow a = -1$

Now, the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots

$\Rightarrow x^2 + 6x + b = 0$ has equal roots

$\Rightarrow D = 36 - 4b = 0$

$\Rightarrow b = 9$

21. Question

Find the value of p for which the quadratic equation: $(p+1)x^2 - 6(p+1)x + 3(p + 9) = 0$, where, $p \neq -1$ has equal roots. Hence, find the roots of the equation.

Answer

Note: For a quadratic equation, $ax^2 + bx + c = 0$, we have $D = b^2 - 4ac$.

If D = 0, then the roots of the quadratic equation are equal.

Therefore, $(p+1)x^2 - 6(p+1)x + 3(p + 9) = 0$ will have equal roots when,

$\Rightarrow D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

Here, $b = -6(p+1)$,

$a = (p+1)$

and, $c = 3(p+9) \Rightarrow \{-6(p + 1)\}^2 = 4\times(p + 1)\times3(p + 9)$

$\Rightarrow 36(p+1)(p+1) = 12(p + 1)(p + 9) \Rightarrow 3(p+1)=(p + 9)$

$\Rightarrow 3p + 3 - p - 9 = 0$

$\Rightarrow 2p - 6 = 0$

$\Rightarrow p = 6/2$

$\Rightarrow p = 3$

Thus, the value of p is 3

Now, putting the value of p in $(p+1)x^2 - 6(p+1)x + 3(p + 9) = 0$, we get,

$\Rightarrow 4x^2 - 24x + 36 = 0$

On taking 4 common, we get,

$\Rightarrow x^2 – 6x + 9 = 0$

$\Rightarrow (x - 3)^2 = 0$

$\Rightarrow x = 3$

Thus, the root of the given equation is $x = 3$

Exercise 8.7

1. Question

Find two consecutive numbers whose squares have the sum of 85

Answer

Let the consecutive numbers be ‘a’ and a + 1.

Given, sum of squares is 85

$\Rightarrow a^2 + (a+ 1)^2 = 85$

$\Rightarrow a^2 + a^2 + 2a + 1 = 85$

$\Rightarrow a^2 + a – 42 = 0$

$\Rightarrow a^2 + 7a – 6a – 42 = 0$

$\Rightarrow a(a + 7) – 6(a + 7) = 0$

$\Rightarrow (a – 6)(a + 7) = 0$

$\Rightarrow a = 6, -7$

Numbers are, 6, 7 or -7, -6

2. Question

Divide 29 into two parts so that the sum of the squares of the parts is 425.

Answer

Let one of the number be ‘a’.

Given, sum of two numbers is 29 and the sum of their squares is 425

$\Rightarrow a^2 + (29 - a)^2 = 425$

$\Rightarrow a^2 + 841 + a^2 - 58a = 425$

$\Rightarrow a^2 - 29a + 416 = 0$

$\Rightarrow a^2 - 16a - 13a + 208 = 0$

$\Rightarrow a(a - 16) - 13(a - 16) = 0$

$\Rightarrow (a - 13)(a - 16) = 0$

$\Rightarrow a = 13, 16$

3. Question

Two squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. Find the sides of the squares.

Answer

Area of a square = side × side

Given, squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2

$\Rightarrow x^2 + (x + 4)^2 = 656$

$\Rightarrow x^2 + x^2 + 16 + 8x = 656$

$\Rightarrow x^2 + 4x - 320 = 0$

$\Rightarrow x^2 - 16x + 20x - 320 = 0$

$\Rightarrow x(x - 16) + 20(x - 16) = 0$

$\Rightarrow (x - 16)(x + 20) = 0$

$\Rightarrow x = 16, -20$

The sides are 16, 20.

4. Question

The sum of two numbers is 48 and their product is 432. Find the numbers.

Answer

Let the numbers be ‘a’ and ‘b’.

Given, sum of two numbers is 48 and their product is 432.

$\Rightarrow a + b = 48$

$\Rightarrow a = 48 - b$

Also, $ab = 432$

$\Rightarrow 48b - b^2 = 432$

$\Rightarrow b^2 - 48b + 432 = 0$

$\Rightarrow b^2 - 36b - 12b + 432 = 0$

$\Rightarrow b(b - 36) - 12(b - 36) = 0$

$\Rightarrow (b - 12)(b - 36) = 0$

$\Rightarrow b = 12, 36$

5. Question

If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

Answer

Let the integer be ‘a’.

Given, an integer is added to its square, the sum is 90

$\Rightarrow a + a^2 = 90$

$\Rightarrow a^2 + 10a - 9a - 90 = 0$

$\Rightarrow a(a + 10) - 9(a + 10) = 0$

$\Rightarrow (a - 9)(a + 10) = 0$

$\Rightarrow a = -10, 9$

6. Question

Find the whole number which when decreased by 20 is equal to 69 times the reciprocal of the number.

Answer

Let the number be ‘a’

$\Rightarrow a - 20 = 69/a$

$\Rightarrow a^2 - 20a - 69 = 0$

$\Rightarrow a^2 - 23a + 3a - 69 = 0$

$\Rightarrow a(a - 23) + 3(a - 23) = 0$

$\Rightarrow (a + 3)(a - 23) = 0$

$\Rightarrow a = 23$ or -3

Whole number is 23

7. Question

Find two consecutive natural numbers whose product is 20.

Answer

Let the consecutive numbers be a, a + 1.

$\Rightarrow a(a + 1) = 20$

$\Rightarrow a^2 + a - 20 = 0$

$\Rightarrow a^2 + 5a - 4a - 20 = 0$

$\Rightarrow a(a + 5) - 4(a + 5) = 0$

$\Rightarrow (a - 4)(a + 5) = 0$

$\Rightarrow a = 4$ as a is a natural number

Thus the numbers are 4 and 5.

8. Question

The sum of the squares of two consecutive odd positive integers is 394. Find then.

Answer

Let the consecutive odd integers be a, a + 2

$\Rightarrow a^2 + (a + 2)^2 = 394$

$\Rightarrow a^2 + a^2 + 4a + 4 = 394$

$\Rightarrow a^2 + 2a - 195 = 0$

$\Rightarrow a^2 + 15a - 13a - 195 = 0$

$\Rightarrow a(a + 15) - 13(a + 15) = 0$

$\Rightarrow (a - 13)(a + 15) = 0$

$\Rightarrow a = 13, -15$

The numbers are 13 and 15

9. Question

The sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8. Find the numbers.

Answer

Let the numbers be 'a' and 'b'.

Given, sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8.

$\Rightarrow a + b = 8$

$\Rightarrow a = 8 - b$

Also, $15 \times (1/a + 1/b) = 8$

$\Rightarrow 1/a + 1/b = 8/15$

$\Rightarrow 1/a + 1/(8 - a) = 8/15$

$\Rightarrow 15(8 - a + a) = 8(8a - a^2)$

$\Rightarrow a^2 - 8a + 15 = 0$

$\Rightarrow a^2 -5a - 3a + 15 = 0$

$\Rightarrow a(a - 5) -3(a - 5) = 0$

$\Rightarrow (a - 3)(a - 5) = 0$

$\Rightarrow a = 3, 5$

10. Question

The sum of a number and its positive square root is 6/25. Find the number.

Answer

Given: The sum of a number and its positive square root is 6/25. **To find:** the number.**Solution:**Let the number be 'a'.

$\Rightarrow a + \sqrt{a} = 6/25$

$\Rightarrow \sqrt{a} = (6/25) - a$

Squaring both sides

$\Rightarrow a = 36/625 + a^2 - 12a/25$

$\Rightarrow a^2 - 37a/25 + 36/625 = 0$factorise by splitting the middle term.

$\Rightarrow a^2 - a/25 - 36a/25 + 36/625 = 0$

$\Rightarrow a(a - 1/25) - (36/25) \times (a - 1/25) = 0$

$\Rightarrow (a - 36/25)(a - 1/25) = 0$

$\Rightarrow a = 36/25 , 1/25$

But only 1/25 is possible as its sum with its positive root is 6/25.Hence the number is 1/25.

11. Question

The sum of a number and its square is 63/4, find the numbers.

Answer

Let the number be 'a'

$\Rightarrow a + a^2 = 63/4$

$\Rightarrow 4a^2 + 4a - 63 = 0$

$\Rightarrow 4a^2 + 18a - 14a - 63 = 0$

$\Rightarrow 2a(2a + 9) - 7(2a + 9) = 0$

$\Rightarrow (2a - 7)(2a + 9) = 0$

$\Rightarrow a = 7/2$ or $- 9/2$

12. Question

There are three consecutive integers such that the square of the first increased by the product of the other two gives 154. What are the integers?

Answer

Let the three consecutive numbers be a, a + 1, a + 2

Given, there are three consecutive integers such that the square of the first increased by the product of the other two gives 154.

$\Rightarrow a^2 + (a + 1)(a + 2) = 154$

$\Rightarrow 2a^2 + 3a + 2 = 154$

$\Rightarrow 2a^2 + 3a - 152 = 0$

$\Rightarrow 2a^2 + 19a - 16a - 152 = 0$

$\Rightarrow a(2a + 19) - 8(2a + 19) = 0$

$\Rightarrow (a - 8)(2a + 19) = 0$

Thus, a = 8

Numbers are 8, 9, 10

13. Question

The product of two successive integral multiples of 5 is 300. Determine the multiples.

Answer

Let the successive integral multiples of 5 be a, a + 5.

$\Rightarrow a(a + 5) = 300$

$\Rightarrow a^2 + 5a - 300 = 0$

$\Rightarrow a^2 + 20a - 15a - 300 = 0$

$\Rightarrow a(a + 20) - 15(a + 20) = 0$

$\Rightarrow (a + 20)(a - 15) = 0$

$\Rightarrow a = -20, 15$

Numbers are, -20, -15 or 15, 20

14. Question

The sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number. Find the numbers.

Answer

Given: The sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number.**To find:** the numbers.**Solution:**Let one of the numbers be a.

Given, sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number.

2^{nd} number = 2a – 3

According to given condition,

$a^2 + (2a - 3)^2 = 233$Apply the formula $(x - y)^2 = x^2 + y^2 - 2xy$ on $(2a - 3)^2$

$\Rightarrow a^2 + 4a^2 + 9 - 12a = 233 \Rightarrow a^2 + 4a^2 + 9 - 12a - 233 = 0$

$\Rightarrow 5a^2 - 12a - 224 = 0$

$\Rightarrow 5a^2 - 40a + 28a - 224 = 0$

$\Rightarrow 5a(a - 8) + 28(a - 8) = 0$

$\Rightarrow (5a + 28)(a - 8) = 0 \Rightarrow (5a + 28) = 0$ and $(a - 8) = 0$

$\Rightarrow a = -28/5$ and $a=8$To satisfy the given conditions a will be 8.2^{nd} number = 2(8) – 3 = 16-3 = 13

Thus the numbers are 8, 13.

15. Question

Find the consecutive even integers whose squares have the sum 340.

Answer

Let the consecutive even integers be ‘a’ and a + 2

$\Rightarrow a^2 + (a + 2)^2 = 340$

$\Rightarrow 2a^2 + 4a - 336 = 0$

$\Rightarrow a^2 + 2a - 168 = 0$

$\Rightarrow a^2 + 14a - 12a - 168 = 0$

$\Rightarrow a(a + 14) - 12(a + 14) = 0$

$\Rightarrow (a - 12)(a + 14) = 0$

Thus, a = 12 or – 14

Consecutive even integers are 12, 14 or -14, - 12

16. Question

The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the numbers.

Answer

Let the numbers be ‘a’ and ‘b’.

Given, difference of two numbers is 4 and difference of their reciprocals is $\frac{4}{21}$

$\Rightarrow a - b = 4$

$\Rightarrow a = b + 4$

and $1/b - 1/a = 4/21$

$\Rightarrow 1/(b + 4) - 1/b = -4/21$

$\Rightarrow 21(b - b - 4) = -4(b^2 + 4b)$

$\Rightarrow b^2 + 4b - 21 = 0$

$\Rightarrow b^2 + 7b - 3b - 21 = 0$

$\Rightarrow b(b + 7) - 3(b + 7) = 0$

$\Rightarrow (b - 3)(b + 7) = 0$

$\Rightarrow b = 3, -7$

Numbers are , 3, 7 or -7, -3

17. Question

Find two natural numbers which differ by 3 and whose squares have the sum 117.

Answer

Let one of the natural numbers be ‘a’

Given, the numbers differ by 3.

$\Rightarrow 2^{nd}$ number $= a + 3$

$\Rightarrow a^2 + (a + 3)^2 = 117$

$\Rightarrow a^2 + a^2 + 6a + 9 = 117$

$\Rightarrow a^2 + 3a - 54 = 0$

$\Rightarrow a^2 + 9a - 6a - 54 = 0$

$\Rightarrow a(a + 9) - 6(a + 9) = 0$

$\Rightarrow (a - 6)(a + 9) = 0$

$\Rightarrow a = 6, -9$

Thus, the numbers are 6, 9

18. Question

The sum of the squares of three consecutive natural numbers is 149. Find the numbers.

Answer

Let the three consecutive natural numbers be ‘a’, ‘a + 1’ and ‘a + 2’

$\Rightarrow a^2 + (a + 1)^2 + (a + 2)^2 = 149$

$\Rightarrow a^2 + a^2 + 2a + 1 + a^2 + 4a + 4 = 149$

$\Rightarrow 3a^2 + 6a - 144 = 0$

$\Rightarrow a^2 + 2a - 48 = 0$

$\Rightarrow a^2 + 8a - 6a - 48 = 0$

$\Rightarrow a(a + 8) -6(a + 8) = 0$

$\Rightarrow (a - 6)(a + 8) = 0$

$\Rightarrow a = 6$ or $a = -8$, however $a = -8$ is not possible as -8 is not a natural number

Numbers are 6, 7, 8

19. Question

The sum of two numbers is 16. The sum of their reciprocals is 1/3. Find the numbers.

Answer

Let the numbers be 'a' and 'b'

Given, sum of two numbers is 16. The sum of their reciprocals is 1/3.

$\Rightarrow a + b = 16$

$\Rightarrow a = 16 - b$

Also, $1/a + 1/b = 1/3$

$\Rightarrow 1/(16 - b) + 1/b = 1/3$

$\Rightarrow 3(b + 16 - b) = 16b - b^2$

$\Rightarrow b^2 - 16b + 48 = 0$

$\Rightarrow b^2 - 12b - 4b + 48 = 0$

$\Rightarrow b(b - 12) - 4(b - 12) = 0$

$\Rightarrow (b - 4)(b - 12) = 0$

$\Rightarrow b = 4, 12$

Numbers are 4 , 12

20. Question

Determine two consecutive multiples of 3 whose product is 270.

Answer

Let the consecutive multiples of 3 be a, a + 3

$\Rightarrow a(a + 3) = 270$

$\Rightarrow a^2 + 3a - 270 = 0$

$\Rightarrow a^2 + 18a - 15a - 270 = 0$

$\Rightarrow a(a + 18) -15(a + 18) = 0$

$\Rightarrow (a - 15)(a + 18) = 0$

$\Rightarrow a = 15$

Numbers are 15, 18

21. Question

The sum of a number and its reciprocal is 17/4. Find the number.

Answer

Let the number be a.

$$a + \frac{1}{a} = \frac{17}{4}$$

$$\Rightarrow 4\left(a + \frac{1}{a}\right) = 1$$

$$\Rightarrow 4\left(\frac{a^2 + 1}{a}\right) = 17$$

$\Rightarrow 4a^2 + 4 - 17a = 0$

$\Rightarrow 4a^2 - 16a - a + 4 = 0$

$\Rightarrow 4a(a - 4) - (a - 4) = 0$

$\Rightarrow (4a - 1)(a - 4) = 0$

$\Rightarrow a = 1/4$ or 4

22. Question

A two-digit number is such that the product of its digits is 8. When 18 is subtracted from the number, the digits interchange their places. Find number.

Answer

Let the ones digit be 'a' and tens digit be 'b'.

Given, two-digit number is such that the product of its digits is 8.

$\Rightarrow ab = 8$ --- (1)

Also, when 18 is subtracted from the number, the digits interchange their places

$\Rightarrow 10b + a - 18 = 10a + b$

$\Rightarrow 9b - 9a = 18$

$\Rightarrow b - a = 2$

$\Rightarrow b = 2 + a$

Substituting in 1

$\Rightarrow a \times (2 + a) = 8$

$\Rightarrow a^2 + 2a - 8 = 0$

$\Rightarrow a^2 + 4a - 2a - 8 = 0$

$\Rightarrow a(a + 4) - 2(a + 4) = 0$

$\Rightarrow (a - 2)(a + 4) = 0$

$\Rightarrow a = 2$

Thus, b = 4

Number is 42

23. Question

A two-digit number is such that the product of the digits is 12. When 36 is added to the number the digits interchange their places. Determine the number.

Answer

Let the ones digit be 'a' and tens digit be 'b'.

Given, two-digit number is such that the product of its digits is 12.

$\Rightarrow ab = 12$ --- (1)

Also, when 36 is added to the number, the digits interchange their places

$\Rightarrow 10b + a + 36 = 10a + b$

$\Rightarrow 9a - 9b = 36$

$\Rightarrow a - b = 4$

$\Rightarrow a = 4 + b$

Substituting in 1

$\Rightarrow b \times (4 + b) = 12$

$\Rightarrow b^2 + 4b - 12 = 0$

$\Rightarrow b^2 + 6b - 2b - 12 = 0$

$\Rightarrow b(b + 6) - 2(b + 6) = 0$

$\Rightarrow (b - 2)(b + 4) = 0$

$\Rightarrow b = 2$

Thus, $a = 6$

Number is 26

24. Question

A two-digit number is such that the product of the digits is 16. When 54 is subtracted from the number, the digits are interchanged. Find the number.

Answer

Let the ones digit be 'a' and tens digit be 'b'.

Given, two-digit number is such that the product of its digits is 16.

$\Rightarrow ab = 16$ --- (1)

Also, when 54 is subtracted from the number, the digits interchange their places

$\Rightarrow 10b + a - 54 = 10a + b$

$\Rightarrow 9b - 9a = 54$

$\Rightarrow b - a = 6$

$\Rightarrow b = 6 + a$

Substituting in 1

$\Rightarrow a \times (6 + a) = 16$

$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow a^2 + 8a - 2a - 16 = 0$

$\Rightarrow a(a + 8) - 2(a + 8) = 0$

$\Rightarrow (a - 2)(a + 8) = 0$

$\Rightarrow a = 2$

Thus, b = 8

Number is 82

25. Question

Two numbers differ by 3 and their product is 504. Find the numbers.

Answer

Given, two numbers differ by 3.

Let one of the numbers be 'a'.

Second number = a − 3

Also, their product is 504.

$\Rightarrow a(a - 3) = 504$

$\Rightarrow a^2 - 3a - 504 = 0$

$\Rightarrow a^2 - 24a + 21a - 504 = 0$

$\Rightarrow a(a - 24) + 21(a - 24) = 0$

$\Rightarrow (a + 21)(a - 24) = 0$

$\Rightarrow a = -21, 24$

Thus numbers are -21, -24 or 24, 21

26. Question

Two numbers differ by 4 and their product is 192. Find the numbers.

Answer

Given, two numbers differ by 4.

Let one of the numbers be 'a'.

Second number = a − 4

Also, their product is 192.

$\Rightarrow a(a - 4) = 192$

$\Rightarrow a^2 - 4a - 192 = 0$

$\Rightarrow a^2 - 16a + 12a - 192 = 0$

$\Rightarrow a(a - 16) + 12(a - 16) = 0$

$\Rightarrow (a + 12)(a - 16) = 0$

$\Rightarrow a = -12, 16$

Thus numbers are -12, -16 or 12, 16

27. Question

A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the number.

Answer

Let the ones and tens digits be ‘a’ and ‘b’ respectively.

$10b + a = 4 \times (a + b)$

$\Rightarrow 6b = 3a$

$\Rightarrow a = 2b$

Also, $10b + a = 2ab$

$\Rightarrow 10b + 2b = 2 \times 2b \times b$

$\Rightarrow 4b^2 = 12b$

$\Rightarrow b = 3$

Thus, $a = 6$

Number is 36.

28. Question

The difference of the squares of two positive integers is 180. The square of the smaller number is 8 times the larger number, find the numbers.

Answer

Let the positive integers be ‘a’ and ‘b’.

Given, difference of the squares of two positive integers is 180.

$\Rightarrow a^2 - b^2 = 180$

Also, square of the smaller number is 8 times the larger.

$\Rightarrow b^2 = 8a$

Thus, $a^2 - 8a - 180 = 0$

$\Rightarrow a^2 - 18a + 10a - 180 = 0$

$\Rightarrow a(a - 18) + 10(a - 18) = 0$

$\Rightarrow (a + 10)(a - 18) = 0$

$\Rightarrow a = -10, 18$

Thus, the other number is

$324 - 180 = b^2$

$\Rightarrow b = 12$

Numbers are 12, 18

29. Question

The sum of two numbers is 18. The sum of their reciprocals is 1/4. Find the numbers.

Answer

Let the numbers be ‘a’ and ‘b’

Given, sum of two numbers is 18. The sum of their reciprocals is 1/4

$\Rightarrow a + b = 18$

$\Rightarrow b = 18 - a$

Also, 1/a + 1/b = 1/4

⇒ 1/a + 1/(18 – a) = 1/4

⇒ 18 × 4 = 18a – a^2

⇒ a^2 – 18a + 72 = 0

⇒ a^2 – 12a – 6a + 72 = 0

⇒ a(a – 12) – 6(a – 12) = 0

⇒ (a – 6)(a – 12) = 0

⇒ a = 6, 12

Numbers are are 6,12 or 12, 6

30. Question

The sum of two numbers a and b is 15, and the sum of their reciprocals $\frac{1}{a}$ *and* $\frac{1}{b}$ is 3/10. Find the numbers a and b.

Answer

Let the numbers be 'a' and 'b'

Given, sum of two numbers is 15. The sum of their reciprocals is 1/4

⇒ a + b = 15

⇒ b = 15 – a

Also, 1/a + 1/b = 3/10

⇒ 1/a + 1/(15 – a) = 3/10

⇒ 15 × 10 = 45a – 3a^2

⇒ a^2 – 15a + 50 = 0

⇒ a^2 – 15a – 5a + 50 = 0

⇒ a(a – 10) – 5(a – 10) = 0

⇒ (a – 5)(a – 10) = 0

⇒ a = 5, 10

Numbers are are 5,10 or 10, 5

31. Question

The sum of two numbers is 9. The sum of their reciprocals is 1/2. Find the numbers.

Answer

Let the numbers be 'a' and 'b'

Given, sum of two numbers is 18. The sum of their reciprocals is 1/4

⇒ a + b = 18

⇒ b = 18 – a

Also, 1/a + 1/b = 1/4

⇒ 1/a + 1/(18 – a) = 1/4

⇒ 18 × 4 = 18a – a^2

$\Rightarrow a^2 - 18a + 72 = 0$

$\Rightarrow a^2 - 12a - 6a + 72 = 0$

$\Rightarrow a(a - 12) - 6(a - 12) = 0$

$\Rightarrow (a - 6)(a - 12) = 0$

$\Rightarrow a = 6, 12$

Numbers are are 6,12 or 12, 6

32. Question

Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers.

Answer

Let the three consecutive numbers be a, a + 1, a + 2

Given, there are three consecutive integers such that the sum of square of the first and the product of the other two is 46.

$\Rightarrow a^2 + (a + 1)(a + 2) = 46$

$\Rightarrow 2a^2 + 3a + 2 = 46$

$\Rightarrow 2a^2 + 3a - 44 = 0$

$\Rightarrow 2a^2 + 11a - 8a - 44 = 0$

$\Rightarrow a(2a + 11) - 4(2a + 11) = 0$

$\Rightarrow (a - 4)(2a + 11) = 0$

Thus, a = 4

Numbers are 4, 5, 6

33. Question

The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers.

Answer

Let the numbers be ‘a’ and ‘b’.

Given, difference of squares of two numbers is 88.

$\Rightarrow a^2 - b^2 = 88$

Also, the larger number is 5 less than twice the smaller number.

$\Rightarrow a = 2b - 5$

Thus, $(2b - 5)^2 - b^2 = 88$

$\Rightarrow 4b^2 + 25 - 20b - b^2 = 88$

$\Rightarrow 3b^2 - 20b - 63 = 0$

$\Rightarrow 3b^2 - 27b + 7b - 63 = 0$

$\Rightarrow 3b(b - 9) + 7(b - 9) = 0$

$\Rightarrow (3b + 7)(b - 9) = 0$

$\Rightarrow b = 9$

Thus, $a = 2 \times 9 - 5 = 13$

34. Question

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find two numbers.

Answer

Let the numbers be 'a' and 'b'.

Given, difference of the squares of two numbers is 180.

$\Rightarrow a^2 - b^2 = 180$

Also, square of the smaller number is 8 times the larger.

$\Rightarrow b^2 = 8a$

Thus, $a^2 - 8a - 180 = 0$

$\Rightarrow a^2 - 18a + 10a - 180 = 0$

$\Rightarrow a(a - 18) + 10(a - 18) = 0$

$\Rightarrow (a + 10)(a - 18) = 0$

$\Rightarrow a = -10, 18$

Thus, the other number is

$324 - 180 = b^2$

$\Rightarrow b = \pm 12$

Numbers are 12, 18 or -12, 18

35. Question

Find two consecutive odd positive integers, sum of whose squares is 970.

Answer

Let the consecutive odd positive integers be 'a' and a + 2

$\Rightarrow a^2 + (a + 2)^2 = 970$

$\Rightarrow 2a^2 + 4a - 966 = 0$

$\Rightarrow a^2 + 2a - 483 = 0$

$\Rightarrow a^2 + 23a - 21a - 483 = 0$

$\Rightarrow a(a + 23) - 21(a + 23) = 0$

$\Rightarrow (a - 21)(a + 23) = 0$

Thus, $a = 21$

Consecutive odd positive integers are 21, 23

36. Question

The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers.

Answer

Let the natural numbers be ‘a’ and ‘b’.

Given, difference of two natural numbers is 3 and difference of their reciprocals is 3/28

$\Rightarrow a - b = 3$

$\Rightarrow a = b + 3$

and $1/b - 1/a = 3/28$

$\Rightarrow 1/b - 1/(b + 3) = 3/28$

$\Rightarrow 28(b - b - 3) = -3(b^2 + 3b)$

$\Rightarrow b^2 + 3b - 28 = 0$

$\Rightarrow b^2 + 7b - 4b - 28 = 0$

$\Rightarrow b(b + 7) - 4(b + 7) = 0$

$\Rightarrow (b - 4)(b + 7) = 0$

$\Rightarrow b = 4$

Numbers are, 4, 7

37. Question

The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Answer

given: The sum of the squares of two consecutive odd numbers is 394.

To find: the numbers.

Solution: Let the consecutive odd number be ‘a’ and a + 2According to given condition,

$a^2 + (a + 2)^2 = 394$Use the formula $(x+y)^2=x^2+y^2+2xy$ in $(a + 2)^2$Here x=a and y=2,$\Rightarrow a^2 + a^2 + 4 + 4a=394$

$\Rightarrow 2a^2 + 4a + 4 - 394 = 0 \Rightarrow 2a^2 + 4a - 390 = 0$Take 2 common out of the above equation,

$\Rightarrow a^2 + 2a - 195 = 0$Factorise by splitting the middle term.

$\Rightarrow a^2 + 15a - 13a - 195 = 0$

$\Rightarrow a(a + 15) - 13(a + 15) = 0$

$\Rightarrow (a - 13)(a + 15) = 0$

Thus, a = 13, - 15When a=13 then a+2=15And when a = -15 then a+2 = -13

So Consecutive odd numbers are 13, 15 and -15,-13.

38. Question

The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

Answer

Let the consecutive multiples of 7 be ‘a’ and a + 7

$\Rightarrow a^2 + (a + 7)^2 = 637$

$\Rightarrow 2a^2 + 14a - 588 = 0$

$\Rightarrow 2a^2 + 42a - 28a - 588 = 0$

$\Rightarrow 2a(a + 21) - 28(a + 21) = 0$

⇒ (2a – 28)(a + 21) = 0

Thus, a = 14

Consecutive multiples of 7 are 14, 21

39. Question

The sum of the squares of two consecutive even numbers is 340. Find the numbers.

Answer

Let the consecutive even integers be 'a' and a + 2

⇒ $a^2 + (a + 2)^2 = 340$

⇒ $2a^2 + 4a – 336 = 0$

⇒ $a^2 + 2a – 168 = 0$

⇒ $a^2 + 14a – 12a – 168 = 0$

⇒ a(a + 14) – 12(a + 14) = 0

⇒ (a – 12)(a + 14) = 0

Thus, a = 12 or – 14

Consecutive even integers are 12, 14 or -14, - 12

40. Question

The numerator of a fraction is 3 less than the denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and the original fraction is $\frac{29}{20}$, find the original fraction.

Answer

Let the denominator be 'a'.

Numerator = a – 3

[As numerator is 3 less than denominator]Now, if 2 is added to both the numerator and the denominator, then the sum of the new fraction and the original fraction is $\frac{29}{20}$.

$$\Rightarrow \frac{a-3}{a}+\frac{a-3+2}{a+2}=\frac{29}{20}$$

$$\Rightarrow (a-3)(a+2)+(a-1)(a)=\frac{29a(a+2)}{20}$$

⇒ $20(a^2 – a – 6) + 20a^2 – 20a = 29a^2 + 58a$

⇒ $11a^2 – 98a – 120 = 0$

⇒ $11a^2 – 110a + 12a – 120 = 0$

⇒ 11a(a – 10) + 12(a – 10) = 0

⇒ (11a + 12)(a – 10) = 0

⇒ a = 10 or a = -12/11Since, denominator can't be a fraction⇒ a = 10

and numerator = a - 3 = 10 - 3 = 7

Thus the original fraction is 7/10.

Exercise 8.8

1. Question

The speed of a boat in still water is 8 km / hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

Answer

Given: The speed of a boat in still water is 8 km / hr. It can go 15 km upstream and 22 km downstream in 5 hours.

To find: the speed of the stream.

Solution: Let the speed of stream be 'a' km/hr.

Given, speed of a boat in still water is 8 km / hr. It can go 15 km upstream and 22 km downstream in 5 hours.

Going upstream means that boat is going in opposite direction of the stream so speeds will be added and going downstream means that the boat is going in the same direction of the stream.So,

Relative speed of boat going upstream = 8 – a

Relative speed of boat going downstream = 8 + a

Time = distance/speedTotal time is given to be 5 hrs.

$$\Rightarrow \frac{15}{8-a}+\frac{22}{8+a}=5$$

$$\Rightarrow \frac{15(8+a)+22(8-a)}{(8-a)(8+a)}=5$$

⇒15(8+a) + 22(8-a) = 5(8-a)(8+a)Apply the formula (a-b)(a+b)= a^2-b^2 in (8-a)(8+a)Here a=8 and b=a.

⇒ 120 + 15a + 176 – 22a = 5(64 – a^2)⇒ 296 – 7a = -5a^2 + 320

⇒ 5a^2 – 7a – 24 = 0Factorize the equation by splitting the middle term

⇒ 5a^2 – 15a + 8a – 24 = 0

⇒ 5a(a – 3) + 8(a – 3) = 0

⇒ (5a + 8)(a – 3) = 0⇒ (5a + 8) = 0 and (a – 3) = 0

$$\Rightarrow a=\frac{-8}{5} \; and \; a=3$$

$$Since \; a=\frac{-8}{5} \; is \; not \; possible$$

⇒ a = 3 km/hrHence speed of the stream is 3 km/hr.

2. Question

A passenger train takes 3 hours less for a journey of 360 km, if its speed is increased by 10 km / hr from its usual speed. What is the usual speed?

Answer

Distance = speed × time

Given, passenger train takes 3 hours less for a journey of 360 km, if its speed is increased by 10 km / hr from its usual speed.

Let the speed be 's' and time be 't'.

⇒ st = 360

⇒ t = 360/s

Also, 360 = (s + 10)(t − 3)

$$\Rightarrow 360 = (s + 10)\left(\frac{360}{s} - 3\right)$$

⇒ 360s = 360s + 3600 -3s^2 − 30s

⇒ s^2 + 10s − 1200 = 0

⇒ s^2 + 40s − 30s − 1200 = 0

⇒ s(s + 40) − 30(s + 40) = 0

⇒ (s − 30)(s + 40) = 0

⇒ s = 30 km/hr

3. Question

A fast train takes one hour less than a slow train for a journey of 200 km. If the speed of the slow train is 10 km / hr less than that of the fast rain, find the speed of the two trains.

Answer

Speed = distance/time

Let the speed of the faster train be 'a' km/hr.

Speed of the slow train = a − 10 km/hr

Also, fast train takes one hour less than a slow train for a journey of 200 km.

$$\Rightarrow \frac{200}{a - 10} - \frac{200}{a} = 1$$

⇒ 200a + 2000 − 200a = a^2 − 10a

⇒ a^2 − 10a − 2000 = 0

⇒ a^2 − 50a + 40a − 2000 = 0

⇒ a(a − 50) + 40(a − 50) = 0

⇒ (a + 40)(a − 50) = 0

⇒ a = 50 km/hr

Speed of the trains is 50 km/hr, 40 km/hr

4. Question

A passenger train takes one hour less for a journey of 150 km if its speed is increased by

5 km/hr from the usual speed. Find the usual speed of the train.

Answer

Time = distance/speed

Let the speed of the train be 'a' km/hr.

Given, passenger train takes one hour less for a journey of 150 km if its speed is increased by

5 km/hr from the usual speed.

$$\Rightarrow \frac{150}{a} - \frac{150}{a+5} = 1$$

$\Rightarrow 150(a + 5 - a) = a^2 + 5a$

$\Rightarrow a^2 + 5a - 750 = 0$

$\Rightarrow a^2 + 30a - 25a - 750 = 0$

$\Rightarrow a(a + 30) - 25(a + 30) = 0$

$\Rightarrow (a - 25)(a + 30) = 0$

$\Rightarrow a = 25$ km/hr

5. Question

The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at a speed of 10 km / hr more than the speed of going, what was the speed per hour in each direction?

Answer

Time = distance/speed

Let the speed of person on onward journey be 'a' km/hr

Speed at which he returned = a – 10 km/hr

Given, time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey.

$$\Rightarrow \frac{150}{a-10} - \frac{150}{a} = 2.5$$

$\Rightarrow 150(a - a + 10) = 2.5a(a - 10)$

$\Rightarrow 1500 = 2.5a^2 - 25a$

$\Rightarrow a^2 - 10a - 600 = 0$

$\Rightarrow a^2 - 30a + 20a - 600 = 0$

$\Rightarrow a(a - 30) + 20(a - 30) = 0$

$\Rightarrow (a + 20)(a - 30) = 0$

$\Rightarrow a = 30$ km/hr

6. Question

A plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km / hr from its usual speed. Find the usual speed of the plane.

Answer

Time = distance/speed

Given, plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km / hr from its usual speed.

Let the usual speed be 'a'.

$$\frac{1600}{a} - \frac{1600}{a+400} = \frac{40}{60}$$

$\Rightarrow 3(1600 \times 400) = 2(a^2 + 400a)$

$\Rightarrow a^2 + 400a - 960000 = 0$

⇒ a^2 + 1200a – 800a – 960000 = 0

⇒ a(a + 1200) – 800(a + 1200) = 0

⇒ (a + 1200)(a – 800) = 0

⇒ a = 800 km/hr

7. Question

An areoplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km / hr from its usual speed. Find its usual speed.

Answer

Time = distance/speed

Given, areoplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km / hr from its usual speed.

Let the usual speed be 'a'.

$$\frac{1200}{a}-\frac{1200}{a+100}=1$$

⇒ 1200 ×(a + 100 –a) = a^2 + 100a

⇒ a^2 + 100a – 120000 = 0

⇒ a^2 + 400a – 300a – 120000 = 0

⇒ a(a + 400) – 300(a + 400) = 0

⇒ (a + 400)(a – 300) = 0

⇒ a = 300 km/hr

8. Question

A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km / hr from its usual speed. Find the usual speed of the train.

Answer

Given, passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed.

Let the usual speed be 'a' km/hr

Increased speed = (a + 5) km/hrwe know, $\text{time}=\frac{\text{distance}}{\text{speed}}$

⇒ time taken by train to cover with usual speed to travel 300 km $=\frac{300}{\text{a}}$ km/h

⇒ time taken by train to cover with increased speed to travel 300 km $=\frac{300}{\text{a}+5}$ km/h

Therefore, According to question

$$\Rightarrow \frac{300}{a}-\frac{300}{a+5}=2$$

⇒ 300 ×(a + 5 –a) = 2(a^2 + 5a)

⇒ a^2 + 5a – 750 = 0

⇒ $a^2 + 30a - 25a - 750 = 0$

⇒ $a(a + 30) - 25(a + 30) = 0$

⇒ $(a + 30)(a - 25) = 0$

⇒ a = 25 km/hr

9. Question

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km / hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Answer

Time = distance/speed

Given, train covers a distance of 90 km at a uniform speed. Had the speed been 15 km / hour more, it would have taken 30 minutes less for the journey.

Let the usual speed be 'a'.

$$\frac{90}{a} - \frac{90}{a+15} = \frac{30}{60}$$

⇒ $90 \times (a + 15 - a) = (a^2 + 15a)/2$

⇒ $a^2 + 15a - 2700 = 0$

⇒ $a^2 + 60a - 45a - 2700 = 0$

⇒ $a(a + 60) - 45(a + 60) = 0$

⇒ $(a + 60)(a - 45) = 0$

⇒ a = 45 km/hr

10. Question

A train travels 360 km at a uniform speed. If the speed had been 5 km / hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer

To find: Speed of the train

Method 1: Let the speed of the train be *x* km/hr.

Time taken to cover 360 km = $\frac{360}{x}$ hr,

As

$$time = \frac{distance}{speed}$$

Now, given that if the speed would be 5 km/hr more, the same distance would be covered in 1 hour less, i.e.if speed = x + 5, and

$$time = (\frac{360}{x} - 1)\ hours$$

then, using distance = speed x time, we have

$$(x + 5)\left(\frac{360}{x} - 1\right) = 360$$

Now we can form the quadratic equation from this equation

$$360 - x + \frac{1800}{x} - 5 = 360$$

$$\frac{360\ x\ -\ x^2\ +\ 1800\ -\ 5x}{x}\ =\ 360$$

Now, cross multiplying we get

360 x - x^2 + 1800 - 5 x = 360 x

x^2 + 5 x – 1800 = 0

Now we have to factorize in such a way that the product of the two numbers is 1800 and the difference is $5x^2$ + 45 x – 40 x – 1800 = 0

x(x + 45) – 40(x+45) = 0

(x+45)(x – 40) = 0

x = - 45, 40

since, the speed of train can't be negative,so, speed will be 40 km/hour.

11. Question

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express trains is 11 km / hr more than that of the passenger train, find the average speeds of the two trains.

Answer

Time = distance/speed

Now, express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. The average speed of the express trains is 11 km / hr more than that of the passenger train.

Let the average speed of passenger train be 'a'.

$$\frac{132}{a} - \frac{132}{a+11} = 1$$

⇒ 132 ×(a + 11 –a) = a^2 + 11a

⇒ a^2 + 11a – 1452 = 0

⇒ a^2 + 44a – 33a – 1452 = 0

⇒ a(a + 44) – 33(a + 44) = 0

⇒ (a + 44)(a – 33) = 0

⇒ a = 33 km/hr

Speed of express train = 44 km/hr

12. Question

An aeroplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed by 250 km / hr from its usual speed. Find its usual speed.

Answer

Given: An aeroplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed by 250 km / hr from its usual speed.

To find: its usual speed.

Solution:

Given, aeroplane left 50 minutes later than its schedule time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed to 250 km / hr from its usual speed.

Time = distance/speed

Let the usual speed be ‘a’.

Since the speed of the aeroplane has been increased the time taken by the now will be less. So we will subtract the time taken by the aeroplane when the speed has been increased from the time it would have taken originally and this time is equal to 50 min.Since the speed has been given in km/h so we need to convert 50 min into hours.To convert 50 min into hours divide it by 60 as1 hr = 60 min

$\frac{1250}{a} - \frac{1250}{a+250} = \frac{50}{60}$Take 1250 common from the LHS,So, $\Rightarrow 1250\left(\frac{1}{a} - \frac{1}{a+250}\right) = \frac{5}{6}$

$\Rightarrow 1250\left(\frac{a+250-a}{a(a+250)}\right) = \frac{5}{6}$ ⇒ (6 × 1250) (a + 250 –a) = 5a(a+250)

⇒ (6 × 1250) (a + 250 –a) = 5(a^2 + 250a)⇒1875000= 5a^2 + 1250a⇒ 5a^2 + 1250a - 1875000 = 0Take 5 common out of the above equation,

⇒ a^2 + 250a – 375000 = 0Factorise the equation by splitting the middle term as:

⇒ a^2 + 750a – 500a – 375000 = 0

⇒ a(a + 750) – 500(a + 750) = 0

⇒ (a + 750)(a – 500) = 0⇒ (a + 750) = 0 and (a – 500) = 0

⇒ a = - 750 and a = 500Since the speed cannot be negative.

⇒ a = 500 km/hrHence the speed of aeroplane is 500 km/hr.

13. Question

While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalize the injured and so the plane started late by 30 minutes to reach the destination, 1500 km away in time, the pilot increased the speed by 100 km / hr. Find the original speed / hour of the plane.

Answer

Time = distance/speed

Given, while boarding an aeroplane, a passenger got hurt and the plane started late by 30 minutes to reach the destination, 1500 km away in time, the pilot increased the speed by 100km/hr.

Let the usual speed be 'a'.

$$\frac{1500}{a} - \frac{1500}{a+100} = \frac{30}{60}$$

⇒ 2 × 1500 × (a + 100 –a) = a^2 + 100a

⇒ a^2 + 100a – 300000 = 0

⇒ a^2 + 600a – 500a – 300000 = 0

⇒ a(a + 600) – 500(a + 600) = 0

⇒ (a + 750)(a – 500) = 0

⇒ a = 500 km/hr

14. Question

A motorboat whose speed in still water is 18 km/hr takes 1 hour more to go 24 km upstream that to return downstream to the same spot. Find the speed of the stream.

Answer

Time = Distance/speed

Given, motorboat whose speed in still water is 18 km/hr takes 1 hour more to go 24 km upstream that to return downstream to the same spot.

Let the speed of stream be 'a' km/hr.

Relative speed of boat going upstream = 18 – a km/hr

Relative speed of boat going downstream = 18 + a km/hr

$$\Rightarrow \frac{24}{18-a} - \frac{24}{18+a} = 1$$

⇒ 24(18 + a – 18 + a) = -a^2 + 324

⇒ a^2 + 48a – 324 = 0

⇒ a^2 + 54a - 6a – 324 = 0

⇒ a(a + 54) – 6(a + 54) = 0

⇒ (a – 6)(a + 54) = 0

⇒ a = 6 km/hr

Exercise 8.9

1. Question

Ashu is x years old while his mother Mrs. Veena is x^2 years old. Five years hence Mrs. Veena will be three times old as Ashu. Find their present ages.

Answer

Given, Ashu is x years old while his mother Mrs. Veena is x^2 years old.

After 5 years, Mrs. Veena will be three times old as Ashu.

$\Rightarrow x^2 + 5 = 3(x + 5)$

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x + 2)(x - 5) = 0$

$\Rightarrow x = 5$ km/hr

2. Question

The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present ages.

Answer

Let the present ages of the man and son be 'a' and 'b' respectively.

Given, sum of the ages of a man and his son is 45 years.

$\Rightarrow a + b = 45$ .....(1)

Ans, five years ago, the product of their ages was four times the man's age at the time.

$\Rightarrow (a - 5)(b - 5) = 4(a - 5)$

$\Rightarrow b - 5 = 4$

$\Rightarrow b = 9$ years

Thus, $a = 45 - 9 = 36$ years

3. Question

The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years. Find her present age.

Answer

Let the present age of Shikha be 'a' years.

Given, product of Shikha's age five years ago and her age 8 years later is 30

$\Rightarrow (a - 5)(a + 8) = 30$

$\Rightarrow a^2 + 3a - 40 - 30 = 0$

$\Rightarrow a^2 + 10a - 7a - 70 = 0$

$\Rightarrow a(a + 10) - 7(a + 10) = 0$

$\Rightarrow (a - 7)(a + 10) = 0$

$\Rightarrow a = 7$ years

4. Question

The product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15. Determine Ramu's present age.

Answer

Let the present age of Ramu be 'a' years.

Given, product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15.

$\Rightarrow (a - 5)(a + 9) = 15$

$\Rightarrow a^2 + 4a - 45 - 15 = 0$

$\Rightarrow a^2 + 10a - 6a - 60 = 0$

$\Rightarrow a(a + 10) - 6(a + 10) = 0$

$\Rightarrow (a - 6)(a + 10) = 0$

$\Rightarrow a = 6$ years

5. Question

Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer

Given, sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48

Let the age of one of the friends be 'a'

Age of the other friend = 20 – a

$\Rightarrow (a - 4)(20 - a - 4) = 48$

$\Rightarrow (a - 4)(16 - a) = 48$

$\Rightarrow a^2 - 20a + 64 + 48 = 0$

$\Rightarrow a^2 - 20a + 112 = 0$

$D = b^2 - 4ac$

$\Rightarrow D = 400 - 4 \times 112 = -48$

Thus, roots are not real as $D < 0$

The following situation is not possible

6. Question

A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Answer

Let the present ages of the younger sister be 'a'.

Given, girl is twice as old as her sister.

Age of elder sister = 2a

Also, four years ago, the product of their ages (in years) will be 160.

$\Rightarrow (a + 4)(2a + 4) = 160$

$\Rightarrow 2a^2 + 12a + 16 - 160 = 0$

$\Rightarrow a^2 + 6a - 72 = 0$

$\Rightarrow a^2 + 12a - 6a - 72 = 0$

$\Rightarrow a(a + 12) - 6(a + 12) = 0$

$\Rightarrow (a - 6)(a + 12) = 0$

$\Rightarrow a = 6$ years

Age of sisters – 6 years and 12 years

7. Question

The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3. Find the present age.

Answer

Let Rehman's present age be 'a'.

Given, sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3.

$1/(a - 3) + 1/(a + 5) = 1/3$

$\Rightarrow 3(a - 3 + a + 5) = a^2 + 2a - 15$

$\Rightarrow a^2 - 4a - 21 = 0$

$\Rightarrow a^2 - 7a + 3a - 21 = 0$

$\Rightarrow a(a - 7) + 3(a - 7) = 0$

$\Rightarrow (a + 3)(a - 7) = 0$

$\Rightarrow a = 7$

Exercise 8.10

1. Question

The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Answer

Given: The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm.

To find: the lengths of these sides.

Solution: We know$(\text{Hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$ ..... (1)

Given, hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm

Let the base be 'b'.

$\Rightarrow$ Perpendicular = b – 5Put the known values in (1),

$\Rightarrow 25^2 = b^2 + (b - 5)^2$Apply the formula $(x - y)^2 = x^2 + y^2 - 2xy$ in $(b - 5)^2$.Here x=b and y=5

$\Rightarrow 625 = b^2 + b^2 + 25 - 10b \Rightarrow 625 = 2b^2 + 25 - 10b \Rightarrow 2b^2 + 25 - 10b = 625 \Rightarrow 2b^2 + 25 - 10b - 625 = 0 \Rightarrow 2b^2 - 10b - 600 = 0$Take out 2 common of the above equation.

$\Rightarrow b^2 - 5b - 300 = 0$

Factorise the above quadratic equation by splitting the middle term.

$\Rightarrow b^2 - 20b + 15b - 300 = 0$

⇒ b(b – 20) + 15(b – 20) = 0⇒ (b - 20) (b + 15) = 0⇒ (b - 20) =0 and (b + 15) = 0

⇒ b=20 and b=-15Since the length of any side cannot be negative,We will ignore -15.

⇒ b = 20

Perpendicular = 20 – 5 = 15

Hence,Sides are 15 and 20

2. Question

The hypotenuse of a right triangle is $3\sqrt{10}$ cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be $9\sqrt{5}$ cm. How long are the legs of the triangle?

Answer

Let the smaller leg be 'a' and longer leg be 'b'.

Hypotenuse^2 = length^2 + breadth^2

Given, hypotenuse of a right triangle is $3\sqrt{10}$ cm

⇒ 9 × 10 = $a^2 + b^2$

⇒ $a^2 + b^2$ = 90 -------- (1)

Now, the smaller leg is tripled and the longer leg doubled, new hypotenuse is $9\sqrt{5}$ cm.

⇒ $(3a)^2 + (2b)^2 = 81 \times 5$

⇒ $9a^2 + 4b^2$ = 405 -------- (2)

Multiplying (1) by 4 and subtracting from eq 2

⇒ $5a^2$ = 45

⇒ a^2 = 9

⇒ a = 3

Thus, 9 + b^2 = 90

⇒ b^2 = 81

⇒ b = 9

3. Question

A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it the possible to do so? If yes, at what distances from the two gates should the pole be erected?

Answer

Let the distance of pole from gate A be 'a'.

Difference of the distance of the pole from two diametrically opposite fixed gates A and B on the boundary is 7 metres.

Distance of pole from gate B = a – 7 m

Diameter of the park = 13 m

Hypotenuse^2 = length^2 + breadth^2

⇒ $13^2 = a^2 + (a - 7)^2$

⇒ $169 = 2a^2 + 49 - 14a$

$\Rightarrow a^2 - 7a - 60 = 0$

$\Rightarrow a^2 - 12a + 5a - 60 = 0$

$\Rightarrow a(a - 12) + 5(a - 12) = 0$

$\Rightarrow (a + 5)(a - 12) = 0$

$\Rightarrow a = 12$ m

Thus distance of pole is 12 m from gate A and 5 m from gate B

4. Question

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field

Answer

Let the shorter side be 'a'.

Given, diagonal of a rectangular field is 60 metres more than the shorter side

Diagonal = a + 60

Also, longer side is 30 metres more than the shorter side

Longer side = a + 30

Hypotenuse^2 = length^2 + breadth^2

$\Rightarrow (a + 60)^2 = (a + 30)^2 + a^2$

$\Rightarrow a^2 + 120a + 3600 = a^2 + 60a + 900 + a^2$

$\Rightarrow a^2 - 60a - 2700 = 0$

$\Rightarrow a^2 - 90a + 30a - 2700 = 0$

$\Rightarrow a(a - 90) + 30(a - 90) = 0$

$\Rightarrow (a + 30)(a - 90) = 0$

$\Rightarrow a = 90$m

Length of sides = 90m, 120 m

Exercise 8.11

1. Question

The perimeter of a rectangular field is 82 m and its area is 400 m^2. Find the breadth of the rectangle.

Answer

Perimeter of a rectangle = 2(l + b)

Area of the rectangle = l × b

Given, perimeter of a rectangular field is 82 m and its area is 400 m^2

Let the breadth be 'a' m and length be 'b' m

$\Rightarrow 2(a + b) = 82$

$\Rightarrow b = 41 - a$

Also, a × b = 400

$\Rightarrow a \times (41 - a) = 400$

$\Rightarrow a^2 - 41a + 400 = 0$

$\Rightarrow a^2 - 25a - 16a + 400 = 0$

$\Rightarrow a(a - 25) - 16(a - 25) = 0$

$\Rightarrow (a - 16)(a - 25) = 0$

$\Rightarrow a = 16, 25$

Assuming breadth to smaller, thus breadth = 16m

2. Question

The length of a hall is 5 m more than its breadth. If the area of the floor of the hall is 84 m^2, what are the length and breadth of the hall?

Answer

Let the breadth of the hall be 'a'

Length = a + 5

Given, area of the floor of the hall is 84 m^2

$\Rightarrow a(a + 5) = 84$

$\Rightarrow a^2 + 5a - 84 = 0$

$\Rightarrow a^2 + 12a - 7a - 84 = 0$

$\Rightarrow a(a + 12) - 7(a + 12) = 0$

$\Rightarrow (a - 7)(a + 12) = 0$

$\Rightarrow a = 7$ m

Length of the hall = 7 + 5 = 12 m

3. Question

Two squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. Find the sides of the squares.

Answer

Area of a square = side × side

Given, squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2.

$\Rightarrow x^2 + (x + 4)^2 = 656$

$\Rightarrow x^2 + x^2 + 8x + 16 = 656$

$\Rightarrow x^2 + 4x - 320 = 0$

$\Rightarrow x^2 + 20x - 16x - 320 = 0$

$\Rightarrow x(x + 20) - 16(x + 20) = 0$

$\Rightarrow (x - 16)(x + 20) = 0$

$\Rightarrow x = 16$ cm

The other side = 16 + 4 = 20 cm

4. Question

The area of a right angled triangle is 165 m^2. Determine its base and altitude if the latter exceeds the former by 7 m.

Answer

Let the base of the triangle be 'a'.

Given, altitude exceeds the base by 7 m.

⇒ Altitude = a + 7 m

Area of a right angled triangle = 1/2 × base × height

⇒ 1/2 × a(a + 7) = 165

⇒ a^2 + 7a – 330 = 0

⇒ a^2 + 22a – 15a – 330 = 0

⇒ a(a + 22) – 15(a + 22) = 0

⇒ (a – 15)(a + 22) = 0

⇒ a = 15 m

Altitude = 15 + 7 = 22 m

5. Question

Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2? If so, find its length and breadth.

Answer

Let the breadth be 'a' m

Given, rectangular mango grove whose length is twice its breadth.

Length = 2a

Area = 800 m^2

⇒ 2a × a = 800

⇒ a^2 = 400

⇒ a = 20 m

Thus, length = 40m

6. Question

Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Answer

Let the length and breadth of park be 'a' and 'b' m

Given, rectangular park of perimeter 80 m and area 400 m^2.

⇒ 2(a + b) = 80

⇒ a + b = 40

⇒ a = 40 – b

⇒ a × b = 400

⇒ (40 – b)b = 400

⇒ b^2 – 40b + 400 = 0

$\Rightarrow (b - 20)^2 = 0$

$\Rightarrow b = 20$ m

Thus, it is possible to design a rectangular park with length = 20m and breadth = 20 m

7. Question

Sum of the areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of the two squares.

Answer

Area of a square = s^2

Perimeter of a square = 4s

Let the sides of the square be a and b respectively.

Given, sum of the areas of two squares is 640 m^2 and the difference of their perimeters is 64m.

$\Rightarrow a^2 + b^2 = 640$ and

$4a - 4b = 64$

$\Rightarrow a - b = 16$

$\Rightarrow a = 16 + b$

$\Rightarrow (16 + b)^2 + b^2 = 640$

$\Rightarrow 2b^2 + 32b + 256 = 640$

$\Rightarrow b^2 + 16b - 192 = 0$

$\Rightarrow b^2 + 24b - 16b - 192 = 0$

$\Rightarrow (b - 8)(b + 24) = 0$

$\Rightarrow b = 8$ m

Thus, a = 24 m

8. Question

Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of two squares.

Answer

Area of a square = s^2

Perimeter of a square = 4s

Let the sides of the square be a and b respectively.

Given, sum of the areas of two squares is 400 cm^2 and the difference of their perimeters is 16 cm.

$\Rightarrow a^2 + b^2 = 400$ and

$4a - 4b = 16$

$\Rightarrow a - b = 4$

$\Rightarrow a = 4 + b$

$\Rightarrow (4 + b)^2 + b^2 = 400$

$\Rightarrow 2b^2 + 8b + 16 = 400$

$\Rightarrow b^2 + 4b - 192 = 0$

$\Rightarrow b^2 + 16b - 12b - 192 = 0$

$\Rightarrow (b + 16)(b - 12) = 0$

$\Rightarrow b = 12$ m

Thus, a = 16 m

9. Question

The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one metre more then twice its breadth. Find the length and the breadth of the plot.

Answer

Given: The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one metre more then twice its breadth.

To find: the length and the breadth of the plot.

Solution: Let the breadth be 'a' m

Given, length of the plot (in metres) is one metre more then twice its breadth.

$\Rightarrow$ length = (2a + 1) m

Now, area of a rectangular plot is 528 m^2

Area of rectangle = length × breadth

$\Rightarrow a \times (2a + 1) = 528$

$\Rightarrow 2a^2 + a - 528 = 0$In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\Rightarrow 2a^2 + 33a - 32a - 528 = 0$

$\Rightarrow a(2a + 33) - 16(2a + 33) = 0$

$\Rightarrow (2a + 33)(a - 16) = 0 \Rightarrow (2a + 33) = 0$ and $(a - 16) = 0 \Rightarrow 2a = -33$ and $a = 16 \Rightarrow a = -33/2$ and $a = 16$

As any side can never be negative,

$\Rightarrow a = 16$ mlength = (2a + 1) = 2 × 16 + 1 = 33 mHence breadth = 16 m and length = 33 m

Exercise 8.12

1. Question

A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Answer

Let the number of days in which B finishes the work be 'b'.

∴ Number of days in which A finishes the work = b − 10

In 1 day,

B finishes 1/b of the work

A finishes 1/(b − 10) of the work

Now, both A and B together can finish the work in 12 days

$$\Rightarrow \frac{1}{b}+\frac{1}{b-10}=\frac{1}{12}$$

$\Rightarrow 12(b - 10 + b) = b^2 - 10b$

$\Rightarrow 24b - 120 = b^2 - 10b$

$\Rightarrow b^2 - 34b + 120 = 0$

$\Rightarrow b^2 - 30b - 4b + 120 = 0$

$\Rightarrow b(b - 30) - 4(b - 30) = 0$

$\Rightarrow b = 4$, 30 b can't be 4 as A takes 10 days less than B

Thus number of days in which B alone finishes the work is 30 days.

2. Question

If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?

Answer

Let the slower pipe fill the reservoir in 'a' hours

Faster pipe fills it in 'a – 10' hours.

Given, the two pipes will fill the reservoir together in 12 hours.

In 1 hour, part of reservoir filled = 1/12

$$\Rightarrow \frac{1}{a}+\frac{1}{a-10}=\frac{1}{12}$$

$\Rightarrow 12(a + a - 10) = a^2 - 10a$

$\Rightarrow 24a - 120 = a^2 - 10a$

$\Rightarrow a^2 - 34a + 120 = 0$

$\Rightarrow a^2 - 30a - 4a + 120 = 0$

$\Rightarrow a(a - 30) - 4(a - 30) = 0$

$\Rightarrow (a - 4)(a - 30) = 0$

Value of a can't be 4 as (a – 10) will be negative

Thus a = 30

3. Question

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer

Let the smaller diameter tap fill the reservoir in 'a' hours

Larger diameter tap fills it in 'a – 10' hours.

Given, two water taps together can fill a tank in $9\frac{3}{8}$ = 75/8 hours.

In 1 hour, part of tank filled = 8/75

$$\Rightarrow \frac{1}{a}+\frac{1}{a-10}=\frac{8}{75}$$

⇒ 75(a + a – 10) = $8a^2$ – 80a

⇒ 150a – 750 = $8a^2$ – 80a

⇒ $8a^2$ – 230a + 750 = 0

⇒ $4a^2$ – 115a + 375 = 0

⇒ $4a^2$ – 100a – 15a + 375 = 0

⇒ 4a(a – 25) – 15(a – 25) = 0

⇒ (4a – 15)(a – 25) = 0

Value of a can't be 15/4 as (a – 10) will be negative

Thus a = 25

Time taken by faster tap = 25 – 10 = 15 hours

4. Question

Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Answer

Let the faster pipe fill the tank in 'a' min

Slower pipe fills it in 'a + 5' min.

Given, the pipes running together can fill a tank in $11\frac{1}{9}$ = 100/9 minutes.

In 1 min, part of tank filled = 9/100

$$\Rightarrow \frac{1}{a}+\frac{1}{a+5}=\frac{9}{100}$$

⇒ 100(a + a + 5) = 9(a^2 + 5a)

⇒ 200a + 500 = $9a^2$ + 45a

⇒ $9a^2$ – 155a - 500 = 0

⇒ $9a^2$ – 180a + 25a - 500 = 0

⇒ 9a(a – 20) + 25(a – 20) = 0

⇒ (9a + 25)(a – 20) = 0

⇒ a = 20 mins

Slower pipe will fill it in 25 min

5. Question

To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?

Answer

Let the larger diameter pipe fill it in 'a' hours

The smaller diameter pipe fills it in ‘a + 10’ hours

In 1 hour, larger diameter pipe fills 1/a part of the pool.

In 1 hour, smaller diameter pipe fills 1/(a + 10) part of the pool.

Given, the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled.

⇒ 4 × 1/a + 9 × 1/(a+10) = 1/2

⇒ 2(4a + 40 + 9a) = a^2 + 10a

⇒ 26a + 80 = a^2 + 10a

⇒ a^2 – 16a – 80 = 0

⇒ a^2 – 20a + 4a – 80 = 0

⇒ a(a – 20) + 4(a – 20) = 0

⇒ (a + 4)(a – 20) = 0

⇒ a = 20 hours

Time in which smaller diameter pipe fills the pool = 20 + 10 = 30 hours

Exercise 8.13

1. Question

A piece of cloth costs Rs. 35. If the piece were 4 m longer and each metre costs Rs. 1 less, the cost would remain unchanged. How long is the piece?

Answer

Let the length of cloth be ‘a’ m.

Given, piece of cloth costs Rs. 35 and if the piece were 4 m longer and each metre costs Rs. 1 less, the cost remains unchanged.

Cost of 1m of cloth = 35/a

$$\Rightarrow (a + 4) \times (\frac{35}{a} - 1) = 35$$

⇒ (a + 4)(35 – a) = 35a

⇒ 35a + 140 – a^2 – 4a = 35a

⇒ a^2 + 4a – 140 = 0

⇒ a^2 + 14a – 10a – 140 = 0

⇒ a(a + 14) – 10(a + 14) = 0

⇒ (a – 10)(a + 14) = 0

⇒ a = 10 m

2. Question

Some students planned a picnic. The budget for food was Rs. 480. But eight of these failed to go and thus the cost of food for each member increased by Rs. 10. How many students attended the picnic?

Answer

Let the number of students who planned the picnic be ‘a’.

Budget for the food was Rs. 480

Cost of food for each member = 480/a

Given, eight of these failed to go and thus the cost of food for each member increased by Rs. 10

$$\Rightarrow (a-8) \times (\frac{480}{a} + 10) = 480$$

⇒ (a – 8)(480 + 10a) = 480a

⇒ 480a + $10a^2$ – 3840 – 80a = 480a

⇒ a^2 – 8a – 384 = 0

⇒ a^2 – 24a + 16a – 384 = 0

⇒ a(a – 24) + 16(a – 24) = 0

⇒ (a + 16)(a – 24) = 0

⇒ a = 24

Number of students who attended the picnic = 24 – 8 = 16

3. Question

A dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article. Find the cost price of the article.

Answer

Let the cost price be Rs a.

Given, the dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article.

It's given that he gains as much as the cost price of the article, thus, Gain% = a%

Gain% = $\frac{SP-CP}{CP} \times 100$

⇒ $a = \frac{24-a}{a} \times 100$

⇒ a^2 = (24-a) ×100

⇒ a^2 + 100a – 2400 = 0

⇒ a^2 + 120a – 20a – 2400 = 0

⇒ (a + 120)(a – 20) = 0

⇒ a = 20 or -120

Since money cannot be negative so, negalecting -120, we get,

⇒ a = 20

Thus, the cost price of the article is Rs 20

4. Question

Out of a group of swans, 7/2 times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water. Find the total number of swans.

Answer

Let the number of swans in the pond be ‘a’.

Given, out of a group of swans, 7/2 times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water.

$$\Rightarrow \frac{7}{2}\sqrt{a} + 2 = a$$

⇒ 7√a = 2a – 4

Squaring both sides

⇒ 49a = $4a^2$ + 16 – 16a

⇒ $4a^2$ – 65a + 16 = 0

⇒ $4a^2$ – 64a – a + 16 = 0

⇒ 4a(a – 16) – (a – 16) = 0

⇒ (4a – 1)(a – 16) = 0

⇒ a can’t be 1/4, thus a = 16

5. Question

If the list price of a toy is reduced by Rs. 2, a person can buy 2 toys more for Rs. 360. Find the original price of the toy.

Answer

Let the original price of the toy be ‘a’.

Given, when the list price of a toy is reduced by Rs. 2, a person can buy 2 toys more for Rs. 360.

The number of toys he can buy at the original price for Rs. 360 = 360/a

According to the question,

$$\Rightarrow \frac{360}{a-2} = \frac{360}{a} + 2$$

⇒ 360a = (a – 2)(360 + 2a)

⇒ 360a = 360a + $2a^2$ – 720 – 4a

⇒ a^2 – 2a – 360 = 0

⇒ a^2 – 20a + 18a – 360 = 0

⇒ a(a – 20) + 18(a – 20) = 0

⇒ (a + 18)(a – 20) = 0⇒ a + 18 = 0 or a - 20 = 0⇒ a = -18 or a = 20As, price can't be negative, a = -18 is not possibleTherefore,a = Rs. 20

6. Question

Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less. Find the original number of persons.

Answer

Let the original number of people be ‘a’.

Given, Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less.

Amount which each receives= 9000/a

$$\Rightarrow \frac{9000}{a+20} = \frac{9000}{a} - 160$$

⇒ 9000a = (9000 – 160a)(a + 20)

⇒ 9000a = 9000a + 180000 – $160a^2$ – 3200a

⇒ a^2 + 20a – 1125 = 0

⇒ a^2 + 45a – 25a – 1125 = 0

⇒ a(a + 45) – 25(a + 45) = 0

⇒ (a – 25)(a + 45) = 0

⇒ a = 25

7. Question

Some students planned a picnic. The budget for food was Rs. 500. But, 5 of them failed to go and thus the cost of food for each member increased by Rs. 5. How many students attended the picnic?

Answer

Let the number of students who planned the picnic be 'a'.

Budget for the food was Rs. 500

Cost of food for each member = 500/a

Given, 5 of these failed to go and thus the cost of food for each member increased by Rs. 5

$$\Rightarrow (a-5) \times \left(\frac{500}{a} + 5\right) = 500$$

⇒ (a – 5)(100 + a) = 100a

⇒ 100a + a^2 – 500 – 5a = 100a

⇒ a^2 – 5a – 500 = 0

⇒ a^2 – 25a + 20a – 500 = 0

⇒ a(a – 25) + 20(a – 25) = 0

⇒ (a + 20)(a – 25) = 0

⇒ a = 25

Number of students who attended the picnic = 25 – 5 = 20

8. Question

A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it the possible to do so? If yes, at what distances from the two gates should the pole be erected?

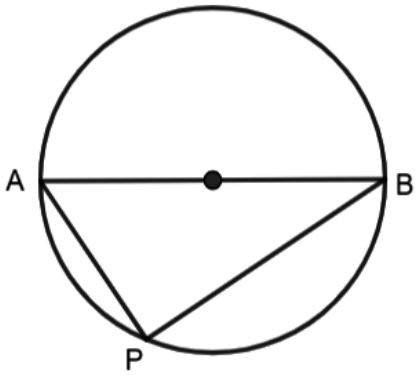
Answer

Let the distance of pole from gate A be 'a'.

⇒ Difference of the distance of the pole from two diametrically opposite fixed gates A and B on the boundary is 7 metres.

⇒ Distance of pole from gate B = a – 7 m

⇒ Diameter of the park = 13 m



Now, Angle in a semicircle is a right angle, therefore ABP is a triangle right-angled at P, Therefore, By Pythagoras theorem i.e.Hypotenuse^2 = length^2 + breadth^2

$\Rightarrow 13^2 = a^2 + (a - 7)^2$

$\Rightarrow 169 = 2a^2 + 49 - 14a$

$\Rightarrow a^2 - 7a - 60 = 0$

$\Rightarrow a^2 - 12a + 5a - 60 = 0$

$\Rightarrow a(a - 12) + 5(a - 12) = 0$

$\Rightarrow (a + 5)(a - 12) = 0$

$\Rightarrow a = -5$ or $a = 12$ m

but distance can't be negative, hence a = 12

Thus, distance of pole is 12 m from gate A and (12 - 7) = 5 meters from gate B

9. Question

In a class test, the sum of the marks obtained by P in Mathematics and science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science. The product of his marks, would have been 180. Find his marks in the two subjects.

Answer

Given: In a class test, the sum of the marks obtained by P in Mathematics and science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science. The product of his marks, would have been 180.

To find: his marks in the two subjects.

Solution: Let the marks obtained in Mathematics by P be 'a'.

Given, sum of the marks obtained by P in Mathematics and science is 28.

$\Rightarrow$ Marks obtained in science = 28 – a

Also, if he got 3 marks more in Mathematics and 4 marks less in Science, product of his marks, would have been 180.

$\Rightarrow (a + 3)(28 - a - 4) = 180 \Rightarrow (a + 3)(24 - a) = 180$

$\Rightarrow 24a - a^2 + 72 - 21a = 180$

$\Rightarrow -a^2 + 21a + 72 = 180 \Rightarrow -a^2 + 21a + 72 - 180 = 0 \Rightarrow -a^2 + 21a - 108 = 0$

$\Rightarrow a^2 - 21a + 108 = 0$

Factorise the above quadratic equation by splitting the middle term:

$\Rightarrow a^2 - 12a - 9a + 108 = 0$

⇒ a(a – 12) – 9(a – 12) = 0

⇒ (a – 9)(a – 12) = 0

⇒ a = 9 or 12If marks obtained in mathematics is 9, the marks obtained in science is 28-a = 28 - 9 = 19

⇒Marks in Mathematics = 9, Marks in Science = 19

Or

If marks obtained in mathematics is 12, the marks obtained in science is 28-a = 28 - 12 = 16

Marks in Mathematics = 12, Marks in Science = 16

10. Question

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in two subjects.

Answer

Given : In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

To find: her marks in two subjects.

Solution: Let the marks obtained in Mathematics by Shefali be 'a'.

Given, sum of the marks obtained by Shefali in Mathematics and English is 30.

Marks obtained in english = 30 – a

Also, she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

⇒ (a + 2)(30 – a – 3) = 210⇒ (a + 2)(27 – a) = 210

⇒ 27a -a^2 + 54 - 2a = 210

⇒ -a^2 + 25a + 54 = 210⇒ -a^2 + 25a + 54 - 210 = 0⇒ -a^2 + 25a - 156 = 0

⇒ a^2 – 25a + 156 = 0

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

⇒ a^2 – 13a – 12a + 156 = 0

⇒ a(a – 13) – 12(a – 13) = 0

⇒ (a – 12)(a – 13) = 0

⇒ a = 12 or 13

If marks in mathematics is 12marks in english is 30 - a = 30 - 12 = 18If marks in mathematics is 13marks in english is 30 - a = 30 - 13 = 17

Hence

Marks in Mathematics = 12, Marks in English = 18

Or

Marks in Mathematics = 13, Marks in English = 17

11. Question

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Answer

Let the number of article produced on the day be 'a'.

Given, it was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day.

Cost of production of each article = 2a + 3

Given, cost of production was Rs. 90

$\Rightarrow a(2a + 3) = 90$

$\Rightarrow 2a^2 + 3a - 90 = 0$

$\Rightarrow 2a - 12a + 15a - 90 = 0$

$\Rightarrow 2a(a - 6) + 15(a - 6) = 0$

$\Rightarrow (2a + 15)(a - 6) = 0$

$\Rightarrow a = 6$

Cost of each article = $2 \times 6 + 3$ = Rs. 15

CCE - Formative Assessment

1. Question

Write the value of k for which the quadratic equation $x^2 - kx + 4 = 0$ has equal roots.

Answer

Quadratic equation has equal roots then $d = b^2 - 4ac = 0$

Here a = 1, b = k and c = 4

So $b^2 - 4ac = 0$

$\Rightarrow k^2 - 4 \times 1 \times 4 = 0$

$\Rightarrow k^2 - 16 = 0$

$\Rightarrow k = \pm 4$

2. Question

What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

Answer

Consider the equation $4x^2 - 12x - 9 = 0$,To check the roots of the equation we will check the value of $d = b^2 - 4ac$

Here a = 4, b = 12 and c = − 9

So $d = (12)^2 - 4 \times 4 \times -9$

= 144 + 144

= 288>0 which is real

So the roots of the given equation are Real and distinct.

3. Question

If $1 + \sqrt{2}$ is a root of a quadratic equation with rational coefficients, write its other root.

Answer

$1 + \sqrt{2}$ Is a root of quadratic equation with rational coefficients that is the sum of the roots is rational and the product of the roots is also rational.

Since the rational roots occurs in conjugate pairs so the other root of the equation is $1 - \sqrt{2}$

4. Question

Write the number of real roots of the equation $x^2 + 3|x| + 2 = 0$.

Answer

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

If $D = b^2 - 4ac \geq 0$ then the values of x are real

If $D = b^2 - 4ac < 0$ then the values of x are complex

$|z|$ is always a positive real number regardless of x being a real number or complex number.

Given eqn. is $|x|^2 + 3|x| + 2 = 0$ and $a = 1$, $b = 3$ and $c = 2$

$|x|^2 + 3|x| + 2 = 0$

$\Rightarrow |x| = \frac{-3 \pm \sqrt{9-8}}{2}$

$\Rightarrow |x| = -2$ or -1

But $|x|$ cannot be negative

No real root for the equation.

5. Question

Write the sum of real roots of the equation $x^2 + |x| - 6 = 0$.

Answer

First of all, the equation is $x^2 + |x| - 6 = 0$

CASE 1: $x>0$ then $|x| = x$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow (x + 3)(x-2) = 0$

$\Rightarrow x = 2, -3$

CASE 2: $x<0$ $|x| = -x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x-3)(x + 2) = 0$

$\Rightarrow x = -2, 3$

So the sum of the roots is $2 + (-3) + (-2) + 3$

$= 0$

6. Question

Write the set of values of 'a' for which the equation $x^2 + ax - 1 = 0$ has real roots.

Answer

Consider $x^2 + ax - 1 = 0$,For the quadratic equation to have real roots $D \geq 0$

Here $a = 1$, $b = a$ and $c = 1$

In the given equation $D = a^2 - 4 \geq o$

$\Rightarrow a^2 \geq 4$

So for all the real values of 'a' which are greater than or equal to 2 and − 2 the equation will have the real roots.

7. Question

Is there any real value of 'a' for which the equation

$x^2 + 2x + (a^2 + 1) = 0$ has real roots?

Answer

A quadratic equation has two real roots if discriminant = 0

For the given equation, we have:

$d = b^2 - 4\ a\ c$

$d = (2)^2 - 4\ (1)\ (a^2 + 1)$

$d = 4 - 4(a^2 + 1)$

$d = 4(1 - a^2 - 1)$

$d = - 4a^2$

Now, D = 0 when a = 0. So, the equation will have real and equal roots if a = 0. And for all other values of a, the equation will have no real roots.

No, there is no real value of 'a' for which the given equation has real roots.

8. Question

Write the value of λ for which $x^2 + 4x + \lambda$, is a perfect square.

Answer

For being the perfect square, the roots are equal

So, $d = b^2 - 4ac = 0$

Here $a = 1$, $b = 4$ and $c = \lambda$

$\Rightarrow d = 16 - 4\ \lambda = 0$

$\Rightarrow \lambda = 4$

9. Question

Write the condition to be satisfied for which equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}\,x + b = 0$ have equal roots.

Answer

Given the roots of both the equations are real

For first equation $ax^2 + 2bx + c = 0$

Its discriminant; $d \geq 0$

$D = b^2 - 4ac$

$D = (2b)^2 - 4 \times a \times c \geq 0$

$4b^2 \geq 4ac$

$b^2 \geq ac$ …1

For second equation

$bx^2 - 2\sqrt{ac}\,x + b = 0$

$d = b^2 - 4ac \geq 0$

$= (2\sqrt{ac})^2 - 4\ b\ xb \geq 0$

$= 4ac - 4\ b^2 \geq 0$

$ac \geq b^2$ …2

From 1 and 2 we get only one case where $b^2 = ac$

10. Question

Write the set of values of k for which the quadratic equation has $2x^2 + kx + 8 = 0$ has real roots.

Answer

To have the real roots $D = b^2 - 4ac \geq 0$

Here a = 2, b = k and c = 8

$D = k^2 - 4 \times 2 \times 8 \geq 0$

$\Rightarrow K^2 \geq 64$

$\Rightarrow K \geq \pm 8$

So for all the values of k greater than or equal to 8 and − 8, the given quadratic equation will have real roots.

11. Question

Write a quadratic polynomial, sum of whose zeros is $2\sqrt{3}$ and their product is 2.

Answer

The sum of the two zeros of the quadratic equation is given by $-b/a$

Here it's given $-b/a = 2\sqrt{3}$

The product of the quadratic equation is c/a

Here $c/a = 2$

the quadratic equation is of the form $ax^2 + b\,x + c = 0$

or x^2 + (sum of the roots) x + product of the roots = 0

$= x^2 - 2\sqrt{3}\,x + 2$

$f(x) = k(x^2 - 2\sqrt{3}\,x + 2)$, where k is any real number

12. Question

Show that x = – 3 is a solution of $x^2 + 6x + 9 = 0$.

Answer

To be the solution of the equation the value x = – 3 should satisfy the given equation $x^2 + 6x + 9 = 0$

Putting value of x on L.H.S

$(– 3)^2 + 6 \text{ x } (– 3) + 9$

⇒ 9 – 18 + 9 = 0 = R.H.S

Hence x = – 3 is the solution of given equation.

13. Question

Show that x = – 2 is a solution of $3x^2 + 13x + 14 = 0$.

Answer

To be solution of the equation x = – 2 should satisfy the given equation

L.H.S $3x (– 2)^2 + 13 \text{ x } – 2 + 14$

⇒ 12 – 26 + 14 = 0 = R.H.S

Hence x = – 2 is the solution of the equation.

14. Question

Find the discriminant of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Answer

$d = b^2 – 4 ac$

Here a = $3\sqrt{3}$

B = 10 and c = $\sqrt{3}$

D = $(10)^2 – 4 (3\sqrt{3}) \sqrt{3}$

D = 100 – 36

D = 64

15. Question

If $x = \frac{-1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx – 3 = 0$, find the value of k.

Answer

Since x = $\frac{-1}{2}$ is the solution of the equation it should satisfy the equation

Putting value of x in the given equation

$3(– \frac{1}{2})^2 + 2 \text{ k } (— \frac{1}{2}) – 3 = 0$

$\frac{3}{4}$ – k – 3 = 0

$\frac{9}{4}$ = k

$\frac{9}{4}$

1. Question

If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then

A. $k < 4$

B. $k > 4$

C. $k \geq 4$

D. $k \leq 4$

Answer

If roots of given equation are real and distinct then $D = b^2 - 4ac > 0$

Here $a = 1$, $b = 4$ and $c = k$

So, $4^2 - 4\ (1)\ (k) > 0$

$16 - 4k > 0$

$16 > 4k$

$K < 4$

2. Question

If the equation $x^2 - ax + 1 = 0$ has two distinct roots, then

A. $|a| = 2$

B. $|a| < 2$

C. $|a| > 2$

D. None of these

Answer

If roots of given equation are distinct then

$d = b^2 - 4ac > 0$

Here $a = 1$, $b = a$, $c = 1$

So, $a^2 - 4(1)\ (1) > 0$

$a^2 - 4 > 0$

$a^2 > 4$

$|a| > 2$

3. Question

If the equation $9x^2 + 6kx + 4 = o$ has equal roots, then the roots are both equal to ?

A. $\pm\frac{3}{2}$

B. $\pm\frac{3}{2}$

C. 0

D. ± 3

Answer

Given: the equation $9x^2 + 6kx + 4 = o$ has equal roots.

To find: the roots are both equal to ?

Solution: If roots of given equation are equal then $D = b^2 - 4ac = 0$

$\Rightarrow (6k)^2 - 4(9)(4) = 0$

$\Rightarrow 36k^2 - 144 = 0$

$\Rightarrow 36k^2 = 144$

$\Rightarrow K^2 = 4$

$\Rightarrow K = \pm 2$

Case 1 :- when k = 2

In equation $9x^2 + 6kx + 4 = 0$

$9x^2 + 6(2)x + 4 = 0$

$9x^2 + 12x + 4 = 0$

$(3x)^2 + 2 \times 2 \times 3x + (2)^2 = 0$

$(3x + 2)^2 = 0$

$3x + 2 = 0$

$3x = -2$

$$x = -\frac{2}{3}$$

Case 2 :- when k = − 2

In equation $9x^2 + 6kx + 4 = 0$

$9x^2 + 6(-2)x + 4 = 0$

$9x^2 - 12x + 4 = 0$

$(3x)^2 - 2 \times 2 \times 3x + (2)^2 = 0$

$(3x - 2)^2 = 0$

$3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$ So the roots of the given quadratic equation are $\pm\frac{2}{3}$

4. Question

If $ax^2 + bx + c = 0$ has equal roots, then c =

A. –b/2a

B. $\frac{b}{2a}$

C. $\frac{-b^2}{4a}$

D. $\frac{b^2}{4a}$

Answer

Let the roots of given equation be m and n

According to the question M = n

Sum of roots = m + n = – b / a

$2m = -b/a$

$\Rightarrow m = -b/2a$

Product of roots = m × n = c/a

$m^2 = c/a$

$(-b/2a)^2 = c/a$

$b^2 / 4a^2 = c/a$

$b^2 / 4a^2 = c$

Therefore c = $\frac{b^2}{4a}$

5. Question

If the equation $ax^2 + 2x + a = 0$ has two distinct roots, if

A. a = ±1

B. a = 0

C. a = 0, 1

D. a = – 1, 0

Answer

If the roots of given equation are distinct then

$d = b^2 - 4ac = 0$

$\Rightarrow d = b^2 - 4ac = 0$

$\Rightarrow 2^2 - 4(a)(a) = 0$

$\Rightarrow 4 - 4a^2 = 0$

$\Rightarrow 4a^2 = 4$

$\Rightarrow a^2 = 1$

⇒ a = $\pm$1

6. Question

The positive value of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots, is

A. 4

B. 8

C. 12

D. 16

Answer

If the given equation $x^2 + kx + 64$ has real roots then D ≥ 0

$D = b^2 - 4ac \geq 0$

Here a = 1, b = k, c = 64

$d = K^2 - 4\ (1)\ (64) \geq 0$

$d = K^2 - 256 \geq 0$

K ≥ 16…… (1)

If the given equation $x^2 - 8x + k = 0$ has real roots then then

d≥0

$D = b^2 - 4ac \geq 0$

$8^2 - 4(1)\ (k) \geq 0$

64– 4k ≥ 0

64 ≥ 4k

K ≤ 16…… (2)

From (1) and (2) we can conclude that k = 16

7. Question

The value of $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ is

A. 4

B. 3

C. – 2

D. 3.5

Answer

In given equation let x = $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$

So, x = √(6 + x)

Now squaring both side

$x^2 = 6 + x$

$X^2 - x - 6 = 0$

$X^2 - 3x + 2x - 6 = 0$

$x(x - 3) + 2(x - 3) = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or -2

x cannot be equal to -2 as root can never be negative.

$x = 3$

8. Question

If 2 is a root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots, then q =

A. 8

B. – 8

C. 16

D. – 16

Answer

2 is the root of given equation $x^2 + bx + 12 = 0$

So $2^2 + 2b + 12 = 0$

$16 + 2b = 0$

$b = -8$……..1

Now, $d = b^2 - 4ac = 0$ of second equation is

$d = b^2 - 4(1)(q) = 0$ here $a = 1$, $b = -8$ (from 1) and $c = q$

$(-8)^2 - 4q = 0$

$64 - 4q = 0$

$q = 16$

Hence value of q is 16.

9. Question

If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$ has equal roots, then

A. ab = cd

B. ad = bc

C. $ad = \sqrt{bc}$

D. $ab = \sqrt{cd}$

Answer

If the roots are equal then $d = b^2 - 4ac = 0$

Here $a = (a^2 + b^2)$, $b = 2(ac + bd)$, $c = (c^2 + d^2)$

$D = b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

$\Rightarrow \{-2(ac + bd)\}^2 = 4\{(a^2 + b^2)(c^2 + d^2)\}$

$\Rightarrow 4(a^2c^2 + b^2d^2 + 2acbd) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$\Rightarrow 2acbd = a^2d^2 + b^2c^2$

$\Rightarrow a^2d^2 + b^2c^2 - 2abcd = 0$

$\Rightarrow (ad - bc)^2 = 0$

$\Rightarrow ad - bc = 0$

$\Rightarrow ad = bc$

10. Question

If the roots of the equation $(a^2\ b^2)\ x^2 - 2b\ (a + c)\ x + (b^2 + c^2) = 0$ are equal, then

A. $2b = a + c$

B. $b^2 = ac$

C. $b = \frac{2ac}{a + c}$

D. $b = ac$

Answer

The roots of the equation are equal so $d = b^2 - 4ac = 0$

Here $a = (a^2 + b^2)$, $b = -2b\ (a + c)$, $c = (b^2 + c^2)$

$d = (-2b\ (a + c))^2 = 4\ (a^2 + b^2)\ (b^2 + c^2)$

$\Rightarrow b^2\ (a^2 + 2ac + c^2) = a^2b^2 + a^2c^2 + b^4 + b^2c^2$

$\Rightarrow (ac)^2 - 2(ac)\ (b^2) + (b^2)^2 = 0$

$\Rightarrow (ac - b^2)^2 = 0$

$\Rightarrow (ac - b^2) = 0$

$\Rightarrow a\ c = b^2$

11. Question

If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then

A. $-3 < b < 3$

B. $-2 < b < 2$

C. $b > 2$

D. $b < -2$

Answer

If the equation does not possess real roots then

$d = b^2 - 4ac\ 0$

Here $a = 1$, $b = -b$, $c = 1$

$d = b^2 - 4 < 0$

$b^2 < 4$

$b < \pm 2$

$-2 < b < 2$

12. Question

If x = 1 is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then ab =

A. 3

B. 3.5

C. 6

D. – 3

Answer

Since x = 1 is root of equations, it will satisfy both the equations.

Putting x = 1 in $ax^2 + ax + 3 = 0$

1a + a + 3 = 0

2a + 3 = 0

A = – 3/2

Putting x = 1 in $x^2 + x + b = 0$

1 + 1 + b = 0

b = – 2

$$ab = \frac{-3}{2} x - 2$$

ab = 3

13. Question

If p and q are the roots of the equation $x^2 + px + q = 0$, then

A. p = 1, q = – 2

B. q = 0, p = 1

C. p = – 2, q = 0

D. p = – 2, q = 1

Answer

Since p and q are roots of the equations then

Sum of the roots is $p + q = -b/a = -(p) = -p$

Here a = 1, b = p and c = q

Products of the root = $p \times q = c/a = q$

$\therefore p \times q = q$

p = 1

Putting value of ‘p’ in $p + q = -p$

$1 + q = -1$

$q = -2$

14. Question

If a and b can take values 1, 2, 3, 4. Then the number of the equations of the form $ax^2 + bx + 1 = 0$ having real roots is

A. 10

B. 7

C. 6

D. 12

Answer

For quadratic equation to have real roots,

$d \geq 0$

$b^2 - 4a \geq 0$

$b^2 \geq 4a$

For a = 1, 4a = 4, b = 2, 3, 4 (3 equations)

With values of (a,b) as (1,2), (1,3), (1,4)

a = 2, 4a = 8, b = 3, 4 (2 equations)

With values of a,b as (2,3), (2,4)

a = 3, 4a = 12, b = 4 (1 equation)

With value of (a,b) as (3,4)

a = 4, 4a = 16, b = 4 (1 equation)

With values of (a,b) as (4,16)

Thus, total 7 equations are possible.

15. Question

The number of quadratic equations having real roots and which do not change by squaring their roots is

A. 4

B. 3

C. 2

D. 1

Answer

The roots of the equation are real (given)

Let α and β be the two roots according to the given condition

$\alpha = \alpha^2$

$\beta = \beta^2$

Sum of the roots = $\alpha + \beta = \alpha^2 + \beta^2$

Product of the roots = $\alpha\ \beta = \alpha^2\ \beta^2$

There are only two number who does not change on squaring them that is 0 and 1

So the number of equations could be 2 by being the roots as

(0,1) and (1,0)

16. Question

If $(a^2 + b^2)\ x^2 + 2\ (ac + bd)\ x + c^2 + d^2 = 0$ has no real roots, then

A. ad = bc

B. ab = cd

C. ac = bd

D. $ad \neq bc$

Answer

Since the equation

$(a^2 + b^2)\ x^2 + 2\ (ac + bd)\ x + c^2 + d^2 = 0$ has no real root

$D < 0$

$b^2 - 4ac < 0$

$b^2 < 4ac$

Here $a = (a^2 + b^2)$, $b = 2\ (ab + bd)$, $c = c^2 + d^2$

$4(ac + bd)^2 - 4\ (a^2 + b^2)(c^2 + d^2) < 0$

$4a^2c^2 + 4b^2d^2 + 8abcd - 4(a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2) < 0$

$- 4(a^2d^2 + b^2c^2 - 2abcd) < 0$

$- 4(ad + bc)^2 < 0$

$\therefore$ d is always negative

And $ad \neq bc$

17. Question

If the sum of the roots of the equation $x^2 - x = \lambda\ (2x - 1)$ is zero, then λ =

A. − 2

B. 2

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

Answer

equation is $x^2 - x = \lambda\ (2x - 1)$

$x^2 - x - \lambda\ (2x - 1) = 0$

$x^2 - (2\lambda + 1)x + \lambda = 0$

Here a = 1, b = − (2λ + 1), c = λ

Sum of the roots = − b/a

⇒ −(− (2λ + 1)) = 0

⇒ λ = − 1/2

18. Question

If x = 1 is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ then, ab =

A. 1

B. 2

C. 4

D. 3

Answer

Since x = 1 is root of equation

Then it satisfy the equation

Putting x = 1 in first equation

a + a + 2 = 0

2a + 2 = 0

a = − 1

Putting x = 1 in equation second

1 + 1 + b = 0

2 + b = 0

b = − 2

ab = − 1 x − 2

ab = 2

19. Question

The value of c for which the equation $ax^2 + 2bx + c = 0$ has equal roots is

A. $\frac{a^2}{4b}$

B. $\frac{b^2}{4a}$

C. $\frac{a^2}{b}$

D. $\frac{a^2}{4b}$

Answer

The equation has equal root which means d = 0

d = b^2 – 4ac = 0

Here a = a, b = 2b, c = c

$(2b)^2$ – 4 ac = 0

b^2 – ac = 0

c = b^2/a

20. Question

If x^2 + k (4x + k – 1) + 2 = 0 has equal roots, then k =

A. $-\frac{2}{3}, 1$

B. $\frac{2}{3}, -1$

C. $\frac{3}{2}, \frac{1}{3}$

D. $-\frac{3}{2}, -\frac{1}{3}$

Answer

Equation x^2 + k (4x + k – 1) + 2 = 0 has equal roots

d = 0

d = b^2 – 4ac = 0

Here a = 1, b = 4k, c = k^2 – k + 2

⇒ 16 k^2 – 4(k^2 – k + 2) = 0

⇒ 12 k^2 + 4k – 8 = 0

⇒ 3k^2 + k – 2 = 0

⇒ (3k – 2) (k + 1) = 0

⇒ K = 2/3, – 1

21. Question

If the sum and product of the roots of the equation kx^2 + 6x + 4k = 0 are equal, then k =

A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

Answer

In the given equation $kx^2 + 6x + 4k = 0$

Sum of the roots = product of the roots (given)

– b/a = c/a

Here a = k, b = 6 and c = 4k

$-6/k = 4k/k$

K = $-3/2$

22. Question

If sin α and cos α are the roots of the equation $ax^2 + bx + c = 0$, then b^2 =

A. $a^2 - 2ac$

B. $a^2 + 2ac$

C. $a^2 - ac$

D. $a^2 + ac$

Answer

Equation $ax^2 + bx + c = 0$ has $\cos\alpha$ and $\cos\alpha$ as two roots

sin α + cos α = $-\frac{b}{a}$

sin α × cos α = c/a …eq(1)

$(\sin\alpha + \cos\alpha)^2 = \frac{b^2}{a^2}$

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha.\cos\alpha = \frac{b^2}{a^2}$ …..eq (2)

But $\sin^2 \alpha + \cos^2 \alpha = 1$

∴ a^2 (1 + 2 sinα.cos α) = b^2

Putting sin α × cos α = c/a, we get,

⇒ $b^2 = a^2 + 2ac$.

23. Question

If 2 is a root of the equation $x^2 + ax + 12 = 0$ and the quadratic equation $x^2 + ax + q = 0$ has equal roots, then q

A. 12

B. 8

C. 20

D. 16

Answer

The given equation $x^2 + ax + 12 = 0$ has a root = 2

So it will satisfy the equation

$4 + 2a + 12 = 0$

$2a + 16 = 0$

$a = -8$

Putting value of a in second equation, it becomes

$x^2 + ax + q = 0$

$x^2 - 8x + q = 0$

Roots are equal so d = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow 64 - 4q = 0$

$\Rightarrow q = 64/4$

$\Rightarrow q = 16$

24. Question

If the sum of the roots of the equation $x^2 - (k + 6)x + 2 (2k - 1) = 0$ is equal to half of their product, then k =

A. 6

B. 7

C. 1

D. 5

Answer

In the given equation $x^2 - (k + 6)x + 2 (2k - 1) = 0$

$a = 1, b = -(k + 6), c = 2 (2k - 1)$

Sum of the roots = 1/2 (product of roots) (given)

$K + 6 = 2k - 1$

$K = 7$

25. Question

If a and b are roots of the equation $x^2 + a x + b = 0$, then a + b =

A. 1

B. 2

C. – 2

D. – 1

Answer

Given a and b are roots of the equation $x^2 + a x + b = 0$

Here $a = 1, b = a, c = b$

Sum of the roots = $a + b = -a/1$

b = − 2a

Product of the roots

ab = b

a = 1

∴ b = − 2 x 1 = − 2

Now a + b = 1 + (− 2) = − 1

26. Question

A quadratic equation whose one root is 2 and the sum of whose roots is zero, is

A. $x^2 + 4 = 0$

B. $x^2 - 4 = 0$

C. $4x^2 - 1 = 0$

D. $x^2 - 2 = 0$

Answer

Let Root of an equation = α = 2

Sum of the roots = α + β = 0, where α and β are two roots of the equation

β = − 2

α β = 2 x − 2 = − 4

the general equation is of the form

x^2 + (sum of the roots)x + product of the roots = 0

$x^2 - 4 = 0$ is the required equation

27. Question

If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then b^2 :ac =

A. 3: 1

B. 3 : 16

C. 16 : 3

D. 16 : 1

Answer

In the given equation $ax^2 + bx + c = 0$

Let α and β be the two roots

Given α = 3β ……..1

α β = c/a (product of the roots)

3 β^2 = c / a ……. by using 1

$\beta^2 = c/3a$ ……..2

α + β = $-b/a$

9. Arithmetic Progressions

Exercise 9.1

1. Question

Write the first five terms of each of the following sequences whose nth terms are :

(i) $a_n = 3n + 2$

(ii) $a_n = \frac{n-2}{3}$

(iii) $a_n = 3^n$

(iv) $a_n = \frac{3n-2}{5}$

(v) $a_n = (-1)^n.2^n$

(vi) $a_n = \frac{n(n-2)}{2}$

(vii) $a_n = n^2 - n + 1$

(viii) $a_n = 2n^2 - 3n + 1$

(ix) $a_n = \frac{2n-3}{6}$

Answer

(i) $a_n = 3n + 2$

Put n = 1

$A_1 = 3(1) + 2 = 3+2=5$

Put n = 2

$A_2 = 3(2) + 2=8$

Put n = 3

$A_3 = 3(3) + 2 = 9+2=11$

Put n = 4

$A_4 = 3(4) +2 = 12+2=14$

Put n =5

$A_5 = 3(5) + 2 = 15+2=17$

(ii) $a_n = \frac{n-2}{3}$

Put n = 1

$A_1=\frac{1-2}{3} = \frac{-1}{3}$

Put n = 2

$A_2= \frac{2-2}{3} = 0$

Put n = 3

$A_3= \frac{3-2}{3} = \frac{1}{3}$

Put n = 4

$A_4= \frac{4-2}{3} = \frac{2}{3}$

Put n = 5

$A_5= \frac{5-2}{3} = \frac{3}{3} = 1$

(iii) $a_n = 3^n$

Put n = 1

$A_1= 31= 3$

Put n = 2

$A_2= 32= 9$

Put n = 3

$A_3= 33= 27$

Put n = 4

$A_4= 34= 81$

Put n = 5

$A_5= 35= 243$

(iv) $a_n = \frac{3n-2}{5}$

Put n = 1

$A_1= \frac{3(1)-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

Put n = 2

$A_2 = \frac{3(2)-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

Put n = 3

$A_3 = \frac{3(3)-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

Put n = 4

$A_4 = \frac{3(4)-2}{5} = \frac{12-2}{5} = \frac{10}{5} = 2$

Put n = 5

$A_5 = \frac{3(5)-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

(v) $a_n = (-1)^n.2^n$

Put n = 1

A_1= (-1)1.21 = (-1).2 = -2

Put n = 2

A_2= (-1)2.22 = (1).4 = 4

Put n = 3

A_3= (-1)3.23 = (-1).8 = -8

Put n = 4

A_4= (-1)4.24 = (1).16 = 16

Put n = 5

A_5= (-1)5.25 = (-1).32 = -32

(vi) $a_n = \frac{n(n-2)}{2}$

Put n = 1

$A_1 = \frac{1\,(1-2)}{2} = \frac{-1}{2}$

Put n = 2

$A_2 = \frac{2(2-2)}{2} = 0$

Put n = 3

$A_3 = \frac{3(3-2)}{2} = \frac{3}{2}$

Put n = 4

$A_4 = \frac{4(4-2)}{2} = \frac{8}{2} = 4$

Put n = 5

$A_5 = \frac{5(5-2)}{2} = \frac{15}{2}$

(vii) $a_n = n^2 - n + 1$

Put n = 1

$A_1 = (1)^2 - 1+1 = 1$

Put n = 2

$A_2 = (2)^2 -2+1 = 3$

Put n = 3

$A_3 = (3)^2 -3+1 = 9-2 = 7$

Put n = 4

$A_4 = (4)^2 -4+1 = 16-3 = 13$

Put n = 5

$A_5 = (5)^2 - 5+1 = 25-4 = 21$

(viii) $a_n = 2n^2 - 3n + 1$

Put n = 1

$A_1 = 2(1)^2 - 3(1) + 1 = 2-3+1 = 0$

Put n = 2

$A_2 = 2(2)^2 - 3(2) + 1 = 8-6+1 = 3$

Put n = 3

$A_3 = 2(3)^2 - 3(3) + 1 = 18-9+1 = 10$

Put n = 4

$A_4 = 2(4)^2 - 3(4) + 1 = 32-12+1 = 21$

Put n = 5

$A_5 = 2(5)^2 - 3(5) + 1 = 50-15+1 = 36$

(ix) $a_n = \frac{2n-3}{6}$

Put n = 1

$A_1 = \frac{2(1)-3}{6} = \frac{-1}{6}$

Put n = 2

$A_2 = \frac{2(2)-3}{6} = \frac{1}{6}$

Put n = 3

$A_3 = \frac{2(3)-3}{6} = \frac{3}{6} = \frac{1}{2}$

Put n = 4

$A_4 = \frac{2(4)-3}{6} = \frac{5}{6}$

Put n = 5

$A_5 = \frac{2(5)-3}{6} = \frac{7}{6}$

2. Question

Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 5n - 4; a_{12} \text{ and } a_{15}$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7 \text{ and } a_8$

(iii) $a_n = n(n-1)(n-2); a_5 \text{ and } a_8$

(iv) $a_n = n(n-1)(2-n)(3+n); a_1, a_2, a_2$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

Answer

(i) $a_n = 5n - 4; a_{12} \text{ and } a_{15}$

Put n = 12

A_{12}= 5(12) – 4

= 60-4 = 56

Put n = 15

A_{15}= 5(15) – 4

= 75-4 = 71

(ii) $a_n = \frac{3n-2}{4n+5}; a_7 \text{ and } a_8$

Put n = 7

$A_7 = \frac{3(7)-2}{4(7)+5} = \frac{21-2}{28+5} = \frac{19}{33}$

Put n = 8

$A_8 = \frac{3(8)-2}{4(8)+5} = \frac{24-2}{32+5} = \frac{22}{37}$

(iii) $a_n = n(n-1)(n-2)$; a_5 and a_8

Put n = 5

A_5= 5(5-1) (5-2)

= 5(4)(3) = 60

Put n = 8

A_8= 8(8-1) (8-2)

= 8(7)(6) = 336

(iv) $a_n = n(n-1)(2-n)(3+n)$; a_1, a_2, a_2

Put n = 1

A_1= 1(1-1) (2-1) (2-2) (3+2)

= 0

Put n = 2

A_2= 2(2-1) (2-2) (3+2)

= 0

Put n = 3

A_3= 3(3-1) (2-3) (3+3)

= 3 (2) (-1) (6)

= 36

(v) $a_n = (-1)^n n$; a_3, a_5, a_8

Put n = 3

A_3= $(-1)^3$ (3) = -3

Put n = 5

A_5= $(-1)^5$ (5) = -5

Put n = 8

A_8= $(-1)^8$ (8) = 8

3. Question

Find the next five terms of each of the following sequences given by:

(i) $a_1 = 1, a_n = a_{n-1} + 2, n \geq 2$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

(iv) $a_1 = 4, a_n = 4a_{n-1} + 3, n > 1$

Answer

(i) $a_1 = 1, a_n = a_{n-1} + 2, n \geq 2$

Put n = 2

$A_2 = a_{2-1} + 2$

$A_2 = a_1 + 2$

$= 1+2 = 3$

Put n = 3

$A_3 = a_{3-1} + 2$

$= a_2 + 2$

$= 3+2 = 5$

Put n = 4

$A_4 = a_{4-1} + 2$

$= a3 + 2$

$= 5+2 = 7$

Put n = 5

$A_5 = a_{5-1} + 2$

$= a_4 + 2$

$= 7+2 = 9$

Put n = 6

$A_6 = a_{6-1} + 2$

$= a_5 + 2$

$= 9+2 = 11$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

Put n = 3

$A_3 = a_{3-1} - 3$

$= a_2 - 3$

$= 2-3 = -1$

Put n = 4

$A_4 = a_{4-1} - 3$

$= a_3 - 3$

$= -1 - 3 = -4$

Put n = 5

$A_5 = a_{5-1} - 3$

$= a_4 - 3$

$= -4 - 3 = -7$

Put n = 6

$A_6 = a_{6-1} - 3$

$= a_5 - 3$

$= -7 - 3 = -10$

Put n = 7

$A_7 = a_{7-1} - 3$

$= a_6 - 3$

$= -10 - 3 = -13$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

Put n = 2

$A2 = a_{2-1} / 2$

$= -1/2$

Put n = 3

$A_3 = a_{3-1} / 3$

$= -1/6$

Put n = 4

$A_4 = a_{4-1} / 3$

$= -1/24$

Put n = 5

$A_5 = a_{5-1} / 3$

= -1/120

Put n = 6

$A_6 = a_{6-1} / 3$

= -1/720

(iv) $a_1 = 4, a_n = 4a_{n-1} + 3, n > 1$

Put n = 2

$A_2 = 4a_{2-1} + 3$

$= 4a_1 + 3 = 19$

Put n = 3

$A_3 = 4a_{3-1} + 3$

$= 4a_2 + 3 = 79$

Put n = 4

$A_4 = 4a_{4-1} + 3$

$= 4a_3 + 3$

= 316 + 3

= 319

Put n = 5

$A_5 = 4a_{5-1} + 3$

$= 4a_4 + 3$

= 1276 + 3

= 1279

Put n = 6

$A_6 = 4a_{6-1} + 3$

$= 4a_5 + 3$

= 5116 + 3

= 5119

Exercise 9.2

1. Question

For the following arithmetic progressions write the first term a and the common difference d :

(i) $-5,-1,3,7,...$

(ii) $\frac{1}{5},\frac{3}{5},\frac{5}{5},\frac{7}{5},...$

(iii) $0.3,0.55,0.80,1.05,...$

(iv) $-1.1,-3.1,-5.1,-7.1,...$

Answer

(i) $-5,-1,3,7,...$

First term, a = -5

Common difference, d = -1 – (-5)

= -1 + 5 = 4

(ii) $\frac{1}{5},\frac{3}{5},\frac{5}{5},\frac{7}{5},...$

First term, a = $\frac{1}{5}$

Common difference, d = $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

(iii) $0.3,0.55,0.80,1.05,...$

First term, a = 0.3

Common difference, d = 0.55 – 0.30 = 0.25

(iv) $-1.1,-3.1,-5.1,-7.1,...$

First term, a = -1.1

Common difference, d = -3.1 – (-1.1)

= -2

2. Question

Write the arithmetic progression when first term a and common difference of d are as follows:

(i) $a = 4, \quad d = -3$

(ii) $a = -1, \quad d = \frac{1}{2}$

(iii) $a = -1.5, d = -0.5$

Answer

(i) $a = 4, \quad d = -3$

a_1= 4, d= -3

$A_2 = a_1 + d = 4-3 = 1$

$A_3 = a_1 + 2d = 4 + 2(-3) = 4-6 = -2$

$A_4 = a_1 = 3d = 4 + 3(-3) = 4-9 = -5$

Series = 4, 1, -2, -5,…

(ii) $a = -1, \quad d = \frac{1}{2}$

$a_1 = -1, d = \frac{1}{2}$

$A_2 = a_1 + d = -1 + \frac{1}{2} = \frac{-1}{2}$

$A_3 = a_1 + 2d = -1 + 1 = 0$

$A_4 = a_1 + 3d = -1 + \frac{3}{2} = \frac{1}{2}$

Series = $-1, \frac{-1}{2}, 0, \frac{1}{2}, \ldots$

(iii) $a = -1.5, d = -0.5$

$a_1 = -1.5, d = -0.5$

$A_2 = a_1 + d = -1.5 - 0.5 = -2$

$A_3 = a_1 + 2d = -1.5 + 2(-0.5) = -1.5 - 1 = -2.5$

$A_4 = a_1 + 3d = -1.5 + 3(-0.5) = -1.5 - 1.5 = -3$

Series = $-1.5, -2, -2.5, -3, \ldots$

3. Question

In which of the following situation, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs. 150 and rises by Rs.20 for each succeeding metre.

(ii)The mount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder.

Answer

(i) The cost of digging a well for the first metre = Rs. 150

The cost of digging a well for the second metre = 150 + 20 = Rs. 170

The cost of digging a well for the third metre = 170 + 20 = Rs. 190

The cost of digging a well for the fourth metre = 190 + 20 = Rs. 210

Difference between first and second metre = 170 – 150 = 20

Difference between second and third metre = 190 – 170 = 20

Difference between third and fourth metre = 210 – 190 = 20

Since, all the differences are equal. Therefore, it's an A.P.

(ii) Let the amount of air present in the cylinder first time = x

Amount of air present in the cylinder second time = $x - \frac{x}{4} = \frac{3x}{4}$

Amount of air present in the cylinder third time = $\frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$

Difference in the amount of air present in the cylinder between first and the second time = $\frac{3x}{4} - x = \frac{-x}{4}$

Difference in the amount of air present in the cylinder between second and the third time = $\frac{9x}{16} - \frac{3x}{4}$

$= \frac{-3x}{16}$

Since, the differences are unequal. Therefore, it's not an A.P.

4. Question

Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.

Answer

Put n = 1

$A_1 = 5(1) - 7$

$= -2$

Put n = 2

$A_2 = 5(2) - 7$

$= 10 - 7 = 3$

Put n = 3

$A_3 = 5(3) - 7$

$= 15 - 7 = 8$

Common difference, $d_1 = a_2 - a_1 = 3 - (-2) = 5$

Common difference, $d_2 = a_3 - a_2 = 8 - 3 = 5$

Since, $d_1 = d_2$ and Common difference = 5

Therefore, it's an A.P.

5. Question

Show that the sequence defined by $a_n = 3n^2 - 5$ is not A.P.

Answer

Given: The sequence $a_n = 3n^2 - 5$.

To prove: the sequence defined by $a_n = 3n^2 - 5$ is not A.P.

Proof: Consider the sequence $a_n = 3n^2 - 5$,Put n = 1

$A_1 = 3(1)^2 - 5 = 3 - 5 = -2$

Put n = 2

$A_2 = 3(2)^2 - 5 = 12 - 5 = 7$

Put n = 3

$A_3 = 3(3)^2 - 5 = 27 - 5 = 22$In an A.P the difference of consecutive terms should be same.

So,

Common difference, $d_1 = a_2 - a_1 = 7 - (-2) = 9$

Common difference, $d_2 = a_3 - a_2 = 22 - 9 = 13$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

6. Question

The general term of a sequence is given by $a_n = -4n^2 + 15$.Is the sequence an A.P..? If so, find its 15^{th} term and the common difference.

Answer

Put n = 1

$A_1 = -4(1)^2 + 15 = 11$

Put n = 2

$A_2 = -4(2)^2 + 15 = -1$

Put n = 3

$A_3 = -4(3)^2 + 15 = -21$

Common difference, $d_1 = a_2 - a_1 = -1 - 11 = -12$

Common difference, $d_2 = a_3 - a_2 = -21 + 1 = 20$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

7. Question

Find the common difference and write the next four terms of each of the following arithmetic progressions:

(i) $1, -2, -5, -8, \ldots$

(ii) $0, -3, -6, -9, \ldots$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots$

(iv) $-1, -\frac{5}{6}, -\frac{2}{3}, \ldots$

Answer

(i) $1, -2, -5, -8, \ldots$

Common difference, d = -2 – 1 = -3

A_5= -8 – 3 = -11

A_6= -11 – 3 = -14

A_7= -14 – 3 = -17

A_8= -17 – 3 = -20

(ii) $0, -3, -6, -9, \ldots$

Common difference, d = -3 – 0 = -3

A_5= -9 – 3 = -12

A_6= -12 – 3 = -15

A_7= -15 – 3 = -18

A_8= -18 – 3 = -21

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots$

Common difference, d = $\frac{1}{4} + 1 = \frac{5}{4}$

$A_4 = \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$

$A_5 = \frac{11}{4} + \frac{5}{4} = \frac{16}{4} = 4$

$A_6 = 4 + \frac{5}{4} = \frac{21}{4}$

$A_7 = \frac{21}{4} + \frac{5}{4} = \frac{26}{4} = \frac{13}{2}$

(iv) $-1, -\frac{5}{6}, -\frac{2}{3}, \ldots$

Common difference, d = $\frac{-5}{6} + 1 = \frac{1}{6}$

$A_4 = \frac{-2}{3} + \frac{1}{6} = \frac{-1}{2}$

$A_5 = \frac{-1}{2} + \frac{1}{6} = \frac{-1}{3}$

$A_6 = \frac{-1}{3} + \frac{1}{6} = \frac{-1}{6}$

$A_7 = \frac{-1}{6} + \frac{1}{6} = 0$

8. Question

Prove that no matter what is the real number a and b are, the sequence with nth term a+nb is always an A.P. What is the common difference?

Answer

Put n = 1

$A_1 = a + b$

Put n = 2

$A_2 = a + 2b$

Put n = 3

$A_3 = a + 3b$

Put n = 4

$A_4 = a + 4b$

Common difference, $d_1 = a_2 - a_1 = a + 2b - a - b = b$

Common difference, $d_2 = a_3 - a_2 = a + 3b - a - 2b = b$

Common difference, $d_3 = a_4 - a_3 = a + 4b - a - 3b = b$

Since, $d_1 = d_2 = d_3$

Therefore, it's an A.P. with common difference 'b'

9. Question

Write the sequence with nth term:

(i) $a_n = 3 + 4n$

(ii) $a_n = 5 + 2n$

(iii) $a_n = 6 - n$

(iv) $a_n = 9 - 5n$

Show that all of the above sequences form A.P.

Answer

(i) Put n = 1

$A_1 = 3 + 4(1) = 7$

Put n = 2

$A_2 = 3 + 4(2) = 11$

Put n = 3

$A_3 = 3 + 4(3) = 15$

Common difference, $d_1 = a_2 - a_1 = 11 - 7 = 4$

Common difference, $d_2 = a_3 - a_2 = 15 - 11 = 4$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 7, 11, 15,…

(ii) Put n = 1

$A_1 = 5 + 2(1) = 7$

Put n = 2

$A_2 = 5 + 2(2) = 9$

Put n = 3

$A_3 = 5 + 2(3) = 11$

Common difference, $d_1 = a_2 - a_1 = 9 - 7 = 2$

Common difference, $d_2 = a_3 - a_2 = 11 - 9 = 2$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 7, 9, 11,…

(iii) Put n = 1

$A_1 = 6 - 1 = 5$

Put n = 2

$A_2 = 6 - 2 = 4$

Put n = 3

$A_3 = 6 - 3 = 3$

Common difference, $d_1 = a_2 - a_1 = 4 - 5 = -1$

Common difference, $d_2 = a_3 - a_2 = 3 - 4 = -1$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 5, 4 , 3,…

(iv) Put n = 1

$A_1 = 9 - 5(1) = 4$

Put n = 2

$A_2 = 9 - 5(2) = -1$

Put n = 3

$A_3 = 9 - 5(3) = -6$

Common difference, $d_1 = a_2 - a_1 = -1 - 4 = -5$

Common difference, $d_2 = a_3 - a_2 = -6 - (-1) = -5$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 4, -1, -6,…

10. Question

Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common difference.

(i) $3, 6, 12, 24, \ldots$

(ii) $0, -4, -8, -12, \ldots$

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

(iv) $12, 2, -8, -18, \ldots$

(v) $3, 3, 3, 3, \ldots$

(vi) $p, p+90, p+180, p+270 \ldots where\ p = (999)^{999}$

(vii) $1.0, 1.7, 2.4, 3.1, \ldots$

(viii) $-225, -425, -625, -825, \ldots$

(ix) $10, 10+2^6, 10+2^7$

(x) $a+b, (a+1)+b, (a+1)+(b+1), (a+2)+(b+1), (a+2)+(b+2), \ldots$

(xi) $1^2, 3^2, 5^2, 7^2, \ldots$

(xii) $1^2, 5^2, 7^2, 73, \ldots$

Answer

(i) $3, 6, 12, 24, \ldots$

Common difference, $d_1 = 6 - 3 = 3$

Common difference, $d_2 = 12 - 6 = 6$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

(ii) $0, -4, -8, -12, \ldots$

Common difference, $d_1 = -4 - 0 = -4$

Common difference, $d_2 = -8 - (-4) = -4$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, d = -4

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

Common difference, $d_1 = \frac{1}{4} - \frac{1}{2} = \frac{-1}{2}$

Common difference, $d_2 = \frac{1}{6} - \frac{1}{4} = \frac{-1}{12}$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

(iv) $12, 2, -8, -18, \ldots$

Common difference, $d_1 = 2 - 12 = -10$

Common difference, $d_2 = -8 - 2 = -10$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, d = -10

(v) $3, 3, 3, 3, \ldots$

Common difference, $d_1 = 3 - 3 = 0$

Common difference, $d_2 = 3 - 3 = 0$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, d = 0

(vi) $p, p + 90, p + 180, p + 270 \ldots where\ p = (999)^{999}$

Common difference, $d_1 = p + 90 - p = 90$

Common difference, d_2= p + 180 – p – 90 = 90

Since, $d_1=d_2$

Therefore, it's an A.P. with common difference, d = 90

(vii) $1.0, 1.7, 2.4, 3.1, \ldots$

Common difference, d_1= 1.7 – 1.0 = 0.7

Common difference, d_2= 2.4 – 1.7 = 0.7

Since, $d_1=d_2$

Therefore, it's an A.P. with common difference, d = 0.7

(viii) $-225, -425, -625, -825, \ldots$

Common difference, d_1= -425 + 225 = -200

Common difference, d_2= -625 + 425 = -200

Since, $d_1=d_2$

Therefore, it's an A.P. with common difference, d = -200

(ix) $10, 10+2^6, 10+2^7$

Common difference, d_1= 10 + 2^6 – 10 = 2^6 = 64

Common difference, d_2 = 10 + 2^7 – 10 – 2^6 = 2^6 (2 – 1) = 64

Since, $d_1=d_2$

Therefore, it's an A.P. with common difference, d = 64

(x) $a+b, (a+1)+b, (a+1)+(b+1), (a+2)+(b+1), (a+2)+(b+2), \ldots$

Common difference, d_1 = (a + 1) + b – a – b = 1

Common difference, d_2 = (a + 1) + (b + 1) – (a + 1) – b = 1

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, d = 1

(xi) $1^2, 3^2, 5^2, 7^2, \ldots$

Common difference, d_1= 3^2 – 1^2 = 8

Common difference, d_2 = 5^2 – 3^2 = 25 – 9 = 16

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

(xii) $1^2, 5^2, 7^2, 73, \ldots$

Common difference, $d_1 = 5^2 - 1^2 = 24$

Common difference, $d_2 = 7^2 - 5^2 = 24$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, d = 24

11. Question

Find the common difference of the A.P. and write the next two terms:

(i) $51, 59, 67, 75, \ldots$

(ii) $75, 67, 59, 51, \ldots$

(iii) $1.8, 2.0, 2.2, 2.4, \ldots$

(iv) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

(v) $119, 136, 153, 170, \ldots$

Answer

(i) $51, 59, 67, 75, \ldots$

Common difference, d = 59 – 51 = 8

$A_5 = 75 + 8 = 83$

$A_6 = 83 + 8 = 91$

(ii) $75, 67, 59, 51, \ldots$

Common difference, d = 67 – 75 = -8

$A_5 = 51 - 8 = 43$

$A_6 = 43 - 8 = 35$

(iii) $1.8, 2.0, 2.2, 2.4, \ldots$

Common difference, d = 2.0 – 1.8 = 0.2

$A_5 = 2.4 + 0.2 = 2.6$

$A_6 = 2.6 + 0.2 = 2.8$

(iv) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

Common difference, $d = \frac{1}{4} - 0 = \frac{1}{4}$

$A_5 = \frac{3}{4} + \frac{1}{4} = 4$

$A_6 = 4 + \frac{1}{4} = \frac{5}{4}$

(v) $119, 136, 153, 170, \ldots$

Common difference, d = 136 – 119 = 17

$A_5 = 170 + 17 = 187$

$A_6 = 187 + 17 = 204$

12. Question

The nth term of an A.P. is 6n+2. Find the common difference.

Answer

$a_n = 6n + 2$

Put n = 1

$A_1 = 6(1) + 2 = 8$

Put n = 2

$A_2 = 6(2) + 2 = 14$

Common difference, $d = a_2 - a_1 = 14 - 8 = 6$

Exercise 9.3

1. Question

Find:

(i) 10^{th} term of the A.P. $1, 4, 7, 10, \ldots$

(ii) 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

(iii) n^{th} term of the A.P. $13, 8, 3, -2, \ldots$

(iv) 10^{th} term of the A.P. $-40, -15, 10, 35, \ldots$

(v) 8^{th} term of the A.P. $117, 104, 91, 78, \ldots$

(vi) 11^{th} term of the A.P. $10.0, 10.5, 11.0, 11.5, \ldots$

(vii) 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

Answer

(i) 10^{th} term of the A.P. $1, 4, 7, 10, \ldots$

a = 1, d = 4 – 1 = 3

$A_{10} = a + (10 - 1) d$

= 1 + (9)3 = 28

(ii) 18th term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

a = √2, d = 3√2 - √2 = 2√2

A_{18}= a + (18 – 1) d

= √2 + (17) 2√2 = 35√2

(iii) n^{th} term of the A.P. $13, 8, 3, -2, \ldots$

a = 13, d = -5

A_n= a + (n-1)d = 13 + (n-1) (-5)

= 13 -5n + 5 = 18 – 5n

(iv) 10^{th} term of the A.P. $-40, -15, 10, 35, \ldots$

a = -40, d = 25

A_{10}= a + (10-1) d = -40 + 9 (25)

= -40 + 225 = 185

(v) 8^{th} term of the A.P. $117, 104, 91, 78, \ldots$

a = 117, d = -13

A_8= a + (8 – 1) d = 117 + 7 (-13)

= 117 – 91 = 26

(vi) 11^{th} term of the A.P. $10.0, 10.5, 11.0, 11.5, \ldots$

a = 10.0, d = 10.5 – 10.0 = 0.5

A_{11}= a + (11 – 1) d = 10.0 + 10 (0.5)

= 10.0 + 5 = 15.0

(vii) 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

a = $\frac{3}{4}$, d = $\frac{5}{4} - \frac{3}{4} = \frac{1}{2}$

A_9= a + (9 – 1) d = $\frac{3}{4} + 8 (\frac{1}{2}) = \frac{19}{4}$

2 A. Question

(i) Which term of the A.P. $3, 8, 13, \ldots$ is 248?

Answer

a = 3, d = 5

A_n= a + (n − 1) d

248 = 3 + (n − 1) 5 = 3 + 5n − 5

248 = -2 + 5n

250 = 5n

n = 50

2 B. Question

Which term of the A.P. 84,80,76...is 0?

Answer

a = 84, d = -4,A_n = 0

We know,A_n= a + (n − 1) d

0 = 84 + (n − 1) -4

0 = 84 − 4n + 4

0 = 88 − 4n

n = 22

2 C. Question

Which term of the A.P. 4,9,14,...is 254?

Answer

a = 4, d = 5

A_n= a + (n − 1) d

254 = 4 + (n − 1) 5

254 = 4 + 5n − 5

254 = -1 + 5n

255 = 5n

n = 51

2 D. Question

Which term of the A.P. 21,42,63,84,...is 420?

Answer

a = 21, d = 21

A_n= a + (n − 1) d

420 = 21 + (n − 1) 21

420 = 21 + 21n − 21

420 = 21n

n = 20

2 E. Question

Which term of the A.P. $121,117,113,...$is its first negative term?

Answer

a = 121, d = -4

A_n = -ve

A_n= a + (n − 1) d

= 121 + (n − 1) -4

= 125 − 4n

Since, we want −ve term

Therefore, 4n should be a number greater than 125

Hence, 128 is the number greater than 125 and divisible by 4

4n = 128

n = 32

Therefore, 32^{nd} term of the given A.P. is −ve

3 A. Question

Is 68 a term of the A.P. $7,10,13,...$?

Answer

a = 7, d = 3

A_n= a + (n − 1) d

68= 7 + (n − 1) 3

68= 4 + 3n

3n = 64

Since, 64 is not divisible by 3

Therefore, 68 isn't the term of the given A.P.

3 B. Question

Is 302 a term of the A.P. $3,8,13,...$?

Answer

a = 3, d = 5

A_n= a + (n – 1) d

302 = 3 + (n – 1) 5

302 = -2 + 5n

304 = 5n

Since, 304 is not divisible by 5

Therefore, 302 isn't the term of the given A.P.

3. Question

Is -150 a term of the A.P. $11,8,5,2,...$?

Answer

a = 11, d = -3

A_n= a + (n – 1) d

-150 = 11 + (n – 1) (-3)

-150 = 11 – 3n + 3

-150 = 14 – 3n

-164 = 3n

Since, -164 is not divisible by 3

Therefore, -150 isn't the term of the given A.P.

4. Question

How many terms are there in the A.P.?

(i) $7,10,13,...43.$

(ii) $-1,-\frac{5}{6},-\frac{2}{3},-\frac{1}{2},...,\frac{10}{3}.$

(iii) $7,13,19,...,205.$

(iv) $18,15\frac{1}{2},13,...,-47$

Answer

(i) $7,10,13,...43.$

a = 7, d = 3

A_n = 43

a + (n – 1) d = 43

$7 + (n - 1)\ 3 = 43$

$3n = 39$

$n = 13$

Therefore, there are total 13 terms in the A.P.

(ii) $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}.$

$a = -1, d = \frac{1}{6}$

$A_n = \frac{10}{3}$

$a + (n - 1)\ d = \frac{10}{3}$

$-1 + (n - 1)\ \frac{1}{6} = \frac{10}{3}$

$n = 26$

(iii) $7, 13, 19, \ldots, 205.$

$a = 7, d = 6$

$A_n = 205$

$a + (n - 1)\ d = 205$

$7 + (n - 1)\ 6 = 205$

$6n = 204$

$n = 34$

(iv) $18, 15\frac{1}{2}, 13, \ldots, -47$

$a = 18, d = \frac{-5}{2}$

$A_n = -47$

$a + (n - 1)\ d = -47$

$18 + (n - 1)\ \frac{-5}{2} = -47$

$18 - \frac{5n}{2} + \frac{5}{2} = -47$

$n = 27$

5. Question

The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.

Answer

Given: The first term of an A.P. is 5, the common difference is 3 and the last term is 80**To find:** the number of terms.**Solution :**We have a = 5, d = 3

a_n = 80From the formula a_n = a + (n-1) d Number of terms is : $n = \frac{a_n - a}{d} + 1$

$= \frac{(80-5)}{3} + 1$

$= \frac{75}{3} + 1$

= 25 + 1

= 26Hence the number of terms in a given A.P. is 26.

6. Question

The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.

Answer

a_6= 19 = a + (n – 1) d

=19 = a + 5d …(i)

A_{17} = 41 = a + (n – 1) d

= 41 = a + 16d …(ii)

Subtracting (i) from (ii), we get

22 = 11d

d = 2

Now substituting the value of d in (i): 19 = a + 10

= a = 9

A_{40}= a + (40 – 1) 2

= 9 + 78

= 87

7. Question

If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term.

Answer

a_9 = 0

a + (9 – 1)d =0

a + 8d = 0

$a = -8d$ …(i)

To Prove: $a_{29} = 2a_{19}$

Proof: LHS= $a_{29} = a + 28d = -8d + 28d = 20d$

RHS= $2a_{19} = 2[a + (18)d] = 2(-8d + 18d) = 2(10d) = 20d$

Since, LHS=RHS

Hence, proved

8. Question

If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.

Answer

$10a_{10} = 15a_{15}$

$10[a + (10 – 1)d] = 15[a + (15 – 1)d]$

$2a + 2(9)d = 3a + 3(14)d$

$-a = 42d – 18d$

$-a = 24d$

$a = -24d$

$a_{25} = a + 24d$

$= -24d + 24d$

$= 0$

9. Question

The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.

Answer

$a_{10} = 41$

$a + (10 – 1)d = 41$

$a + 9d = 41$ …(i)

$a_{18} = 73$

$a + (18 – 1)d = 73$

Substituting the value of a from (i),

$41 – 9d + 17d = 73$

$8d = 32$

d =4

a = 41 – 9(4)

a =41 – 36

a =5

a_{26} =a + (26 – 1)d

=5 + 25(4)

=5 + 100

=105

10. Question

In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Answer

Given: $a_{24} = 2a_{10}$

To Prove: $a_{72} = 2a_{34}$

Proof: $a_{24} = 2a_{10}$

a + (24 –1)d = 2a + 2(10 –1)d

23d – 18d = a

a = 5d

LHS= a_{72} =a + 71d =5d +71d =76d

RHS= a_{34} =2a + 2(33)d =10d + 66d =76d

Since, LHS=RHS

Hence, proved

11. Question

If $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term, prove that $(3m+1)^{th}$ term is twice $(m+n+1)^{th}$ term.

Answer

Given: $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term$a_{(m+1)} =2a_{(n+1)}$

To Prove:$(3m+1)^{th}$ term is twice $(m+n+1)^{th}$ term $a_{(3m+1)} =2a_{(m+n+1)}$

Proof: $a_{(m+1)} =2a_{(n+1)}$

⇒ a + (m + 1 – 1) d = 2a + 2(n +1 -1)d

⇒ -a = 2nd – md

$\Rightarrow a = md - 2nd$ (i)

LHS: $a_{3m+1} = a + (3m + 1 - 1)d = md - 2nd + 3md = 2d(2m - n)$

RHS: $2a_{(m + n + 1)} = 2[a + (m + n + 1 - 1)d] = 2[md - 2nd + md + nd] = 2d(2m - n)$

LHS = RHS

Hence, proved

12. Question

If the n^{th} term of the A.P. $9, 7, 5, \ldots$ is same as the n^{th} term of the A.P. $15, 12, 9, \ldots$ find n.

Answer

A=9, D=7-9=-2

a=15, d=-3

$A_n = a_n$

$A + (n - 1)D = a + (n - 1)d$

$9 - (n - 1)2 = 15 - (n - 1)3$

$(n - 1)(3 - 2) = 6$

$n - 1 = 6$

$n = 7$

13. Question

Find the 12^{th} term from the end of the following arithmetic progressions:

(i) $3, 5, 7, 9, \ldots 201$

(ii) $3, 8, 13, \ldots, 253$

(iii) $1, 4, 7, 10, \ldots, 88$

Answer

(i) $3, 5, 7, 9, \ldots 201$

a=3, d=5 -3=2, $a_n = 201$

$a_n = a + (n - 1)d$

$201 = 3 + 2n - 2$

$N = 100$

Now, we have to find 12^{th} term from the last that means,

$100^{th} - 11 = 89^{th}$ term

Then,

a_{89} =a + (89 – 1)d

=3 +88 $\times 2$

=179

Hence, the 12^{th} term from the end of the A.P. is 179.

(ii) $3, 8, 13, \ldots, 253$

a=3, d= 8 -3 =5

a_n= 253

a + (n -1)d =253

3 + (n -1)5 =253

n=51

Now, we have to find 12^{th} term from the last that means,

51th -11 = 40^{th} term

Then,

a_{40} = a + (n -1)d

= 3 + 39(5)

=198

Hence, the 12^{th} term from the end of the A.P. is 198

(iii) $1, 4, 7, 10, \ldots, 88$

a=1, d= 4 -1 =3

a_n= 88

a + (n -1)d =88

1 + (n -1)3 =88

n =30

Now, we have to find 12^{th} term from the last that means,

30^{th} -11 = 19^{th} term

a_{19} = a + 18d

= 1 + 18(3)

=55

Hence, the 12^{th} term from the end of the A.P. is 198.

14. Question

The 4^{th} term of an A.P. is three times the first and the 7^{th} term exceeds twice the third term by 1. Find the first term and the common difference.

Answer

Given:

$a_4 = 3a_1$ …(i)

$a_7 = 2a_3 + 1$ …(ii)We know $a_n = a + (n-1)\,d$

So,

$a_3 = a + 2d$

$a_4 = a + 3d$

$a_7 = a + 6d$

Put all the values of a_1 and a_4 in (i),

$a + 3d = 3a$

$\Rightarrow 3d = 3a - a \Rightarrow 3d = 2a$

$\Rightarrow d = \frac{2a}{3}$

Put all the values of a_7 and a_3 in (ii),

$\Rightarrow a + 6d = 2(a + 2d) + 1 \Rightarrow a + 6d = 2a + 4d + 1$

$\Rightarrow 6d - 4d = 2a - a + 1$

$\Rightarrow 2d = a + 1$

Put $d = \frac{2a}{3}$,

$\Rightarrow 2(\frac{2a}{3}) = a + 1$

$\Rightarrow \frac{4a}{3} = a + 1$

$$\Rightarrow \frac{4a}{3} - a = 1$$

$$\Rightarrow \frac{4a - 3a}{3} = 1$$

$$\Rightarrow \frac{a}{3} = 1$$

$\Rightarrow a = 3$

Now, difference, d= $\frac{2a}{3}$

$$\Rightarrow d = \frac{2(3)}{3}$$

$$d = \frac{6}{3}$$

⇒ d= 2

Hence, first term is 3 and the common difference is 2.

15. Question

Find the second term and n^{th} term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22.

Answer

a_6= a + 5d

12= a + 5d …(i)

a_8= a + 7d

22= a + 7d …(ii)

Subtracting (i) from (ii), we get,

10= 2d

d= 5

from (i)

12 = a + 5(5)

12= a + 25

a= -13

a_2= a +d

=-13 +5

=-8

a_n= a + (n-1)d

= -13 + (n-1)5

=-18 +5n

16. Question

How many numbers of two digit are divisible by 3?

Answer

Two digit numbers divisible by 3 are 12, 15, 18,… ,99

Hence, a= 12, ,d=3

a_n= a + (n -1)d

99= 12(n -1)3

99= 9 = 3n

n=30

Hence, the total numbers of two digit divisible by 3 are 30.

17. Question

An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Answer

n =60, a=7, a_{60} =l =125

a_{60} = a + 59d

125 = 7 + 59d

d= $\frac{118}{59}$

d=2

a_{32}= a + 31d

= 7 + 31(2)

=69

Hence, 32th term is 69 in the given A.P.

18. Question

The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 34. Find the first term and the common difference of the A.P.?

Answer

Given: $a_4 + a_8 = 24$ …(i)

$a_6 + a_{10} = 34$ …(ii)

a_4 = a + 3d

a_8 =a + 7d

a_6 =a + 5d

a_{10}=a + 9d

Put the value of a_4 and a_8 in (i)

(a + 3d) + (a + 7d) = 24

a + 5d =12 …(iii)

Put the value of a_6 and a_{10} in (ii)

(a +5d) + (a +9d) =34

a +7d =17 …(iv)

Subtracting (iii) from (iv), we get

2d = 5

d = $\frac{5}{2}$

Putting the value of d in (iii), we get

a = 12 - $\frac{25}{2} = \frac{-1}{2}$

Hence, first term is $\frac{-1}{2}$ and common difference is $\frac{5}{2}$

19. Question

The first term of an A.P. is 5 and its 100^{th} term is - 292. Find the 50^{th} term of this A.P.

Answer

Given, a = 5

a_{100}= -292

5 – 99d = -292

d = $\frac{-297}{-99}$ = 3

Now, 50^{th} term, a_{50} = a + (50 – 1) d

= 5 + (49) 3 = 5 + 147 = 152

Hence, 50^{th} term of given A.P. is 152

20. Question

Find $a_{30} - a_{20}$ for the A.P.

(i) $-9, -14, -19, -24, \ldots$

(ii) $a, a+d, a+2d, a+3d, \ldots$

Answer

(i) $-9, -14, -19, -24, \ldots$

a = -9, d = -14 + 9 = -5

a_{30}= a + 29d

= -9 + 29(-5) = -154

a_{20}= a + 19d

= -9 + 19(-5) = -104

$a_{30} - a_{20}$= -154 – (-104)

= -154 + 104

= -50

(ii) $a, a+d, a+2d, a+3d, \ldots$

a = a, d = a + d – a = d

a_{30}= a + 29d

a_{20}= a+ 19d

$a_{30} - a_{20}$ = a + 29d – a – 19d

= 10d

21. Question

Write the expression $a_n - a_k$ for the A.P. $a, a+d, a+2d, \ldots$

Hence, find the common difference of the A.P. for which

(i)11^{th} terms is 5 and 13^{th} term is 79.

(ii) $a_{10} - a_5 = 200$

(iii) 20^{th} term is 10 more than the 18^{th} term.

Answer

Let n^{th} term a_n = a + (n – 1) d

= a + nd – d

k^{th} term, a_k = a + (k – 1) d

= a + kd – d

Now,

a_n - a_k = (a + nd – d) – (a + kd – d)

= nd – kd = d (n – k)

(i) a_{11} = 5

a + 10d = 5 (i)

$a_{13} = 79$

$a + 12d = 79$ (ii)

By subtracting (i) from (ii), we get

$2d = 74$

$d = 37$

Hence, the common difference is 37

(ii)$a_{10} = a + 9d$ (i)

$a_5 = a + 4d$ (ii)

$a_{10} - a_5 = a + 9d - a - 4d$

$200 = 5d$

$d = 40$

Hence, common difference is 40

(iii) $a_{20} = a + 19d$ (i)

$a_{18} = a + 17d$ (ii)

Given that $a_{20} = a_{18} + 10$

$a + 19d = a + 17d + 10$

$2d = 10$

$d = 5$

Hence, common difference is 5

22. Question

Find n if the given value of x is the n^{th} term of the given A.P.

(i) $25, 50, 75, 100, \ldots; x = 1000$

(ii) $-1, -3, -5, -7, \ldots; x = -151$

(iii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots; x = 550$

(iv) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots, x = \frac{171}{11}$

Answer

(i) $25, 50, 75, 100, \ldots; x = 1000$

$a = 25$, $d = 50 - 25 = 25$

Last term, $l = 1000$

Number of terms, n = $\frac{l-a}{d}$ + 1

= $\frac{1000-25}{25}$ + 1 = 40

Hence, the value of n is 40

(ii) $-1,-3,-5,-7,\ldots;x=-151$

a = -1, d = -2

Last term, l = -151

Number of terms, n = $\frac{l-a}{d}$ + 1

= $\frac{-151-(-1)}{-2}$ + 1 = 76

Hence, the value of n is 76

(iii) $5\frac{1}{2},11,16\frac{1}{2},22,\ldots;x=550$

a = $\frac{11}{2}$, d = $\frac{11}{2}$

Last term, l = 550

l = a + (n − 1) d

550 = $\frac{11}{2}$ + (n − 1) $\frac{11}{2}$

550 = $\frac{11n}{2}$

1100 = 11n

n = 100

Hence, number of terms, n = 100

(iv) $1,\frac{21}{11},\frac{31}{11},\frac{41}{11},\ldots,x=\frac{171}{11}$

a = 1, d = $\frac{21}{11}$ − 1 = $\frac{10}{11}$

Last term, l = $\frac{171}{11}$

a + (n − 1) d = $\frac{171}{11}$

1 + (n − 1) $\frac{10}{11}$ =$\frac{171}{11}$

$\frac{10n}{11}$ = $\frac{171}{11}$ − 1 + $\frac{10}{11}$

$\frac{10n}{11} = \frac{171-11+10}{11}$

10n = 170

n = 17

Hence, value of n is 17

23. Question

If an A.P. consists of n terms with first term a and nth term ls how that the sum of the m^{th} term from the beginning and the m^{th} term from the end is (a+l).

Answer

Given, a = a

Last term, l = l

We have to prove that the sum of the m^{th} term from the beginning and m^{th} term from the end is (a + 1)

Now, m^{th} term from the beginning,

$a_{m(b)}$ = a + (m – 1) l

= a + ml – l (i)

Again m^{th} term from the end

$a_{m(e)}$ = l – (m – 1)l

= l – ml + l = 2l – ml (ii)

By adding (i) and (ii), we get

a + ml – l + 2l – ml = a + l

Hence, proved

24. Question

Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

Answer

Given, a_3 = 16

a + 2d = 16 (i)

a_5 = a + (5 – 1) d

= a + 4d

a_7 = a + (7 -1)d

= a + 6d

Acc. To question,

$a_7 = 12 + a_5$ (ii)

Put the value of a_5 and a_7 in (ii), we get

$a + 6d = 12 + a + 4d$

$2d = 12 + a - a$

$2d = 12$

$d = 6$

From equation (i),

$16 = a + 2(6)$

$16 = a + 12$

$a = 4$

We have to find A.P.

$a_1 = 4$

$a_2 = a + d = 4 + 6 = 10$

$a_3 = a + 2d = 4 + 2(6) = 16$

$a_4 = a + 3d = 4 + 3(6) = 22$

Hence, the A.P. is 4 , 10, 16 , 22,…

25. Question

The 7^{th} term of an A.P. is 32 and its 13^{th} term is 62. Find the A.P.

Answer

$a_7 = 32$

$a + 6d = 32$ (i)

$a_{13} = a + 12d = 62$ (ii)

By subtracting (i) from (ii), we get

$6d = 30$

$d= 5$

From (i), $a = 32 - 6(5) = 2$

We have to find A.P.

$a_1 = 2$

$a_2 = a + d = 2 + 5 = 7$

$a_3 = a + 2d = 2 + 2(5) = 12$

$a_4 = a + 3d = 2 + 3(5) = 17$

Hence, A.P. is 2, 7, 12, 17,…

26. Question

Which term of the A.P. $3,10,17,...$ will be 84 more than its 13^{th} term?

Answer

$a = 3, d = 7$

$a_{13} = 3 + (13 - 1)7$

$= 3 + 84 = 87$

Now, n^{th} term is 84 more than the 13^{th} term

$a_n = 84 + a_{13}$

$= 84 + 87$

$= 171$

Now we have to find n,

$a_n = a + (n - 1)\, d$

$171 = 3 + (n - 1)\, 7$

$7n = 175$

$n = 25$

Hence, 25^{th} term of the given A,.P. is 84 more than its 13^{th} term

27. Question

Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms?

Answer

Two arithmetic progressions have the same common difference.

Let, common difference = d

First term of first A.P. = a

First term of second A.P. = a’

Given that difference between their 100^{th} term is 100

100^{th} term of first A.P., $a_{100} = a + 99d$

100^{th} term of second A.P., $a'_{100} = a' + 99d$

Acc. To question,

$a_{100} - a'_{100} = 100$

$a + 99d - a' - 99d = 100$

$a - a' = 100$ (i)

1000^{th} term of first A.P., $a_{1000} = a + 999d$

1000^{th} term of second A.P., $a'_{1000} = a' + 999d$

Acc. To question,

$a_{1000} - a'_{1000} = a + 999d - a' - 999d$

$= a - a'$

$a_{1000} - a'_{1000} = 100$ [from (i)]

Hence, difference between 1000^{th} term of two A.P. is 100

28. Question

For what value of n, the nth terms of the arithmetic progressions $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ are equal?

Answer

Given, A.P. = 63, 65, 67,…

A.P.' = 3, 10, 17,…

Let for A.P.,

First term, a = 63, d = 65 – 63 = 2

And n^{th} term = a_n

$a_n = 63 + (n - 1)\ 2$

$= 63 + 2n - 2$

$= 61 + 2n$

For A.P.'

First term, a = 3, d = 10 – 3 = 7

n^{th} term = a_n

Now n^{th} term of both A.P. are equal

$61 + 2n = -4 + 7n$

$7n - 2n = 61 + 4$

$n = 13$

Hence, 13^{th} term of both the A.P. are equal

29. Question

How many multiples of 4 lie between 10 and 250?

Answer

Let, multiple of 4 lies between 10 and 250

12, 16, 20, 24,…248

We know, $a_n = a + (n - 1)d$

First term, a = 12

C0mmon difference, d = 16 – 12 = 4

Last term, $a_n = 248$

$a_n = a+ (n - 1)d$

$248 = 12 + (n - 1)\ 4$

$248 = 12 + 4n - 4$

$4n = 248 - 12 + 4$

$4n = 240$

$n = 60$

Hence, multiple of 4 lies between 10 and 250 is 60

30. Question

How many three digit numbers are divisible by 7?

Answer

Let, three digit number divisible by 7 are

105, 112, 119,….994

Here, First term, a = 105

Common difference, d = 112 – 105 = 7

And last term, $a_n = 994$

$a_n = a+ (n - 1)\ d$

$994 = 105 + (n - 1)\ 7$

$994 = 105 + 7n - 7$

$994 = 98 + 7n$

$7n = 896$

n = 128

Hence, three digit number that are divisible by 7 are 128

31. Question

Which term of the arithmetic progression $8,14,20,26,...$ will be72 more than its 41th term?

Answer

a = 8, d = 6,

Last term = a_n

a_n= a + (n – 1) d

= 8 + (n – 1) 6

= 2 + 6n (i)

Let, 41^{st} term, a_{41} = 8 + (41 – 1) 6

= 8 + 40 * 6 = 248

Now the term is 72 more than its 41^{st} term

a_n = 72 + a_{41}

= 72 + 248 = 320

Putting this value in (i), we get

320 = 2 + 6n

6n = 318

n = 53

Hence, 53^{rd} term of the given A.P. is 72 more than its 41^{st} term

32. Question

Find the term of the arithmetic progression $9,12,15,18,...$ which is 39 more than its 36^{th} term.

Answer

First term, a = 9, d = 12 – 9 = 3

Let, last term be a_n

a_n = a + (n – 1) d

= 9 + (n – 1)3

= 9 + 3n – 3 = 6 + 3n (i)

Let, 36^{th} term, a_{36} = a + 35d

= 9 + 35 (3) = 114

Now the term is 39 more than its 36^{th} term

$a_n = 39 + a_{36}$

$= 39 + 114 = 153$

Putting the value in (i), we get

$153 = 6 + 3N$

$3n = 153 – 6$

$3n = 147$

$n = 49$

Hence, 49^{th} term of the given A.P. is 39 more than its 36^{th} term

33. Question

Find the 8^{th} term from the end of the A.P. $7,10,13,...,184$

Answer

$a = 7, d = 3$

Last term, $a_n = 184$

$a_n = a + (n – 1) d$

$184 = 7 + (n - 1) 3$

$184 = 7 + 3n – 3$

$180 = 3n$

$n = 60$

Now we have to find last 8^{th} term, it means = $60^{th} – 7 = 53^{rd}$ term

$a_{53} = 7 + (53 – 1)3$

$= 7 + 52 * 3$

$= 7 + 156 = 163$

Hence, 8^{th} term from the end of given is 163

34. Question

Find the 10^{th} term from the end of the A.P. 8, 10, 12, ..., 126

Answer

Given,First term, a = 8Common difference, d = 2

Let the number of terms be 'n'We know, nth term of an AP is$a_n = a + (n – 1)$dwhere ‘a’ and ‘d’ are first term and common difference of AP respectivelyLast term, $a_n = 126$

$\Rightarrow a_n = a + (n - 1)\, d$

$\Rightarrow 126 = 8 + (n - 1)\, 2$

$\Rightarrow 120 = 2n$

$\Rightarrow n = 60$

Now we have to find last 10^{th} term, means = 60^{th} term – 10 + 1 = 51^{th} term

Now, $a_{51} = 8 + (51 - 1)2$

$= 8 + 50(2) = 8 + 100 = 108$

Hence, 10^{th} term from the last of given A.P. is 108

35. Question

The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of 6^{th} and 10^{th} terms is 44. Find the A.P.

Answer

Given, $a_4 + a_8 = 24$ (i)

$a_6 + a_{10} = 44$ (ii)

We know, $a_n = a + (n - 1)\, d$

4^{th} term, $a_4 = a + 3d$

6^{th} term, $a_6 = a + 5d$

8^{th} term, $a_8 = a + 7d$

10^{th} term, $a_{10} = a + 9d$

Putting the value of a_4 and a_8 in (i), we get

$a + 3d + a + 7d = 24$

$2a + 10d = 24$ (iii)

Put the value of a_6 and a_{10} in (ii), we get

$a + 5d + a + 9d = 44$

$2a + 14d = 44$ (iv)

By subtracting (iii) from (iv), we get

$4d = 20$

$d = 5$

Now putting value of d in (iii), we get

$2a = 24 - 10(5)$

a = -13

a_1 = -13

a_2 = a + d = -13 + 5 = - 8

a_3 = a + 2d = -13 + 10 = -3

Hence, the A.P. is -13, -8, -3,…

36. Question

Which term of the A.P. $3,15,27,39,\ldots$ will be 120 more than its 21^{st} term?

Answer

a = 3, d = 15 – 3 = 12

Let last term be a_n

a_n = a + (n – 1) d

= 3 + (n – 1) 12

= 12n – 9 (i)

a_{21} = a + 20d

= 3 + 20(12) = 243

Now, the term is 120 more than the 21^{st} term

a_n = 120 + a_{21}

= 120 + 243

= 363

Putting this value in (i), we get

363 = 12n – 9

12n = 363 + 9

n = 31

Hence, 31^{st} term of given A.P. is 120 more than its 21^{st} term

37. Question

The 17^{th} term of an A.P. is 5 more than twice its 8^{th} term. If the 11^{th} term of the A.P. is 43, find the n^{th} term.

Answer

Given: The 17^{th} term of an A.P. is 5 more than twice its 8^{th} term and the 11^{th} term of the A.P. is 43.

To find: the n^{th} term.

Solution: Given, $a_{17} = 5 + 2(a_8)$ .....(i)

And$a_{11} = 43$As $a_n = a + (n-1)\ d$ For n=11,

$a + 10d = 43a = 43 - 10d$ .....(ii)For n = 8,

$a_8 = a + 7d$For n=17,

$a_{17} = a + 16d$Put the value of a from (ii)

$\Rightarrow a_{17} = 43 - 10d + 16d$

$\Rightarrow a_{17} = 43 + 6d$

Putting the value of a_8 and a_{17} in (i), we get

$\Rightarrow 43 + 6d = 5 + 2(a + 7d) \Rightarrow 43 + 6d = 5 + 2a + 14d$

$\Rightarrow 43 - 5 = 2a + 14d - 6d$

$\Rightarrow 38 = 2a + 8d$

$\Rightarrow 38 = 2(43 - 10d) + 8d$ [from (ii)]

$\Rightarrow 38 = 86 - 20d + 8d$

$\Rightarrow 38 = 86 - 12d$

$\Rightarrow 12d = 86 - 38 \Rightarrow 12d = 48$

$\Rightarrow d = 4$

From (ii), $a = 43 - 10d$

$\Rightarrow 43 - (10 \times 4) = 43 - 40 = 3$

We know, n^{th} term of A.P., $a_n = a + (n -1)\ d$

$a_n = 3 + (n - 1)\ 4$

$a_n = 3 + 4n - 4\ a_n = 4n - 1$

Hence, n^{th} term is $4n - 1$.

38. Question

Find the number of all three digit natural numbers which are divisible by 9.

Answer

Let the three digit numbers divisible by 9 are

108, 117, 126,… ,999

a=108, d=9, last term, l= 999

Number of terms, n $= \frac{l-a}{d} + 1$

$=\frac{999-108}{9}+1$

$=\frac{891}{9}+1$

$=100$

Hence, the number of all three digit natural numbers which are divisible by 9 are 100

39. Question

The 19^{th} term of an A.P. is equal to three times its sixth term. If its 9^{th} term is 19, find the A.P.

Answer

Given: $a_9 = 19$

$a + 8d = 19$ (i)

Acc. to question,

$a_{19} = 3a_6$ (ii)

$a_{19} = a + 18d$

$a_6 = a + 5d$

Putting the value of a_{19} and a_6 in (ii)

$a + 18d = 3(a + 5d)$

$3d = 2a$

$3d = 2(19 - 8d)$ (from (i))

$19d = 38$

$d = 2$

Now, putting the value of d in (i)

$a = 19 - 8(2) = 3$

$a_1 = a = 3$

$a_2 = a + d = 3 + 2 = 5$

$a_3 = a + 2d = 3 + 2(2) = 7$

Hence, the A.P. is $3, 5, 7, 9, \ldots$

40. Question

The 9^{th} term of an A.P. is equal to 6 times its second term. If its 5^{th} term is 22, find the A.P.

Answer

Given: $a_5 = 22$

$a + 4d=22$ (i)

Acc. to question,

$a_9 = 6a_2$ (ii)

$a_9= a +8d$

$a_2=a+d$

Putting the value of a_9 and a_2 in (ii)

$a +8d= 6(a +d)$

$2d=5a$

$2d=5(22 -4d)$

$22d=110$

$d=5$

Now, putting the value of d in (i)

$a=22 -4(5)=2$

$a_1 = a=2$

$a_2 =a +d=2 +5=7$

$a_3 =a +2d=2 + 2(5)=12$

Hence, the A.P. is $2,7,12,17,...$

41. Question

The 24^{th} term of an A.P. is twice its 10^{th} term. Show that its 72^{nd} term is 4 times its 15^{th} term.

Answer

Given: $a_{24}=2a_{10}$

$a +23d= 2(a +9d)$

$5d=a$ (i)

To Prove:$a_{72} =4a_{15}$

Proof: L.H.S. $=a_{72}$

$= a +71d$

$=5d +71d$ [from (i)]

$=76d$

R.H.S= $4a_{15}$

$=4(a +14d)$

=4a +56d

=4(5d) +56d [from (i)]

=20d +56d

=76d

Since, L.H.S=R.H.S.

Hence proved

42. Question

Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Answer

We have to find the numbers between 101 and 999 which are divisible by both 2 and 5

That means the numbers divisible by 10 between 101 and 999.

Let, the required divisible numbers be,

110, 120, 130, …, 990

a=110, d=10, last term, l=990

Number of terms, n $=\frac{l-a}{d}$ +1

$=\frac{990-110}{10}$ + 1

$=\frac{880}{10}$ +1

=89

Hence, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 are 89.

43. Question

If the seventh term of an A.P. is 1/9 and its ninth term is 1/7, find its $(63)^{rd}$ term.

Answer

We know, nth term of an AP isa_n = a + (n − 1)dwhere ‘a’ and ‘d’ are first term and common difference of AP respectively

Given,

$$7\text{th term of AP} = \frac{1}{9}$$

$$\Rightarrow a_7 = \frac{1}{9}$$

$$\Rightarrow a + 6d = \frac{1}{9} \quad [1]$$

$$9\text{th term of AP} = \frac{1}{7}$$

$$\Rightarrow a_9 = \frac{1}{9}$$

$$\Rightarrow a + 8d = \frac{1}{7} \quad [2]$$

Subtracting equation [1] from [2], we get

$$\Rightarrow a + 8d - a - 6d = \frac{1}{7} - \frac{1}{9}$$

$$\Rightarrow 2d = \frac{2}{63}$$

$$\Rightarrow d = \frac{1}{63}$$

Putting this value of d in equation [1], we get

$$\Rightarrow a + 6\left(\frac{1}{63}\right) = \frac{1}{9}$$

$$\Rightarrow a + \frac{2}{21} = \frac{1}{9}$$

$$\Rightarrow a = \frac{1}{9} - \frac{2}{21}$$

$$\Rightarrow a = \frac{1}{63}$$

Now, 63rd term = a + 62d

$$= \frac{1}{63} + 62\left(\frac{1}{63}\right)$$

$$= 1$$

Hence Proved!

44. Question

The sum of 5^{th} and 9^{th} terms of an AP is 30. If its 25^{th} term is three times its 8^{th} term, find the AP.

Answer

$a_5 + a_9 = 30$

$(a + 4d) + (a + 8d) = 30$

$2a + 12d = 30$

$a + 6d = 15$ (i)

Acc. To question,

$a_{25} = 3a_8$

$a + 24d = 3(a + 7d)$

$3d = 2a$

$3d = 2(15 - 6d)$ [from(i)]

$15d = 30$

$d = 2$

Putting the value of d in (i),

$a = 15 - 6(2) = 3$

$a_1 = a = 3$

$a_2 = a + d = 3 + 2 = 5$

$a_3 = a + 2d = 3 + 2(2) = 7$

Hence, the A.P. is $3, 5, 7, 9, \ldots$

45. Question

Find where 0 (zero) is a term of the A.P. $40, 37, 34, 31, \ldots$

Answer

$a = 40$, $d = 37 - 40 = -3$

Let, $a_n = 0$

$a + (n - 1)d = 0$

$40 + (n - 1)(-3) = 0$

$-3n = -43$

$n = 43/3$

Since, 'n' can't be a fraction

Hence, the answer is no

46. Question

Find the middle term of the A.P. $213, 205, 197, ..., 37$.

Answer

a=213, l=37

Middle term of A.P. = $\frac{1}{2}(a + l)$

$=\frac{1}{2}(213 + 37)$

$=\frac{250}{2}$

=125

47. Question

If the 5^{th} term of an A.P. is 31 and 25^{th} term is 140 more than the 5^{th} term, find the A.P.

Answer

a_5=31

a +4d =31 (i)

a_{25}= 140 + a_5

a + 24d= 140 + 31

31 -4d +24d=171 [from (i)]

20d=140

d=7

Putting the value of d in (i)

a= 31 – 4(7) = 3

a_1 = a=3

a_2 =a +d=3 +7=10

a_3 =a +2d=3 + 2(7) =17

Hence, the A.P. is 3, 10, 17,…

Exercise 9.4

1. Question

The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds these term by 6, find three terms.

Answer

Let the three terms be a - d, a, a + d

a – d + a + a – d = 21 (given)

3a = 21

a = 7

According to question,

(a – d) (a + d) = a + 6

$a^2 – d^2 = a + 6$

$7^2 – d^2 = 7 + 6$

$49 – d^2 = 13$

$d^2 = 36$

d = 6

Therefore, a – d = 7 – 6 = 1

a + d = 7 + 6 = 13

Thus, the three terms are 1, 7 and 13.

2. Question

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Answer

Let the three numbers be a – d, a, a + d

a – d + a + a + d = 27

3a = 27

a = 9

According to question,

(a – d) (a) (a + d) = 648

$(a^2 – d^2)$ (a) = 648

$(9^2 – d^2)$ 9 = 648

$(81 – d^2) = 72$

$d^2 = 9$

d = 3

Therefore, a – d = 9 – 3 = 6

A + d = 9 + 3 = 12

3. Question

Find the four numbers in A.P. whose sum is 50 and in which the greatest number is 4 times the least.

Answer

Let the four numbers be (a – 3d), (a – d), (a + d) and (a + 3d)

a – 3d + a – d + a + d + a + 3d = 50

4a = 50

$a = \frac{25}{2}$

According to question,

(a + 3d) = 4(a – 3d)

a + 3d = 4a – 12d

3a = 15d

$5d = \frac{25}{2}$

$d = \frac{5}{2}$

Therefore,

$a - 3d = \frac{25}{2} - \frac{15}{2} = 5$

$a - d = \frac{25}{2} - \frac{5}{2} = 10$

$a + d = \frac{25}{2} + \frac{5}{2} = 15$

$a + 3d = \frac{25}{2} + \frac{15}{2} = 20$

4. Question

The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.

Answer

Let the angles be (a – 3d), (a – d), (a + d) and (a + 3d)

a – 3d + a – d + a + d + a + 3d = 360^o (sum of angles of quadrilateral)

4a = 360^o

a = 90^o

According to question,

$(a + d) - (a - d) = 10^o$

$2d = 10^o$

$d = 5^o$

Therefore, $(a - 3d) = 90^o - 15^o = 75^o$

$(a - d) = 90^o - 5^o = 85^o$

$(a + d) = 90^o + 5^o = 95^o$

$(a + 3d) = 90^o + 15^o = 105^o$

5. Question

The sum of three numbers in A.P. is 12 and the sum of their cube is 288. Find the numbers.

Answer

Let the numbers be (a – d), a, (a+ d)

A – d + a + a+ d = 12

3a = 12

a = 4

According to question,

$(a - d)^3 + a^3 + (a + d)^3 = 288$

$a^3 - d^3 - 3ad\ (a - d) + a^3 + a^3 + d^3 + 3ad\ (a + d) = 288$

$3a^3 - 3a^2d + 3ad^2 + 3a^2d + 3ad^2 = 288$

$3(4)^3 + 6(4)\ d^2 = 288$

$24d^2 = 96$

$d^2 = 4$

d = ±4

Therefore, when a = 4 and d = 2

a – d = 4 – 2 = 2

a + d = 4 + 2 = 6

When a = 4 and d = -2

a – d = 4 + 2 = 6

a + d = 4 – 2 = 2

6. Question

Find the value of x for which (8x+4), (6x-2), (2x+7) are in A.P.

Answer

We know that, if three numbers are a,b,c are in AP then, 2b = a + c

According to question, (8x+4), (6x-2), (2x+7) are in AP.

Hence,

2 (6x – 2) = 8x + 4 + 2x + 7

12x – 4 = 10x + 11

2x = 15

x = 15/2 = 7.5

7. Question

If x+1, 3x and 4x+2 are in A.P, find the value of x.

Answer

Given terms: x+1, 3x and 4x+2

Since, the given terms are in A.P.

Therefore, their common difference will be same

3x – (x + 1) = (4x + 2) – 3x

3x – x - 1 = 4x + 2 – 3x

2x - 1 = x + 2

x = 3

8. Question

Show that $(a–b)^2$, (a^2+b^2) and $(a + b)^2$ are in A.P.

Answer

The terms given below are : $(a–b)^2$, (a^2+b^2) and $(a + b)^2$

Common difference, $d_1 = a^2 + b^2 – (a – b)^2$

$d_1 = a^2 + b^2 – (a^2 + b^2 - 2ab)$

$d_1 = a^2 + b^2 – a^2 - b^2 + 2ab$

$d_1 = 2ab$

Common difference, $d_2 = (a + b)^2 – (a^2 + b^2)$

$d_2 = a^2 + b^2 + 2ab – a^2 –b^2$

$d_2 = 2ab$

Since, $d_1 = d_2$ i.e. the common difference is same.

Therefore, the given terms are in A.P.

Exercise 9.5

1. Question

Find the sum of the following arithmetic progressions:

(i) $50, 46, 42, \ldots$ to 10 terms

(ii) $1, 3, 5, 7, \ldots$ to 12 terms

(iii) $3, 9/2, 6, 15/2, \ldots$ to 25 terms

(iv) $41, 36, 31, \ldots$ to 12 terms

(v) $a+b, a-b, a-3b, \ldots$ to 22 terms

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots,$ to n terms

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots$ to n terms

(viii) $-26, -24, -22, \ldots$ to 36 terms

Answer

(i) $50, 46, 42, \ldots$ to 10 terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$S_{10} = \frac{10}{2}$ [100 + 9(-4)]

= 5 [100 – 36]

= 5(64) = 320

(ii) $1, 3, 5, 7, \ldots$ to 12 terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{12}{2}$ [2 + (12 – 1) 2]

= 6 (2 + 22)

= 144

(iii) $3, 9/2, 6, 15/2, \ldots$ to 25 terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{25}{2}$ [2 (3) + 24($\frac{3}{2}$)]

$= \frac{25}{2}$ [6 + 36]

= 525

(iv) $41, 36, 31, \ldots$ to 12 terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{12}{2}$ [82 - 55]

= 6 [27] = 162

(v) $a+b, a-b, a-3b, \ldots$ to 22 terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{22}{2}$ [2(a + b) + (21) (-2b)]

= 11 [2a + 2b – 42b]

= 11 [2a – 40b]

= 22a – 440b

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots,$ to n terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [2(x – y)^2 + (n – 1) (2xy)]

$= \frac{n}{2}$ (2) [(x – y)^2 + (n – 1) xy]

= n [(x – y)^2 + (n – 1) xy]

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots$ to n terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [2($\frac{x-y}{x+y}$) + (n – 1) xy]

$= \frac{n}{2(x+y)}$ {2(x – y) + (n – 1) (2x – y)}

$= \frac{n}{2(x+y)}$ {n (2x – y) – y}

(viii) $-26, -24, -22, \ldots$ to 36 terms

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{36}{2}$ [-52 + (35) 2]

= 18 [-52 + 70]

= 18 [18] = 324

2. Question

Find the sum on n term of the A.P. $5, 2, -1, -4, -7, \ldots,$

Answer

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [10 + (n – 1) -3]

$= \frac{n}{2}$ [13 – 3n]

3. Question

Find the sum of n terms of an A.P. whose n^{th} term is given by $a_n = 5 - 6n$

Answer

Put n = 1

a_1 = 5 – 6(1) = 5 – 6 = -1

Put n = 2

a_2= 5 – 6(2) = 5 – 12 = -7

a = -1, d = -7 + 1 = -6

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [2(-1) + (n -1) (-6)]

$= \frac{n}{2}$ [-1 + 3n + 3]

= n [2 -3n]

4. Question

If the n sum of a certain number of terms starting from first term of an A.P. is $25, 22, 19, \ldots,$ is 116. Find the last term.

Answer

a = 25, d = -3

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$116 = \frac{n}{2}$ [50 – 3n +3]

$116 = \frac{n}{2}[53 - 3n]$

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

$n = 8$ or $n = \frac{29}{3}$, which isn't possible as n must be a natural number.

Therefore, $n = 8$

$\frac{n}{2}[a + l] = 116$

$\frac{8}{2}[25 + l] = 116$

$25 + l = 29$

$l = 4$

Hence, the last term is 4

5 A. Question

How many terms of the sequence $18, 16, 14, \ldots$ should be taken so that their sum is zero?

Answer

$S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$0 = \frac{n}{2}[2(18) + (n - 1)(-2)]$

$38n - 2n^2 = 0$

$n = 0$ (which is not possible)

$n = 19$

Therefore, $n = 19$ terms

5 B. Question

How many terms are there in the A.P. whose first and fifth terms are 14 and 2 respectively and the sum of the terms is 40?

Answer

$a_5 = a + (n - 1)\, d$

$2 = -14 + 4d$

$16 = 4d$

$d = 4$

$S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$40 = \frac{n}{2}[2(-14) + (n -1)(4)]$

$40 = 12n^2 - 16n$

$2n^2 - 16n -40 = 0$

$n^2 - 8n -20 = 0$

$n^2 - 10n + 2n - 20 = 0$

$(n - 10)(n + 2) = 0$

n = 10 and n = -2 (not possible)

5 C. Question

How many terms of the A.P. $9, 17, 25, ...$ must be taken so that their sum is 636?

Answer

$S_n = \frac{n}{2}[2a + (n - 1) d]$

$636 = \frac{n}{2}[2(9) + (n - 1) 8]$

$636 = n [9 + 4n -4]$

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

$n = 12$ or $n = \frac{-53}{4}$ (not possible)

5 D. Question

How many terms of the A.P. $63, 60, 57, ...$ must be taken so that their sum is 693?

Answer

$S_n = \frac{n}{2}[2a + (n - 1) d]$

$693 = \frac{n}{2}[2(63) + (n - 1)(-3)]$

$693 = \frac{n}{2}[126 -3n + 3]$

$462 = n [43 - n]$

$n^2 - 43n + 462 = 0$

$n = 22$ or $n = 21$

6. Question

The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer

a = 17, l = 350, d = 9

Number of terms, n = $\frac{l-a}{d}$ + 1

= $\frac{350-17}{9}$ + 1

= $\frac{333+9}{9}$ = $\frac{342}{9}$ = 38

S_n= $\frac{n}{2}$ (a + l)

= $\frac{38}{2}$ (350 + 17)

= 19 (367) = 6973

7. Question

The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Answer

We know, nth term of an AP isa_n = a + (n − 1)dwhere 'a' and 'd' are first term and common difference of AP respectivelyGiven, third term of an A.P. is 7a_3 = 7⇒ a + 2d = 7 (i)the seventh term exceeds three times the third term by 2

⇒ a_7 = 3a_3 + 2⇒ a + 6d = 3(7) + 2⇒ 7 − 2d +6d = 21 + 2⇒ 4d = 16⇒ d = 4

From (i), a = 7 − 2(4)

= -1

Also, we know sum of first 'n' terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$ Sum of first 20 terms is

$\Rightarrow S_{20} = \frac{20}{2}[2(-1) + 19(4)]$ = 10[-2 + 76]= 740

8. Question

The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Answer

a = 2, l = 50

S_n= $\frac{n}{2}$ [a + l]

442 = $\frac{n}{2}$ [50 +2]

n = $\frac{884}{52}$ = 17

$n = \frac{l-a}{d} + 1$

$17 = \frac{50-2}{d} + 1$

$16d = 48$

$d = 3$

9. Question

If 12^{th} term of an A.P. is 13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Answer

$a_{12} = -13$

$a + (12 - 1)\, d = -13$

$a + 11d = -13$

$S_4 = \frac{4}{2}\,[2a + (4 - 1)\, d]$

$24 = 2\,[2a + 3d]$

$12 = 2a + 3d$

$12 = 2\,(-11d - 13) + 3d$

$12 = -22d - 26 + 3d$

$38 = -19d$

$d = -2$

$a = -13 + 11(2)$

$= -13 + 22$

$= 9$

$S_{10} = \frac{10}{2}\,[2(9) + (10 - 1)\,(-2)]$

$= 5\,[18 - 18]$

$= 0$

10. Question

Find the sum of first 22 terms of an A.P. in which d = 22 and $a_{22} = 149$.

Answer

$a_{22} = a + 21d$

$149 = a + 21(22)$

a = 149 – 462

a = -313

$S_{22}= \frac{22}{2}$ [2a + (22 – 1) d]

= 11 [-626 + 462]

= 11 (-164)

= -1804

11. Question

Find the sum of all natural numbers between a and 100, which are divisible by 3.

Answer

a = 3, l = 99, n = 33

$S_{33}= \frac{33}{2}$ (a + l)

$= \frac{33}{2}$ (3 + 99)

$= \frac{33}{2}$ (102)

= 33 (51) = 1683

12. Question

Find the sum of first n odd natural numbers.

Answer

a = 1, d = 2

Sum of first n odd numbers, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [2(1) + (n – 1) 2]

= n [1 + n – 1]

$= n^2$

13. Question

Find the sum of all odd numbers between

(i) 0 and 50

(ii) 100 and 200

Answer

(i) 0 and 50

a = 1, l = 49, n = 25

$S_{25} = \frac{25}{2}[a + l]$

$= \frac{25}{2}[1 + 49]$

$= \frac{25}{2}(50) = 625$

(ii) 100 and 200

a = 101, l = 199, n = 50

$S_{50} = \frac{50}{2}[a + l]$

= 25 (101 + 199)

= 25 (300) = 7500

14. Question

Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Answer

a = 3, l = 999, n = $\frac{333+1}{2}$ = 167

$S_n = \frac{167}{2}(a + l)$

$= \frac{167}{2}(3 + 999)$

$= \frac{167}{2}(1002)$

= 167 (501) = 83667

15. Question

Find the sum of all integers between 84 and 719, which are multiples of 5.

Answer

To find: the sum of all integers between 84 and 719, which are multiples of 5.

Solution: The smallest and largest digits between 84 and 719 which are divisible by 5 is 85 and 715.So the sequence will be 85,90,..... 715. a = 85, d = 5, l = 715From the formula l=a+(n-1)dThe number of terms are:

$n = \frac{l-a}{d} + 1$

$n = \frac{715-85}{5} + 1$

$n = \frac{635}{5}$ n= 127

So there are 127 terms between 84 and 719.Now to calculate the sum of 127 terms use the formula:

$$s_n = \frac{n}{2}(a + l)$$

⇒ $S_{127} = \frac{127}{2}$ (a + l)

⇒ $S_{127} = \frac{127}{2}$ (85 + 715)

⇒ $S_{127} = \frac{127}{2}$ (800) = 50800So the sum of the terms between 84 and 719 is 50800.

16. Question

Find the sum of all integers between 50 and 500, which are multiples of 7.

Answer

a = 56, d = 7, l = 497

n = $\frac{l-a}{d}$ + 1

= $\frac{497-56}{7}$ + 1

= $\frac{441}{7}$ + 1 = $\frac{448}{7}$

= 64

$S_{64} = \frac{64}{2}$ (a + l)

= 32 (56 + 497)

= 32 (553) = 17696

17. Question

Find the sum of all even integers between 101 and 999.

Answer

a = 102, l = 998

n = $\frac{999-101}{2}$ = 449

$S_{449} = \frac{449}{2}$ (a + l)

= $\frac{449}{2}$ (102 + 998)

= $\frac{449(1100)}{2}$ = 246950

18. Question

Find the sum of all integers between 100 and 550, which are multiples of 9.

Answer

a = 108

L = 549

D = 9

549 = 108 + (n - 1)9

549 = 99 + 9n

9n = 450

n = 50

$S_n = \frac{50}{2}$ [216 + 49 (9)]

= 16425

19. Question

In an A.P. if the first term is 22, the common difference is – 4 and the sum to n terms is 64, find n.

Answer

a = 22, d = -4

S_n = 64

64 = $\frac{n}{2}$ (2a + (n – 1) d)

n [44 – (n – 1)4] = 128

n (44 – 4n + 4) = 128

$48n - 4n^2 = 128$

$n^2 - 12n + 32 = 0$

$n^2 - 8n - 4n + 32 = 0$

(n – 8) (n – 4) = 0

n = 8 or n = 4

20. Question

In an A.P. If the 5^{th} and 12^{th} terms are 30 and 65 respectively, what is the sum of first 20 terms?

Answer

According to question,

a_5= 30

a + 4d = 30

a = 30 – 4d (i)

a_{12}= 65

a + 11d = 65

30 – 4d + 11d = 65 [from (i)]

7d = 35

d = 5

Put d in (i)

a = 30 – 4(5)

= 10

$S_{20}= \frac{20}{2}$ [2(a) + (20 – 1) d]

= 10 [2(10) + 19(5)]

= 10 {20 + 95}

= 1150

21. Question

Find the sum of the first

(i) 11 terms of the A.P : $2,6,10,14,...$

(ii) 13 terms of the A.P : $-6,0,6,12,...$

(iii) 51 terms of the A.P. whose second term is 2 and fourth term is 8.

Answer

(i) 11 terms of the A.P : $2,6,10,14,...$

a = 2, d = 6 – 2 = 4

$S_{11}= \frac{11}{2}$ [2(a) + 10d]

$= \frac{11}{2}$ [2(2) + 10(4)]

= 11 [2 + 20]

= 242

(ii) 13 terms of the A.P : $-6,0,6,12,...$

a = -6, d = 0 + 6 = 6

$S_{13}= \frac{13}{2}$ [2(a) + 12d]

= 13 [-6 + 6(6)]

= 13 [-6 + 36]

= 13 (30) = 390

(iii) 51 terms of the A.P. whose second term is 2 and fourth term is 8.

$a_2 = 2$

a + d = 2 (i)

$a_4 = 8$

a + 3d = 8

2 – d + 3d = 8

2 + 2d = 8

d = 3

a = -1

$S_{51} = \frac{51}{2}[2(a) + 50d]$

= 51 [-1 + 25(3)]

= 51 (74)

= 3774

22. Question

Find the sum of

(i) The first 15 multiples of 8

(ii) The first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(iii) All 3 – digit natural numbers which are divisible by 13.

(iv) All 3 – digit natural numbers, which are multiples of 11.

Answer

(i) The first 15 multiples of 8

a = 8, d = 8

$S_{15} = \frac{15}{2}[2(8) + 14(8)]$

= 15[8 + 56]= 15[64]

= 960

(ii) The first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(a) a = 3, d = 3

$S_{40} = \frac{40}{2}[2(a) + 39(d)]$

= 20 [2(3) + 39(3)]

= 60 (41) = 2460

(b) a = 6, d = 5

$S_{40} = \frac{40}{2}[2(5) + 39(5)]$

= 100 [41]

= 4100

(iii) All 3 – digit natural numbers which are divisible by 13.

a = 6, d = 6

$S_{40} = \frac{40}{2}[2(6) + 39(6)]$

= 120 (41) = 4920

(iv) All 3 – digit natural numbers, which are multiples of 11.

a = 110, d = 11, l = 990

$n = \frac{l-a}{d} + 1$

$= \frac{990-110}{11} + 1$

$= \frac{880+11}{11} = \frac{891}{11}$

= 81

$S_{81} = \frac{81}{2}[2(110) + 80(11)]$

= 8910 [1 + 4]

= 44550

23. Question

Find the sum :

(i) $2 + 4 + 6 + ... + 200$

(ii) $3 + 11 + 19 + ... + 803$

(iii) $(-5) + (-8) + (-11) + ... + (-230)$

(iv) $1 + 3 + 5 + 7 + ... + 199$

(v) $7 + 10\frac{1}{2} + 14 + ... + 84$

(vi) $34 + 32 + 30 + ... + 10$

(vii) $25 + 28 + 31 + ... + 100$

(viii) $18 + 15\frac{1}{2} + 13 + ... + \left(-49\frac{1}{2}\right)$

Answer

(i) $2 + 4 + 6 + ... + 200$

a = 2, d = 4 – 2 = 2, l = 200

n = $\frac{l-a}{d}$ + 1

= $\frac{200-2}{2}$ + 1 = 100

S_{100}= $\frac{100}{2}$ [2(2) + 99(2)]

= 100 [101] = 10100

(ii) $3 + 11 + 19 + ... + 803$

a = 3, d = 11 – 3 = 8, l = 803

S_n= $\frac{l-a}{d}$ + 1 = $\frac{803-3}{8}$ + 1

= 101

S_{101}= $\frac{101}{2}$ [2(3) + 100(8)]

= 101 (403) = 40703

(iii) $(-5) + (-8) + (-11) + ... + (-230)$

a = -5, d = -8 + 5 = -3, l = -230

n = $\frac{-230+5}{-3}$ + 1

= $\frac{228}{3}$ = 76

S_n= 38(5) [-2 – 45]

= 190 (-47) = -8930

(iv) $1 + 3 + 5 + 7 + ... + 199$

a = 1, d = 3 – 1 = 2, l = 199

n = $\frac{199-1}{2}$ + 1

= 100

$S_{100} = \frac{100}{2}[2(1) + 99(2)]$

$= 100\ (100) = 10000$

(v) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

$a = 7,\ d = \frac{21}{2} - 7 = \frac{7}{2}$

Last term, $a_n = 84$

$a_n = a + (n - 1)d$

$84 = 7 + (n - 1)\frac{7}{2}$

$84 = \frac{14 + 7n - 7}{2}$

$84 * 2 = 7 + 7n$

$168 = 7 + 7n$

$7n = 168 - 7$

$7n = 161$

$n = 23$

Sum of n^{th} term, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{23} = \frac{23}{2}[2(7) + (23 - 1)\frac{7}{2}]$

$= \frac{23}{2}[14 + 22 * \frac{7}{2}]$

$= \frac{23}{2}[14 + 77]$

$= \frac{23}{2} * 91 = \frac{2093}{2}$

Hence, sum of given A.P. is $\frac{2093}{2}$

(vi) $34 + 32 + 30 + \ldots + 10$

$a = 34,\ d = 32 - 34 = -2$

Last term, $a_n = 10$

$a_n = a + (n - 1)d$

$10 = 34 + (n - 1)\ (-2)$

$10 = 34 - 2n + 2$

$2n = 34 - 10 + 2$

2n = 26

n = 13

Sum of n terms,

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$S_{13} = \frac{13}{2}$ [2(34) + (13 – 1) (-2)]

$= \frac{13}{2}$ [68 + 12(-2)]

$= \frac{13}{2}$ [68 – 24]

$= \frac{13}{2}$ * 44 = 286

Hence, Sum of given A.P. is 286

(vii) $25 + 28 + 31 + ... + 100$

a = 25, d = 3

Last term, a_n = 100

A_n = a + (n -1) d

100 = 25 (n – 1)3

75 = 3n – 3

78 = 3n

n = 26

Sum of n terms, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{26}{2}$ [2(25) + (26 – 1) 3]

= 13 [50 + 25 * 3]

= 13 * 125 = 1625

Hence, Sum of given A.P. is 1625

(viii) $18 + 15\frac{1}{2} + 13 + ... + \left(-49\frac{1}{2}\right)$

a = 18, d = $\frac{31}{2}$ - 18 = $\frac{-5}{2}$

Last term, $a_n = \frac{-99}{2}$

a_n = a+ (n – 1)d

$\frac{-99}{2}$ = 18 + (n – 1) ($\frac{-5}{2}$)

-99 = 36 + 5 – 5n

-140 = -5n

n = 28

S_{28}= $\frac{28}{2}$ [a + l]

= 14 (18 - $\frac{99}{2}$)

= 7 (36 – 99)

= 7 (-63) = -441

24. Question

Find the sum of the first 15 terms of each of the following sequences having n^{th} term as

(i) $a_n = 3 + 4n$

(ii) $b_n = 5 + 2n$

(iii) $x_n = 6 - n$

(iv) $y_n = 9 - 5n$

Answer

(i) $a_n = 3 + 4n$

Put n = 1

a_1 = 3 + 4(1) = 7

Put n = 15

a_{15} = 3 + 4(15) = 63 = l

Sum of 15 terms, S_{15} = $\frac{15}{2}$ [a + l]

= $\frac{15}{2}$ [7 + 63] = 525

(ii) $b_n = 5 + 2n$

Put n = 1

b_1 = 5 + 2(1) = 7

Put n = 15

b_{15} = 5 + 2(15) = 35 = l

Sum of 15 terms, $S_{15}= \frac{15}{2}[a + l]$

$= \frac{15}{2}[7 + 35] = 315$

(iii) $x_n = 6 - n$

Put n = 1

$x_1 = 6 - 1 = 5$

Put n = 15

$x_{15} = 6 - 15 = -9$

Sum of 15 terms, $S_{15}= \frac{15}{2}[a + l]$

$= \frac{15}{2}[5 - 9] = -30$

(iv) $y_n = 9 - 5n$

Put n = 1

$y_1= 9 - 5(1) = 4$

Put n = 15

$y_{15}= 9 - 5(15) = -66$

Sum of 15 terms, $S_{15}= \frac{15}{2}[a + l]$

$= \frac{15}{2}[4 - 66] = -465$

25. Question

Find the sum of first 20 terms of these sequence whose n^{th} term is $a_n = An + B$.

Answer

Given, $a_n= An + B$

Put n = 1, $a_1= A + B$

Put n = 20, $a_{20}= 20A + B$

$S_{20}= \frac{20}{2}[a + l]$

$= 10 [A + B + 20A + B]$

$= 10 [21A + 2B]$

$= 210A + 20B$

26. Question

Find the sum of first 25 terms of an A.P. whose n^{th} term is given by $a_n = 2 - 3n$.

Answer

Given, n^{th} term, $a_n = 2 - 3n$

Put n = 1, $a_1 = 2 - 3(1) = -1$

Put n = 25, $a_{15} = 2 - 3(15) = -43$

Therefore, $S_{25} = \frac{25}{2}(-1 - 43)$

$= \frac{25}{2}(-44) = -925$

27. Question

Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 7 - 3n$.

Answer

Given, $a_n = 7 - 3n$

Put n = 1

$a_1 = 7 - 3(1) = 4$

Put n = 25

$a_{25} = 7 - 3(25) = -68 = l$

$S_{25} = \frac{25}{2}[4 - 68]$

$= 25[-32] = -800$

28. Question

Find the sum of the first 25 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Answer

$a_2 = 14$

$a + d = 14$ (i)

$a_3 = 18$

$a + 2d = 18$ (ii)

Subtracting (i) from (ii), we get

$d = 4$

Putting the value of d in (i), we get

$a = 14 - 4 = 10$

$S_{25} = \frac{25}{2}[2(a) + 24(d)]$

$= 25\ [58]$

$= 1450$

29. Question

If the sum of 7 terms of an A.P. is 49 and that of 17 term is 289, find the sum of n terms.

Answer

$S_7 = 49$

$\frac{7}{2}[2a + 6d] = 49$

$a + 3d = 7$ (i)

$S_{17} = 289$

$\frac{17}{2}[2a + 16d] = 289$

$a + 8d = 17$ (ii)

Subtract (i) from (ii), we get

$5d = 10$

$d = 2$

Put d = 2 in (i), we get

$a = 7 - 6 = 1$

$S_n = \frac{n}{2}[2(1) + (n - 1)2]$

$= n\ [1 + n - 1]$

$= n^2$

30. Question

The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer

$a = 5,\ l = 45$

$S_n = 400$

$\frac{n}{2}[5 + 45] = 400$

$\frac{n}{2}[50] = 400$

$n = 16$

16^{th} term is 45

a_{16}= 45

5 + 15d = 45

15d = 40

d = $\frac{8}{3}$

31. Question

In an A.P., the sum of first n terms is $\frac{3n^2}{2}+\frac{13n}{2}$. Find its 25^{th} term.

Answer

Given, $S_n=\frac{3n^2}{2}+\frac{13}{2}n$

Let a_n be the n^{th} term of the A.P.

a_n= S_n – S_{n-1}

= $3n^2/2 + 13n/2 – 3(n – 1)^2/2 – 13(n – 1)/2$

= $\frac{3}{2}\{n^2 – (n – 1)^2\} + \frac{13}{2}\{n – (n – 1)\}$

= $3n - \frac{3}{2} + \frac{13}{2}$

= $3n + \frac{10}{2}$ = 3n + 5

Put n = 25

a_{25}= 3(25) + 5

= 75 + 5 = 80

Therefore, 25^{th} term is a_{25} = 80

32. Question

Let there be an A.P. with first term ‘a’, common difference, d. If a_n denotes its n^{th} term and S_n the sum of first n terms, find.

(i) n and S_n, if $a = 5, d = 3$ and $a_n = 50$.

(ii) n and a, if $a_n = 4, d = 2$ and $S_n = -14$.

(iii) d, if $a = 3, n = 8$ and $S_n = 192$.

(iv) a, if $a_n = 28, S_n = 144$ and $n = 9$.

(v) n and d, if $a = 8, a_n = 62$ and $S_n = 210$.

(vi) n and a_n, if $a = 2, d = 8$ and $S_n = 90$.

Answer

(i) n and S_n, if $a = 5, d = 3$ and $a_n = 50$.

Given, $a_n = 50$

$a + (n - 1) d = 50$

$5 + (n - 1) 3 = 50$

$(n - 1)3 = 45$

$(n - 1) = 15$

$n = 16$

$S_n = \frac{n}{2} [a + l]$

$= 8 [5 + 50] = 8 [55] = 440$

(ii) n and a, if $a_n = 4, d = 2$ and $S_n = -14$.

$a_n = 4$, $d = 2$, $S_n = -14$

$a + (n - 1) 2 = 4$

$a + 2n = 6$, and

$\frac{n}{2} [2a + (n - 1)2] = -14$

$n [2a + 2n - 2] = -14$, or

$\frac{n}{2} [a + a_n] = -14$

$\frac{n}{2} [a + 4] = -14$

$n [6 -2n +4] = -28$

$n [10 -2n] = -28$

$2n^2 - 10n - 28 = 0$

$2(n^2 - 5n - 14) = 0$

$(n + 2) (n - 7) = 0$

$n = -2, n = 7$

Therefore, $n = -2$ is not a natural number. So, $n = 7$

(iii) d, if $a = 3, n = 8$ and $S_n = 192$.

Given, $a = 3$, $n = 8$, $S_n = 192$

$S_n = \frac{n}{2}$ {2a + (n – 1) d]

192 * 2 = 8 [6 + (8 – 1)d]

$\frac{192*2}{8}$ = 6 + 7d

48 = 6 + 7d

7d = 42

d = 6

(iv) a, if $a_n = 28, S_n = 144$ and $n = 9$.

a_n= 28, S_n = 144, n = 9

$S_n = \frac{n}{2}$ [a + l]

144 = $\frac{9}{2}$ [a + 28]

$\frac{144*2}{9}$ = a = 28

a + 28 = 32

a = 4

(v) n and d, if $a = 8, a_n = 62$ and $S_n = 210$.

Given, a = 8, a_n= 62 and S_n= 210

$S_n = \frac{n}{2}$ [a + l]

210 = $\frac{n}{2}$ [8 + 62]

210 * 2 = n (70)

n = $\frac{210*2}{40}$ = 6

a + (n – 1) d = 62

8 + (6 – 1) d = 62

5d = 54

d = 10.8

(vi) n and a_n, if $a = 2, d = 8$ and $S_n = 90$.

Given, a = 2, d = 8 and S_n = 90

90 = $\frac{n}{2}$ [4 + (n – 1)d] {Therefore, $S_n = \frac{n}{2}$ [2a + (n – 1)d]}

180 = n [4 + 8n – 8]

$8n^2 - 4n - 180 = 0$

$4(2n^2 - n - 45) = 0$

$2n^2 - n - 45 = 0$

$(2n + 1)(n - 5) = 0$

Therefore, $n = \frac{-1}{2}$ is not a natural number

$n = 5$

$a_n = a + (n - 1)d$

$= 2 + 4(8) = 32$

33. Question

Aman saved Rs. 16500 in ten years. In each year after the first he saved Rs.100 more than he did in the preceding year. How much did he save in the first year?

Answer

Given,

Man saved in 10 years, $S_{10} = 16500$

In each year after first he saved, $d = 100$

We know, sum of n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

For 10 years

$S_{10} = \frac{10}{2}[2a + (10 - 1)100]$

$16500 = 5[2a + 9 * 100]$

$\frac{16500}{5} = 2a + 900$

$3300 = 2a + 900$

$2a = 3300 - 900$

$2a = 2400$

$a = 1200$

Hence, Rs. 1200 saved by him in first year

34. Question

Aman saved Rs.32 during the first year, Rs.36 in the second year and in this way he increases his savings by Rs.4 every year. Find in what time his saving will be Rs.200.

Answer

Given, A man saved in first year, $a = 32$

A man saved in second year, $a_2 = 36$

Increase saving, d = 4

In n years his saving will be 200, $S_n = 200$

We know,

$S_n = \frac{n}{2} [2a + (n - 1) d]$

$200 = \frac{n}{2} [2(32) + (n - 1) 4]$

$400 = n [64 + 4n - 4]$

$400 = n [60 + 4n]$

$400 = 4n [15 + n]$

$100 = 15n + n^2$

$n^2 + 15n - 100 = 0$

$n^2 + 20n - 5n - 100 = 0$

$n (n + 20) - 5 (n + 20) = 0$

$(n - 5) (n + 20) = 0$

Here, $n - 5 = 0$, $n = 5$

$n + 20 = 0$, $n = -20$

The term can never be negative. So, we consider n = 5

Hence, in 5 years his saving will be Rs. 200

35. Question

Aman arrange stop ay off a debt of Rs.3600 by 40 annual installments which for man arithmetic series. When 30 of the installments are paid, he dies leaving one - third of the debt unpaid, find the value of the first installment.

Answer

Given, Total amount of payable in 40 annual installments, $S_{40} = 3600$

After 30 installment he died and leaving $\frac{1}{3}$ of the debt unpaid

Which means, total in 30 installment, $S_{30} = \frac{2}{3}$ of the debt

$S_{30} = \frac{2}{3} * 3600 = 2400$

We know sum of n terms, $S_n = \frac{n}{2} [2a + (n - 1) d]$

For 30 installments, $S_{30} = \frac{30}{2}[2a + (30 - 1)\, d]$

$2400 = 15\,[2a + 29d]$

$160 = 2a + 29d$

$2a = 160 - 29d$

$a = \frac{160-29d}{2}$ (i)

For 40 installments, $S_{40} = \frac{n}{2}[2a + (n - 1)\, d]$

$3600 = \frac{40}{2}[2a + (40 - 1)\, d]$

$180 = 2a + 39d$

$2a = 180 - 39d$

$a = \frac{180-39d}{2}$ (ii)

From (i) and (ii), we get

$\frac{160-29d}{2} = \frac{180-39d}{2}$

$160 - 29d = 180 - 39d$

$39d - 29d = 180 - 160$

$10d = 20$

$d = 2$

Putting the value of d in (i), we get

$a = \frac{160-29(2)}{2}$

$= \frac{160-58}{2} = \frac{102}{2} = 51$

Hence, value of first installment is 51

36. Question

There are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and here turns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

Answer

Total numbers of trees = 25

Distance between trees = 5

Total distance between well and first tree is = 10 m

Now, gardener return back to well after watering each tree then,

Distance between well and second tree is = 10 + 10 + 5 = 25

Distance between well and third tree = 25 + 5 + 5 = 35

Distance between well and fourth tree = 35 + 5 + 5 = 45

Distance between well and last tree without return back = 10 + 24(5) = 10 + 120 = 130

So, common difference = 35 – 25 = 10

Now, Total distance covered by 24 trees,

We knew, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{24}{2}$ [2(25) + (24 – 1) 10]

= 12 [50 + 23 * 10]

= 12 [50 + 230]

= 12 * 280

= 3360

Now, distance covered by 25 trees = 10 + 3360 = 3370

Total distance covered by gardener with return back = 3370 + 130 = 3500 m

Hence, total distance will covered by gardener is 3500.

37. Question

A man is employed to count Rs.10710. He counts at there of Rs.180 per minute for half an hour. After this, he counts at the rate of Rs.3 less every minute than the preceding minute. Find the time taken by him to count the entire amount.

Answer

Total amount for counting = Rs. 10710

In 1 min he counts =Rs. 180

In half an hour (30 min) he count = Rs.180×30 =Rs.5400

Amount left count after half an hour, S_n= Rs.10710 –Rs. 5400 = 5310

Now, in 31st min he count 3 less than preceding minute = Rs.180 – Rs.3 = Rs.177

In 32nd min he count 3 less than preceding minute = Rs. 177 –Rs. 3 =Rs. 174

Then, Arithmetic progression formed is 177, 174,…

Here, difference between minutes = 174 – 177 = -3

We know, sum of n terms, S_n= (n/2) [2a + (n – 1) d]

$5310 = (n/2) [2(177) + (n – 1) -3]$

$5310 \times 2 = n [354 – 3n + 3]$

$10620 = 354n – 3n^2 + 3n$

$10620 = 357n – 3n^2$

$\Rightarrow 3n^2 – 357n + 10620 = 0$

On taking 3 common from the complete equation, we get,

$\Rightarrow 3 (n^2 – 119n + 3540) = 0$

$\Rightarrow n^2 – 119n + 3540 = 0$

Now, from factoring by splitting the middle term method, we get,

$\Rightarrow n^2 – 60n – 59n + 3540 = 0$

$\Rightarrow n (n – 60) – 59 (n -60) = 0$

$\Rightarrow (n – 59) (n – 60) = 0$

Therefore, either n = 59 or n = 60We will use 59 as these are minutesSo The total time to calculate whole amount is 59 + 30 = 89 min

= 1 hr 29 min

38. Question

A piece of equipment cost a certain factory Rs.600,000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and soon. What will be its value at the end of 10 years, all percentages applying to the original cost?

Answer

Given: A piece of equipment cost a certain factory Rs.600,000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and soon.

To find: The value at the end of 10 years if all percentages applying to the original cost.

Solution: Cost of equipment = 6, 00, 000

In 1 year the value depreciate by 15%

$\Rightarrow$ The value of the equipment after first year= 6, 00, 000 * $\frac{15}{100}$ = 90, 000

In 2 year depreciate by 13.5%

$\Rightarrow$ The value of the equipment after second year= 6, 00, 000 * $\frac{13.5}{100}$ = 81, 000

In 3 year depreciate by 12%

$\Rightarrow$ The value of the equipment after third year= 6, 00, 000 * $\frac{12}{100}$ = 72, 000

Now A.P. is 90000, 81000, 72000,…

So, common difference = 81000 – 90000 = -9, 000

We have to find total depreciation for 10 years,Since the value of depreciation is constant in any consecutive years i.e its value is -9000. So to find the depreciation after ten years we will use the formula:$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$S_{10} = \frac{10}{2}$ [2(90, 000) + (10 – 1) (-9000)]

S_{10}= 5 [180000 + 9 * (-9000)]

S_{10}= 5 [180000 – 81000]

S_{10}= 5 [99000] = 495000

Hence, cost of equipment at the end of 10 years = original cost – depreciation

= 6, 00, 000 – 4, 95, 000 = Rs. 1, 05, 000

39. Question

A sum of Rs.700 is to be used to gives even cash prizes to students of a school for their overall academic performance. If each prize is Rs.20 less than its preceding prize, find the value of each prize.

Answer

Amount of money = 700

Total number of prize = 7

Each prize is Rs. 20 less than its preceding prize

Let, Prize of first prize = a

Prize of second prize = a – 20

Prize of third prize = a – 20 – 20 = a – 40

Here, A.P. is a, a – 20, a – 40,…

So, common difference, d = a – 20 – a = -20

We know, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

700 = $\frac{7}{2}$ [2a + (7 – 1) -20]

$\frac{1400}{7}$ = 2a + 6 (-20)

$2a = 200 + 120$

2a = 320

a = 160

So, value of first prize = 160

Value of second prize, 160 – 20 = 140

Value of third prize, 140 – 20 = 120

Value of fourth prize, 120 – 20 = 100

Value of fifth prize, 100 – 20 =80

Value of sixth prize, 80 – 20 = 60

And value of seventh prize, 60 – 20 = 40

Hence, value of prizes, Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, Rs 40.

40. Question

In an A.P. the first term is 8, nth term is 33 and the sum to first n terms is 123. Find n and d, the common differences.

Answer

Given, First term, a = 8

N^{th} term, a_n = 33 = l

And sum of n terms, $S_n = \frac{n}{2}$ [a + l]

$123 = \frac{n}{2}$ [8 + 33]

123 * 2 = n * 41

$n = \frac{123*2}{41} = \frac{246}{41} = 6$

We know,

a_n= a + (n – 1) d

33 = 8 + (6 – 1) d

33 – 8 = 5d

25 = 5d

d = 5

Hence, number of terms n = 6 and common difference, d = 5

41. Question

In an A.P., the first term is 22, nth term is -11 and the sum to first n terms is 66. Find n and d, the common difference.

Answer

Given, First term, a = 22

n^{th} term, a_n= -11 = l

and sum of n terms, S_n= 66

We know sum of n terms, $S_n = \frac{n}{2}$ [a + l]

$66 = \frac{n}{2}$ [22 – 11]

66 * 2 = n * 11

$n = \frac{66*2}{11} = 12$

we know, a_n= a + (n – 1) d

-11 = 22 + (12 – 1) d

-11 – 22 = 11d

-33 = 11d

d = -3

Hence, number of terms, n = 12 and common difference, d = -3

42. Question

If the sum of the first n terms of an A.P. is $4n – n^2$, which is the first term? What is the sum of first two terms? What is the second term? Similarly, find the third, the tenth and the nth terms.

Answer

Given, $S_n = 4n – n^2$

Putting n = 1, 2

$S_1 = 4(1) – (1)^2 = 3$

$S_2 = 4(2) – (2)^2 = 4$

We know, $a_n = S_n – S_{n-1}$

For first term, $a_1 = S_1 – S_0 = 3 – 0 = 3$

For term $a_2 = S_2 – S_1 = 4 – 3 = 1$

Now fore term a_3,

$S_2 = 4(2) – (2)^2 = 8 – 4 = 4$

$S_3 = 4(3) – (3)^2 = 12 – 9 = 3$

$a_3 = S_3 – S_2 = 3 – 4 = -1$

43. Question

The first and the last term of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer

Given,

First term, $a_1 = 17$

Last term, $a_n = 350 = l$

And difference, d = 9

We know,

$a_n = a + (n - 1)d$

$350 = 17 + (n - 1)(9)$

$350 = 8 + 9n$

$342 = 9n$

$n = 38$

We know sum of n terms, $S_n = \frac{n}{2}[a + l]$

$S_{38} = \frac{38}{2}[17 + 350]$

$= 19 * 367 = 6973$

Hence, number of terms, n = 38

Sum of n terms, $S_n = 6973$

44. Question

In an A.P., the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P.

Answer

Given, First term, a = 2

Last term, $a_n = l = 29$

And, Sum of terms, $S_n = \frac{n}{2}[a + l]$

$155 = \frac{n}{2}[2 + 29]$

$155 * 2 = n * 31$

$n = \frac{155*2}{31} = \frac{310}{31} = 10$

We know, $a_n = a + (n - 1)\ d$

$29 = 2 = (10 -1)\ d$

$29 - 2 = 9d$

27 = 9d

d = 3

Hence, common difference, d = 3

45. Question

In an A.P., the sum of first ten terms is – 150 and the sum of its next ten terms is-550. Find the A.P.

Answer

Given: Sum of first ten terms that is, S_{10} = -150 and Sum of next ten terms is -550.

To find: The A.P.

Solution: Let, first term = a

Last term = a_nWe know in an A.P,a_n = a + (n – 1) d

For n=10,

a_n = a + (10 -1) d

a_n = a + 9d

We know, Sum of n terms

$S_n = \frac{n}{2}$ [a + l] ..... (1)

l and a_n means the last term.

Put the value of l in (1),So,

$S_{10} = \frac{10}{2}$ [a + a + 9d]

Substitute the known values,

-150 = 5 (2a + 9d)

-150 = 5 (2a + 9d)-150= 10a + 45d

10a = -150 – 45d

$a = \frac{-150-45d}{10}$ ..... (2)

For next 10 terms, First term = a_{11}

Last term = a_{20}

Since a_n = a + (n – 1) d,

For 11^{th} term,a_{11} = a + 10d

For 20^{th} term,a_{20} = a +19d

And $S'_n = \frac{n}{2}$ [a + l] ..... (3)

Let the sum of next 10 terms to be S'_{10} ,

Now a will be equal to a_{11} in this case and l will be equal to a + 19d

Put the value of a and l in the equation (3).$S'_{10} = \frac{10}{2}$ [a + 10d + a + 19d]

Substitute the known values,

-550 = 5 [2a + 29d]

-550 = 10a + 145d

10a = -550 – 145d

a = $\frac{-550-145d}{10}$ .....(4)

From (2) and (4), we get

$\frac{-150-45d}{10} = \frac{-550-145d}{10}$

-150 – 45d = -550 – 145d

-45d + 145d = -550 + 150

100d = -400

d = -4

Put the value of d in (2), we get

a = $\frac{-150-45(-4)}{10}$

a= $\frac{-150+180}{10} = \frac{30}{10} = 3$

We know A.P is of the form,a_1 , a_2 ,..... ,a_n

Where a_1 is the first term of an A.P,

d is the common difference.anda_n = a+ (n-1)dSo,

a_1 = 3

a_2 = a + d = 3 + (-4) = -1

a_3 = a + 2d = 3 + 2(-4) = -5

Conclusion : The A.P. is 3, -2, -5,...

and a = 3, d = -4

46. Question

Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25th term.

Answer

Given, a = 10

S_{14} = 1505

$S_{14} = \frac{n}{2}[2a + (n - 1)\, d]$

$1505 = \frac{14}{2}[2(10) + (14 - 1)\, d]$

1505 = 7 [20 + 13d]

215 = 20 + 13d

13d = 215 – 20

13d = 195

d = 15

Now, a_{25} = a + (n – 1) d

= 10 + (25 – 1) 15

= 10 + 24 * 15

= 10 + 360 = 370

Hence, value of 25^{th} term is 370

47. Question

The sum of first n terms of an A.P. is $5n^2 + 3n$. If its *m*th term is 168, find the value of m. Also, find the 20^{th} term of this A.P.

Answer

Given that

$S_n = 5n^2 + 3n$

Put n = 1

$S_1 = T_1 = 5 + 3 = 8$

Put n = 2

$S_2 = 5(2)^2 + 3 + 2 = 26$

$T_2 = S_2 - S_1 = 26 - 8 = 18$

$S_3 = 5(3)^2 + 3 + 3 = 54$

$T_3 = S_3 - S_2 = 54 - 26$

= 28

Therefore, first term, a = 8 and common difference = 18 – 8 = 10

$T_m = a + (m - 1) d$

168 = 8 + (m – 1) 10

168 = 8 + 10m – 10

170 = 10m

m = 17

$T_{20} = 8 + (20 - 1) 10$

= 8 + 19 * 10 = 198

48. Question

The sum of first q terms of an A.P. is $63q - 3q^2$. If its pth term is-60, find the value of p. Also, find the 11^{th} term of this A.P.

Answer

Given that: $S_q = 63q - 3q^2$

Put q = 1

$S_1 = T_1 = 63 - 3 = 60$

Put q = 2

$S_2 = 63 * 2 - 3 * (2)^2$

= 126 – 12 = 114

$T_2 = S_2 - S_1 = 114 - 60 = 54$

Put q = 3

$S_3 = 63 * 3 - 3(3)^2$

= 189 – 27 = 162

$T_3 = S_3 - S_2$

= 162 – 114 = 48

Therefore, first term of this A.P. is 60 and Common difference is 54 - 60 = -6

$T_p = a + (p - 1) d$

-60 = 60 + (p – 1) (-6)

-120 = (p – 1) (-6)

(p – 1) = 20

p = 21

Now 11^{th} term of this A.P

$T_{11} = 60 + (11 - 1)(-6)$

$= 60 - 60 = 0$

49. Question

The sum of first m terms of an A.P. is $4\ m^2$ - m. If its nth term is 107, find the value of n. Also, find the 21^{st} term of this A.P.

Answer

$S_m = 4m^2 - m$

Put m = 1

$S_1 = T_1 = 4 - 1 = 1$

Put m = 2

$S_2 = 4(2)^2 - 2 = 14$

$T_2 = S_2 - S_1 = 14 - 3 = 11$

Put m = 3

$S_3 = 4(3)^2 - 3 = 33$

$T_3 = S_3 - S_2$

$= 33 - 14 = 19$

The first term of given A.P. is 3 and common difference, d = 11 – 3 = 8

n^{th} term of the given A.P. is 107

$107 = 3 + (n - 1)8$

$104 = (n - 1)8$

$(n - 1) = 13$

$n = 14$

the 21^{st} term of the given A.P., $T_{21} = 3 + (21 - 1)8$

$= 3 + 160 = 163$

50. Question

The nth term of an A.P. is given by (-4n+15). Find the sum of first 20 terms of this A.P.

Answer

Given: The nth term of an A.P. (-4n+15).

To find: the sum of first 20 terms of this A.P.

Solution: We have,

$T_n = (-4n + 15)$

$T_1 = -4 + 15 = 11$

$T_2 =(-4 \times 2)+ 15 = 7$

$T_3 =(-4 \times 3) + 15 = 3$Hence the A.P is 11,7,3,.....

The first term is 11 and the common difference is, $d = T_2 - T_1 = 7 - 11 = -4$

We calculate the sum of terms of A.P by using the formula:$S_n= \frac{n}{2} [2a + (n - 1) d]$

Substitute the known values to get the sum.

The sum of first 20 terms, $S_{20} = \frac{20}{2} [(2 \times 11) + (20 - 1) (-4)]$

$S_{20} = \frac{20}{2} [(22) + (19) (-4)]$

$S_{20} = 10 (22 - 76)$ $S_{20 = 10(-54)}S_{20 = -540}$

51. Question

Find the number of terms of the A.P. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained.

Answer

The given A.P. is -12, -9, -6,…21

Here, a = -12, d = -9 – (-12) = 3

Let the number of terms be n

$T_n = a + (n - 1) d$

$21 = -12 + (n - 1) (3)$

$21 = -15 + 3n$

$36 = 3n$

$n = 12$

If 1 is added to each term than A.P. is

-11, -8, -5…20

Here, a = -11, d = 3

$S_{12}= \frac{12}{2} [2(-11) + (12 -1) * 3]$

$= 6 (-22 + 33)$

= 66

52. Question

The sum of first n terms of an A.P. is $3n^2 + 4n$. Find the 25^{th} term of this A.P.

Answer

We have sum of n terms, $S_n = 3n^2 + 4n$

Put n = 1

$S_1 = T_1 = 3(1)^2 + 4\ (1) = 7$

Put n = 2

$S_2 = 3(2)^2 + 4\ (2) = 20$

$T_2 = S_2 - S_1 = 20 - 7 = 13$

Put n = 3

$S_3 = 3(3)^2 + 4(3) = 39$

$T_3 = S_3 - S_2 = 39 - 20 = 19$

Therefore, first term is 7 and common difference, d = 13 – 7 = 6

The 25^{th} term is, $T_n = a + (n - 1)\ d$

$T_{25} = 7 + (25 - 1) * 6$

$= 7 + 24 * 6 = 151$

53. Question

In a school, students decide to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students.

Answer

Each class has to plant double the class they are studying

Therefore, first class will plant = 2

Second class will plant = 4

Same as, 12^{th} class will plant = 24

Total number of plants to be planted = sum of all plants

$= \frac{n}{2}$ [a + l]

$= \frac{12}{2}$ [2 + 24]

= 6 * 26 = 156

Therefore, there are 2 sections in each class

So, total trees = 2 * 156 = 312

54. Question

The sum of first seven terms of an A.P. is 182. If its 4^{th} and the 17^{th} terms are in the ratio 1:5, find the A.P.

Answer

$S_7 = \frac{7}{2}[2a + (7 - 1)\, d]$

$182 = \frac{7}{2}[2a + 6d]$

$52 = 2a + 6d$

$a + 3d = 26$ (i)

Acc. To question,

$a_{17} = 5a_4$

$a + 16d = 5(a + 3d)$

$d = 4a$

$d = 4(26 - 3d)$ [from (i)]

$13d = 104$

$d = 8$

Putting the value of d in (i), we get

$a = 26 - 3*8) = 2$

$a_1 = a = 2$

$a_2 = a + d = 2 + 8 = 10$

$a_3 = a + 2d = 2 + 2(8) = 18$

Hence, The A.P. is 2, 10, 18,…

55. Question

The sum of the first n terms of an A.P. is $3n^2 + 6n$. Find the nth term of this A.P.

Answer

Let 'a' be the first term and 'd' be the common difference

$S_n = 3n^2 + 6n$

Firsdt term, a, $S_1 = 3(1)^2 + 6(1)$

= 3 + 6 = 9

$S_2 = 3(2)^2 + 6(2)$

a + a + d = 12 + 12

9 + 9 + d = 24

18 + d = 24

d = 6

Therefore, $a_n = a + (n - 1)\ d$

= 9 + (n – 1) 6

= 6n + 3

56. Question

The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P.

Answer

$S_7 = 63$

$\frac{7}{2}$ [2(a) + 6d] = 639

$a + 3d = 9$ (i)

$a8 = a + 7d$

Hence, for next 7 terms first term will be the 8th term i.e. a + 7d

Sum of next 7 terms, $S'_7 = \frac{7}{2}$ [2(a + 7d) + 6d]

161 = 7 [a + 7d + 3d]

23 = a + 10d

23 = 9 – 3d + 10d [From (i)]

14 = 7d

d = 2

Putting the value of d in (i), we get

A = 9 – 3(2) = 3

Now, a_{28}= a + 27d

= 3 + 27(2)

= 3 + 54 = 57

57. Question

If demotes the sum of the first n terms of an A.P., prove that $S_{30}= 3 (S_{20} – S_{10})$.

Answer

Proof: $S_{20} = \frac{20}{2} [2a + 19d]$

$= 10 [2a + 19d]$ (i)

$S_{10} = \frac{10}{2} [2a + 9d]$

$= 5 [2a + 9d]$ (ii)

L.H.S: $S_{30} = \frac{30}{2} [2a + 29d]$

$= 15 [2a + 29d]$

R.H.S: $3 (S_{20} – S_{10})$

$= 3 [10 (2a + 19d) – 5 (2a + 9d)]$ {From (i) and (ii)}

$= 15 [4a + 38d – 2a – 9d]$

$= 15 [2a + 29d]$

Since, L.H.S = R.H.S

Hence, proved

58. Question

The sum of first n terms of an A.P. is $5n – n^2$. Find the nth term of this A.P.

Answer

$S_n = 5n – n^2$

First term, $a = 5(1) – (1)^2$

$= 5 – 1 = 4$

$S_2 = 5(2) – (2)^2$

$a + (a + d) = 10 – 4$

$4 + 4 + d = 6$

$d = -2$

Now, the n^{th} term, $a_n = a + (n - 1) d$

$= 4 + (n – 1) (-2)$

$= 4 + 2 – 2n = 6 – 2n$

59. Question

The sum of the first n terms of an A.P. is $4n^2 + 2n$. Find the nth term of this A.P.

Answer

$S_n = 4n^2 + 2n$

First term, $a = 4(1)^2 + 2(1)$

$= 4 + 2 = 6$

Sum of first two terms, $S_2 = 4(2)^2 + 2(2)$

$a + a + d = 16 + 4$

$6 + 6 + d = 20$

$d = 8$

Now the n^{th} term, $a_n = a + (n - 1)\, d$

$= 6 + (n - 1)\, 8$

$= 6 + 8n - 8$

$= 8n - 2$

60. Question

If the 10^{th} term of an A.P. is 21 and the sum of its first ten terms is 120, find its nth term.

Answer

$a_{10} = a + 9d$

$a + 9d = 21$ (i)

$S_{10} = \frac{10}{2}[2a + 9d]$

$120 = 5\,[2a + 9d]$

$24 = 2a + 9d$

$24 = 2\,(21 - 9d) + 9d$ [From (i)]

$-18 = -9d$

$d = 2$

Putting the value of d in (i), we get

$a = 21 - 9(2)$

$= 3$

The n^{th} term, $a_n = a + (n - 1)\, d$

$= 3 + (n - 1)\, 2$

$= 3n + 2n - 2$

= 2n + 1

61. Question

Ram kali would need Rs.1800 for admission fee and books etc., for her daughter to start going to school from next year. She saved Rs.50 in the first month of this year and increased her monthly saving by Rs.20. After a year, how much money will she save? Will she be able to fulfill her dream of sending her daughter to school?

[CBSE2005,2014]

Answer

a = 50, d = 20

a_n = a + (n – 1) d = l

a_{12} = 50 + 11 * 20 = 270

$S_n = \frac{n}{2}$ [a + l]

$= \frac{12}{2}$ [50 + 270]

= 12 * 160 = 1920

Yes, she will be able to fulfill her dream of sending her daughter to school.

62. Question

The first and the last terms of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Answer

First term, a = 5

Last term, l = 45

S_n = 400

$\frac{n}{2}$ [a + l] = 400

$n\,[5 + 45] = 800$

n = 16

Now, n = $\frac{l-a}{d}$ + 1

16 = $\frac{45-5}{d}$ + 1

16d = 40 + d

15d = 40

d = $\frac{8}{3}$

Hence, the common difference is $\frac{8}{3}$

63. Question

The first and the last terms of an AP are 7 and 49 respectively. If sum of all its terms is 420, find its common difference.

Answer

First term, a = 7

Last term, l = 49

$S_n = \frac{n}{2}$ [a + l]

420 = $\frac{n}{2}$ [7 + 49]

840 = 56n

n = 15

Now, n = $\frac{l-a}{d}$ + 1

15 = $\frac{49-7}{d}$ + 1

15d = 42 + d

14d = 42

d = 3

Hence, it's common difference is 3

64. Question

If demotes the sum of first n terms of an A.P., prove that S_{12} = 3 (s_8 – S_4) .

Answer

$S_8 = \frac{8}{2}$ [2a + 7d]

= 4 (2a + 7d) (i)

$S_4 = \frac{4}{2}$ [2a + 7d]

= 2 (2a + 7d) (ii)

L.H.S = $S_{12} = \frac{12}{2}$ [2a + 11d]

= 6 [2a + 11d]

R.H.S = 3 (S_8 – S_4)

= 3 [4 (2a + 7d) – 2 (2a + 3d)] [From (i) and (ii)]

= 6 [4a + 14d – 2a – 3d]

= 6 [2a + 11d]

Since, L.H.S = R.H.S

Hence, proved

65. Question

If the sum of first n terms of an A.P. is $\frac{1}{2}(3n^2 + 7n)$, then find its nth term. Hence write its 20^{th} term.

Answer

$S_n = \frac{1}{2}(3n^2 + 7n)$

First term, $a = S_1 = \frac{1}{2}[3(1)^2 + 7(1)]$

$= \frac{1}{2}[3 + 7] = 5$

$S_2 = \frac{1}{2}[3(2)^2 + 7(2)]$

$a + a + d = \frac{1}{2}[12 + 14]$

$5 + 5 + d = 13$

$d = 3$

$a_n = a + (n - 1)d$

$= 5 + (n - 1)\ 3$

$= 5 + 3n - 3$

$= 3n + 2$

$a_{20} = 3(20) + 2$

$= 60 + 2 = 62$

Hence, it's 20^{th} term is 62

66. Question

The sum of first 9 terms of an A.P. is 162. The ratio of its 6^{th} term to its 13^{th} term is 1:2. Find the first and 15^{th} term of the A.P.

Answer

Let a be the first term and d be the common difference

Now, $a_6 / a_{13} = \frac{1}{2}$

$\frac{a+5d}{a+12d} = \frac{1}{2}$

$2a + 10d = a + 12d$

$a = 2d$ (i)

Now sum of first n terms of A.P

$S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$S_9 = \frac{9}{2}[2a + 8d]$

$162 = 9\,(a + 4d)$

$a + 4d = 18$

$2d + 4d = 18$ [Using (i)]

$6d = 18$

$d = 3$

Now from (i), we get

$a = 2 * 3 = 6$

So, first term, $a_1 = a = 6$

15^{th} term, $a_{15} = a + 14d = 6 + 14(3)$

$= 6 + 42 = 48$

Therefore, First term is 6 and 15^{th} term is 48

CCE - Formative Assessment

1. Question

Define an arithmetic progression.

Answer

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called the common difference of the AP.

2. Question

Write the common difference of an A.P. whose nth term is $a_n = 3n + 7$.

Answer

If $a_n = 3n + 7$

Then $a_1 = 3\,(1) + 7 = 10$

$a_2 = 3(2) + 7 = 13$

$a_3 = 3(3) + 7 = 16$

d= common difference = $a_n - a_{n-1}$

$= a_3 - a_2$

$= 16 - 13 = 3$

3. Question

Which tem of the sequence 114, 109, 104, ... is the first negative term?

Answer

Here a = 114, d is common difference

$d = a_3 - a_2 = a_2 - a_1 = -5$

For finding first negative term

$T_n < 0$

$a + (n - 1) d < 0$

$114 + (n - 1) (- 5) < 0$

$114 - 5n + 5 < 0$

$119 - 5n < 0$

$- 5n < - 119$

$5n > 119$

$n > 119/5$

$n > 23.8$

Therefore first negative term is 24^{th} term

4. Question

Write the value of $a_{30} - a_{10}$ for the A.P. 4, 9, 14, 19, …….

Answer

Here $a_1 = 4$

$a_2 = 9$

$a_3 = 14$

d = common difference $= a_3 - a_2 = a_2 - a_1 = 14 - 9 = 9 - 4 = 5$

$a_{30} = a + (n-1) d$

$a_{30} = 4 + (30 -1) 5$

$a_{30}= 4 + 29 X 5$

$a_{30}= 149$

Similarly, $a_{10} = 4 + 9x5$

$a_{10} = 49$

$a_{30} - a_{10} = 149 - 49 = 100$

5. Question

Write 5th term from the end of the A.P. 3, 5, 7, 9, ..., 201.

Answer

Here $a = 201$, $a_2 = 5$, $a_3 = 7$

$d = a_3 - a_2 = a_2 - a_1$

$= 7 - 5 = 5 - 3 = 2$

$t_n = a + (n - 1)d$

$t_n = 3 + (n-1)2 = 201$

$(n - 1)\ 2 = 201 - 3 = 198$

$n - 1 = \frac{198}{2} = 99$

$n = 99 + 1 = 100$

5^{th} term from end = 96th term

$T_{95} = 3 + (96 - 1)2$

$T_{95} = 3 + 95 \times 2$

$T_{95} = 3 + 190 = 193$

6. Question

Write the value of x for which 2x, x + 10 and 3x + 2 are in A.P.

Answer

Terms are in A.P. if common difference (d) is same between two continuous numbers.

$x + 10 - 2x = 3x + 2 - (x + 10)$

$10 - x = 2x - 8$

$18 = 3x$

$x = 6$

So the terms in A.P. are: 2(6), 6 + 10, 3(6) + 2

= 12, 16, 20

7. Question

Write the nth term of an A.P. the sum of whose n terms is S_n.

Answer

First term = a

Sum up to first term = a

Last term (nth term) = a_n

Sum up to n terms = S_n

Second last term = a_{n-1}

Sum up to $(n-1)^{th}$ term = s_{n-1}

Therefore, $a_n = S_n - S_{n-1}$

8. Question

Write the sum of first n odd natural numbers.

Answer

First n odd numbers are 1, 3, 5, 7, 9....., (2n–1) which forms an A.P

a= 1

l= 2n – 1 where l be the last term

$S_n = \frac{n}{2}(2a + (n-1)\,d)$ or $n/2(a + l)$

$= n/2\ (1 + 2n - 1)$

$= \frac{n}{2}(2n)$

$=n\ (n)$

$=n^2$

9. Question

Write the sum of first n even natural numbers.

Answer

First n even Natural numbers are 2, 4, 6,……, 2n

This forms an A.P where

a= 2

l= 2n where l is last term of the A.P

$S_n = n/2\ (2a + (n-1)\,d)$ or $n/2\ (a + l)$

$= n/2\ (2 + 2n)$

$=n/2$ [2(1 + n)]

=n (n + 1)

10. Question

1f the sum of n terms of an A.P. is $S_n = 3n^2 + 5n$. Write its common difference.

Answer

Let a1, a2, a3…….. an be the given A.P

Given, sum of n terms = $3n^2 + 5n$

$S_n=3n^2 + 5n$…….(1)

Putting n =1 in (1)

$S_n= 3x1^2 + 5x1$

= 3 + 5 = 8

Sum of first 1 terms = first term

∴ first term = a = S_1 = 8

$S_n = 3n^2 + 5n$

Putting n= 2 in….. (1)

$S_2= 3x\ 2^2 + 5x2$

$S_2= 22$

Sum of first two terms = first term + second term

$S_2= a_1 + a_2$

$S_2 – a_1 = a_2$

$a_2= 22 – 8 = 14$

Thus $a_1= 8$, $a_2 = 14$

d= common difference = 14 – 8= 6

11. Question

Write the expression for the common difference of an A.P. whose first term is a and nth term is b.

Answer

The n^{th} term of the A.P whose first term a_1 and common difference is d is given by

$a_n = a + (n–1)\ d$

Here a_n is given as b

so b = a + (n−1) d

$\frac{b-a}{n-1}$ = d

⇒ d= $\frac{b-a}{n-1}$

12. Question

The first term of an A.P. is p and its common difference is q. Find its 10th term.

Answer

Given: The first term of an A.P. is p and its common difference is q.

To find: its 10th term.

Solution: The first term of A.P is p.Common difference = q

We know a_n = a + (n−1)dWhere a is first term and d is common difference.

So,a_{10} = p + (10-1)q

a_{10} = p + 9q

13. Question

For what values of p are 2p + 1, 13, 5p – 3 are three consecutive terms of an A.P.?

Answer

Given: 2p + 1, 13, 5p – 3 are three consecutive terms of an A.P.

To find: The value of p

Solution: Consider a_1 = 2p + 1 , a_2 = 13 and a_3 = 5p – 3The A.P will be of the form 2p + 1, 13, 5p – 3,.....Since the terms are in A.P so the common differences in them is same.⇒ a_2 - a_1 = a_3 - a_2

⇒ 13– (2p + 1) = 5p –3 –13⇒ 13– 2p - 1 = 5p –3 –13

⇒12– 2p = 5p –16

⇒12+16 = 5p +2p

⇒ 28 = 7p⇒ p= 4

14. Question

If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a.

Answer

$\frac{4}{5}$, a, 2 are three terms in an A.P so there common difference will be same

a–$4/5$ = 2–a

2a= 2 + $\frac{4}{5}$

2a= $\frac{14}{5}$

a = $7/5$

15. Question

If the sum of first p term of an A.P. is $ap^2 + bp$, find its common difference.

Answer

Here, $s_p = ap^2 + bp$

S_1= a + b

S_2= $ax2^2 + bx2$

=4a + 2b

$S_2 = a_1 + a_2$

$4a + 2b = a_1 + a_2$

$A_2 = 4a + 2b – (a + b)$

$A_2 = 3a + b$

Now, d = common difference = $a_2 – a_1$

= 3a + b – (a + b)

= 2a

1. Question

If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is

A. 87

B. 88

C. 89

D. 90

Answer

Here, $a_7 = 34$

$a_{13} = 64$

$a_7 = a + 6d = 34$…….(1)

a_{13} = a + 12d = 64……(2)

Subtracting (1) from (2)

6d = 30

D = 5

Multiplying (1) by 2

2a + 12d = 68…….(3)

Subtracting (2) from (3)

a = 4

a_{18} = a + (n–1) d

a_{18} = 4 + (17) 5

a_{18} = 89 = C

2. Question

If the sum of P terms of an A.P. is q and the sum of q terms is p, then the sum of p + q terms will be

A. 0

B. p – q

C. p + q

D. – (p + q)

Answer

Let a be the first term and d is common difference of the A.P

then sum of n terms in A.P is $S_n = (\frac{n}{2})[2a + (n - 1) d]$

Here, s_p= q

S_q =p

$S_p = (\frac{p}{2}) [2a + (p - 1) d]$

$q = \frac{p}{2} [2a + (p - 1) d]$

$\frac{2q}{p}$ = [2a + (p – 1) d] --------(1)

$S_q = p = (\frac{q}{2}) [2a + (q - 1) d]$

$\frac{2p}{q}$ = [2a + (q – 1) d] ……..(2)

Subtract (1) from (2) we get

$$(q - p)d = \left(\frac{2p}{q}\right) - \left(\frac{2q}{p}\right)$$

(q – p) d = $(2p^2 - 2q^2)$ / pq ----------(3)

d = -2(q + p) / pq ----------(3)

Sum of first (p + q) terms

$$Sp + q = \frac{p + q}{2}[2a + (p + q - 1)\,d]$$

$$Sp + q = \frac{p + q}{2}[2a + (p - 1)\,d + qd]$$

$$Sp + q = \frac{p+q}{2}\left[\left(\frac{2q}{p}\right) + q\left(-\frac{2(q+p)}{pq}\right)\right]$$

[from (1) and (3)]

$$Sp + q = \frac{p + q}{2}\left[\frac{2q - 2q - 2p}{p}\right)$$

$$Sp + q = \frac{p + q}{2}\frac{(-2p)}{p}$$

$$Sp + q = -(p + q)$$

3. Question

If the sum of n terms of an A.P. be $3n^2$ + n and its common difference is 6, then its first term is

A. 2

B. 3

C. 1

D. 4

Answer

Here, S_n= $3n^2$ + n

d = 6

Putting n= 1

S_1 = 3 + 1 = 4

Sum of first 1 term = first term = 4

4. Question

4. Triangles

Exercise 4.1

1. Question

Fill in the blanks using the correct word given in brackets :

(i) All circles are……..(congruent, similar).

(ii) All squares are……(similar, congruent).

(iii) All……triangles are similar (isosceles, equilaterals).

(iv) Two triangles are similar, if heir corresponding angles are……(proportional, equal)

(v) Two triangles are similar, if their corresponding sides are……(proportional, equal)

(vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles ae and (b) heir corresponding sides are……(equal, proportional)

Answer

(i) similar (ii) similar

(iii) equilateral (iv) equal

(v) proportional (vi) equal, proportional

2. Question

Write the truth value (T/F) of each of the following statements:

(i) Any two similar figures are congruent.

(ii) Any two congruent figures are similar.

(iii) Two polygons are similar, if their corresponding sides are proportional.

(iv) Two polygons are similar if their corresponding angles are proportional.

(v) Two triangles are similar if their corresponding sides are proportional.

(vi) Two triangles are similar if their corresponding angles are proportional.

Answer

(i) False (ii) True

(iii) False (iv) False

(v) True (vi) True

Exercise 4.2

1. Question

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$

(i) If AD = 6 cm, DB = 9 cm and AE = 8 cm, find AC.

(ii) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, find AE.

(iii) If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, find AE.

(iv) If AD = 4, AE = 8, DB = x – 4, and EC = 3x – 19, find x.

(v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.

(vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.

(vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.

(viii) If $\frac{AD}{BD} = \frac{4}{5}$ and EC = 2.5 cm, find AE.

(ix) If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.

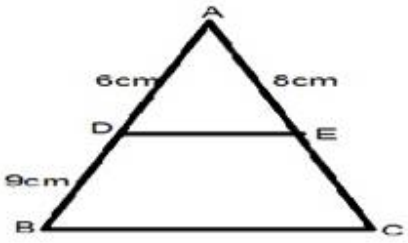
(x) If AD = 8x - 7, DB = 5x – 3, AE = 4x - 3 and EC = (3x – 1), find the value of x.

(xi) If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x - 3, find the volume x.

(xii) If AD = 2.5 cm, BD = 3.0 cm and AE = 3.75 cm, find the length of AC.

Answer

(i)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

6/9=8/EC

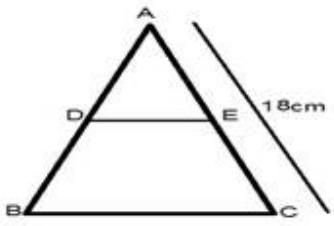
2/3=8/EC

EC=3x8/2

EC=3x4

EC=12 cm

(ii)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

Adding 1 both side

AD/DB +1=AE/EC +1

3/4 +1=AE+BC/BC

3+4/4=AC/EC [AE+EC=AC]

7/4= 15/EC

EC=15x4/7

EC=60/7

Now AE+EC=AC

AE+60/7=15

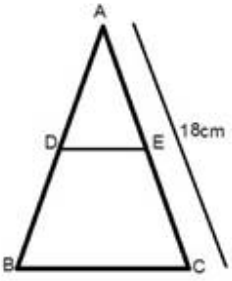
AE=15-60/7

AE=105-60/7

AE=45/7

AE=6.43 cm

(iii)



we have

DE‖BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

Adding 1 both side

AD/DB +1=AE/EC +1

$\frac{3}{2}$ +1=$\frac{EC}{AE}$ +1

$\frac{3+2}{2} = \frac{EC+AE}{AE}$

$\frac{5}{2}$ = AC/AE [AE+EC=AC]

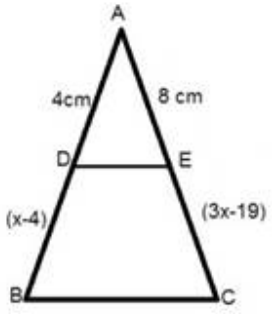
5/2=18/AE

AE= $\frac{18x2}{5}$

AE=36/5

AE=7.2 cm

(iv)



we have

DE‖BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{4}{x-4} = \frac{8}{3x-19}$

4(3x-19)=8(x-4)

12x-76=8x-32

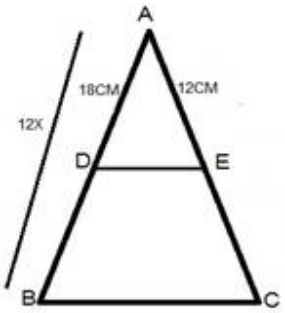
12x-8x=76-32

4x=44

x=44/4

x=11 cm

(v)



AD=8cm,AB=12cm

since BD=AB-AC

BD=12-8

BD=4 cm

DE||BC

Therefore by basic proportionally theorem

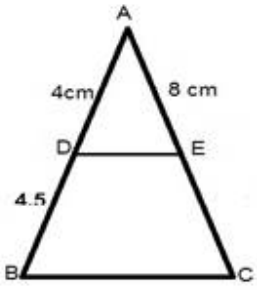
AD/DB=AE/EC

8/4=12/EC

EC=$\frac{12x4}{8}$

EC =6 cm

(vi)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

4/4.5=8/EC

$EC=\frac{8x4.5}{4}$

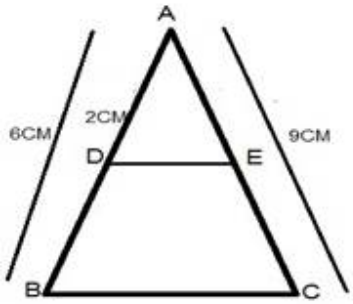
EC=9cm

Now AE+EC=AC

AC=8+9

AC=17 cm

(vii)



AD=2cm, AB=6cm

Since BD=AB-AC

BD=6-2

BD=4 cm

DE‖BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

Taking reciprocal on both side

DB/AD=EC/AE

4/2=EC/AE

Adding 1 both side

AD/DB +1=AE/EC +1

$\frac{4}{2}+1=\frac{EC}{AE}+1$

$\frac{4+2}{2}=\frac{EC+AE}{AE}$

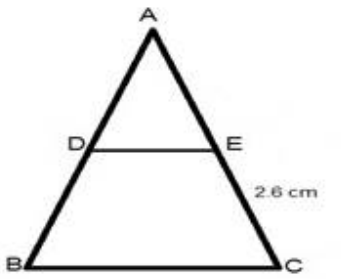
$\frac{6}{2}$ = AC/AE [AE+EC=AC]

3=9/AE

AE= $\frac{9}{3}$

AE=3 cm

(viii) we have



DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

4/5=AE/2.5

AE=4x2.5/5

AE=10/5

AE=2 cm

(ix) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{x}{x-2} = \frac{x+2}{x-1}$

x(x-1)=(x+2)(x-2)

$x^2-x=x^2-2^2$

-x=-4

x=4 cm

(x) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

(8x-7)(3x-1)=(4x-3)(5x-3)

$8x(3x-1)-7(3x-1)=4x(5x-3)-3(5x-3)$

$24x^2-8x-21x+7=20x^2-12x-15x+9$

$24x^2-20x^2-29x+27x+7-9=0$

$4x^2-2x-2=0$

$2[2x^2-x-1]=0$

$2x^2-x-1=0$

$2x^2-2x-x-1=0$

$2x(x-1)+1(x-1)=0$

$(x-1)(2x+1)=0$

$x-1=0$

$x=1$

or $2x+1=0$

or $x=-1/2$

-1/2 is not possible.

So $x=1$

(xi) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{4x-3}{3x-1}=\frac{8x-7}{5x-3}$

$(8x-7)(3x-1)=(4x-3)(5x-3)$

$24x^2-8x-21x+7=20x^2-12x-15x+9$

$24x^2-20x^2-29x+27x+7-9=0$

$4x^2-2x-2=0$

$2[2x^2-x-1]=0$

$2x^2-x-1=0$

$2x^2-2x-x-1=0$

$2x(x-1)+1(x-1)=0$

$(x-1)(2x+1)=0$

x-1=0

x=1

or 2x+1=0

or x=-1/2

-1/2 is not possible.

So x=1

(xii) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

2.5/3=3.75/EC

EC=3.75x3/2.5

EC=375x3/250

EC=15x3/10

EC=9/2

EC=4.5 cm

Now AC=AE+EC

AC=3.75+4.5

AC=8.25 cm

2. Question

In a ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$:

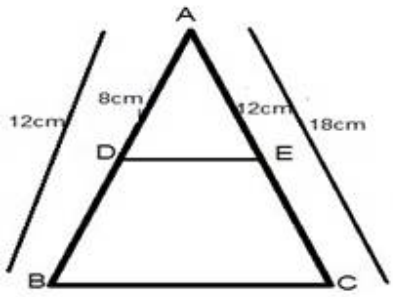
(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

(ii) AB = 5.6 cm, AD = 1.4 cm, AE = 7.2 cm and AC = 1.8 cm.

(iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm.

(iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm.

Answer



(i) AB = 12 cm, AD = 8 cm, and AC = 18 cm.

∴ DB=AB-AD

= 12-8

=4 cm

EC=AC-AE

= 18-12

= 6 cm

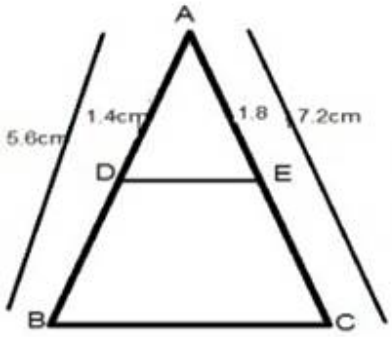
Now AD/DB=8/4=2

AE/EC=12/6=2

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

(ii)



AB = 5.6 cm, AD = 1.4 cm, AE = 1.8 cm and AC = 7.2 cm

∴ DB=AB-AD

DB=5.6-1.4

DB= 4.2 cm

And EC=AC-AE

EC= 7.2-1.8

EC=5.4

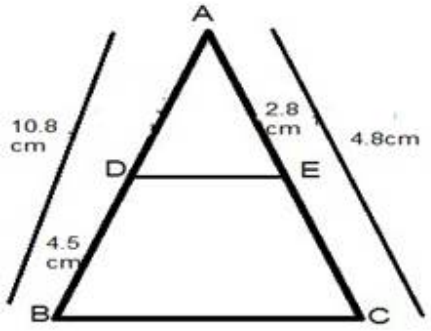
Now AD/DB=1.4/4.2=1/3

AE/EC=1.8/5.4=1/3

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

(iii)



we have

AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm

∴ AD=AB-DB

AD=10.8-4.5

AD= 6.3 cm

And EC=AC-AE

EC= 4.8-2.8

EC=2 cm

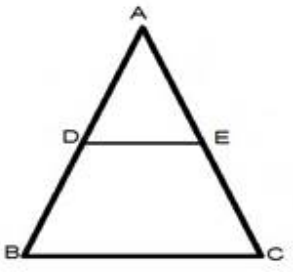
Now AD/DB=6.3/4.5=7/5

AE/EC=2.8/2=28/20=7/5

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

(iv)



DE∥BC

We have,

AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm

Now AD/DB=5.7/9.5=57/95 =3/5

AE/EC=3.3/5.5=33/55=3/5

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

3. Question

In a ΔABC, P and Q are points on sides AB and AC respectively, such that $PQ \parallel BC$. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm, find AB and PQ.

Answer

WE have,

PQ∥BC

We have AP/PB=AQ/QC

2.4/PB=2/3

PB=3x2.4/2

PB=3x1.2

PB=3.6 cm

Now AB=AP+PB

AB=2.4+3.6

AB=6 cm

Now IN ⊿ APQ and ⊿ ABC

∠A=∠A [Common]

∠APQ=∠ABC [PQ∥BC]

⊿ APQ ~⊿ ABC [By AA criteria]

AB/AP=BC/PQ

PQ=6x2.4/6

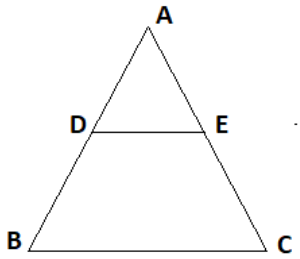
PQ=2.4 cm

4. Question

In a ΔABC, D and E are points on AB and AC respectively such that DE||BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

Answer

In the figure given below,



Given: AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm

Let BD be x cm and CE be y cm,

Then, from, ΔADE and ΔABC, DE || BC, so by basic proportionality theorem we can write,

$$\frac{AD}{AB} = \frac{DE}{BC}$$

Or $\frac{AD}{AD + BD} = \frac{DE}{BC}$

or $\frac{2.4}{2.4 + x} = \frac{2}{5}$

or 12 = 4.8 + 2x

or x = 7.2/2

or x = DB = 3.6cm

Similarly, from ΔADE and ΔABC, we can write,

$$\frac{AE}{AC} = \frac{DE}{BC}$$

Or $\frac{AE}{AE + EC} = \frac{DE}{BC}$

or $\frac{3.2}{3.2 + y} = \frac{2}{5}$

or 16 = 6.4 +2y

or y = 9.6/2

or y = CE = 4.8 cm

Thus, the lengths of BD and CE are 3.6 cm and 4.8 cm respectively.

5. Question

In Fig. 4.35, state if $PQ \parallel EF$.

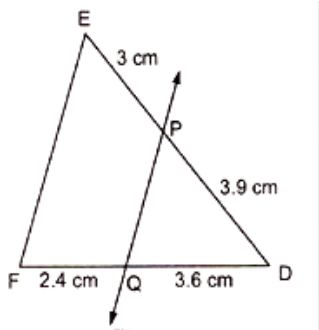


Fig. 4.35

Answer

DP/PE=3.9/3=1.3/1=13/10

DQ/QF=3.6/2.4=36/24=3/2

DP/PE≠DQ/QF

So PQ is not parallel to EF

6. Question

M and N are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $MN \parallel QR$:

(i) PM = 4 cm, QM = 4.5 cm, PN = 4 cm, NR = 4.5 cm

(ii) PQ = 1.28 cm, PR = 2.56 cm, PM = 0.16 cm, PN = 0.32 cm

Answer

(i) we have PM=4cm, QM=4.5 cm,PN=4 cm and NR=4.5 cm

Hence PM/QM=4/4.5=40/45=8/9

PN/NR=4/4.5=40/45=8/9

PM/QM= PN/NR

by the converse of proportionality theorem

MN∥QR

(ii) we have PQ=1.28cm, PR=2.56 cm,PM=0.16 cm and PN=0.32 cm

Hence PQ/PR=1.28/2.56=128/256=1/2

PM/PN=0.16/0.32=16/32=1/2

PQ/PR = PM/PN

by the converse of proportionality theorem

MN∥QR

7. Question

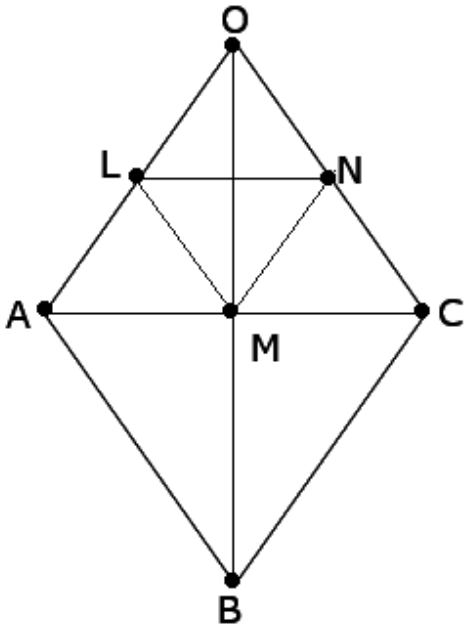
In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear. Show that $LN \parallel AC$.

Answer

Given: In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear.

To show : $LN \parallel AC$

Solution:



We have LM∥AB and MN∥BC

by the basic proportionality theorem

OL/AL=OM/MB …….(i)

ON/NC=OM/MB ……(ii)

Comparing equ.(i)and(ii)

OL/AL=ON/NC

Thus LN divides side OA and OC of ⊿ OAC in same ratio

Then by the converse of basic proportionality theorem

$LN \parallel AC$

8. Question

If D and E are points on sides AB and AC respectively of a ΔABC such that $DE \parallel BC$ and BD = CE. Prove that ΔABC is isosceles.

Answer

We have DE∥BC

by the converse of proportionality theorem

AD/DB=AE/EC

AD/DB=AE/DB [BD=CE]

AD=AE

Adding D both sides

AD+BD=AE+DB

AD+BD=AE+EC [BD=CE]

AB=AC

⊿ABC is isosceles

Exercise 4.3

1. Question

In a ΔABC, AD is the bisector of $\angle A$, meeting side BC at D.

(i) If BD = 2.5 cm, AB = 5 cm and AV = 4.2 cm, find DC.

(ii) If BD = 2 cm, AB = 5 cm and DC = 3 cm, find AC.

(iii) If AB = 3.5 cm, AC = 4.2 cm and DC = 2.8 cm, find BD.

(iv) If AB = 10 cm, AC = 14 cm and BC = 6 cm, find BD and DC.

(v) If AC = 4.2 cm, DC = 6 cm and BC = 10 cm, find AB.

(vi) If AB = 5.6 cm, AC = 6 cm and DC = 6 cm, find BC.

(vii) If AD = 5.6 cm, BC = 6 cm and BD = 3.2 cm, find AC.

(viii) If AB = 10 cm, AC = 6 cm and BC = 12 cm, find BD and DC.

Answer

(i) we have

Angle BAD=CAD

Here AD bisects ∠A

BD/DC=AB/AC

2.5/DC=5/4.2

DC=2.5*4.2/5

DC=2.1 cm

(ii) Here AD bisects ∠A

AB/DC=AB/AC

2/3=5/AC

AC=15/2

AC=7.5 cm

(iii) in △ ABC A bisects ∠A

BD/DC=AB/BC

BD/2.8=3.5/4.2

BD=3.5*2.8/4.2

BD=7/3

BD=2.33 cm

(iv) In△ABC, AD bisects ∠A

BD/DC=AB/AC

X/6-x =10/14

14x=60-10x

14x+10x=60

24x=60

x= 60/24

x=5/2

x=2.5

BD=2.5

DC= 6-2.5

DC=3.5

(v) AB/AC=BD/DC

AB/4.2=BC-DC/DC

AB/4.2=10-6/6

AB/4.2=4/6

AB=4*4.2/6

AB=2.8 cm

(vi) BD/DC=AB/AC

BD/6=5.6/6

BD=5.6

BC= BD+DC

BC=5.6+6

BC=11.6 cm

(viii) In△ABC, AD bisects ∠A

AB/AC=BD/DC

5.6/AC=3.2/BC-BD

5.6/AC=3.2/6-3.2

5.6/AC=3.2/2.8

AC*3.2=2.8*5.6

AC=2.8*5.6/3.2

AC=7*0.7

AC=4.9 cm

(ix) let BD=x,then DC=12-X

BD/DC=AB/BC

x/12-x= 10/6

6x=120-10x

6x+10x=120

16x=120

x=120/16

x= 7.5

BD=7.5 cm

DC =12-x

DC=12-7.5

DC=4.5 cm

2. Question

In Fig. 4.57, AE is the bisector of the exterior $\angle CAD$ meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, find CE.

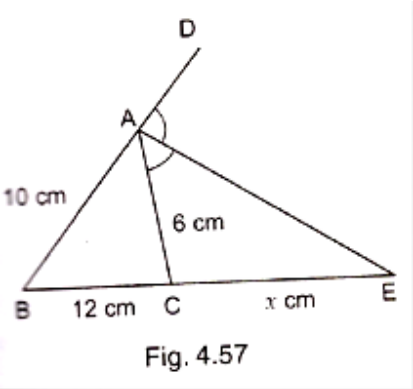


Fig. 4.57

Answer

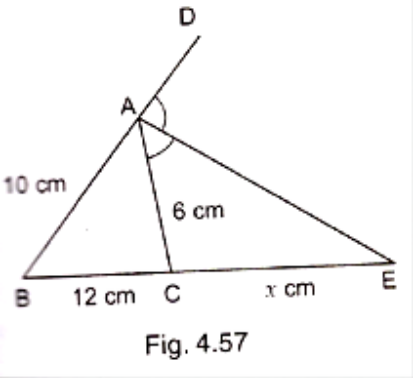


Fig. 4.57

AE is the bisector of ∠A

We know that external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angles.

$$\frac{BE}{CE} = \frac{AB}{AC}$$

$$\Rightarrow \frac{12+x}{x} = \frac{10}{6}$$

⇒ 10X=6(12+x)

⇒ 10X=72+6X

⇒ 10X-6X=72

⇒ 4X =72

⇒ x=72/4

⇒ x=18

3. Question

In Fig. 4.58, $\triangle ABC$ is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70^0$, $\angle C = 50^0$. Find $\angle BAD$.

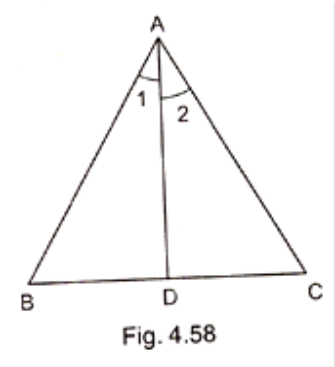


Fig. 4.58

Answer

We have

AB/AC=BD/DC

∴ ∠1=∠2

IN ⊿ABC

∠A+∠B+∠C=180

∠A+70+50=180

∠A+120=180

∠A=180-120

∠A=60

∠1+∠2=60 (∠1+∠2=∠A)

∠1+∠1=60 (∠1=∠2)

2∠1=60

∠1=60/2

∠1=30

∠BAD=30

4. Question

In $\triangle ABC$ (fig. 4.59), if $\angle 1 = \angle 2$, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

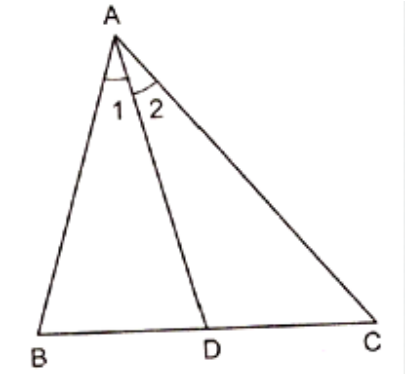


Fig. 4.59

Answer

∠1=∠2 (Given)

Draw a line EC∥AD

AC bisects them

∴∠2=∠3 (by alternate angle) ……. (i)

∠1=∠4 (corresponding angle) ……..(ii)

∠1=∠2 (given)

From equ (i) and equ (ii)

∠3=∠4

or AE=AC ……..(III)

Now ,⊿ BCE

BD/DC=BA/AE (BY PROPORTIONALITY THEORAM)

BD/DC=AB/AC (∵ BA=AB AND AE=AC from equ (iii))

Hence AB/AC=BA/DC Proved

5. Question

D, E and F are the points on sides BC, CA and AB respectively of ΔABC such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. If AB = 5 cm, BC = 8 cm and CA = 4 cm, determine AF, CE and BD.

Answer

in⊿ ABC

CF bisects ∠A

∴ AF/FB=AE/AC

AF/5-AF=4/8

2AF=5-AF

2AF+AF=5

3AF=5

AF=5/3 cm

⊿ABC, BE bisects ∠B

∴ AE/AC=AB/BC

4-CE/CE=5/8

5CE=32-8CE

5CE+8CE=32

13CE=32

CE=32/13 cm

Similarly

BD/DC=AB/AC

BD/8-BD=5/4

4BD=40-5BD

4BD+5BD=40

9BD=40

BD=40/9 cm

6. Question

In Fig. 4.60, check whether AD is the bisector of $\angle A$ of ΔABC in each of the following:

(i) AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm

(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

(iii) AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24 cm

(iv) AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 2 cm

(v) AB = 5 cm, AC = 12 cm, BD = 2.5 cm and BC = 9 cm

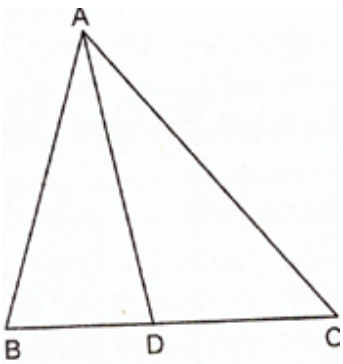


Fig. 4.60

Answer

(i) BD/DC=AB/AC

1.5/3.5=5/10

15/35*10/10=1/2

3/7=1/2

Not bisects

(ii) 1.6/2.4=4/6

16/24=2/3

2/3=2/3

bisects

(iii) BD/CD=AB/AC

BD/BC-BD=AB/AC

BD/24-6=8/24

6/18=1/3

1/3=1/3

bisects

(iv) 1.5/2= 6/8

3/4=3/4

bisects

(v) BD/CD=AB/AC

BD/BC-BD=AB/AC

BD/9-2.5=5/12

2.5/6.5=5/12

5/13=5/12

Not bisects

7. Question

In Fig. 4.60, AD bisects $\angle A$, AB = 12 cm, AC = 20 cm and BD = 5 cm, determine CD.

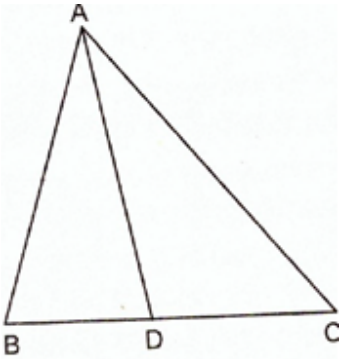


Fig. 4.60

Answer

AD bisects ∠A

∴ AB/AC=BD/CD

12/20=5/CD

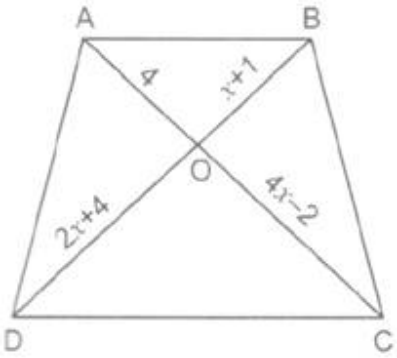
CD =100/12

CD=8.33 cm

Exercise 4.4

1 A. Question

(i) In fig. 4.70, if $AB \parallel CD$, find the value of x.



Answer

Diagonal of trapezium divide each other proportiona

AO/OC=BO/OD

4/4X-2=x+1/2x+4

$4x^2$-2x+4x-2=8x+16

$4x^2$+2x-2-8x-16=0

$4x^2$-6x-18=0

$2(2x^2-3x-9)=0$

$2x^2-3x-9=0$

$2x^2-6x+3x-9=0$

$2x(x-3)+3(x-3)=0$

$(x-3)(2x+3)=0$

$x-3=0$

$x=3$

or,$2x+3=0$

$2x=-3$

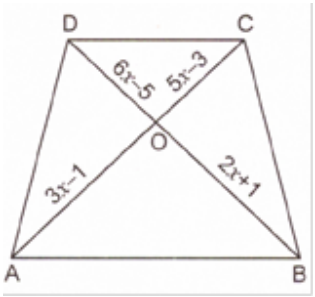
$x= -3/2$

$x=-3/2$ is not possible

So $x=3$

1 B. Question

In Fig. 4.71, if $AB \parallel CD$, find the value of x.



Answer

AO/OC=BO/OD

3x-1/5x-3=2x+1/6x-5

(3x-1)(6x-5)= (2x+1) (5x-3)

$18x^2-15x-6x+5=10x^2-6x+5x-3$

$18x^2-21x+5=10x^2-x-3$

$18x^2-21x+5-10x^2+x+3=0$

$8x^2-20x+8=0$

$4(2x^2-5x+2)=0$

$2x^2-5x+2=0$

$2x^2-4x-x+2=0$

$2x(x-2)-1(x-2)=0$

(x-2)(2x-1)=0

x-2=0

x=2

Or, 2x-1=0

2x=1

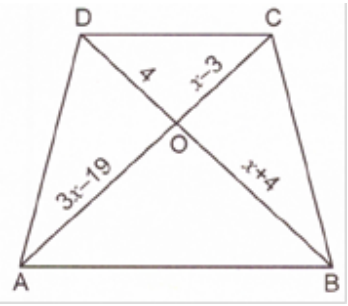
x=1/2

But x=1/2 is not possible

So x=2

1 C. Question

In Fig. 4.72, $AB \parallel CD$. If OA = 3x – 19, OB = x – 4, OC = x – 3 and OD = 4, find x.



Answer

AO/OC=BO/OD

3X-19/X-3=X-4/4

(x-3)(x-4)=4(3x-19)

X^2 -4x-3x+12=12x-76

X^2 -7x+12-12x+76=0

X^2 -19x+88=0

X^2 -11x-8x+88=0

X(x-11)-8(x-11)=0

(x-11)(x-8)=0

x-11=0

x=11

or x-8=0

x=8

x=11 or 8

Exercise 4.5

1. Question

In Fig. 4.136, $\triangle ABC \sim \triangle APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ

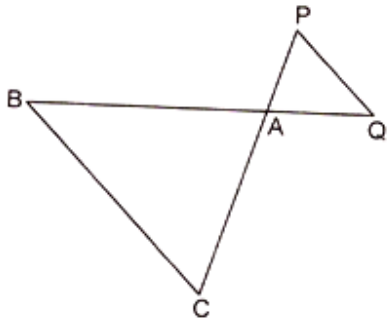


Fig. 4.136

Answer

Given ΔACB ~ ΔAPQ

Then, AC/AP = BC/PQ = AB/AQ

Or AC/2.8 = 8/4 = 6.5/AQ

Or AC/2.8 = 8/4 and 8/4 = 6.5/AQ

Or AC = 8/4 x 2.8 and AQ = 6.5 x4/8

Or AC=5.6cm and AQ = 3.25cm

2. Question

A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a tower casts a shadow 30 m long. Determine the height of the tower.

Answer

Length of stick = 10cm

Length of shadow stick= 8cm

Length of shadow of tower = hcm

In ΔABC and ΔPQR

<B = <C = 90° And <C = <R (Angular elevation of sum)

Then ΔABC ~ ΔPQR (By AA similarty)

So, $\frac{AB}{PQ} = \frac{BC}{QR}$

Or $\frac{10cm}{8cm} = \frac{H}{3000}$

Or h= $\frac{10}{8}$ x 3000

Or 3750cm

Or 37.5m

3. Question

In Fig. 4.137, $AB \parallel QR$. Find the length of PB.

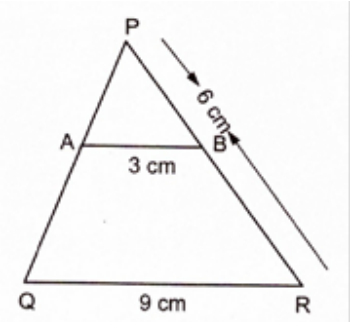


Fig. 4.137

Answer

We have ΔPAB and ΔPQR

<P = <P (Common)

<PAB = <PQR (Corresponding angles)

Then, ΔPAB ~ ΔPQR (BY AA similarity)

So, $\frac{PB}{PR} = \frac{AB}{QR}$ (Corresponding parts of similar triangle area proportion)

Or , $\frac{PB}{6} = \frac{3}{9}$

Or PB = $\frac{3}{9}$ x 6

Or PB= 2cm

4. Question

In Fig. 4.138, $XY \parallel BC$. Find the length of XY.

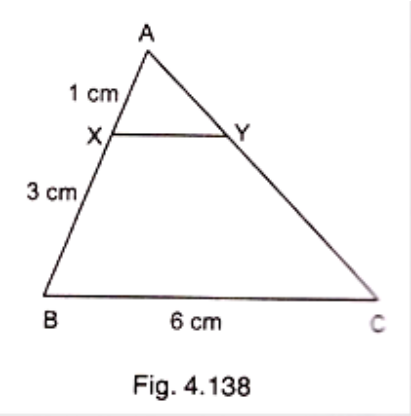


Fig. 4.138

Answer

We have , XY||BC

In Δ AXY and ΔABC

<A = <A (Common)

<AXY = <ABC (Corresponding angles)

Then, Δ AXY ~ΔABC (By AA Similarity)

So, $\frac{AX}{BY} = \frac{XY}{BC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{1}{4} = \frac{XY}{6}$

Or XY = 6/4

Or XY = 1.5cm

5. Question

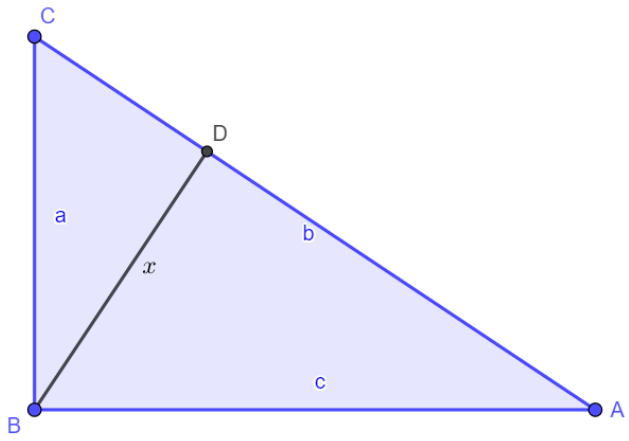
In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx

Answer

Given: In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x.

To prove: ab = cx

Proof: Let in a right-angled triangle ABC at B, a perpendicular from C to AB is drawn such thatBC = aAC = bBA = cBD = x



In ΔABC and ΔCDB

∠B = ∠B (Common)

∠ABC = ∠CDB (Both 90°)

Then, ΔABC ~ ΔCDB (By AA Similarity)

So, $\frac{AC}{CD} = \frac{AB}{CB}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{b}{x} = \frac{c}{a}$

Or ab = cx

6. Question

In Fig. 4.139, $\angle ABC = 90°$ and $BD \perp AC$. If BD = 8 cm and AD = 4 cm, find CD.

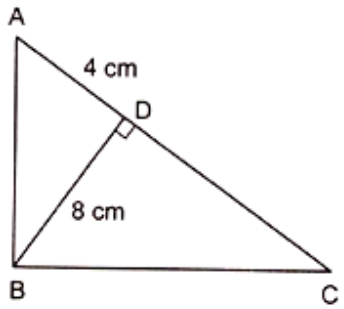


Fig. 4.139

Answer

We have, <ABC = 90° and BD perpendicular AC

Now, <ABD + <DBC – 90° ……..(I) (<ABC – 90°)

And <C + <DBC – 90° ……..(II) (By angle sum Prop. in ΔBCD) Compare equation I &II

<ABD = <C ……..(III)

In ΔABD and ΔBCD

<ABD = <C (From equation I)

<ADB = <BDC (Each 90°)

Then, ΔABD ~ΔBCD (By AA similarity)

So, $\frac{BD}{CD} = \frac{AD}{BD}$ (Corresponding parts of similar triangle area proportion)

Or, $\frac{8}{CD} = \frac{4}{8}$

Or CD = $\frac{8\ X\ 8}{4}$

Or CD = 16cm

7. Question

In Fig. 4.140, $\angle ABC$ = 90° and $BD \perp AC$. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.

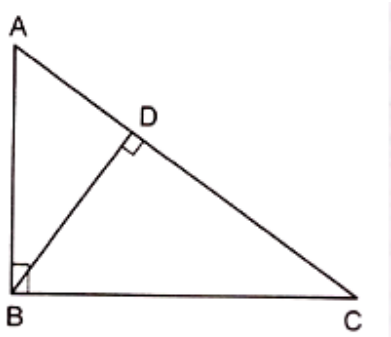


Fig. 4.140

Answer

We have , <ABC = 90° and BD Perpendicular AC

In Δ ABY and ΔBDC

<C = <C (Common)

<ABC = <BDC (Each 90° angles)

Then, Δ ABC ~ΔBDC (By AA Similarity)

So, $\frac{AB}{BD} = \frac{BC}{DC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{5.7}{3.8} = \frac{BC}{5.4}$

Or BC = 5.7/3.8 x 8.1

Or BC = 12.15cm

8. Question

In Fig. 4.141 $DE \parallel BC$ such that AE = (1/4) AC. If AB = 6 cm, find AD.

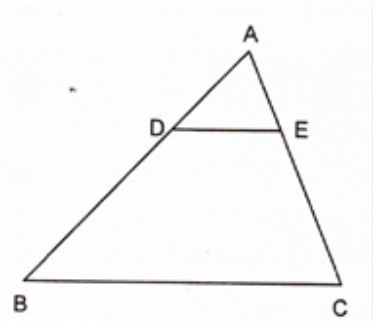


Fig. 4.141

Answer

We have, DE||BC, AB = 6cm and AE = 1/4 AC

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (By AA similarity)

So, $\frac{AD}{AB} = \frac{AE}{AC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$ (AE = 1/4 AC Given)

Or , $\frac{AD}{6} = \frac{1}{4}$

Or, AD = 6/4

Or, AD = 1.5cm

9. Question

In Fig. 4.142, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.

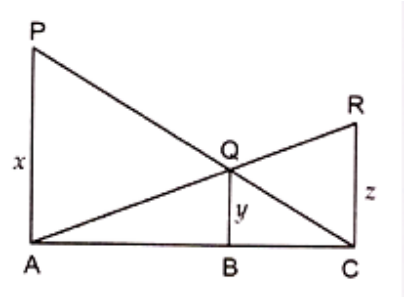


Fig. 4.142

Answer

We have, PA ⊥ AC, and RC ⊥ AC

Let AB = a and BC = b

In ΔCQB and ΔCPA

<QCB = <PCA (Common)

<QBC = <PAC (Each 90°)

Then, ΔCQB ~ ΔCPA (By AA similarity)

So, $\frac{QB}{PA} = \frac{CB}{CA}$ (Corresponding parts of similar triangle area proportion)

Or, $\frac{y}{z} = \frac{b}{a+b}$ ----------(i)

In ΔAQB and ΔARC

<QAB = <RAC (Common)

<ABQ = <ACR (Each 90°)

Then, ΔAQB ~ ΔARC (By AA similarity)

So, $\frac{QB}{RC} = \frac{AB}{CA}$ (Corresponding parts of similar triangle area proportion)

Or, $\frac{y}{x} = \frac{a}{a+b}$ ----------(ii)

Adding equation i & ii

$\frac{y}{x} + \frac{y}{z} = \frac{b}{a+b} = \frac{a}{a+b}$

Or, y $(\frac{1}{x} + \frac{1}{z}) = \frac{b+a}{a+b}$

Or, y $(\frac{1}{x} + \frac{1}{z})$ = 1

Or, $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

10. Question

In Fig. 4.143, $\angle A = \angle CED$, prove that $\Delta CAB \sim \Delta CED$. Also, find the value of x.

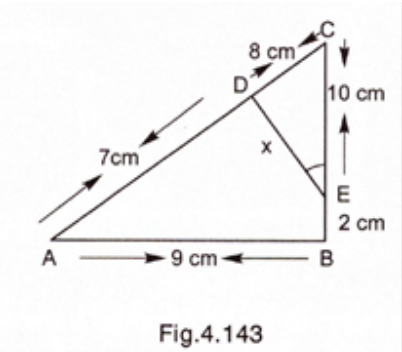


Fig.4.143

Answer

We have, <A = <CED

In ΔCAB and ΔCED

<C = <C (Common)

<A = <CED (Given)

Then, ΔCAB ~ ΔCED (By AA similarity)

So, $\frac{CA}{CE} = \frac{AB}{ED}$ (Corresponding parts of similar triangle area proportion)

Or,15/9 = 9/x

Or, 15x = 90

Or, x = 90/6

Or, x = 6cm.

11. Question

The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

Answer

Assume ABC and PQR to be 2 triangle.

We, have

ΔABC ~ΔPQR

Perimeter of ΔABC = 25cm

Perimeter of ΔPQR = 15cm

AB = 9cm

PQ = ?

Since, ΔABC ~ΔPQR

Then, ratio of perimeter of triangles = ratio of corresponding sides

So, $\frac{25}{15} = \frac{AB}{PQ}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{25}{15} = \frac{9}{PQ}$

Or PQ = 135/25

Or PQ = 5.4 cm

12. Question

In $\Delta ABC \text{ and } \Delta DEF$, it is being given that: AB = 5 cm, BC = 4 cm and CA = 4.2 cm; DE = 10 cm, EF = 8 cm and FD = 8.4 cm. If $AL \perp BC$ and $DM \perp EF$, find AL : DM.

Answer

Since $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$

Then, ΔABC ~ ΔDEF (By SS similarity)

Now, In ΔABL ~ ΔDEM

<B = <E (ΔABC ~ΔDEF)

<ALB =<DME (Each 90°)

Then, ΔABL ~ ΔDEM (By SS similarity)

So, $\frac{AB}{DE} = \frac{AL}{DM}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{5}{10} = \frac{AL}{DM}$

Or, $\frac{1}{2} = \frac{AL}{DM}$

13. Question

D and E are the points on the sides AB and AC respectively of a ΔABC such that AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that BC = 5/2 DE.

Answer

We have ,

$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$

And, $\frac{AD}{EC} = \frac{6}{9} = \frac{2}{3}$

Since, $\frac{AD}{DB} = \frac{AD}{EC}$

Then , by converse of basic proportionality theorem.

DE||BC

In Δ ADE and Δ ABC

<A= <A (Common)

<ADE = <B (Corresponding angles)

Then, Δ ADE ~ Δ ABC (By AA similarity)

$\frac{AD}{AB} = \frac{DE}{BC}$ (Corresponding parts of similar triangle are proportion)

$$\frac{8}{20} = \frac{DE}{BC}$$

$$\frac{2}{5} = \frac{DE}{BC}$$

BC = 5/2 DE

14. Question

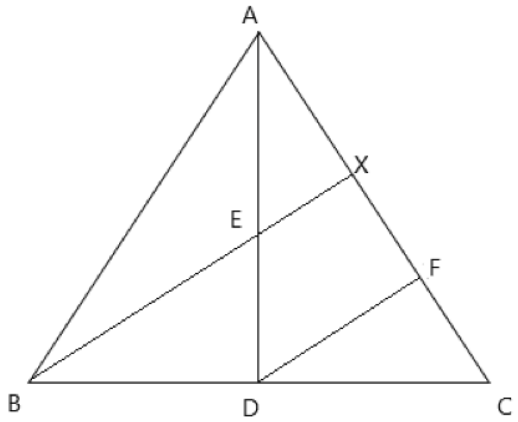
D is the mid-point of side BC of a ΔABC. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE : EX = 3 : 1.

Answer

Given:- In ΔABC, D is the midpoint of BC and E is the midpoint of AD.

To prove:- BE: EX = 3 : 1

Proof: Const:- Through D, Draw DF||BX



In ΔEAX and Δ ADF

∠EAX = ∠DAF (Common)

∠AXE = ∠DFA (Corresponding angles)

By AA similarity,

ΔEAX ~ Δ ADF

So, $\frac{EX}{DF} = \frac{AE}{AD}$ (Corresponding parts of similar triangle are proportion)

As E is mid point of AD

$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE}$

Or, DF = 2EX. ……(i)

In ΔDCF and ΔBCX ∠DCY = ∠BCX (common)∠CFD = ∠CXB (Corresponding angles)By AA similarity,ΔDCF ~ ΔBCX

SO, $\frac{CD}{CB} = \frac{DF}{BX}$ (Corresponding parts of similar triangle area proportion)

As D is mid point of BC and E is mid point of AD.

$$\Rightarrow \frac{CD}{2CD} = \frac{DF}{BE + EX}$$

Or $\frac{1}{2} = \frac{DF}{BE+EX}$

Or BE + EX = 2DFFrom (i)

BE + EX = 4EX

⇒ BE = 4EX – EX

⇒ BE = 4EX – EX

⇒ BE = 3EX

⇒ BE/EX =3/1

⇒ BE:Ex = 3:1

15. Question

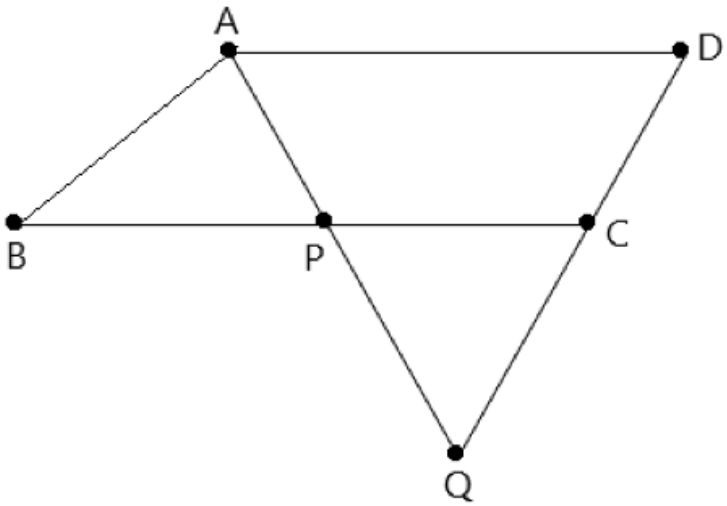
ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC.

Answer

Given :- ABCD is a parallelogram

To prove :- BP x DQ = AB x BC

Proof:-



In ΔABP and ΔQDA

<B = <D (Opposite angles of parallelogram)

<BAP = <AQD (Alternative interior angle)

Then, ΔABP ~ ΔQDA

SO, $\frac{AB}{QD} = \frac{BP}{DA}$ (Corresponding parts of similar triangle area proportion) But, DA = BC (Opposite side of parallelogram)But DA = BC (opposite sides of parallelogram)

Then, $\frac{AB}{QD} = \frac{BP}{BC}$

Or, AB x BC = QD X BPHence proved

16. Question

In $\triangle ABC$, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O, prove that :

(i) $\triangle OMA \sim \triangle OLC$

(ii) $\frac{OA}{OC} = \frac{OM}{OL}$

Answer

We have

AL ⊥ BC and CM ⊥ AB

IN ΔOMA and ΔOLC

<MOA = <LOC (Vertically opposite angles)

<AMO = <LOC (Each 90°)

Then, ΔOMA ~ΔOLC (BY AA Similarity)

SO, $\frac{OA}{OC} = \frac{OM}{OL}$ (Corresponding parts of similar triangle area proportion)

17. Question

In fig. 4.144, we have AB||CD||EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm, calculate the values of x and y.

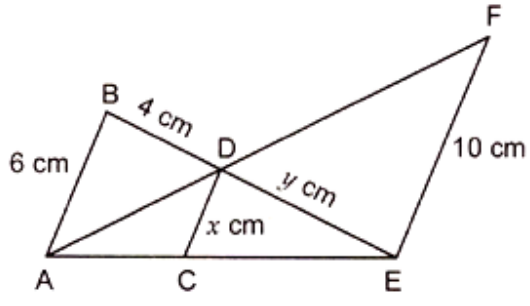


Fig. 4.144

Answer

We have AB||CD. If AB = 6cm, CD = xcm, EF = 10 cm, BD = 4cm and DE = ycm

In ΔECD and ΔEAB

<ECD = <EAB (Corresponding angles)

Then, ΔECD ~ ΔEAB ……(i) (By AA similarity)

SO, $\frac{EC}{EA} = \frac{CD}{AB}$ (Corresponding parts of similar triangle are proportion)

Or $\frac{EC}{EA} = \frac{x}{6}$ ……(ii)

In ΔACD and ΔAEF

<CAD = <EAF (Common)

<ACD = <AEF (Corresponding angles)

Then, ΔACD ~ ΔAEF (By AA similarity)

SO, $\frac{AC}{AE} = \frac{CD}{EF}$

Or, $\frac{AC}{AE} = \frac{x}{10}$…….. (iii)

Adding equation iii & ii

So, $\frac{AC}{AE} + \frac{EC}{EA} = \frac{x}{6} + \frac{x}{10}$

Or, $\frac{AE}{AE} = \frac{5x + 3x}{30}$

Or, $1 = \frac{8x}{30}$

Or, $x = \frac{30}{8}$

Or, x = 3.75cm

From (i)$\frac{DC}{AB} = \frac{CD}{BE}$

Or,$\frac{3.75}{6} = \frac{y}{y+4}$

Or, 6y = 3.75y +15

Or, 2.25y = 15

Or, $y = \frac{15}{2.25}$

Or, y = 6.67cm

18. Question

ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus.

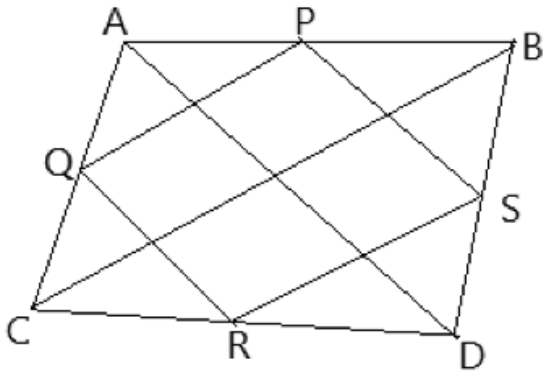
Answer

Given: ABCD is a quadrilateral in which AD = BC. P, Q, R, S be the mid-points of AB, AC, CD and BD respectively.

To show: PQRS is a rhombus.

Solution: So, we have, a quadrilateral ABCD where AD = BC

And P, Q, R and S are the mid-point of the sides AB, AC, and BD.



We need to prove that PQRS is a rhombus.

In ΔBAD, P and S are the mid points of the sides AB and BD respectively,By midpoint theorem which states that the line joining mid-points of a triangle is parallel to third side we get,

PS||AD and PS = 1/2 AD……(i)

In ΔCAD, Q and R are the mid points of the sides CA and CD respectively,by midpoint theorem we get,

QR||AD and QR = 1/2 AD ……..(ii)

Compare (i) and (ii)

PS||QR and PS = QR

Since one pair of opposite sides is equal and parallel,

Then, we can say that PQRS is a parallelogram……(iii)

Now, In ΔABC,P and Q are the mid points of the sides AB and AC respectively,by midpoint theorem,

PQ||BC and PQ = 1/2 BC……..(iv)

And AD = BC ……..(v) (given)

Compare equations (i) (iv) and (v), we get,

PS = PQ ……(vi)

From (iii) and (vi), we get,

PS = QR = PQ Therefore, PQRS is a rhombus.

19. Question

In Fig. 4.145, If $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$, prove that $\Delta CED \sim \Delta ABC$.

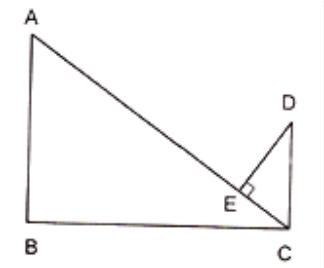


Fig. 4.145

Answer

Given AB⊥BC, DC ⊥ BC and DE ⊥AC

To prove:- ΔCED ~ΔABC

Proof:-

<BAC + <BCA = 90° ……..(i) (By angle sum property)

And, <BCA + <ECD = 90°……(ii) (DC ⊥ BC given)

Compare equation (i) and (ii)

<BAC = <ECD……..(iii)

In ΔCED and ΔABC

<CED = <ABC (Each 90°)

<ECD = <BAC (From equation iii)

Then, ΔCED ~ΔABC.

20. Question

In an isosceles $\triangle ABC$, the base AB is produced both the ways to P and Q such that AP × BQ = AC2. Prove that $\triangle APC \sim \triangle BCQ$.

Answer

Given : In ΔABC , CA – CB and AP x BQ = AC^2

To prove :- ΔAPC ~ BCQ

Proof:-

AP X BQ = AC^2 (Given)

Or, AP x BC = AC x AC

Or, AP x BC = AC x BC (AC = BC given)

Or, AP/BC = AC/PQ ……(i)

Since, CA = CB (Given)

Then, <CAB = <CBA …….(ii) (Opposite angle to equal sides)

NOW, <CAB +<CAP = 180° ……(iii) (Linear pair of angle)

And <CBA + <CBQ = 180° ……..(iv) (Linear pair of angle)

Compare equation (ii) (iii) & (iv)

<CAP = <CBQ……..(v)

In ΔAPC and ΔBCQ

<CAP = < CBQ (From equation v)

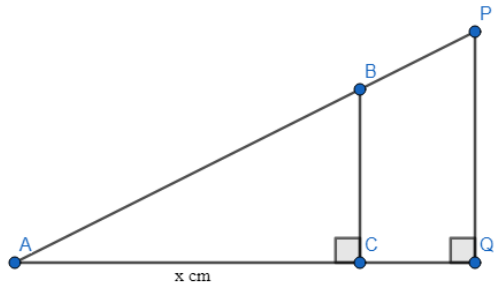
AP/BC = AC/PQ (From equation i)

Then , ΔAPC ~ ΔBCQ (By SAS similarity)

21. Question

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Answer



We have,

Let P be a lamb at a height of 3.6 m above that ground i.e. PQ = 3.6 m

Let BC be a girl, such that CQ is distance she covered and Let AC be her shadow, Height of girl = AB = 90cm = 0.9m

Height of lamp post = PQ = 3.6m

Speed of girl = 1.2 m/sec

So, Distance moved by the girl(CQ) = Speed x time

= 1.2 x 4 = 4.8m

Let length of shadow (AC) = 'x' cm

Then, AQ = AC + CQ = x + 4.8

In ΔABC and ΔAPQ

∠ACB = ∠AQP (Each 90 °)

∠BAC = ∠PAQ (Common)

Then , ΔABC ~ ΔAPQ (By AA similarity)

So, AC/AQ = BC/ PQ(Corresponding parts of similar triangle are proportional)

Or, x/x +4.8 = 0.9/3.6

Or, x/x +4.8 = 1/4

Or, 4x = x + 4.8

Or, 4x – x = 4.8

Or, 3x = 4.8

Or x = 4.8/3

Or x = 1.6mi.e. length of shadow is 1.6 m.

22. Question

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Answer

We have,

ABCD is a trapezium with AB || DC

In ΔAOB and ΔCOD <AOB = <COD (Vertically opposite angle)

<OAB = <OCD (Alternate interior angle)

Then, ΔAOB ~ΔCOD (By AA similarity)

So, OA/OC = OB/OD(Corresponding parts of similar triangle are proportional)

23. Question

If ΔABC and ΔAMP are two right triangles, right angled at B and M respectively such that $\angle MAP = \angle BAC$. Prove that

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer

We have,

<B = <M = 90°

And, <BAC = <MAP

In ΔABC and ΔAMP

<B = <M (each 90°)

<BAC = <MAP (Given)

Then, ΔABC ~ΔAMP (By AA similarity)

So, CA/PM = BC/MP(Corresponding parts of similar triangle are proportional)

24. Question

A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer

Let AB be a tower

CD be a stick , CD = 6m

Shadow of AB is BE = 28cm

Shadow of CD is DF = 4m

At same time light rays from sun will fail on tower and stick at the same angle

So, <DCF = <BAE

And <DFC = <BEA

< CDF = <ABE (Tower and stick are vertically to ground)

Therefore ΔABE ~ ΔCDF (By AAA similarity)

So, AB/CD = BE/DF

AB/6 = 28/4

AB/6 = 7

AB = 7 x 6

AB = 42 m

So, height of tower will be 42 meter.

25. Question

In Fig. 4.145 (a) ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.

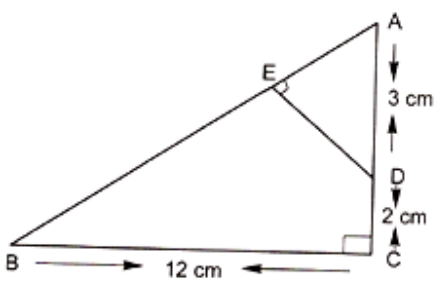


Fig. 4.145 (a)

Answer

In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

Or, $AB^2 = 5^2 + 12^2$

Or, $AB^2 = 25 + 144$

Or, $AB^2 = = 169$

Or AB = 13 (Square root both side)

In Δ AED and Δ ACB

<A = <A (Common)

<AED = <ACB (Each 90°)

Then, Δ AED ~ Δ ACB(Corresponding parts of similar triangle are proportional)

So, AE/AC = DE/ CB =AD/ AB

Or, AE/5 = DE/12 = 3/13

Or, AE/5 = 3/13 and DE/12 = 3/13

Or, AE = 15/13cm and DE = 36/13cm

Exercise 4.6

1. Question

Triangles ABC and DEF are similar.

(i) If area ($\Delta\ ABC$) = 16 cm^2, area ($\Delta\ DEF$) = 25 cm^2 and BC = 2.3 cm, find EF.

(ii) If area ($\Delta\ ABC$) = 9 cm^2, area ($\Delta\ DEF$) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles.

(iv) If area ($\Delta\ ABC$) = 36 cm^2, area ($\Delta\ DEF$) = 64 cm^2 and DE = 6.2 cm, find AB.

(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the areas of $\Delta\ ABC\ and\ \Delta\ DEF$.

Answer

(i) We have

ΔABC ~ΔDEF

Area (ΔABC) = 16cm^2

Area (ΔDEF) = 25cm^2

And BC = 2.3cm

Since, ΔABC ~ΔDEF

Then, Area (ΔABC)/Area (ΔDEF)

= BC^2/EF^2 (By are of similar triangle theorem)

Or, $16/25 = (23)^2/ EF^2$

Or, 4/5 = 2.3/EF (By taking square root)

Or, EF = 11.5/4

Or, EF = 2.875cm

(ii) We have

ΔABC ~ΔDEF

Area (ΔABC) = $9cm^2$

Area (ΔDEF) = $64cm^2$

And BC = 5.1cm

Since, ΔABC ~ΔDEF

Then, Area (ΔABC)/Area (ΔDEF)

= AB^2/DE^2 (By are of similar triangle theorem)

Or, $9/64 = AB^2/(5.1)^2$

Or, AB = 3 x 5.1/8 (By taking square root)

Or, AB = 1.9125cm

(iii) We have,

ΔABC ~ ΔDEF

AC = 19cm and DF = 8cm

By area of similar triangle theorem

Then, Area of ΔABC/Area of ΔDEF = AC^2 /DE^2(Br area of similar triangle theorem)

$(19)^2/(8)^2 = 364/64$

(iv) We have

Area ΔABC = $36cm^2$

Area ΔDEF = 64 cm^2

DE = 6.2 cm

And , ΔABC ~ΔDEF

By area of similar triangle theorem

Area of ΔABC/Area of ΔDEF = AB^2 /DE^2

Or, 36/64 = 6x 6.2/8 (By taking square root)

Or, AB = 4.65cm

(V) We have

ΔABC ~ ΔDEF

AB = 12cm and DF = 1.4 cm

By area of similar triangle theorem

Area of ΔABC/Area of ΔDEF = AB^2 /DE^2

Or, $(1.2)^2/(1.4)^2$ = 1.44x/1.96

2. Question

In Fig. 4.177, $\triangle ACB \sim \triangle APQ$. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ. Also, find the area ($\triangle ACB$) : area ($\triangle APQ$).

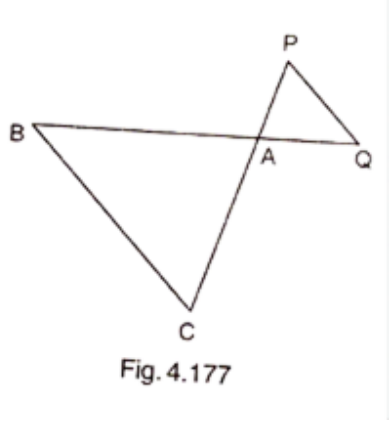


Fig. 4.177

Answer

We have,

ΔACB ~ ΔAPQ

Then, AC/AP = CB/PQ = AB/AQ[Corresponding parts of similar Δ are proportional]

Or, AC/2.8 = 10/5 = 6.5/AQ

Or, AC/2.8 = 10/5 and 10/5 = 6.5/AQ

Or, AC = 5.6cm and AQ = 3.25cm

By area of similar triangle theorem

Area of ΔACB/Area of ΔAPQ = BC^2 /PQ^2

= $(10)^2/(5)^2$

= 100/25

= 4 cm

3. Question

The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?

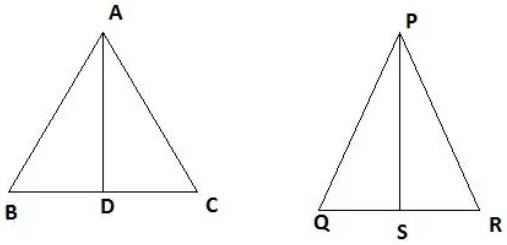
Answer

Given : ΔABC ~ ΔPQR

Area (ΔABC) = 81 cm^2

Area (ΔPQR) = 49 cm^2

Figure:



And AD and PS are the altitudes

By area of similar triangle theorem: The ratio of the areas of two similar triangles equal to the ratio of squares of the corresponding sides of triangles.

$$\frac{Area\ of\ \triangle\ ABC}{Area\ of\ \triangle\ PQR} = \frac{AB^2}{PQ^2}$$

$$\frac{AB^2}{PQ^2} = \frac{81}{49}$$

$$\frac{AB}{PQ} = \sqrt{\frac{81}{49}}$$

$$\frac{AB}{PQ} = \frac{9}{7}$$

We also know that:

$$\frac{AD}{PS} = \frac{AB}{PQ}$$

$$Therefore,\ \frac{AD}{PS} = \frac{9}{7}$$

So, Ratio of altitudes = 9/7

Hence, ratio of altitudes = Ratio of medians = 9:7

4. Question

The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Answer

We have,

ΔABC ~ Δ PQR

Area (ΔABC) = 169cm^2

Area (PQR) = 121 cm^2

And AB = 26 cm

By area of similar triangle theorem

Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2

Or, 169/125 = 26^2/ PQ^2

Or, 13/11 = 26/PQ (Taking square root)

Or, PQ = 11/13 x 26

Or, PQ = 22cm

5. Question

Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25.. Find the ratio of their corresponding heights.

Answer

Given : - AB = AC, PQ = PR and <A = <P

And AD and PS are altitudes

And, Area (ΔABC)/Area of(ΔPQR) = 36/25……..(i)

To find: AD/PS

Proof:- Since, AB = AC and PQ = PR

Then, AB/AC = 1 and PQ/PR = 1

So, AB/AC = PQ/PR

Or, AB/PQ = AC/PR…….(ii)

In ΔABC and ΔPQR

<A = <P (Given)

AB/PQ = AC/PR (From equation ii)

Then, ΔABC ~ ΔPQR (BY AA similarity)

So, Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2…..(iii) (By area of similar triangle)

Compare equation I and II

$AB^2/PQ^2 = 36/25$

Or, AB/PQ = 6/5

In ΔABD and ΔPQS

<B = <Q (ΔABC ~ ΔPQR)

<ADB = <PSO (Each 90°)

Then , ΔABD ~ ΔPQS (By AA similarity)

So, AB/ PQ = AD/PS

6/5 = AD/ PS (From iv)

6. Question

The areas of two similar triangles are 25 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other.

Answer

We have,

ΔABC ~ Δ PQR

Area (ΔABC) = 25 cm^2

Area (PQR) = 36 cm^2

And AD = 2.4 cm

And AD and PS are the altitudes

To find: PS

Proof: Since, ΔABC ~ ΔPQR

Then, by area of similar triangle theorem

Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2

$25/36 = AB^2/PQ^2$

5/6 = AB/PQ……..(i)

In ΔABD and Δ PQS

<B = <Q (ΔABC ~ ΔPQR)

<ADB = <PSQ (Each 90°)

Then, ΔABD ~ Δ PQS (By AA similarity)

So, AB/PS = AD/PS……(ii) (Corresponding parts of similar Δ are proportional)

Compare (i) and (ii)

AD/PS = 5/6

2.4/PS = 5/6

PS = 2.4 x 6/5

PS = 2.88cm

7. Question

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer

We have,

ΔABC ~ ΔPQR

AD = 6cm

PS = 9cm

By area of similar triangle theorem

Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2……(i)

In ΔABD and ΔPQS

<B = <Q (ΔABC ~ ΔPQS)

<ADB = <PSQ (Each 90°)

Then, ΔABD ~ ΔPQS (By AA Similarity)

So, AB/PQ = AD/PS (Corresponding parts of similar Δ are proportional)

Or, AB/PQ = 6/9

Or, AB/PQ = 2/3 ……..(ii)

Compare equation (i) and (ii)

Area of ΔABC/Area of ΔPQR = $(2/3)^2$ = 4/9

8. Question

ABC is a triangle in which $\angle A = 90°$, $AN \perp BC$, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of $\Delta\ ANC$ and $\Delta\ ABC$.

Answer

In Δ ANC and Δ ABC

<C = <C (Common)

<ANC = <BAC (Each 90°)

Then, Δ ANC ~ Δ ABC (By AA similarity)

By area of similarity triangle theorem.

Area of ΔABC/Area of ΔPQR = AC^2 /BC^2

Or, $5^2/12^2$

Or, 25/144

9. Question

In Fig. 4.178, $DE \parallel BC$

(i) If DE = 4 cm, BC = 6 cm and area ($\triangle ADE$) = 16 cm^2, find the area of $\triangle ABC$.

(ii) If DE = 4 cm, BC = 8 cm and area ($\triangle ADE$) = 25 cm^2, find the area of $\triangle ABC$.

(iii) If DE : BC = 3 : 5. Calculate the ratio of the areas of $\triangle ADE$ and the trapezium BCED.

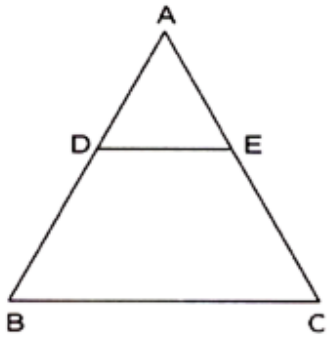


Fig. 4.178

Answer

(i) We have , DE||BC, DE = 4cm, BC = 6cm and area (ΔADE) = 16cm^2

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (BY AA similarity)

So, By area of similar triangle theorem

Area of ΔADE/Area of ΔABC = DE^2 /BC^2

16/Area of ΔABC = $4^2/6^2$

Or, Area (ΔABC) = 16 x 36/16

= 36cm^2

(ii) We have , DE||BC, DE = 4cm, BC = 8cm and area (ΔADE) = 25cm^2

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (BY AA similarity)

So, By area of similar triangle theorem

Area of ΔADE/Area of ΔABC = DE^2 /BC^2

25/Area of ΔABC = $4^2/8^2$

Or, Area (ΔABC) = 25 x 64/16

= 100 cm^2

(iii) We have DE||BC, And DE/BC = 3/5 ……(i)

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (BY AA similarity)

So, By area of similar triangle theorem

Area of ΔADE/Area of ΔABC = DE^2 /BC^2

Area of ΔADE/Area of ΔADE + Area of trap. DECB = $3^2/5^2$

Or, 25 area ΔADE = 9 Area of ΔADE +9 Area of trap. DECB

Or 25 area ΔADE - 9 Area of ΔADE = 9 Area of trap. DECB

Or, 16 area ΔADE = 9 Area of trap. DECB

Or, area ΔADE / Area of trap. DECB = 9/16

10. Question

In ΔABC, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.

Answer

We have, D and E as the midpoint of AB and AC

So, according to the midpoint therom

DE||BC and DE = 1/2 BC……..(i)

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <B (Corresponding angles)

Then, ΔADE ~ ΔABC (By AA similarity)

By area of similar triangle theorem

Area ΔADE/ Area ΔABC = DE^2/BC^2

Or, $(1/2BC)^2/(BC)^2$

Or, 1/4

11. Question

In Fig. 4.179, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD and BC intersect at O. Prove that

$$\frac{Area\,(\triangle ABC)}{Area\,(\triangle DBC)} = \frac{AO}{DO}$$

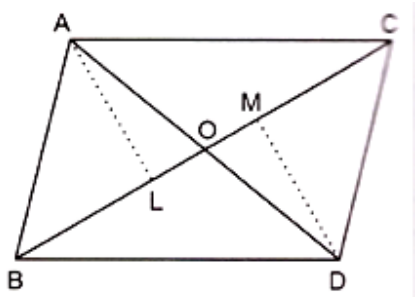


Fig. 4.179

Answer

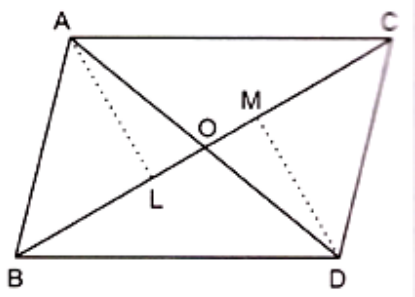


Fig. 4.179

We know that area of a triangle = 1/2 x base x height

Since, ΔABC and ΔDBC are one same base.

Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC

In ΔALO and ΔDMO,

∠ALO =∠DMO (Each is 90°)

∠AOL = ∠DOM (Vertically opposite angle)

∠OAL = ∠ODM (remaining angle)

Therefore ΔALO ~ ΔDMO (By AAA rule)

Therefore AL/DM = AO/DO

Therefore, $\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta\ DBC)} = \frac{AO}{DO}$

12. Question

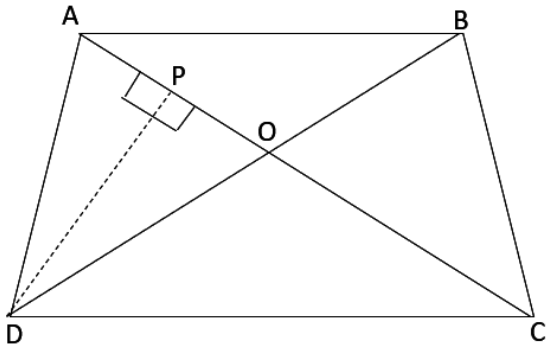
ABCD is a trapezium in which $AB \parallel CD$. The diagonals AC and BD intersect at O. Prove that : (i) $\Delta\ AOB \sim \Delta\ COD$

(ii) If OA = 6 cm, OC = 8 cm, Find:

(a) $\frac{Area\ (\Delta\ AOB)}{Area\ (\Delta\ COD)}$ (b) $\frac{Area\ (\Delta\ AOD)}{Area\ (\Delta\ COD)}$

Answer

We have,



AB||DC

In ΔAOB and ΔCOD ∠AOB =∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angle)

Then , ΔAOB ~ ΔCOD (By AA similarity)

(a) By area of similar triangle theorem.

$$\frac{Area\ of\ \Delta AOB}{Area\ of\ \Delta COD} = \frac{OA^2}{OC^2}$$

$$\Rightarrow \frac{Area\ of\ \Delta AOB}{Area\ of\ \Delta COD} = \frac{6^2}{8^2}$$

$$\Rightarrow \frac{Area\ of\ \Delta AOB}{Area\ of\ \Delta COD} = \frac{36}{64} = \frac{9}{16}$$

b) Draw DP ⊥ AC

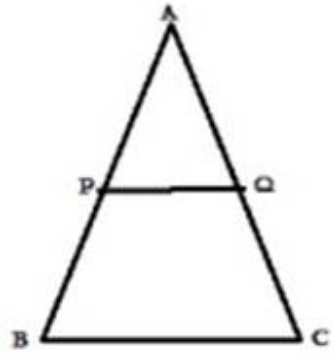
$$\Rightarrow \frac{Area\ of\ \Delta AOD}{Area\ of\ \Delta COD} = \frac{\frac{1}{2} \times OA \times DP}{\frac{1}{2} \times OC \times DP}$$

$$\Rightarrow \frac{Area\ of\ \Delta AOD}{Area\ of\ \Delta COD} = \frac{6}{8} = \frac{3}{4}$$

13. Question

In $\triangle ABC$, P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that $PQ \parallel BC$. Find the ratio of the areas of $\triangle APQ$ and trapezium BPQC.

Answer



We know

PQ∥BC

1= AP

2 PB

In $\Delta APQ\ and\ \Delta ABC$

∠A=∠A [Common]

∠APQ=∠B [Corresponding angle]

ΔABC∼ ΔAPQ

Area(ΔAPQ) =AP^2

Area (ΔABC) AB^2

ar (ΔAPQ)_____ = $1^2/3^2$

ar(ΔAPQ)+ar($trapBPQC$)

9ar(ΔAPQ)= ar(ΔAPQ)+ar($trapBPQC$)

9ar(ΔAPQ)- ar(ΔAPQ)=ar($trapBPQC$)

8ar(ΔAPQ)=ar($trapBPQC$)

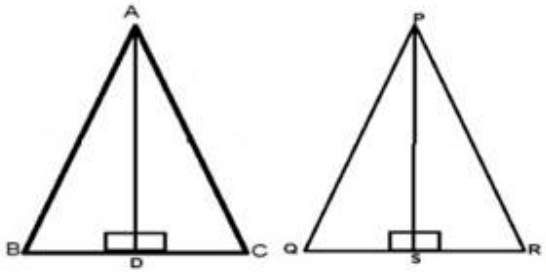
ar(ΔAPQ) = $\frac{1}{8}$

ar($trapBPQC$)

14. Question

The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude of the bigger triangle is 5 cm, find the corresponding altitude of the other.

Answer



We have,

ΔABC$\sim$ ΔPQR

Area (ΔABC) =100cm^2

Area (ΔPQR) =49 cm^2

AD= 5cm

AD and PS are the altitudes

by area of similar triangle theorem

Area(ΔABC) =AB^2

Area (ΔPQR) PQ^2

AB^2 = 100/49

PQ^2

AB/PQ= 10/7 ……..(i)

In ΔABD and ΔPQS

$\angle B=\angle Q$ [ΔABC$\sim$ ΔPQR]

$\angle ADB=\angle PQS=90°$

ΔABD ~ ΔPQS [By AA similarity]

AB/PQ=AD/PS …….(ii)

Compare equ. (i)and(ii)

AD/PS=10/7

5/PS=10/7

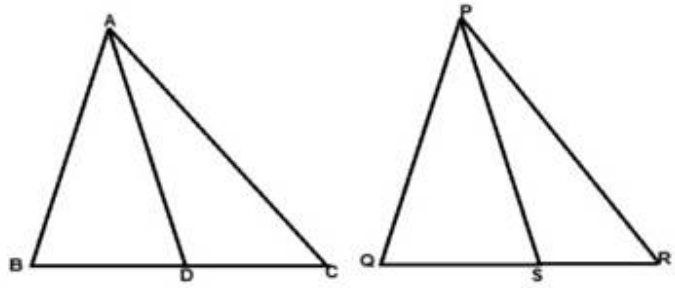
PS=35/10

PS=3.5 cm

15. Question

The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Answer



We have,

ΔABC$\sim$ ΔPQR

Area (ΔABC) =121cm^2

Area (ΔPQR) =64cm^2

AD= 12.1cm

AD and PS are the medians

By area of similar triangle theorem

Area(ΔABC) =AB^2

Area (ΔPQR) PQ^2

AB^2 =121

PQ^2 64

AB =11 …… (i)

PQ 8

ΔABC$\sim$ ΔPQR

AB/PQ=BC/QR [Corresponding parts of similar triangles are proportional] AB/PQ=2BD/2QS [AD and BD are medians]

AB/PQ=BD/QS …… (ii)

In ΔABD and ΔPQS

$\angle B=\angle Q$ [$\Delta ABC \sim \Delta PQS$]

AB/PQ=BD/QS [from (ii)]

ΔABD ~ ΔPQS [By AA similarity]

AB/PQ=AD/PS Compare equ. (i)and(ii)

AD/PS=11/8

12.1/PS=11/8

PS=12.1x8/8

PS= 8.8 cm

16. Question

If $\Delta ABC \sim \Delta DEF$ such that AB = 5 cm, area (ΔABC) = 20 cm^2 and area (ΔDEF) = 45 cm^2 , determine DE.

Answer

We have

ΔABC$\sim$ ΔDEF

Where AB= 5cm

Area (ΔABC) =20cm^2

Area (ΔDEF) =45cm^2

By area of similar triangle theorem

Area (ΔABC) =AB^2

Area (ΔDEF) DE^2

$5^2/DE^2=20/25$

$25/DE^2=4/9$

5/DE=2/3

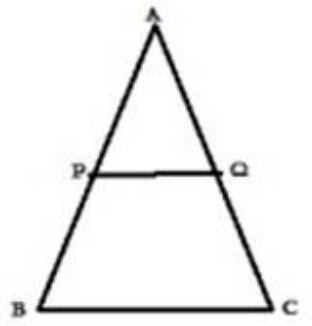
DE=3x5/2

DE=7.5 cm

17. Question

In $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that $PQ \parallel BC$ and PQ divides $\triangle ABC$ into two parts equal in area. Find $\frac{BP}{AB}$.

Answer



We know

PQ∥BC

Area (ΔAPQ) =Area ($trapPQCB$)

Area (ΔAPQ) =Area (ΔABC)- Area (ΔAPQ)

2Area (ΔAPQ)= Area (ΔABC) ……(i)

In $\Delta APQ \; and \; \Delta ABC$

∠A=∠A [Common]

∠APQ=∠B [Corresponding angle]

ΔABC∼ ΔAPQ

Area(ΔAPQ) =AP^2

Area (ΔABC) AB^2

Area(ΔAPQ) =AP^2

Area (ΔAPQ) AB^2 [By using (I)]

1= AP^2

2 AB^2

$\frac{1}{\sqrt{2}}$ =AP/AB

$\frac{1}{\sqrt{2}} = \frac{AB-BP}{AB}$

$\frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$

$\frac{1}{\sqrt{2}}$=1-BP/AB

BP/AB=1-$\frac{1}{\sqrt{2}}$

BP/AB= $\frac{\sqrt{2}-1}{\sqrt{2}}$

18. Question

The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Answer

We have,

ΔABC$\sim$ ΔPQR

Area(ΔABC) =BC^2

Area (ΔPQR) QR^2

$(4.5)^2/QR^2=9/16$

4.5/QR=3/4

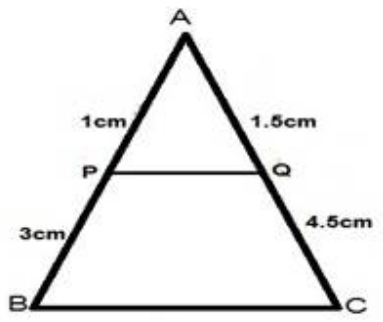
QR=4x4.5/3

QR=6cm

19. Question

ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of $\triangle APQ$ is one-sixteenth of the area of $\triangle ABC$.

Answer



AP=1 cm, PB=3 cm,AQ=1.5cm,and QC=4.5 m

In ΔAPQ and ΔABC

$\angle A=\angle A$ [Common]

AP/AB=AQ/AC [Each equal to 1/4]

ΔAPQ$\sim$ ΔABC [By SAS]

By area of similar triangle theorem

$$\frac{Area\ of\ \triangle APQ}{Area\ of\ \triangle ABC} = (\frac{AP}{AB})^2$$

$$\frac{Area\ of\ \triangle APQ}{Area\ of\ \triangle ABC} = (\frac{1}{4})^2$$

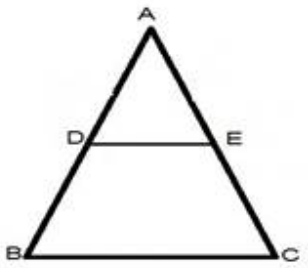
$$\frac{Area\ of\ \triangle APQ}{Area\ of\ \triangle ABC} = \frac{1}{16}$$

Area (ΔABC) = 16 x ar(ΔABC)

20. Question

If D is a point on the side AB of $\triangle ABC$ such that AD : DB = 3.2 and E is a point on BC such that $DE \parallel AC$. Find the ratio of areas of $\triangle ABC$ and $\triangle BDE$.

Answer



We have

AD/DB=3/2

In ΔBDE and ΔBAC

$\angle$B=$\angle$B [Common]

$\angle$BDE=$\angle$A [Corresponding]

ΔBDE$\sim$ ΔBAC

Area(ΔABC) =AB^2

Area (ΔBDE) BD^2

=$5^2/2^2$ [AD/DB=3/2]

=25/4

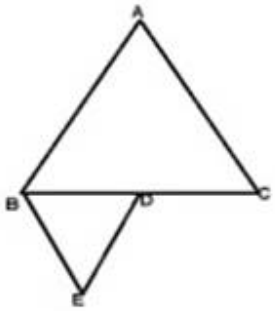
Area(ΔABC)

Area (ΔBDE) =25:4

21. Question

If $\triangle ABC$ *and* $\triangle BDE$ are equilateral triangles, where D is the mid point of BC, find the ratio of areas of $\triangle ABC$ *and* $\triangle BDE$.

Answer



ΔABC and ΔBDE is an equilateral triangles

ΔABC$\sim$ ΔDEF [By SAS]

By area of similar triangle theorem

Area(ΔABC) =AB^2 [D is the midpoint of BC]

Area (ΔBDE) BD^2

=$4BD^2/BD^2$

=4/1

Area(ΔABC) = 4:1 Area (ΔBDE)

22. Question

AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4.

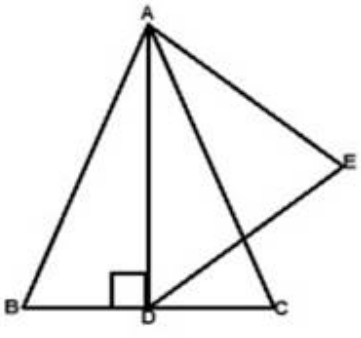
Answer

Given: AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed

To prove: Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4.

Proof:

Construct the figure according to the conditions given.



We have,

ΔABC is an equilateral triangle

Let one side AB be 2XSince in equilateral triangle all the sides are of equal length.

⇒ AB=BC=AC= 2X

∵ AD⊥BCSince perpendicular bisects the given side into two equal parts,then BD=DC=x

Now, In ΔADB

By Pythagoras theorem,$AB^2 = AD^2 + BD^2$

$AD^2=AB^2 - BD^2AD^2 = (2x)^2-(x)^2AD^2 =3x^2$

AD=$\sqrt{3x}$ cm

ΔABC and ΔADE both are equilateral triangles

Since, all the angles of the equilateral triangle are of 60°.

∴ ΔABC$\sim$ ΔADE [By AA similarity]

By the theorem which states that the areas of two similar triangles are in the ratioof the squares of the any two corresponding sides.

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{AD^2}{AB^2}$$

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{\left(\sqrt{3x}\right)^2}{(2x)^2}$$

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{3x^2}{4x^2}$$

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{3}{4}$$

Hence,Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4

Exercise 4.7

1. Question

If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is a right-angled triangle.

Answer

We have,

AB=3cm, BC=4cm, AC=6cm

$\therefore AB^2 = 3^2=9$

$BC^2 = 4^2 =16$

$AC^2=6^2=36$

Since $AB^2+BC^2 \neq AC^2$

SO Triangle is not a right angle.

2. Question

The sides of certain triangles are given below. Determine which of them are right triangles.

(i) a = 7 cm, b = 24 cm and c = 25 cm

(ii) a = 9 cm, b = 16 cm and c = 18 cm

(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm

(iv) a = 8 cm, b = 10 cm and c = 6 cm

Answer

(i) a= 7, b= 24, c=25

Here $a^2=49$, $b^2=576$, $c^2=625$

$=a^2+b^2$

=49+576

$=625=c^2$

$\therefore$ So given triangle is a right angle.

(ii) a=9, b=16, c= 18

Here a^2=81, b^2= 256, c^2= 324

=a^2+b^2

=81+256

=337 ≠ c^2

So given Triangle is not a right angle.

(iii) a=1.6, b=3.8, c= 4

Here a^2=2.56, b^2= 14.44, c^2= 16

=a^2+b^2

=2.56+14.44

=17 ≠ c^2

So given Triangle is not a right angle.

(iv) a=8, b=10, c= 6

Here a^2=64, b^2= 100, c^2= 36

=a^2+c^2

=64+36

=100 = b^2

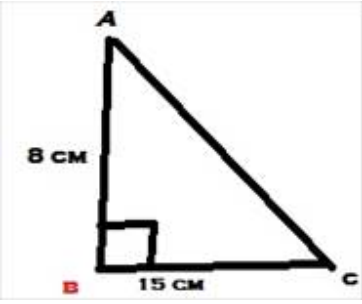
So given Triangle is a right angle.

3. Question

A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Answer

Let the man starts walk from point A and finished at



Point C.

∴ In ⊿ ABC

SO AC^2=AB^2+BC^2

$AC^2=8^2+15^2$

$AC^2=64+225$

$AC^2=289$

$AC=\sqrt{289}$

AC=17 m

The man is 17 m far from the starting point.

4. Question

A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Answer

In ⊿ ABC

$AC^2=AB^2+BC^2$

$17^2=15^2+BC^2$

$289=225+BC^2$

$BC^2=289-225$

$BC^2=64$

$BC=\sqrt{64}$

BC=8 m

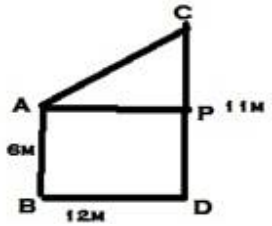
Distance of the foot of ladder is 8 m from the building.

5. Question

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer

Let AB and CD be the poles.



AB=PD= 6m, CD=11m

BD=AP=12m

CP=CD-PD

CP=11-6

CP=5

In ⊿ APC

$AC^2=CP^2+AP^2$

$AC^2=12^2+5^2$

$AC^2=144+25$

$AC^2=169$

AC=$\sqrt{169}$

AC= 13m

6. Question

In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.

Answer

We have,

AB=AC=25cm

BC=14cm

In ⊿ ACD and ⊿ ABD

∠ADB=∠ADB=90

AB=AC=25cm

AD=AD (Common)

⊿ ABD≅∠ACD

∴BD=CD=7cm (By c.p.c.t)

In ⊿ACD

$AB^2=AD^2+BD^2$

$25^2=AD^2+72$

$625=AD^2+49$

$AD^2=625-49$

$AD^2=576$

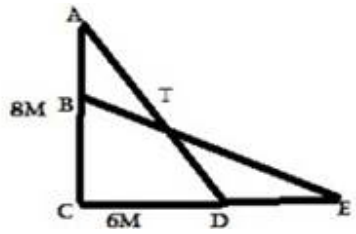
AD=$\sqrt{576}$

AD=24 cm

7. Question

The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Answer



Let length of ladder be AD=BE=1m

InΔ ACD

$AD^2=AC^2+CD^2$

$t^2= 8^2+6^2$ …….(i)

In Δ BCE

$BE^2=BC^2+EC^2$

$t^2= BC^2+8^2$ …….. (II)

From (i) and (ii)

$BC^2+8^2=8^2+6^2$

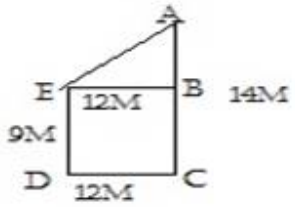
$BC^2=6^2$

BC=6m

8. Question

Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer



We have,

AC=14m, DC=12m, ED=BC=9m

Draw EB ⊥ AC

∴ AB=AC-BC

AB= 14-9=5m

EB=DC=12m

In Δ ABE

$AE^2=AB^2+BE^2$

$AE^2=5^2+12^2$

$AE^2=25+144$

$AE^2=169$

$AE=\sqrt{169}$

AE=13m

9. Question

Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig 4.219.

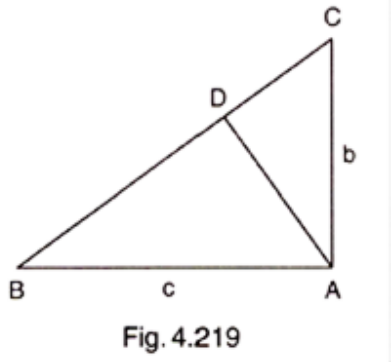


Fig. 4.219

Answer

In Δ ABC

$BC^2=AB^2+AC^2$

$BC^2=c^2+b^2$

BC= $\sqrt{c2+b2}$ .....(i)

In Δ ABC and In Δ CBA

∠B= ∠B (Common)

∠ADB=∠BAC=90°

∴Δ ABD ~ Δ CBA

∴ AB/CB=AD/CA

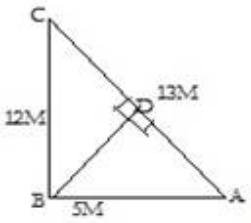
c/$\sqrt{c2 + b2}$ =AD/b

AD=bc/$\sqrt{c2 + b2}$

10. Question

A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Answer



Here AB=5cm,BC=12cm, AC=13cm.

AC2=AB^2+BC^2

Δ ABC is a right angled triangle at $\angle$B.

Area Δ ABC=1/2(BCxBA)

=1/2(12x5)

=1/2x60

=30cm^2

Also Area of Δ ABC=1/2xACxBD

=1/2(13xBD)

30=1/2(13xBD)

13XBD=30x2

BD=60/13

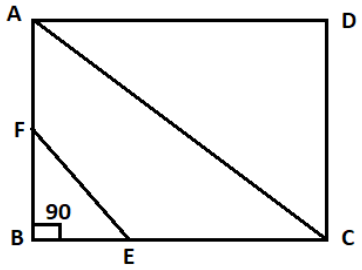
BD=4.6 cm

11. Question

ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of ΔFBE = 108 cm^2, find the length of AC.

Answer

According to the question, the figure is :



∵ ABCD is a square. Hence, AB = BC = CD = DA

∵ F is the midpoint of AB.

∴ Length of BF = AB/2 = BC/2 (∵ AB = BC)

Given that, BE = BC/3

In ΔFBE, ∠B = 90° and Area of ΔFBE = 108 cm^2

$$\therefore \frac{1}{2} \times BE \times BF = 108$$

$$\Rightarrow \frac{1}{2} \times \frac{BC}{3} \times \frac{BC}{2} = 108$$

$\Rightarrow BC^2 = 108 \times 12$

$\Rightarrow BC^2 = 36 \times 36$

$\Rightarrow BC = 36\ cm^2$

AC is the diagonal of the ABCD.

$$\therefore \text{Length of AC} = \sqrt{BC^2 + AB^2}$$

$$\Rightarrow AC = \sqrt{36^2 + 36^2}$$

$$\Rightarrow AC = \sqrt{36^2 + 36^2}$$

$\Rightarrow AC = 36\sqrt{2} = 50.904$ cm

12. Question

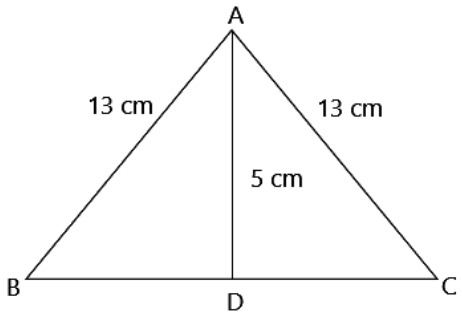
In an isosceles triangle ABC, if AB = AC = 13 cm and the altitude from A on BC is 5 cm, find BC.

Answer

Given: isosceles triangle ABC, where AB = AC = 13 cm and the altitude from A on BC is 5 cm.

To find: The value of BC.

Solution:



In ΔADB

$AD^2+BD^2=AB^2$

$5^2+BD^2=13^2$

$25+BD^2=169$

$BD^2=169-25$

$BD^2=144$

$BD=\sqrt{144}$

BD=12cm

In ΔADB and ΔADC

$\angle ADB=\angle ADC=90^\circ$

AB=AC=13cm

AD=AD (Common)

$\Delta ADB \cong \Delta ADC$ (By RHS condition)

BD=CD=12cm (c.p.c.t)

As BC=BD+DC

BC=12+12

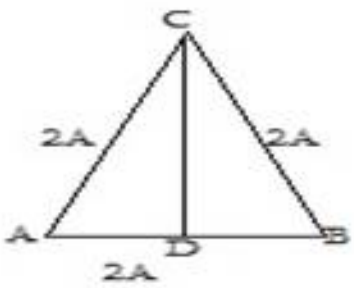
BC = 24cm

13. Question

In a $\triangle ABC$, AB = BC = CA = 2 a and $AD \perp BC$. Prove that

(i) $AD = a\sqrt{3}$ (ii) Area ($\triangle ABC$) = $\sqrt{3}a^2$

Answer



(i) In Δ ABD and Δ ACD

$\angle ADB=\angle ADC=90^{\circ}$

AB=AC (given)

AD=AD (common)

$\Delta ADB\cong \Delta ACD$

BD=CD=a (By c.p.c.t)

In ΔADB

$AD^2+BD^2=AB^2$

$AD^2+a^2=(2a)^2$

$AD^2=4a^2-a^2$

$AD^2=3a^2$

$AD=a\sqrt{3}$

(ii) Area of $\Delta ABC=1/2xBCxAD$

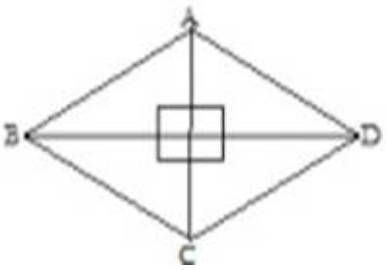
$= 1/2x2axa\sqrt{3}$

$=\sqrt{3a^2}$

14. Question

The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.

Answer



We have,

ABCD is a rhombus

AC and BD are the diagonals with length 10cm and 24 cm respectively.

We know that rhombus of diagonal bisects each other at 90°

∴ AO=OC=5cm and BO=OD=12cm

In ΔAOB

$AB^2=AO^2+BO^2$

$AB^2=5^2+12^2$

$AB^2=25+144$

$AB^2=169$

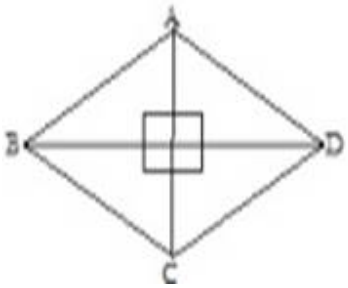
$AB=\sqrt{169}$

AB=13 cm

15. Question

Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

Answer



We have,

ABCD is a rhombus with side 10 cm and diagonal BD=16 CM

We know that rhombus of diagonal bisects each other at 90°

BO=OD=8cm

In ΔAOB

$AB^2=AO^2+BO^2$

$10^2= AO^2 +8^2$

$100= AO^2+64$

$AO^2=100-64$

$AO^2=36$

AO=$\sqrt{36}$

AO=6 cm

∴ AC=AO+OC

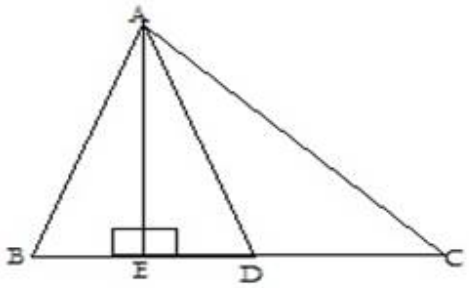
AC=6+6

AC=12 cm

16. Question

In an acute-angled triangle, express a median in terms of its sides.

Answer



We have

In ∆ABC, AD is median

AE⊥BC

In ∆AEB

$AB^2=AE^2+BE^2$

$AB^2=AD^2-DE^2+(BD-DE)^2$

$AB^2=AD^2-DE^2+BD^2-2xBDxDE+DE^2$

$AB^2=AD^2+BD^2-2xBDxDE$

$AB^2=AD^2+BC^2/4-BCxDE$ ……. (I) [GIVEN BC=2BD]

In ∆AEC

$AC^2=AE^2+EC^2$

$AC^2=AD^2-DE^2+ (DE+CD)^2$

$AC^2=AD^2-DE^2+2CDxDE$

$AC^2=AD^2+BC^2/4+BCxDE$ …….(II) [BC=2CD]

By adding equ. (i) and (ii) we get

$AB^2+AC^2=2AD^2+BC^2/2$

$2AB^2+2AC^2=4AD^2+BC^2$ [MULTIPLY BY 2]

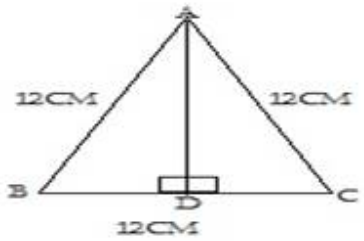
$4AD^2=2AB^2+2AC^2-BC^2$

$AD^2=2AB^2+2AC^2-BC^2$

17. Question

Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

Answer



ΔABC is an equilateral triangle with side 12cm

AE⊥BC

In ΔABD andΔACD

$\angle ADB=\angle ADC=90°$

AB=AC=12cm

AD=AD (COMMON)

$\Delta ABD\cong \Delta ACD$

$AD^2+BD^2=AB^2$

$AD^2+6^2=12^2$

$AD^2+36=144$

$AD^2=144-36$

$AD^2=108$

$AD=\sqrt{108}$

AD=10.39 cm

18. Question

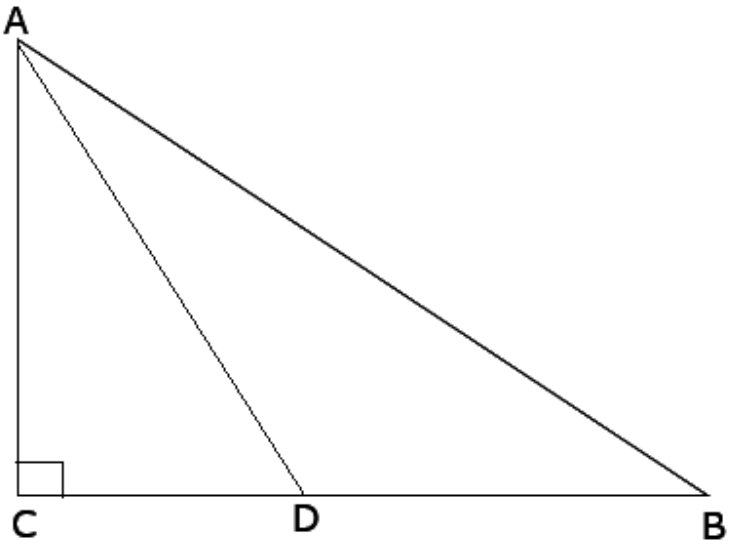
In right-angled triangle ABC in which $\angle C = 90°$, if D is the mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$.

Answer

Given: In right-angled triangle ABC in which $\angle C = 90^\circ$, if D is the mid-point of BC.

To prove: $AB^2 = 4AD^2 - 3AC^2$

Solution:



We have

$\angle C=90^\circ$ and D is the midpoint of BC

In ΔABC

$AB^2=AC^2+BC^2$

As BC = CD + BD D is the mid point of BC$\Rightarrow$ CD = BDSo,$AB^2=AC^2+ (CD + CD)^2$

$\Rightarrow$ $AB^2=AC^2+ (2CD)^2$

$\Rightarrow$ $AB^2=AC^2+4CD^2$

Also In $\Delta ACD AD^2=AC^2+ CD^2\Rightarrow$ $CD^2 = AD^2 - AC^2$So,

$\Rightarrow$ $AB^2=AC^2+4(AD^2-AC^2)$

$AB^2=AC^2+4AD^2-4AC^2$

$AB^2=4AD^2-3AC^2$

19. Question

In Fig. 4.220, D is the mid-point of side BC and $AE \perp BC$. If BC = a, AC = b, AB = c, ED = x, AD = p and AE = h, prove that:

(i) $b^2 = p^2 + a + \frac{a}{4}^2$ (ii) $c^2 = p^2 - ax + \frac{a}{4}^2$

(iii) $b^2 + c^2 = 2p^2 + \frac{a}{2}^2$

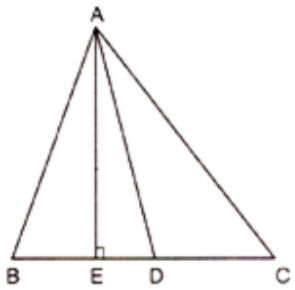


Fig. 4.220

Answer

We have

D is the midpoint of BC

(i) In ΔAEC

$AC^2=AE^2+EC^2$

$b^2=AE^2+(ED+DC)^2$

$b^2=AD^2+DC^2+2xEDxDC$ (Given BC=2CD)

$b^2=p^2+(a/2)^2+2(a/2)x$

$b^2=p^2+a^2/4+ax$

$b^2=p^2+ax+a^2/4$ …….. (i)

(ii) In ΔAEB

$AB^2=AE^2+BE^2$

$c^2=AD^2-ED^2+(BD-ED)^2$

$c^2=p^2-ED^2+BD^2+ED^2-2BDxED$

$c^2=P^2+(a/2)^2-2(a/2)^2x$

$c^2=p^2-ax+a^2/4$ ……..(ii)

(iii) Adding equ. (i)and(ii) we get

$b^2+c^2=2p^2+a^2/2$

20. Question

In Fig. 4.221, $\angle B < 90°$ and segment $AD \perp BC$, show that

(i) $b^2 = h^2 + a^2 + x^2 - 2ax$

(ii) $b^2 = a^2 + c^2 - 2ax$

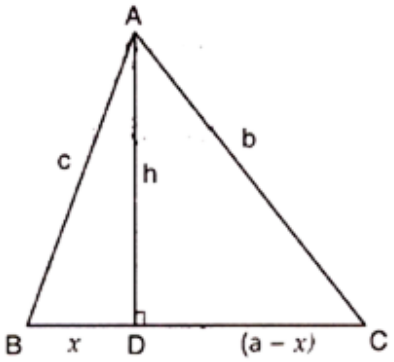


Fig. 4.221

Answer

In ΔADC

$AC^2=AD^2+DC^2$

$b^2=h^2+(a-x)^2$

$b^2=h^2+a^2-2ax+x^2$ ..... (i)

$b^2=h^2+x^2-2ax$

$b^2=a^2+ (h^2+x2)-2ax$ (from equ.i)

$b^2=a^2+c^2-2ax$ [$h^2+x^2=c^2$]

21. Question

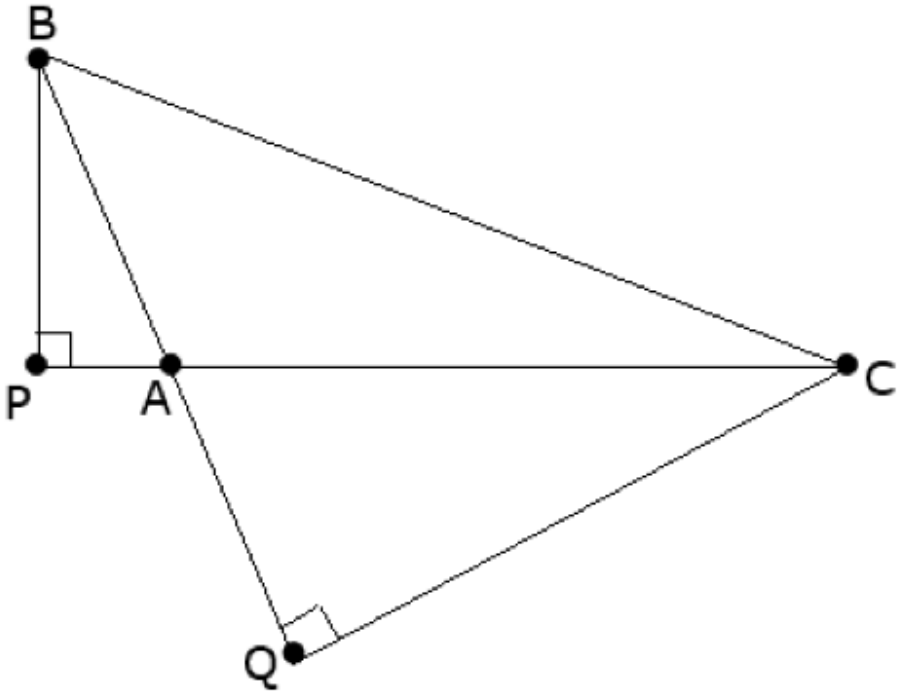
In ΔABC, $\angle A$ is obtuse, $PB \perp AC$ and $QC \perp AB$. Prove that:

(i) $AB \times AQ = AC \times AP$

(ii) $BC^2 = (AC \times CP + AB \times BQ)$

Answer

Draw the diagram according to given questions.



(I) In ⊿APB and ⊿AQC∠A = ∠A (common)∠P = ∠Q (both 90°)∴ ⊿APB~⊿AQC [By AA similarity]

$\Rightarrow \frac{AP}{AQ} = \frac{AB}{AC}$ {Corresponding part of similar triangle are proportional}

AP x AC = AQ x AB …….(1)

(II)

In ⊿BPCBy pythagoras theoram,

$BC^2 = BP^2 + PC^2$Also in ⊿BPA

$BP^2 = AB^2 - AP^2$Also PC = PA + AC

$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2$

Apply the theorem $(a + b)^2 = a^2 + b^2 + 2ab$ in $(AP + AC)^2$

$\Rightarrow BC^2 = AB^2 - AP^2 + AP^2 + AC^2 + 2AP \times AC$

$BC^2 = AB^2 + AC^2 + 2AP \times AC$ ……..(ii)

In ⊿BQC

$BC^2 = CQ^2 + BQ^2$

$BC^2 = AC^2 - AQ^2 + (AB + AQ)^2$

$BC^2 = AC^2 - AQ^2 + AB^2 + 2AB \times AQ$

$BC^2 = AC^2 + AB^2 + AQ^2 + 2AB \times AQ$ …….(iii)

Adding equ. (ii)and(iii)

$BC^2 + BC^2 = AB^2 + AC^2 + 2AP \times AC + AC^2 + AB^2 + AQ^2 + 2AB \times AQ$

$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$

$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$

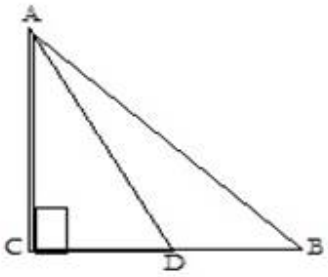
$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$

$\Rightarrow BC^2 = AC \times PC + AB \times BQ$Hence proved.

22. Question

In a right $\triangle ABC$ right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4(AD^2 - AC^2)$.

Answer



We have

$\angle C = 90^\circ$ and D is the midpoint of BC

$LHS = BC^2$

$= (2CD)^2$

$= 4CD^2$

$= 4(AD^2 - AC^2) = RHS$

23. Question

In a quadrilateral ABCD, $\angle B < 90^\circ$, $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^\circ$.

Answer

We have

$\angle B = 90^\circ$ and

$AD^2 = AB^2 + BC^2 + CD^2$ (Given)

But $AB^2 + BC^2 = AC^2$

$AD^2 = AC^2 + CD^2$

By converse of by Pythagoras

$\angle ACD = 90^\circ$

24. Question

In an equilateral ΔABC, $AD \perp BC$, prove that $AD^2 = 3BD^2$.

Answer

We have $\triangle$ ABC is an equilateral triangle and AD⊥BC

In $\triangle$ ADB$\triangle$ ADC

$\angle ADB = \angle ADC = 90^\circ$ AB=AC (Given)

AD=AD (Common)

$\triangle$ ADB≅$\triangle$ ADC (By RHS condition)

∴ BD=CD=BC/2 ……. (i)

In $\triangle$ ABD

$BC^2 = AD^2 + BD^2$

$BC^2 = AD^2 + BD^2$ [Given AB=BC]

$(2BD)^2 = AD^2 + BD^2$ [From (i)]

$4BD^2 - BD^2 = AD^2$

$AD^2 = 3BD^2$

25. Question

ΔABC is a right triangle right-angled at A and $AC \perp BD$. Show that

(i) $AB^2 = BC.BD$ (ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD \cdot CD$ (iv) $\frac{AB^2}{AC^2} = \frac{BD}{DC}$

Answer

(i) In $\triangle$ABD and In $\triangle$CAB

$\angle DAB = \angle ACB = 90^\circ$

$\angle ABD = \angle CBA$ [Common]

$\angle ADB = \angle CAB$ [remaining angle]

So, $\triangle$ADB≅$\triangle$CAB [By AAA Similarity]

∴ AB/CB=BD/AB

$AB^2 = BC \times BD$

(ii)

Let <CAB= x

InΔCBA=180-90°-x

<CBA=90°-x

Similarly in ΔCAD

<CAD=90°-<CAD=90°-x

<CDA=90°-<CAB

=90°-x

<CDA=180°-90°-(90°-x)

<CDA=x

Now in ΔCBA and ΔCAD we may observe that

<CBA=<CAD

<CAB=<CDA

<ACB=<DCA=90°

Therefore ΔCBA~ΔCAD (by AAA rule)

Therefore AC/DC=BC/AC

AC^2=DCxBC

(iii) In DCA and ΔDAB

<DCA=<DAB (both angles are equal to 90°)

<CDA=. <ADB (common)

<DAC=<DBA

ΔDCA= ΔDAB (AAA condition)

Therefore DC/DA=DA/DB

AD^2=BDxCD

(iv) From part (I) AB^2=CBxBD

From part (II) AC^2=DCxBC

Hence AB^2/AC^2=CBxBD/DCxBC

AB^2/AC^2=BD/DC

Hence proved

26. Question

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer

Let OB be the pole and AB be the wire.

$AB^2 = OB^2 + OA^2$

$24^2 = 18^2 + OA^2$

$OA^2 = 576 - 324$

$OA^2 = 252$

$AO = \sqrt{252}$

$AO = 6\sqrt{7}$ m.

Distance from base $= 6\sqrt{7}$ m

27. Question

An aeroplane leaves an airport and files due north at a speed of 1000 km/hr. At the same time, another aeroplane leaves the same airport and files due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer

Distance traveled by the plane flying towards north in 11/2 hrs

$= 1000 \times 1\frac{1}{2} = 1500$km

Similarly Distance traveled by the plane flying towards west in 11/2hrs

$= 1200 \times 1\frac{1}{2} = 1800$ km

Let this distance is represented by OA and OB

Distance between these place after $1^1/_2$hrs AB$= \sqrt{OA^2 + OB^2}$

$= \sqrt{\{1500\}2 + (1000)2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$

$= 300 \times 7.8102$

$= 2343.07$ km

So, distance between these places will be 2343 km (Approx) km, after 1 1/2 hrs

28. Question

Determine whether the triangle having sides (a – 1) cm, $2\sqrt{a}$ cm and (a + 1) cm is a right angled triangle.

Answer

Let ABC be the triangle

Where $AB=(a-1)^2$ cm

$BC=2\sqrt{a}$ cm

CA=(a+1) cm

$AB^2=(a-1)^2=a^2+1-2a$

$BC^2=(2\sqrt{a})^2=4a^2$

$CA^2=(a+1)^2=a^2+1+2a$

Hence $AB^2+BC^2=AC^2$

SO ΔABC is a right angles triangle at B

CCE - Formative Assessment

1. Question

State basic proportionality theorem and its converse.

Answer

Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

2. Question

In the adjoining figure, find AC.

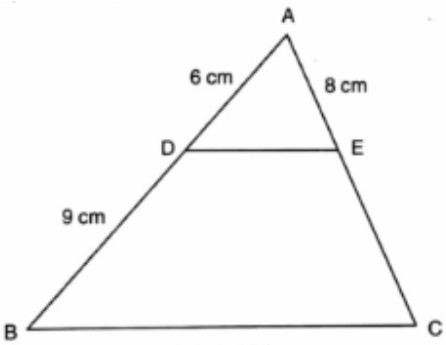


Fig. 4.234

Answer

From the given figure ΔABC, DE || BC.

Let EC = x cm.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the

same ratio.

Then $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{6}{9} = \frac{8}{x}$

$\Rightarrow x = \frac{8(9)}{6}$

⇒ x = 12 cm = EC

Here, AC = AE + EC

⇒ AC = 8 + 12 = 20 cm

∴ AC = 20 cm

3. Question

In the adjoining figure, if AD is the bisector of ∠A, what is AC?

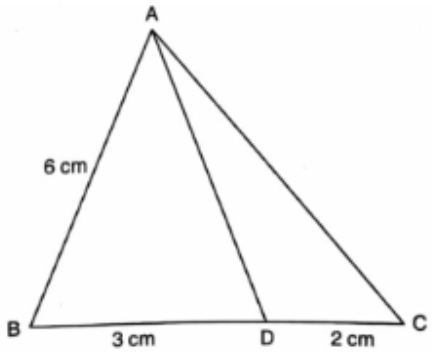


Fig. 4.235

Answer

Given AD is the bisector of ∠A in ΔABC. Let AC be x cm.

We know that the angle bisector theorem states that the internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.

$\Rightarrow \frac{AB}{AC} = \frac{DB}{DC}$

$\Rightarrow \frac{6}{x} = \frac{3}{2}$

$\Rightarrow x = \frac{6(2)}{3}$

⇒ x = 4 cm

∴ AC = 4 cm

4. Question

State AAA similarity criterion.

Answer

AAA similarity criterion: In two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

5. Question

State SSS similarity criterion.

Answer

SSS similarity criterion: If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

6. Question

State SAS similarity criterion.

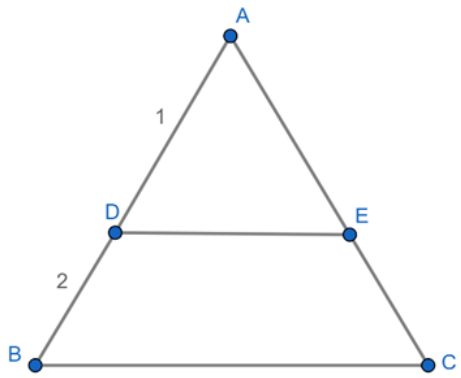
Answer

SAS similarity criterion: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

7. Question

In the adjoining figure, DE is parallel to BC and AD = 1 cm, BD = 2 cm. What is the ratio of the area of A ABC to the area of A ADE?

Answer



Given DE || BC, AD = 1 cm and DB = 2 cm.

So, AB = 3 cm.

In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

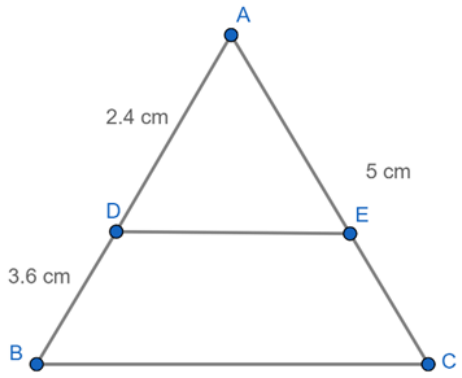
$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{(AB)^2}{(AD)^2} = \frac{3^2}{1^2} = \frac{9}{1}$$

∴ ar (ΔABC): ar (ΔADE) = 9: 1

8. Question

In the figure given below $DE \| BC$. If AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm. Find AE.

Answer



Given DE || BC, AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm.

We have to find AE.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{AC - AE}$$

$$\Rightarrow \frac{2.4}{3.6} = \frac{AE}{5 - AE}$$

⇒ 2.4 (5 – AE) = 3.6 AE

⇒ 12 – 2.4 AE = 3.6 AE

⇒ 12 = 3.6 AE + 2.4 AE

⇒ 12 = 6 AE

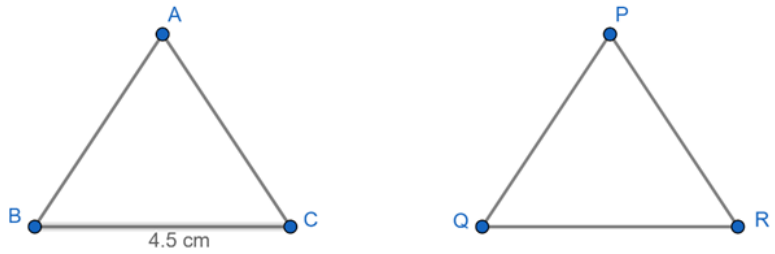
⇒ AE = 12/6

∴ AE = 2 cm

9. Question

If the areas of two similar triangles ABC and PQR are in the ratio 9 : 16 and BC = 4.5 cm, what is the length of QR?

Answer



Given ΔABC ~ ΔPQR, ar (ΔABC): ar (ΔPQR) = 9: 16 and BC = 4.5 cm

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(BC)^2}{(QR)^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{(QR)^2}$$

$$\Rightarrow QR^2 = \frac{20.25(16)}{9}$$

$\Rightarrow QR^2 = 2.25\ (16)$

$\Rightarrow QR^2 = 36$

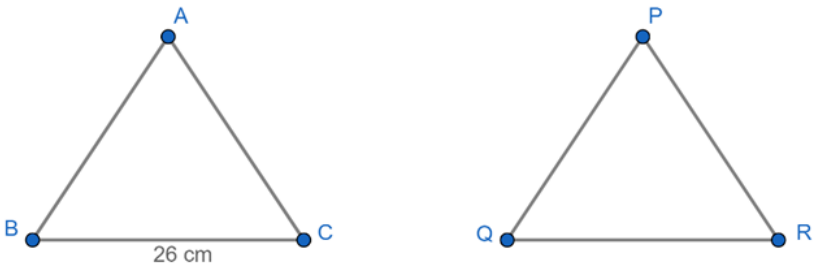
$\Rightarrow QR = 6$

∴ The length of QR is 6 cm.

10. Question

The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, what is the length of the longest side of the smaller triangle?

Answer



Given ΔABC ~ ΔPQR, ar (ΔABC): ar (ΔPQR) = 169: 121 and BC = 26 cm

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(BC)^2}{(QR)^2}$$

$$\Rightarrow \frac{169}{121} = \frac{(26)^2}{(QR)^2}$$

$$\Rightarrow QR^2 = \frac{26(26)(121)}{169}$$

$\Rightarrow QR^2 = 4\ (121)$

$\Rightarrow QR^2 = 484$

$\Rightarrow QR = 22$

∴ The length of QR is 22 cm.

11. Question

If ABC and DEF are similar triangles such that $\angle A$ = 57° and $\angle E$ = 73°, what is the measure of $\angle C$?

Answer

Given ABC and DEF are two similar triangles, ∠A = 57° and ∠E = 73°

We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In ΔABC and ΔDEF,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and ∠A = ∠D, then ΔABC ~ ΔDEF

So, ∠A = ∠D

⇒ ∠D = 57° … (1)

Similarly, ∠B = ∠E

⇒ ∠B = 73° … (2)

We know that the sum of all angles of a triangle is equal to 180°.

⇒ ∠A + ∠B + ∠C = 180°

⇒ 57° + 73° + ∠C = 180°

⇒ 130° + ∠C = 180°

⇒ ∠C = 180° - 130° = 50°

∴ ∠C = 50°

12. Question

If the altitude of two similar triangles are in the ratio 2 : 3, what is the ratio of their areas?

Answer

Given altitudes of two similar triangles are in ratio 2: 3.

Let first triangle be ΔABC and second triangle be ΔPQR.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(2)^2}{(3)^2}$$

∴ ar (ΔABC): ar (ΔPQR) = 4: 9

13. Question

If ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{4}$, then write Area ($\Delta ABC$): Area ($\Delta DEF$).

Answer

Given that ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{4}$

Here, the corresponding sides are given proportional.

We know that two triangles are similar if their corresponding sides are proportional.

And we know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(3)^2}{(4)^2}$$

∴ Area (ΔABC): Area (ΔDEF) = 9: 16

14. Question

If ΔABC and ΔDEF are similar triangles such that AB = 3 cm, BC = 2 cm CA = 2.5 cm and EF = 4 cm, write the perimeter of ΔDEF.

Answer

Given that ΔABC and ΔDEF are similar triangles such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm and EF = 4 cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{3}{2} = \frac{DE}{4}$$

⇒ DE = 6 cm … (1)

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{2.5}{2} = \frac{DF}{4}$$

⇒ DF = 5 cm … (2)

Then, perimeter of ΔDEF = DE + EF + DF = 6 + 4 + 5

∴ Perimeter of ΔDEF = 15 cm

15. Question

State Pythagoras theorem and its converse.

Answer

Pythagoras Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Converse of Pythagoras Theorem: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to first side is a right angle.

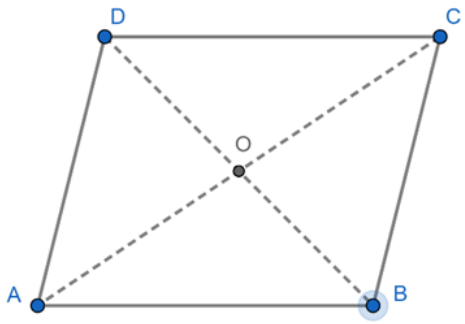
16. Question

The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus. [CBSE 2008]

Answer

Given the lengths of the diagonals of a rhombus are 30 cm and 40 cm.

Let the diagonals AC and BD of the rhombus ABCD meet at point O.



We know that the diagonals of the rhombus bisect each other perpendicularly.

Also we know that Pythagoras theorem states that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Consider right triangle AOD,

$\Rightarrow AD^2 = AO^2 + OD^2$

$= 15^2 + 20^2$

$= 225 + 400$

$= 625$

$\Rightarrow AD = 25$ cm

$\therefore$ The side of the rhombus is 25 cm.

17. Question

In Fig. 4.236, $PQ \| BC$ and AP : PB = 1 : 2. Find $\frac{area\ (\Delta APQ)}{area\ (\Delta ABC)}$ [CBSE 2008]

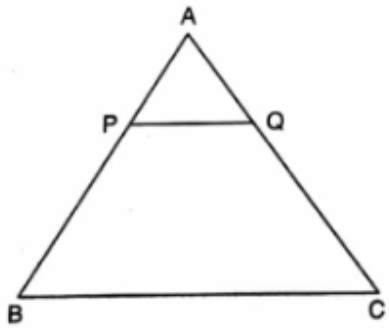


Fig. 4.236

Answer

Given in the given figure PQ || BC and AP: PB = 1: 2

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Since Δ APQ and ΔABC are similar, $\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

Given $\frac{AP}{PB} = \frac{1}{2}$

⇒ PB = 2AP

So, $\frac{AP}{AB} = \frac{AP}{AP+PB} = \frac{AP}{AP+2AP} = \frac{1}{3}$

we know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{(AP)^2}{(AB)^2}$$

$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

∴ Area (ΔAPB): Area (ΔABC) = 1: 9

18. Question

In Fig. 4.237, LM = LN = 46°. Express x in terms of a, b and c where a, b, c are lengths of LM, MN and and NK respectively.

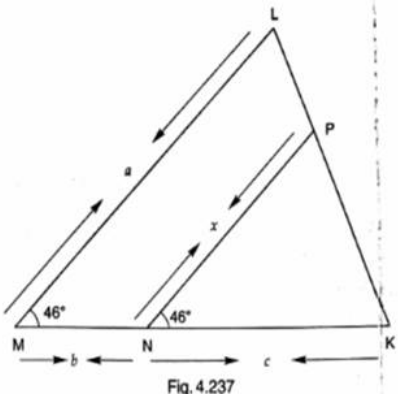


Fig. 4.237

Answer

Given ∠M = ∠N = 46°

It forms a pair of corresponding angles, hence LM || PN.

In ΔLMK and ΔPNK,

∠LMK = ∠PNK [corresponding angles]

∠MLK = ∠NPK [corresponding angles]

∠K = ∠K [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔLMK ~ ΔPNK

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{ML}{NP} = \frac{MK}{NK}$$

$$\Rightarrow \frac{a}{x} = \frac{b+c}{c}$$

$$\therefore x = \frac{ac}{b+c}$$

19. Question

In Fig. 4.238, S and T are points on the sides PQ and PR respectively of A PQR such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR. [CBSE 2010]

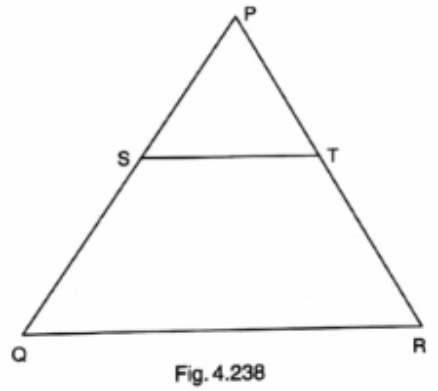


Fig. 4.238

Answer

Given ST || QR, TR = 4 cm and PT = 2 cm.

So, PR = 6 cm.

In ΔPST and ΔPQR,

∠PST = ∠PQR [corresponding angles]

∠PTS = ∠PRQ [corresponding angles]

∠P = ∠P [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔPST ~ ΔPQR

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta PST)}{ar(\Delta PQR)} = \frac{(PT)^2}{(PR)^2} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$$

∴ ar (ΔPST): ar (ΔPQR) = 1: 9

20. Question

In Fig. 4.239, ΔAHK is similar to ΔABC. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. [CBSE 2010]

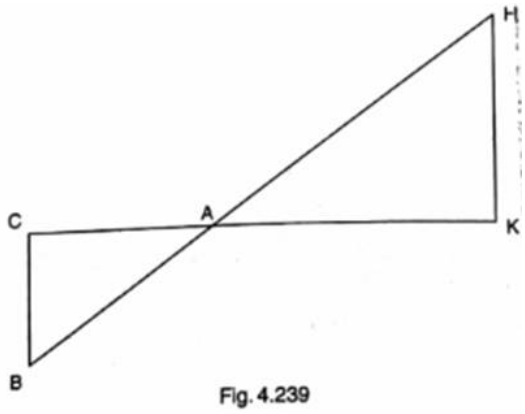


Fig. 4.239

Answer

Given ΔAHK ~ ΔABC, AK = 10 cm, BC = 3.5 cm and HK = 7 cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AC}{AK} = \frac{BC}{HK}$$

$$\Rightarrow \frac{AC}{10} = \frac{3.5}{7}$$

∴ AC = 5 cm

21. Question

In Fig. 4.240, $DE \| BC$ in ΔABC such that BC = 8 cm, AB = 6 cm and DA =1.5 cm. Find DE.

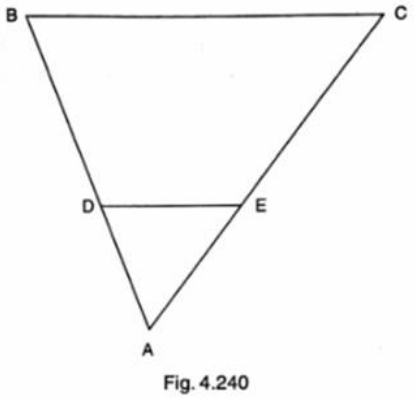


Fig. 4.240

Answer

Given DE || BC, BC = 8 cm, AB = 6 cm and DA = 1.5 cm.

So, PR = 6 cm.

In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{BC}{DE} = \frac{AB}{DA}$$

$$\Rightarrow \frac{8}{DE} = \frac{6}{1.5}$$

∴ DE = 2 cm

22. Question

In Fig. 4.241, $DE \| BC$ and AD = $\frac{1}{2}$ BD. If BC = 4.5 cm, find DE.

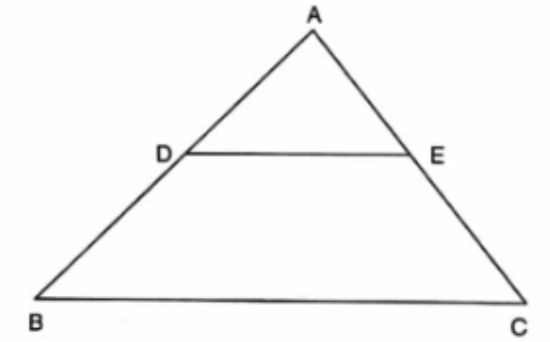


Fig. 4.241

Answer

Given DE || BC, AD = 1/2 BD and BC = 4.5 cm

In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{AD}{AD+BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{\frac{1}{2}BD}{\frac{1}{2}BD+BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1}{3} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1}{3} = \frac{DE}{4.5}$$

∴ DE = 1.5 cm

1. Question

A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is

A. 100 m

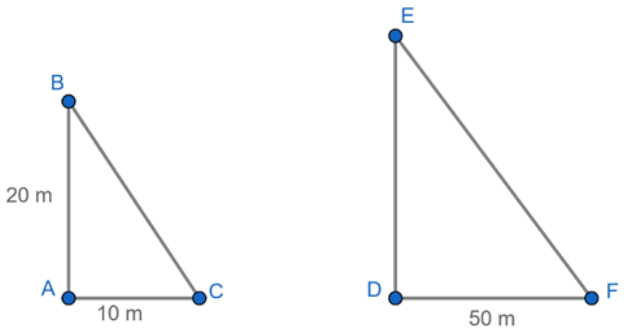
B. 120 m

C. 25 m

D. 200 m.

Answer

Given A vertical stick 20 m long casts a shadow 10 m long on the ground and a tower casts a shadow 50 m long on the ground.



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

In ΔABC and ΔDEF,

∠A = ∠D = 90°, ∠C = ∠F

∴ ΔABC ~ ΔDEF

We know that if two triangles are similar then their sides are proportional.

14. Co-ordinate Geometry

Exercise 14.1

1. Question

On which axis do the following points lie?

(i) P (5, 0) (ii) Q (0 -2)

(iii) R (- 4, 0) (iv) S (0, 5)

Answer

P on x-axis, since ordinate is zero.

Q on y-axis, since abscissa is zero.

R on x-axis, since ordinate is zero.

S on y-axis, since abscissa is zero.

2. Question

Let ABCD be a square of side 2a. Find the coordinates of the vertices of this square when

(i) A coincides with the origin and AB and AD are along OX and OY respectively.

(ii) The centre of the square is at the origin and coordinate axes are parallel to the sides AB and AD respectively.

Answer

(i) Since each side of square is 2a.

Coordinates of A are (0, 0), since it coincides with origin.

Coordinates of B are (2a, 0), for a point along x-axis ordinate is zero.

Coordinates of C are (2a, 2a), since this point is equi-distance from x-axis and y-axis.

Coordinates of D are (0, 2a), since abscissa is zero and ordinate is 2a.

(ii) Each side of square is a units.

Coordinates of A are (a, a), since this point lies in Ist coordinate.

Coordinates of B are (-a, a), since this point lies in IInd coordinate.

Coordinates of C are (-a, -a), since this point lies in IIIrd coordinate.

Coordinates of D are (a, -a), since this point lies in IVth coordinate.

3. Question

The base PQ of two equilateral triangles PQR and PQR' with side 2a lies along y-axis such that the mid-point of PQ is at the origin. Find the coordinates of the vertices R and R' of the triangles.

Answer

$R\ \left(\sqrt{3}a, 0\right)$, $R'(-\sqrt{3}a, 0)$

Since PQ is the base of two equilateral triangles with side 2a and mid-point of PQ is at origin.

Therefore point R lies on positive x-axis and point R' lies on negative y-axis.

$OR^2 = (2a)^2 - a^2$

$OR^2 = 4a^2 - a^2$

$OR = \sqrt{3}a$

Therefore coordinates of R are ($\sqrt{3}a$, 0) and R' (0, $\sqrt{3}a$)

Exercise 14.2

1. Question

Find the distance between the following pair of points:

(i) (-6,7) and (-1, -5)

(ii) (a + b, b + c) and (a - b, c - b)

(iii) (a sin a, - b cos a) and (-a cos a, b sin a)

(iv) (a, 0) and (0, b)

Answer

(i) (-6,7) and (-1, -5)

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance = $\sqrt{(-1 + 6)^2 + (-5 - 7)^2}$

Distance = $\sqrt{(5)^2 + (-12)^2}$

Distance = $\sqrt{25 + 144}$

Distance = $\sqrt{169}$ = 13 units

Thus, the distance between the points (-6,7) and (-1, -5) is 13 units

(ii) (a + b, b + c) and (a - b, c - b)

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance = $\sqrt{\{(a - b) - (a + b)\}^2 + \{(c - b) - (b + c)\}^2}$

Distance = $\sqrt{\{a - b - a - b\}^2 + \{c - b - b - c\}^2}$

Distance = $\sqrt{\{-2b\}^2 + \{-2b\}^2}$

Distance = $\sqrt{8b^2}$ = $2\sqrt{2}b$ units

Thus, the distance between these points is (a + b, b + c) and (a - b, c - b) is $2\sqrt{2}b$ units.

(iii) (a sin a, - b cos a) and (-a cos a, b sin a)

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance = $\sqrt{(-acosa - asina)^2 + (bsina + cosa)^2}$

Distance = $\sqrt{a^2cos^2a + a^2sin^2a + 2a^2sinacosa + b^2sin^2a + b^2cos^2a + 2b^2sinacosa}$

Distance = $\sqrt{a^2(cos^2a + sin^2a) + b^2(sin^2a + cos^2a) + 2sinacosa(a^2 + b^2)}$

Because $cos^2a + sin^2a$ =1 and $2sinacosa = sin2a$, we get,

Distance = $\sqrt{a^2 + b^2 + sin2a(a^2 + b^2)}$

Or Distance = $\sqrt{a^2 + b^2(1 + sin2a)}$

Thus, the distance between points (a sin a, - b cos a) and (-a cos a, b sin a) is $\sqrt{a^2 + b^2(1 + sin2a)}$

(iv) (a, 0) and (0, b)

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance = $\sqrt{(0 - a)^2 + (b - 0)^2}$

Distance = $\sqrt{a^2 + b^2}$Thus, the distance between points (a, 0) and (0, b) is $\sqrt{a^2 + b^2}$

2. Question

Find the value of a when the distance between the points (3, a) and (4, 1) is $\sqrt{10}$.

Answer

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$

$\sqrt{1 + (1-a)^2}$ = 10

$1 + 1 - 2a + a^2 = 10$

a^2 - 2a - 8 = 0

a^2 - 4a + 2a – 8 = 0

a(a - 4) + 2(a - 4) = 0

(a - 4) (a + 2) = 0

a = 4 and a = -2

3. Question

If the points (2, 1) and (1,-2) are equidistant from the point (x, y), show that x + 3y = 0.

Answer

Distance from point (2, 1) = Distance from point (1, -2)

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$

Square roots are cancelled, therefore

$$(x-2)^2+(y-1)^2=(x-1)^2+(y+2)^2$$

$$x^2-4x+4+y^2-2y+1=x^2-2x+1+y^2+4y+4$$

$$-4x+2x-2y-4y=0$$

$$x+3y=0$$

4. Question

Find the values of x, y if the distances of the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

Answer

Given: the distances of the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

To find: the values of x, y

Solution:distances of the point (x, y) from (-3, 0) is Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$4=\sqrt{(-3-x)^2+(0-y)^2}$$

$$\sqrt{(-3-x)^2+(-y)^2}=4$$

$$(-3-x)^2+(-y)^2=16$$

$$9+6x+x^2+y^2=16$$

$$6x+x^2+y^2=7 \text{ ..... (1)}$$

distances of the point (x, y) from (3, 0) is

Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$4=\sqrt{(3-x)^2+(0-y)^2}$$

$$\sqrt{(3-x)^2+(-y)^2}=4$$

$$(3-x)^2+(-y)^2=16$$

$$9-6x+x^2+y^2=16$$

$$-6x+x^2+y^2=7 \text{ ..... (2)}$$

Subtract eq 1 from eq 2 to get,⇒ $-6x + x^2 + y^2 - (6x + x^2 + y^2) = 7 - 7$

⇒ $-6x + x^2 + y^2 - 6x - x^2 - y^2 = 0$

⇒ $-12 x = 0$ ⇒ $x = 0$Putting the value of x in eq 1 we get,$6x + x^2 + y^2 = 7$

⇒ $6(0) + 0^2 + y^2 = 7$

⇒ $y^2 = 7$

⇒ $y=\pm\sqrt{7}$

Hence, X=0, $y=\pm\sqrt{7}$

5. Question

The length of a line segment is of 10 units and the coordinates of one end-point are (2,-3). If the abscissa of the other end is 10, find the ordinate of the other end.

Answer

Let the ordinate of other end is k

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$10 = \sqrt{(10-2)^2 + (k+3)^2}$

On squaring both sides, we get

$100 = (10-2)^2 + (k+3)^2$

$100 = 64 + k^2 + 6k + 9$

$k^2 + 6k - 27 = 0$

$k^2 + 9k - 3k - 27 = 0$

$k(k+9) - 3(k+9) = 0$

$(k-3)(k+9) = 0$

$k = 3;\ k = -9;$

Therefore ordinates are 3, -9

6. Question

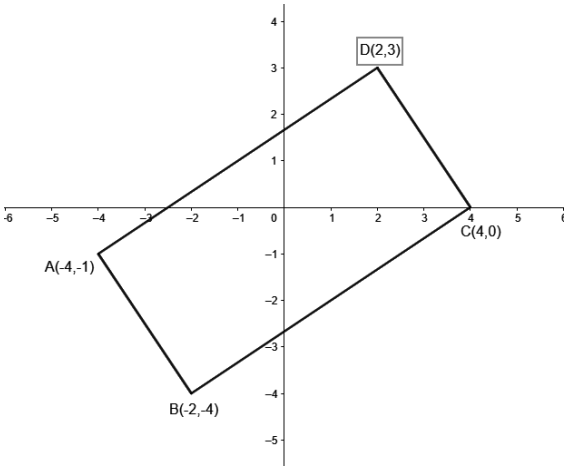
Show that the points A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3) are the vertices points of a rectangle.

Answer

Given: the points A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3)

To prove: the points are the vertices points of a rectangle.

Solution: Vertices of rectangle ABCD are: A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3)



Length of sides= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2+4)^2 + (-4+1)^2}$ = $\sqrt{4+9}$ = $\sqrt{13}$ units

Length of side BC = $\sqrt{(4+2)^2+(0+4)^2}$ = $\sqrt{36+16}$ = $\sqrt{52}$ = $2\sqrt{13}$ units

Length of side CD = $\sqrt{(2-4)^2+(3-0)^2}$ = $\sqrt{4+9}$ = $\sqrt{13}$ units

Length of side AD = $\sqrt{(2+4)^2+(3+1)^2}$ = $\sqrt{36+16}$ = $\sqrt{52}$ = $2\sqrt{13}$ units

Length of diagonal BD = $\sqrt{(2+2)^2+(3+4)^2}$ = $\sqrt{16+49}$ = $\sqrt{65}$ units

Length of diagonal AC = $\sqrt{(4+4)^2+(0+1)^2}$ = $\sqrt{64+1}$ = $\sqrt{65}$ units

Since opposite sides are equal and diagonal are equal. Therefore given vertices are the vertices of a rectangle.

7. Question

Show that the points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram.

Answer

Vertices of a parallelogram ABCD are: A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) Length of side AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(3-1)^2+(6+2)^2}$ = √(4+64)= √68 units

Length of side BC = $\sqrt{(5-3)^2+(10-6)^2}$ = √(4+16) = √20 units

Length of side CD = $\sqrt{(3-5)^2+(2-10)^2}$ = √(4+64)= √68 units

Length of side DA = $\sqrt{(3-1)^2+(2+2)^2}$ = √(4+16) = √20 units

Length of diagonal BD = $\sqrt{(3-3)^2+(2-6)^2}$ = √16= 4 units

Length of diagonal AC = $\sqrt{(5-1)^2+(10+2)^2}$ = √(16+144) = √160 units

Opposite sides of the quadrilateral formed by the given four points are equal i.e. (AB = CD) & (DA = BC)Also, the diagonals BD & AC are unequal.Therefore, the given points form a parallelogram.

8. Question

Prove that the points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) are the vertices of a square.

Answer

Vertices of a square ABCD are: A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) Length of side AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(4-1)^2+(2-7)^2}$ = $\sqrt{9+25}$ = $\sqrt{34}$ units

Length of side BC = $\sqrt{(-1-4)^2+(-2-1)^2}$ = $\sqrt{25+9}$ = $\sqrt{34}$ units

Length of side CD = $\sqrt{(-4+1)^2+(4+1)^2}$ = $\sqrt{9+25}$ = $\sqrt{34}$ units

Length of side DA = $\sqrt{(-4-1)^2+(4-7)^2}$ = $\sqrt{25+9}$ = $\sqrt{34}$ units

Length of diagonal BD = $\sqrt{(-4-4)^2+(4-2)^2}$ = $\sqrt{64+4}$ = $\sqrt{68}$ units

Length of diagonal AC = $\sqrt{(-1-1)^2+(-1-7)^2}$ = $\sqrt{4+64}$ = $\sqrt{68}$ units

Since opposite sides are equal and diagonal are equal. Therefore given vertices are the vertices of a square.

9. Question

Prove that the points (3, 0), (6, 4) and (- 1, 3) are vertices of a right-angled isosceles triangle.

Answer

Vertices of a triangle ABC are: A(3, 0), B(6, 4) and C (- 1, 3)

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(6-3)^2 + (4-0)^2}$ = $\sqrt{9+16}$ = $\sqrt{25}$ units

Length of side BC = $\sqrt{(-1-6)^2 + (3-4)^2}$ = $\sqrt{49+1}$ = $\sqrt{50}$ units

Length of side AC = $\sqrt{(-1-3)^2 + (3-0)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ units

Since AB = AC, therefore triangle is an isosceles.

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

50 = 25 + 25

50 = 50

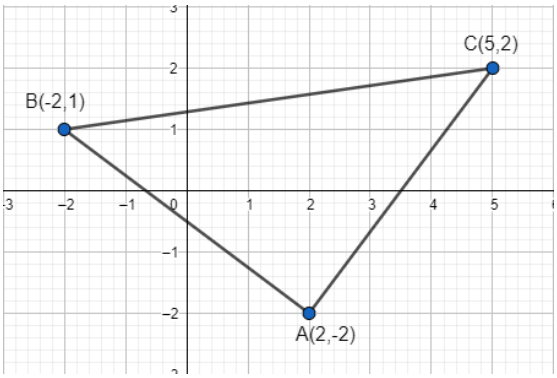
Since $BC^2 = AB^2 + AC^2$; therefore given triangle is right angled triangle.

10. Question

Prove that (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.

Answer

Solution: Vertices of a triangle ABC are: A(2, -2), B(-2, 1) and C(5, 2)



Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2-2)^2 + (1+2)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ units

Length of side BC = $\sqrt{(5+2)^2 + (2-1)^2}$ = $\sqrt{49+1}$ = $\sqrt{50}$ units

Length of side AC = $\sqrt{(5-2)^2 + (2+2)^2}$ = $\sqrt{9+16}$ = $\sqrt{25}$ units

Since AB = AC, therefore triangle is an isosceles.

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

50 = 25 + 25

50 = 50

Since $BC^2 = AB^2 + AC^2$; therefore given triangle is right angled triangle.

Area of right angled triangle = $\frac{1}{2} base \times altitude$

Area of right angled triangle = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ square units

Length of hypotenuse (BC) = $\sqrt{50} = 5\sqrt{2}$ units

11. Question

Prove that the points (2 a, 4 a), (2 a, 6 a) and $\left(2a + \sqrt{3}a, 5a\right)$ are the vertices of an equilateral triangle.

Answer

Vertices of a triangle ABC are: A(2 a, 4 a), B(2 a, 6 a) and C$\left(2a + \sqrt{3}a, 5a\right)$

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(2a - 2a)^2 + (6a - 4a)^2} = \sqrt{(2a)^2}$ = 2a units

Length of side BC = $\sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + (5a - 6a)^2} = \sqrt{3}$ a + a units

Length of side AC = $\sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + (5a - 4a)^2} = \sqrt{3}$ a + a units

The given vertices are not the vertices of an equilateral triangle

12. Question

Prove that the points (2, 3), (-4, -6) and (1, 3/2) do not form a triangle.

Answer

Let the Vertices of a triangle ABC are: A(2, 3), B(-4, -6) and C(1, 3/2)

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-4 - 2)^2 + (-6 - 3)^2} = \sqrt{36 + 81} = \sqrt{117}$ units

Length of side BC = $\sqrt{(1 + 4)^2 + \left(\frac{3}{2} + 6\right)^2} = \sqrt{25 + 56.25} = \sqrt{81.25}$ units

Length of side AC = $\sqrt{(1 - 2)^2 + \left(\frac{3}{2} - 3\right)^2} = \sqrt{1 + 2.25} = \sqrt{2.25}$ units

The given vertices do not form a triangle, since sum of two sides of a triangle are not greater than third side.

13. Question

An equilateral triangle has two vertices at the points (3, 4) and (-2, 3), find the coordinates of the third vertex.

Answer

Given: An equilateral triangle has two vertices at the points (3, 4) and (-2, 3)

To find: the coordinates of the third vertex.

Solution: Let the Vertices of a triangle ABC are A(3, 4) and B (-2, 3), and C(x, y),Since it is equilateral triangle,AB=AC=BCWhere AB,AC and BC are lengths of sides of the given triangle.To find the length of a side use distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$,

Length of side AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(-2-3)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$ units

Length of side BC = $\sqrt{(x+2)^2+(y-3)^2}$ units

Length of side AC = $\sqrt{(x-3)^2+(y-4)^2}$ unitsNow AB=AC

$\Rightarrow (AB)^2 = (BC)^2$

$(\sqrt{26})^2 = (\sqrt{(x+2)^2+(y-3)^2}$

$(x+2)^2+(y-3)^2 = 26$

$x^2+4x+4+y^2-6y+9 = 26$

$x^2+4x+y^2-6y = 13(1)$

$(AB)^2 = (AC)^2$

$(\sqrt{26})^2 = (\sqrt{(x-3)^2+(y-4)^2}$

$(x-3)^2+(y-4)^2 = 26$

$x^2-6x+9+y^2-8y+16 = 26$

$x^2-6x+y^2-8y = 1(2)$

On subtracting eqn (2) from (1), we get

$(x^2+4x+y^2-6y)-(x^2-6x+y^2-8y) = 13-1$

$x^2+4x+y^2-6y-x^2+6x-y^2+8y = 13-1$

$4x-6y+6x-8y = 12 10x+2y = 12 \quad 5x+y = 6$ ..... (3)

$(AC)^2 = (BC)^2$

$(x-3)^2+(y-4)^2 = (x+2)^2+(y-3)^2$

$x^2-6x+9+y^2-8y+16 = x^2+4x+4+y^2-6y+9 -4x-2y = 12$ ..... (4)

Solving equations (3) and (4), we get

$x = \frac{4}{3}$; y = $-\frac{2}{3}$

Therefore coordinates of C are ($\frac{4}{3}$, $-\frac{2}{3}$)

14. Question

Show that the quadrilateral whose vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2) is a rhombus.

Answer

Let the Vertices of a quadrilateral are: A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2)

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(3-2)^2 + (4+1)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side BC = $\sqrt{(-2-3)^2 + (3-4)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of side CD = $\sqrt{(-3+2)^2 + (-2-3)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side DA = $\sqrt{(-3-2)^2 + (-2+1)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Since all sides are of equal length, therefore it is a rhombus.

15. Question

Two vertices of an isosceles triangle are (2, 0) and (2, 5). Find the third vertex if the length of the equal sides is 3.

Answer

Vertices of an isosceles are: A(2, 0) and B(2, 5).

Let the third vertex is P(x, y)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side PA = $\sqrt{(x-2)^2 + (y)^2}$ units

Length of side PB = $\sqrt{(x-2)^2 + (y-5)^2}$ units

Since PA = PB

$$\sqrt{(x-2)^2 + (y)^2} = \sqrt{(x-2)^2 + (y-5)^2}$$

On squaring both sides, we get

$$(x-2)^2 + (y)^2 = (x-2)^2 + (y-5)^2$$

$$x^2 - 4x + 4 + y^2 = x^2 - 4x + 4 + y^2 - 10y + 25$$

$$y = \frac{5}{2},$$

Also, PA = 3

$$\sqrt{(x-2)^2 + (y)^2} = 3$$

On squaring both sides, we get

$$(x-2)^2 + (y)^2 = 9$$

$$x^2 - 4x + 4 + y^2 = 9$$

$$x^2 - 4x + y^2 = 5$$

On substituting $y = \frac{5}{2}$,

$$x^2 - 4x + \frac{25}{4} = 5$$

$$x^2 - 4x + \frac{25}{4} - 5 = 0$$

$$x^2 - 4x + \frac{5}{4} = 0$$

Using quadratic formula:

$$x = \frac{\{-b \pm \sqrt{b^2 - 4ac}\}}{2a}$$

$$x = \frac{\{4 \pm \sqrt{16 - 5}\}}{2}$$

$$x = \frac{\{4 \pm \sqrt{11}\}}{2}$$

Therefore coordinates of third vertex are: $\left(\frac{4+\sqrt{11}}{2}, \frac{5}{2}\right)$; $\left(\frac{4-\sqrt{11}}{2}, \frac{5}{2}\right)$

16. Question

Which point on x-axis is equidistant from (5, 9) and (- 4, 6)?

Answer

Since the point is on x-axis, therefore coordinate of y-axis is zero.

Therefore the point is P(k, 0) which is equidistance from A(5, 9) and B(- 4, 6)

PA = PB

$$\sqrt{(5-k)^2 + 9^2} = \sqrt{(-4-k)^2 + 6^2}$$

On squaring both sides

$$(5-k)^2 + 9^2 = (-4-k)^2 + 6^2$$

$$25 - 10k + k^2 + 81 = 16 - 8k + k^2 + 36$$

$$25 - 2k + 81 = 16 + 36 - 25 - 81$$

$$-2k = 16 + 36 - 25 - 81$$

$$k = 27$$

Therefore coordinate is (27, 0)

17. Question

Prove that the points (- 2, 5), (0, 1) and (2, - 3) are collinear.

Answer

Vertices are: A(- 2, 5), B(0, 1) and C(2, - 3)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(0+2)^2+(1-5)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

Length of side BC = $\sqrt{(2-0)^2+(-3-1)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

Length of side AC = $\sqrt{(2+2)^2+(-3-5)^2}$ = $\sqrt{16+64}$ = $\sqrt{80}$ = $4\sqrt{5}$ units

Since length of AB + BC = AC, therefore points are collinear.

18. Question

The coordinates of the point P are (-3, 2). Find the coordinates of the point Q which lies on the line joining P and origin such that OP = OQ.

Answer

Let the coordinates of Point Q are (x, y) and coordinates of origin O are (0, 0)

Since OP = OQ

Points are: P (-3, 2), Q(x, y) and O (0, 0)

Q is the mid point

$$0 = \frac{-3+x}{2}$$

$$x = 3$$

$$0 = \frac{2+y}{2}$$

$$y = -2$$

Therefore coordinates are (3, -2)

19. Question

Which point on y-axis is equidistant from (2, 3) and (-4, 1)?

Answer

Since the point is on y-axis, therefore coordinate of x-axis is zero.

Therefore the point is P(0, k) which is equidistance from A(2, 3) and B(- 4, 1)

PA = PB

$$\sqrt{(2-0)^2+(3-k)^2} = \sqrt{(-4)^2+(1-k)^2}$$

$$\sqrt{4+9+k^2-6k} = \sqrt{16+1+k^2-2k}$$

On squaring both sides, we get

$$-4k = 4$$

$$k = -1$$

Therefore coordinate is (0, -1)

20. Question

The three vertices of a parallelogram are (3, 4), (3, 8) and (9, 8). Find the fourth vertex.

Answer

Consider A(3, 4), B (3, 8) and C(9, 8).

Let the coordinates of fourth vertex are D (x, y)

In a parallelogram diagonals bisect each other

Coordinate of mid point of AC = $X = \frac{3+9}{2} = \frac{12}{2} = 6$

$$Y = \frac{4+8}{2} = \frac{12}{2} = 6$$

Therefore coordinates of mid point of AC are (6, 6)

Coordinate of mid point of BD = $X = \frac{3+x}{2}$

$$Y = \frac{y+8}{2}$$

Coordinates of point D are

$$\frac{3+x}{2} = 6$$

$$x = 12 - 3 = 9$$

$$\frac{y+8}{2} = 6$$

$$y = 12 - 8 = 4$$

Therefore coordinates of fourth vertex D are (9, 4)

21. Question

Find the circumcentre of the triangle whose vertices are (-2, -3), (- 1, 0), (7, - 6).

Answer

Vertices of triangle are A(-2, -3), B(- 1, 0), C(7, - 6)

Let the coordinates of P are (x, y)

PA = PB =PC

PA = PB

$$\sqrt{(x+2)^2 + (y+3)^2} = \sqrt{(x+1)^2 + (y-0)^2}$$

On squaring both sides, we get

$$x^2 + 4x + 4 + y^2 + 6y + 9 = x^2 + 2x + 1 + y^2$$

$$2x + 6y = -12$$

$$x + 3y = -6 \text{.....(1)}$$

PA = PC

$$\sqrt{(x+2)^2+(y+3)^2} = \sqrt{(x-7)^2+(y+6)^2}$$

On squaring both sides, we get

$$x^2 + 4x + 4 + y^2 + 6y + 9 = x^2 - 14x + 49 + y^2 + 12y + 36$$

$$18x - 6y = 72$$

$$3x - y = 12 \text{.....(2)}$$

On solving equations (1) & (2), We get

X = 3 and y = -3

Therefore coordinates are (3, -3)

22. Question

Find the angle subtended at the origin by the line segment whose end points are (0,100) and (10, 0).

Answer

Since the abscissa of first coordinate is zero, therefore this point lies on y-axis. Ordinate of second point is zero, therefore this point lies on y-axis. We know that both the axes are perpendicular to eachother, therefore the angle between these points is 90°.

23. Question

Find the centre of the circle passing through (2, 1), (5, - 8) and (2, - 9).

Answer

Coordinates of points on a circle are A(2,1), B(5,-8) and C(2,-9).

Let the coordinates of the centre of the circle be O(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since the distance of the points A, B and C will be equal from the center, therefore

⇒ OA = OB

$$\sqrt{(x-2)^2+(y-1)^2} = \sqrt{(x-5)^2+(y+8)^2}$$

On squaring both sides, we get

⇒ x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 25 - 10x + y^2 + 64 + 16y

⇒ 6x - 18y - 84 = 0

⇒ x - 3y - 14 = 0 ---------- (1)

Similarly, OC = OB

$$\sqrt{(x-2)^2+(y+9)^2} = \sqrt{(x-5)^2+(y+8)^2}$$

⇒ x^2 + 4 - 4x + y^2 + 81 + 18y = x^2 + 25 - 10x + y^2 + 64 + 16y

⇒ 6x - 2y - 4 = 0

⇒ 3x - y - 2 = 0 ---------- (2)

By solving equations (1) and (2), we get x = -1, y = -5

So, the coordinates of the centre of the circle is (-1, -5).

Radius of the circle = OA = $\sqrt{(-1-2)^2+(-5-1)^2}$

= $\sqrt{9+36}$

= $\sqrt{45}$

= $3\sqrt{5}$ units

24. Question

Find the value of k, if the point P (0, 2) is equidistant from (3, k) and (k, 5).

Answer

Let the point is P(0, 2) which is equidistance from A(3, k) and B(k, 5)

PA = PB

$$\sqrt{(3-0)^2+(k-2)^2}=\sqrt{(k-0)^2+(5-2)^2}$$

On squaring both sides, we get

$$9+k^2+4-4k=\ k^2+9$$

$$-4k=\ -4$$

$$k=\ 1$$

25. Question

If two opposite vertices of a square are (5, 4) and (1, -6), find the coordinates of its remaining two vertices.

Answer

Let ABCD is a square with A(5, 4) and C(1, -6).

Let the coordinates of B are (x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ AB = BC

$$\sqrt{(x-5)^2+(y-4)^2}=\sqrt{(1-x)^2+(-6-y)^2}$$

On squaring both sides, we get

$$x^2-10x+25+y^2-8y+16=\ x^2+1-2x+y^2+12y+36$$

$$-8x-20y=\ -4$$

$$2x-5y=\ 1$$

$x=\frac{1+5y}{2}$.....(1)

In ΔABC, Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 2BC^2$ [Since AB = BC]

$$\sqrt{(1-5)^2+(-6-4)^2} = 2\sqrt{(x-1)^2+(y+6)^2}$$

On squaring both sides, we get

$$16 + 100 = 2(x^2 + 1 - 2x + y^2 + 12y + 36)$$

$$58 = x^2 + 1 - 2x + y^2 + 12y + 36$$

$$21 = x^2 - 2x + y^2 + 12y$$

$$x^2 - 2x + y^2 + 12y = 21 \text{.....(2)}$$

On substituting value of $x = \frac{1+5y}{2}$ from equation (1) in equation (2), we get

$$\left\{\frac{(1+5y)}{2}\right\}^2 - \left\{\frac{2(1+5y)}{2}\right\} + y^2 + 12y = 21$$

$$\frac{1 + 25y^2 + 10y}{4} + 1 + 5y + y^2 + 12y = 21$$

$$1 + 25y^2 + 10y + 4 + 20y + 4y^2 + 48y = 21$$

$$29y^2 + 78y = 79$$

$$29y^2 + 78y - 79 = 0$$

On solving we get y = -3, -1

Substituting these values of y in eqn 1, we get x = 8, -2

Therefore other coordinates are B(8, -3). And D(-2,1)

26. Question

Show that the points (-3, 2), (-5, -5), (2, -3) and (4, 4) are the vertices of a rhombus. Find the area of this rhombus.

Answer

Vertices of the rhombus are: A(-3, 2), B(-5, -5), C(2, -3) and D(4, 4)

We know that diagonals of a rhombus bisect each other, therefore point of intersection of diagonals is:

Abscissa of Mid point of AC = $\frac{2-3}{2} = -\frac{1}{2}$

Ordinate of Mid point of AC = $\frac{-3+2}{2} = -\frac{1}{2}$

Abscissa of Mid point of BD = $\frac{4-5}{2} = -\frac{1}{2}$

Ordinate of Mid point of BD = $\frac{4-5}{2} = -\frac{1}{2}$

Since the diagonals AC and BD bisect each other at O, therefore it is a rhombus.

Length of diagonal AC = $\sqrt{(2+3)^2+(-3-2)^2}$ = $\sqrt{25+25}$ = $\sqrt{50}$ = $5\sqrt{2}$ units

Length of diagonal BD = $\sqrt{(4+5)^2+(4+5)^2}$ = $\sqrt{81+81}$ = $\sqrt{162}$ = $9\sqrt{2}$ units

Area of rhombus = $\frac{1}{2} \times d1 \times d2$ = $\frac{1}{2} \times 5\sqrt{2} \times 9\sqrt{2}$ = 45 sq units

Area of rhombus is 45 sq units

27. Question

Find the coordinates of the circumcentre of the triangle whose vertices are (3, 0), (-1, -6) and (4,-1). Also, find its circumradius.

Answer

Coordinates of points on a circle are A (3, 0), B(-1, -6) and C(4,-1)

Let the coordinates of the centre of the circle be O(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since the distance of the points A, B and C will be equal from the center, therefore

⇒ OA = OC

$$\sqrt{(x-3)^2+(y-0)^2} = \sqrt{(x+1)^2+(y+6)^2}$$

On squaring both sides, we get

$$(x-3)^2+(y-0)^2 = (x+1)^2+(y+6)^2$$

⇒ x^2 + 9 - 6x + y^2 = x^2 + 2x +1 + y^2 + 36 + 12y

⇒ -8x - 12y = 28

⇒ 2x + 3y = -7---------- (1)

Similarly, OC = OB

$$\sqrt{(x-4)^2+(y+1)^2} = \sqrt{(x+1)^2+(y+6)^2}$$

On squaring both sides, we get

$$(x-4)^2+(y+1)^2 = (x+1)^2+(y+6)^2$$

⇒ x^2 + 16 - 8x + y^2 + 1 + 2y = x^2 + 1 +2x + y^2 + 36 + 12y

⇒ -10x - 10y = 20

⇒ x + y = -2 ---------- (2)

Solving eqn (1) and (2), we get

x = 1; y = -3

Coordinates of circum center are (1, -3)

Circum radius of the circle = OA = $\sqrt{(1-3)^2+(3)^2}$

= $\sqrt{4+13}$

= $\sqrt{13}$ units

28. Question

Find a point on the x-axis which is equidistant from the points (7, 6) and (-3, 4).

Answer

points A(7, 6) and B(-3, 4) are equidistance from point P.

Let the coordinates of point are P(x, 0)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = PB

$$\sqrt{(x-7)^2+(0-6)^2}=\sqrt{(x+3)^2+(0-4)^2}$$

On squaring both sides, we get

$(x-7)^2+(0-6)^2=(x+3)^2+(0-4)^2$

$x^2-14x+49+36=x^2+6x+9+16$

$5x=15$

$x=3$

Therefore coordinates are (3, 0)

29 A. Question

Show that the points A(5, 6), B (1, 5), C(2, 1) and D(6, 2) are the vertices of a square.

Answer

Vertices of a quadrilateral are A(5,6), B(1,5), C(2,1) and D(6,2).

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB = $\sqrt{(1-5)^2+(5-6)^2}=\sqrt{(16+1)}=\sqrt{17}$ units

BC = $\sqrt{(2-1)^2+(1-5)^2}=\sqrt{(1+16)}=\sqrt{17}$ units

CD = $\sqrt{(6-2)^2+(2-1)^2}=\sqrt{(16+1)}=\sqrt{17}$ units

DA = $\sqrt{(6-5)^2+(2-6)^2}=\sqrt{(1+16)}=\sqrt{17}$ units

AB = BC = CD = DA

BD = $\sqrt{(6-1)^2+(2-5)^2}=\sqrt{(25+9)}=\sqrt{34}$ units

AC = $\sqrt{(2-5)^2+(1-6)^2}=\sqrt{(9+25)}=\sqrt{34}$ units

All the four sides of the quadrilateral are equal and diagonals are of equal length. Therefore, the given vertices form a square.

29 B. Question

Prove that the points A (2, 3), B (-2, 2), C (-1, -2), and D (3, -1) are the vertices of a square ABCD.

Answer

The Vertices of a quadrilateral are A(5,6), B(1,5), C(2,1) and D(6,2).

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB = $\sqrt{(-2-2)^2+(2-3)^2}$ = $\sqrt{(16+1)}$ = $\sqrt{17}$ units

BC = $\sqrt{(-1+2)^2+(-2-2)^2}$ = $\sqrt{(1+16)}$ = $\sqrt{17}$ units

CD = $\sqrt{(6-2)^2+(2-1)^2}$ = $\sqrt{(16+1)}$ = $\sqrt{17}$ units

DA = $\sqrt{(6-5)^2+(2-6)^2}$ = $\sqrt{(1+16)}$ = $\sqrt{17}$ units

AB = BC = CD = DA

BD = $\sqrt{(6-1)^2+(2-5)^2}$ = $\sqrt{(25+9)}$ = $\sqrt{34}$ units

AC = $\sqrt{(2-5)^2+(1-6)^2}$ = $\sqrt{(9+25)}$ = $\sqrt{34}$ units

All the four sides of the quadrilateral are equal and diagonals are of equal length. Therefore, the given vertices form a square.

30. Question

Find the point on x-axis which is equidistant from the points (-2, 5) and (2,-3).

Answer

Points A(-2, 5) and B(2, -3) are equidistant from point P.

Let the coordinates of point are P(x, 0)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = PB

$$\sqrt{(x+2)^2+(0-5)^2}=\sqrt{(x-2)^2+(0+3)^2}$$

On squaring both sides, we get

$(x+2)^2+(0-5)^2=(x-2)^2+(0+3)^2$

$x^2+4x+4+25=x^2-4x+4+9$

8x = -16x = -2

Hence, coordinates are (-2, 0).

31. Question

Find the value of x such that $PQ=QR$ where the coordinates of P, Q and R are (6,-1), (1, 3) and (x, 8) respectively.

Answer

Coordinates are P(6,-1), Q(1, 3) and R(x, 8)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PQ = QR

$$\sqrt{(1-6)^2+(3+1)^2} = \sqrt{(x-1)^2+(8-3)^2}$$

On squaring both sides, we get

$$(1-6)^2+(3+1)^2 = (x-1)^2+(8-3)^2$$

$$25+16 = x^2-2x+1+25$$

$$x^2-2x-15=0$$

On solving above equation, we get

$$x^2-5x+3x-15=0$$

$$x(x-5)+3(x-5)=0$$

$$(x+3)(x-5)=0$$

$$x=-3$$

$$x=5$$

Therefore x = -3, 5

32. Question

Prove that the points (0, 0), (5, 5) and (-5, 5) are the vertices of a right isosceles triangle.

Answer

Vertices of a quadrilateral are A (0, 0), B(5, 5) and C(-5, 5)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB = $\sqrt{(5-0)^2+(5-0)^2}$ = $\sqrt{(25+25)}$ = $\sqrt{50}$ units

BC = $\sqrt{(-5-5)^2+(5-5)^2}$ = $\sqrt{(100+0)}$ = $\sqrt{100}$ units

CA = $\sqrt{(-5-0)^2+(5-0)^2}$ = $\sqrt{(25+25)}$ = $\sqrt{50}$ units

Since AB = CA

Using Pythagoras theorem

$BC^2 = AC^2 + AB^2$

100 = 50 + 50

100 = 100

Therefore vertices are of right isosceles triangle.

33. Question

If the point P(x, y) is equidistant from the points A(5, 1) and B (1, 5), prove that x = y.

Answer

Coordinates are P(x, y), A(5, 1) and B (1, 5)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = PB

$$\sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x-1)^2+(y-5)^2}$$

On squaring both sides, we get

$(x-5)^2+(y-1)^2 = (x-1)^2+(y-5)^2$

$x^2-10x+25+y^2-2y+1 = x^2-2x+1+y^2-10y+25$

$-8x+8y = 0$

$x = y$ proved

34. Question

Q (0, 1) is equidistant from P (5, -3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Answer

Coordinates are Q(0, 1), P (5, -3) and R (x, 6),

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ QP = QR

$$\sqrt{(5-0)^2+(-3-1)^2} = \sqrt{(x-0)^2+(6-1)^2}$$

On squaring both sides, we get

$(5-0)^2+(-3-1)^2 = (x-0)^2+(6-1)^2$

$25+16 = x^2+25$

$x^2 = 16$

$x = \pm 4$ proved

QR = $\sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

PR = $\sqrt{(4-5)^2+(6+3)^2} = \sqrt{1+81} = \sqrt{82}$ units

35. Question

Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units.

Answer

Given: the distance between the points P (2, -3) and Q (10, y) is 10 units.

To find: The value of y.

Solution: Coordinates are P (2, -3) and Q (10, y)

We use distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between two points.

Since PQ = 10 unitsS0,

$\sqrt{(10-2)^2+(y+3)^2} = 10$

On squaring both sides, we get

$(10-2)^2+(y+3)^2$ = 100

⇒ 8^2 + $(y+3)^2$ = 100

⇒ $64+y^2+6y+9$ = 100

⇒ 73 + y^2 + 6y = 100

⇒ 73 + y^2 + 6y -100 = 0

⇒ $y^2+6y-27$ = 0

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

⇒ y^2 + 9y - 3y - 27 = 0

⇒y(y + 9) - 3(y + 9) = 0

⇒(y - 3)(y + 9) = 0

⇒y = 3, -9

36. Question

Find the centre of the circle passing through (6, -6), (3, -7) and (3, 3).

Answer

Coordinates of points on a circle are A(6, -6), B(3, -7) and C(3, 3).

Let the coordinates of the centre of the circle be O(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since the distance of the points A, B and C will be equal from the center, therefore

⇒ OA = OC

$$\sqrt{(x-6)^2+(y+6)^2}=\sqrt{(x-3)^2+(y-3)^2}$$

On squaring both sides, we get

$(x-6)^2+(y+6)^2$ = $(x-3)^2+(y-3)^2$

$x^2-12x+36+y^2+12y+36$ = $x^2-6x+9+y^2-6y+9$

$x-3y$ = 9.....(1)

Similarly, OA = OB

$$\sqrt{(x-6)^2+(y+6)^2}=\sqrt{(x-3)^2+(y+7)^2}$$

On squaring both sides, we get

$(x-6)^2+(y+6)^2$ = $(x-3)^2+(y+7)^2$

$x^2-12x+36+y^2+12y+36$ = $x^2-6x+9+y^2+14y+49$

$3x + y = 7$ ……..(1)

Solving eqn (1) and (2), we get

x = 3; y = -2

Coordinates of circum center are (3, -2)

37. Question

Two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of other two vertices.

Answer

The coordinates are A(-1, 2) and C(3, 2).

Let the coordinates of the vertex B are (x, y)

AB = BC

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-2)^2}$$

On squaring both sides, we get

$(x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 - 4y + 4$

$x = 1$

In ΔABC

$AB^2 + BC^2 = AC^2$ [Using Pythagoras theorem]

$2AB^2 = AC^2$ [Since AB = BC]

$2[(x+1)^2 + (y-2)^2] = (3+1)^2 + (2-2)^2$

$2[x^2 + 2x + 1 + y^2 - 4y + 4] = 16$

$x^2 + 2x + 1 + y^2 - 4y + 4 = 8$

$x^2 + 2x + y^2 - 4y = 3$

On substituting x = 1

$1 + 2 \times 1 + y^2 - 4y = 3$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0, 4$

Other coordinates are (1, 0) and (1, 4)

38. Question

Name the quadrilateral formed, if any, by the following points, and give reasons for your answers:

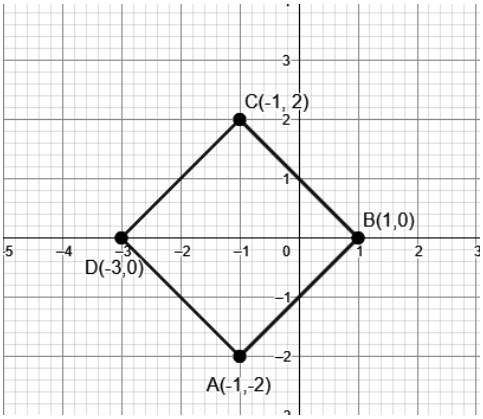
(i) A (-1, - 2), B (1, 0), C (-1, 2), D (-3, 0)

(ii) A (-3, 5), B (3, 1), C (0, 3), D (-1, - 4)

(iii) A (4, 5), B (7, 6), C (4, 3), D (1, 2)

Answer

(i) A (-1, - 2), B (1, 0), C (-1, 2), D (-3, 0)



Using distance formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$AB=\sqrt{(1+1)^2+(0+2)^2}=\sqrt{2^2+2^2}=\sqrt{4+4}=\sqrt{8}\ units$$

$$BC=\sqrt{(-1-1)^2+(-2-0)^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}\ units$$

$$CD=\sqrt{(-3+1)^2+(0-2)^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}\ units$$

$$DA=\sqrt{(-3+1)^2+(0+2)^2}=\sqrt{(-2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}\ units$$

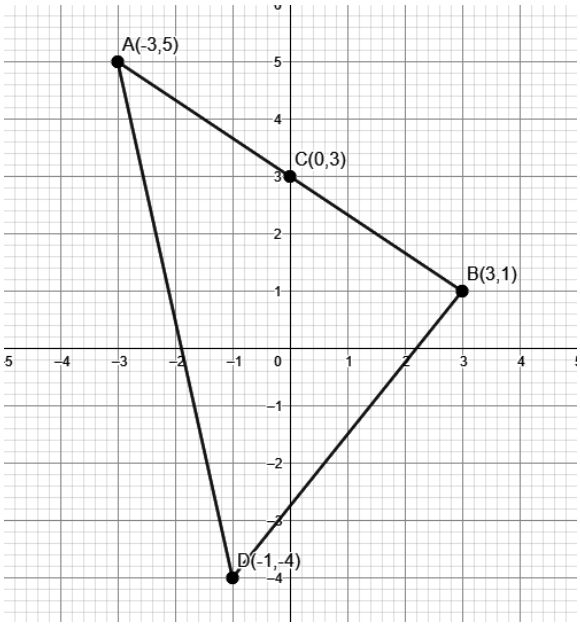
AB = BC = CD = DA

$$BD=\sqrt{(-3-1)^2+(0-0)^2}=\sqrt{(-4)^2+(0)^2}=\sqrt{16+0}=4\ units$$

$$AC=\sqrt{(-1+1)^2+(2+2)^2}=\sqrt{(0)^2+(4)^2}=\sqrt{0+16}=4\ units$$

All the four sides of the quadrilateral are equal and diagonals are of equal length. Therefore, the given vertices form a square.

(ii) A (-3, 5), B (3, 1), C (0, 3), D (-1, - 4)

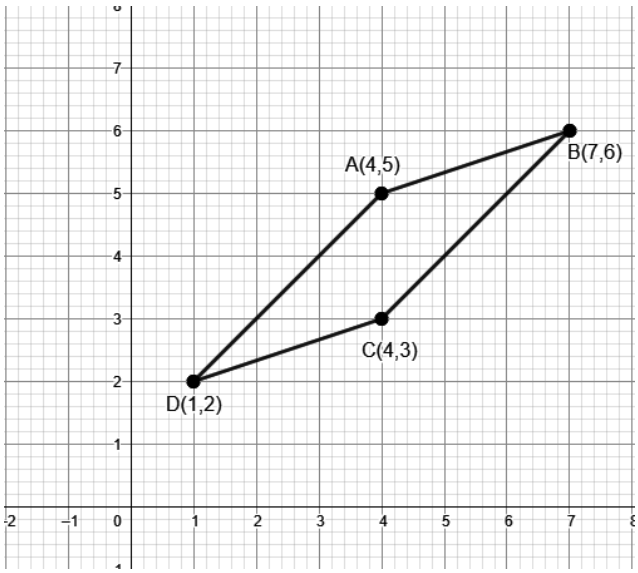


Using distance formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$AB=\sqrt{(3+3)^2+(1-5)^2}=\sqrt{(6)^2+(-4)^2}=\sqrt{36+16}=\sqrt{52}=2\sqrt{13}\,units$$
$$BC=\sqrt{(0-3)^2+(3-1)^2}=\sqrt{(-3)^2+(2)^2}=\sqrt{9+4}=\sqrt{13}\ units$$
$$CD=\sqrt{(-1+0)^2+(-4-3)^2}=\sqrt{(-1)^2+(-7)^2}=\sqrt{1+49}=\sqrt{50}\ units$$
$$DA=\sqrt{(-1+3)^2+(-4-5)^2}=\sqrt{(2)^2+(-9)^2}=\sqrt{4+81}=\sqrt{85}\ units$$

Since all sides are of different length, therefore it is not a particular type of quadrilateral.

(iii) A (4, 5), B (7, 6), C (4, 3), D (1, 2)



Using distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$AB = \sqrt{(7-4)^2+(6-5)^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}\ units$$
$$BC = \sqrt{(4-7)^2+(3-6)^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}\ units$$
$$CD = \sqrt{(1-4)^2+(2-3)^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}\ units$$
$$DA = \sqrt{(1-4)^2+(2-5)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}\ units$$

Coordinates of midpoint of diagonal AC X= $\frac{4+4}{2} = 4$Y = $\frac{5+3}{2} = \frac{8}{2} = 4$

Therefore coordinates of midpoint of AC are (4, 4)

Coordinates of midpoint of diagonal BD= X = $\frac{7+1}{2} = 4$, Y = $\frac{6+2}{2} = \frac{8}{2} = 4$

Therefore coordinates of midpoint of AC are (4, 4)

Since diagonals bisect each other at same point therefore quadrilateral is a parallelogram.

39. Question

Find the equation of the perpendicular bisector of the line segment joining points (7, 1) and (3, 5).

Answer

The points are A(7, 1) and B(3, 5).

Coordinates of midpoint of line AB = X = $\frac{7+3}{2} = 5$, Y = $\frac{5+1}{2} = \frac{6}{2} = 3$

Therefore coordinates of midpoint of AB are (5, 3)

Slope of the line = $\frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{3-1} = \frac{4}{2} = 2$

Negative reciprocal of slope = $-\frac{1}{2}$

Equation of line Y= mX+C

$$y = -\frac{x}{2} + C$$

$$3 = -\frac{5}{2} + C$$

$$C = \frac{11}{2}$$

$$y = -\frac{1}{2}x + \frac{11}{2}$$

$$2y = -x + 11$$

$$x + 2y = 11$$

Since diagonals bisect each other at same point therefore quadrilateral is a parallelogram.

40. Question

Prove that the points (3, 0), (4, 5), (-1, 4) and (-2, -1), taken in order, form a rhombus. Also, find its area.

Answer

Let the Vertices of a quadrilateral are: A(3, 0), B(4, 5), C(-1, 4) and D(-2, -1),

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(4-3)^2 + (5-0)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side BC = $\sqrt{(-1-4)^2 + (4-5)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of side CD = $\sqrt{(-2+1)^2 + (-1-4)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side DA = $\sqrt{(-2-3)^2 + (-1-0)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of diagonal AC = $\sqrt{(-1-3)^2 + (4-0)^2}$ = $\sqrt{16+16}$ = $\sqrt{32}$ units

Length of side BD = $\sqrt{(-2-4)^2 + (-1-5)^2}$ = $\sqrt{36+36}$ = $\sqrt{72}$ units

Since all sides are of equal length, therefore it is a rhombus.

Area of Rhombus = $\frac{1}{2} \times d_1 \times d_2$ = $\frac{1}{2} \times \sqrt{32} \times \sqrt{72}$ = $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ = 24 sq. units

41. Question

In the seating arrangement of desks in a classroom three students Rohini, Sandhya and Bina are seated at A (3, 1), B (6, 4) and C (8, 6). Do you think they are seated in a line?

Answer

Points are A (3, 1), B (6, 4) and C (8, 6)

For sitting in a line three points must be collinear i.e AB + BC = AC

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(6-3)^2 + (4-1)^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$ = $3\sqrt{2}$ units

Length of side BC = $\sqrt{(8-6)^2 + (6-4)^2}$ = $\sqrt{4+4}$ = $\sqrt{8}$ = $2\sqrt{2}$ units

Length of side AC = $\sqrt{(8-3)^2 + (6-1)^2}$ = $\sqrt{25+25}$ = $\sqrt{50}$ = $5\sqrt{2}$ units

AB + BC = AC

$3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

The points are collinear.

42. Question

Find a point on y-axis which is equidistant from the points (5, - 2) and (- 3, 2).

Answer

Points A(5, -2) and B(-3, 2) are equidistance from point P.

Let the coordinates of point are P(0, y)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$$\sqrt{(0-5)^2+(y+2)^2} = \sqrt{(0+3)^2+(y-2)^2}$$

On squaring both sides, we get

$(0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2$

$\Rightarrow (-5)^2 + (y+2)^2 = (3)^2 + (y-2)^2$

$\Rightarrow 25 + y^2 + 4 + 4y = 9 + y^2 + 4 - 4y$

$\Rightarrow 4y + 4y = 9 - 25$

$\Rightarrow 8y = -16$

$\Rightarrow y = -2$

Therefore coordinates are (0, -2).

43. Question

Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (-3, 4).

Answer

Coordinates of the points are A(3, 6) and B(-3, 4)

Let the point P(x, y) is equidistant from A and B

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow$ PA = PB

$$\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$$

On squaring both sides, we get

$$(x-3)^2+(y-6)^2 = (x+3)^2+(y-4)^2$$

$$x^2-6x+9+y^2-12y+36 = x^2+6x+9+y^2-8y+16$$

$$3x+y=5$$

44. Question

If a point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

Answer

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow$ AB = AC

$$\sqrt{(3-0)^2+(P-2)^2} = \sqrt{(P-0)^2+(5-2)^2}$$

On squaring both sides, we get

$$(3-0)^2+(P-2)^2 = (P-0)^2+(5-2)^2$$

$$9+P^2-4P+4 = P^2+9$$

$P = 1$

45. Question

Prove that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

Answer

Vertices of a quadrilateral are A (7, 10), B(-2, 5) and C(3, -4)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(-2-7)^2 + (5-10)^2}$ = $\sqrt{(81+25)}$ = $\sqrt{106}$ units

BC = $\sqrt{(3+2)^2 + (-4-5)^2}$ = $\sqrt{(25+81)}$ = $\sqrt{106}$ units

AC = $\sqrt{(3-7)^2 + (-4-10)^2}$ = $\sqrt{(16+196)}$ = $\sqrt{212}$ units

Since AB = BC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\left(\sqrt{212}\right)^2 = \left(\sqrt{106}\right)^2 + \left(\sqrt{106}\right)^2$

212 = 106 + 106

212 = 212

Therefore vertices are of right isosceles triangle.

46. Question

If the point P (x, 3) is equidistant from the points A (7,-1) and B (6, 8), find the value of x and find the distance AP.

Answer

Coordinates are A (7,-1) and B (6, 8)

The point P (x, 3) is equidistant.

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$\sqrt{(x-7)^2 + (3+1)^2} = \sqrt{(x-6)^2 + (3-8)^2}$

On squaring both sides, we get

$(x-7)^2 + (3+1)^2 = (x-6)^2 + (3-8)^2$

$x^2 - 14x + 49 + 16 = x^2 - 12x + 36 + 25$

$x = 2$

AP = $\sqrt{(2-7)^2 + (3+1)^2}$ = $\sqrt{25+16}$ = $\sqrt{41}$ units

47. Question

If A (3, y) is equidistant from points P (8, -3) and Q (7,6) , find the value of y and find the distance AQ.

Answer

Coordinates are P(8, -3) and Q(7,6)

The point A (3, y) is equidistant.

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = QA

$$\sqrt{(3-8)^2+(y+3)^2}=\sqrt{(3-7)^2+(y-6)^2}$$

On squaring both sides, we get

$$(3-8)^2+(y+3)^2=(3-7)^2+(y-6)^2$$

$$25+y^2+6y+9=16+y^2-12y+36$$

$$y=1$$

AQ = $\sqrt{(3-7)^2+(1-6)^2}=\sqrt{16+25}=\sqrt{41}$ units

48. Question

If (0, - 3) and (0, 3) are the two vertices of an equilateral triangle, find the coordinates of its third vertex.

Answer

Coordinates are A(0, -3) and B(0,3) and C(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ AB = AC

$$\sqrt{(0-0)^2+(3+3)^2}=\sqrt{(x-0)^2+(y+3)^2}$$

On squaring both sides, we get

$$(0-0)^2+(3+3)^2=(x-0)^2+(y+3)^2$$

$$36=x^2+y^2+6y+9$$

$$x^2+y^2+6y=27 \quad (1)$$

⇒ AB = BC

$$\sqrt{(0-0)^2+(3+3)^2}=\sqrt{(x-0)^2+(y-3)^2}$$

On squaring both sides, we get

$$(0-0)^2+(3+3)^2=(x-0)^2+(y-3)^2$$

$$36=x^2+y^2-6y+9$$

$$x^2+y^2-6y=27 \quad (2)$$

On subtracting equation (2) from (1) we get

$12y = 0$

$y = 0$

On substituting y = 0 in equation (1), we get

$x^2 + y^2 + 6y = 27$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

Therefore coordinates of third vertex are $(3\sqrt{3}, 0)$, $(-3\sqrt{3}, 0)$,

49. Question

If the point P (2, 2) is equidistant from the points A (-2, k) and B (-2k, -3), find k. Also, find the length of AP.

Answer

Coordinates of points are P(2, 2) A(-2, k) and B(-2k, -3)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$\sqrt{(-2-2)^2 + (k-2)^2} = \sqrt{(-2k-2)^2 + (-3-2)^2}$

On squaring both sides, we get

$(-2-2)^2 + (k-2)^2 = (-2k-2)^2 + (-3-2)^2$

$16 + k^2 - 4k + 4 = 4k^2 + 8k + 4 + 25$

$k^2 + 4k + 3 = 0$

$k^2 + 3k + k + 3 = 0$

$k(k+3) + 1(k+3) = 0$

$(k+1)(k+3) = 0$

k = -1, -3

AP = $\sqrt{(-2-2)^2 + (-1-2)^2} = \sqrt{16+9}$ = 5 units

50. Question

If the point A (0, 2) is equidistant from the points B (3, p) and C (p, 5) the length of AB.

Answer

Coordinates of points are A(0, 2), B(3, p) and C(p, 5)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ AB = AC

$\sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$

On squaring both sides, we get

$(3-0)^2+(p-2)^2=(p-0)^2+(5-2)^2$

$9+p^2-4p+4=p^2+9$

$-4p=-4$

$p=1$

AB = $\sqrt{(3-0)^2+(1-2)^2}=\sqrt{9+1}=\sqrt{10}$ units

51. Question

If the point P (k -1, 2) is equidistant from the points A (3, k) and B (k,5), find the value of k.

Answer

Coordinates of points are A(3, k), B(k, 5) and P(k-1, 2)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = PB

$\sqrt{(k-1-3)^2+(2-k)^2}=\sqrt{(k-1-k)^2+(2-5)^2}$

On squaring both sides, we get

$(k-1-3)^2+(2-k)^2=(k-1-k)^2+(2-5)^2$

$(k-4)^2+(2-k)^2=(-1)^2+(-3)^2$

$k^2-8k+16+k^2-4k+4=1+9$

$k^2-6k+5=0$

$k^2-5k-k+5=0$

$k(k-5)-1(k-5)=0$

$(k-5)(k-1)=0$

$k=1, 5$

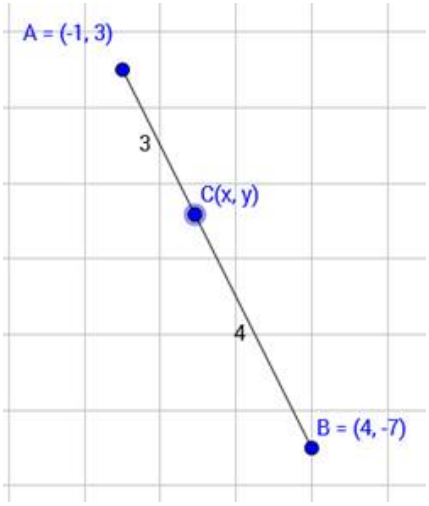
Exercise 14.3

1. Question

Find the coordinates of the point which divides the line segment joining (- 1, 3) and (4, -7) internally in the ratio 3 : 4.

Answer

Let our points be A(-1, 3) and B(4, -7) and required point be C(x, y)



Given that point divides internally in ratio of 3:4.

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

Here, m = 3 and n = 4

$\therefore x = \frac{3\times4+4\times(-1)}{3+4}$, $y = \frac{3\times(-7)+4\times3}{3+4}$

$\therefore x= \frac{12-4}{7}$, $y = \frac{-21+12}{7}$

$\therefore x = \frac{8}{7}$, $y = \frac{-9}{7}$

Hence, the required point is C($\frac{8}{7}$, $\frac{-9}{7}$)

2. Question

Find the points of trisection of the line segment joining the points:

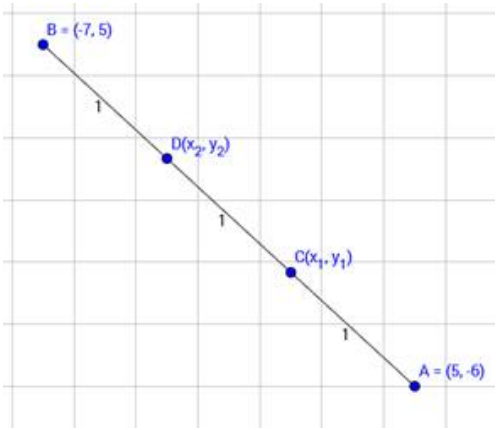
(i) (5, -6) and (- 7, 5), (ii) (3, -2) and (-3, -4), (iii) (2, -2) and (-7, 4)

Answer

(i) (5, -6) and (- 7, 5),

Let our given points be A(5,-6) and B(-7, 5) and required points be C (x_1 , y_1) and D(x_2 , y_2)

The points of trisection of a line are points which divide into the ratio 1:2



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point $C(x_1 , y_1)$

$x_1 = \frac{1\times(-7)+2\times5}{1+2}$,

$y_1 = \frac{1\times5+2\times(-6)}{1+2}$ …Here m = 1 and n = 2

$\therefore x_1 = \frac{3}{3}$, $y_1 = \frac{-7}{3}$

$\therefore C (x_1 , y_1) \equiv (1, \frac{-7}{3})$

For point $D(x_2 , y_2)$

$X_2 = \frac{2\times(-7)+1\times5}{2+1}$, $y_2 = \frac{2\times5+1\times(-6)}{2+1}$ …Here m = 2 and n = 1

$\therefore x_2 = \frac{-9}{3}$, $y_2 = \frac{4}{3}$

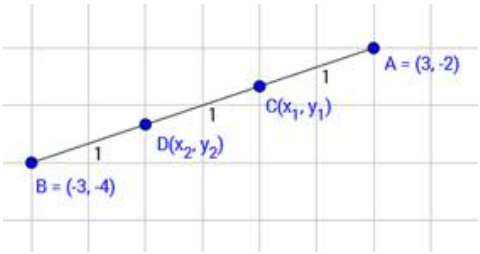
$\therefore D (x_2 , y_2) \equiv (-3, \frac{4}{3})$

Hence, the points of trisection of line joining given points are $(1, \frac{-7}{3})$ and $(-3, \frac{4}{3})$

(ii) (3, -2) and (-3, -4)

Let our given points be A(3,-2) and B(-3, -4) and required points be C (x_1 , y_1) and $D(x_2 , y_2)$

The points of trisection of a line are points which divide into the ratio 1:2



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point $C(x_1 , y_1)$

$x_1 = \frac{1\times(-3)+2\times3}{1+2}$, $y_1 = \frac{1\times(-4)+2\times(-2)}{1+2}$ …Here m = 1 and n = 2

$\therefore x_1 = \frac{3}{3}$, $y_1 = \frac{-8}{3}$

$\therefore C (x_1 , y_1) \equiv (1 , \frac{-8}{3})$

For point $D(x_2 , y_2)$

$X_2 = \frac{2\times(-3)+1\times3}{2+1}$, $y_2 = \frac{2\times(-4)+1\times(-2)}{2+1}$ …Here m = 2 and n = 1

$\therefore x_2 = \frac{-3}{3}$, $y_2 = \frac{-10}{3}$

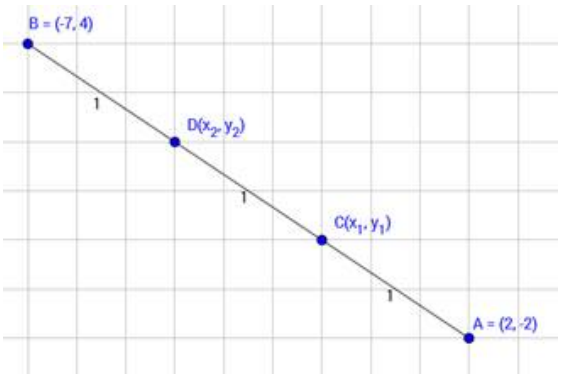
$\therefore$ D $(x_2 , y_2) \equiv (-1, \frac{-10}{3})$

Hence, the points of trisection of line joining given points are $(1 , \frac{-8}{3})$ and $(-1, \frac{-10}{3})$

(iii) (2, -2) and (-7, 4)

Let our given points be A(2,-2) and B(-7, 4) and required points be C (x_1 , y_1) and D(x_2 , y_2)

The points of trisection of a line are points which divide into the ratio 1:2



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(x_1 , y_1)

$x_1 = \frac{1 \times(-7) + 2 \times 2}{1+2}$, $y_1 = \frac{1 \times 4 + 2 \times(-2)}{1+2}$ …Here m = 1 and n = 2

$\therefore x_1 = \frac{-3}{3}$, $y_1 = \frac{0}{3}$

$\therefore$ C $(x_1 , y_1) \equiv (-1 , 0)$

For point D(x_2 , y_2)

$X_2 = \frac{2 \times(-7) + 1 \times 2}{2+1}$, $y_2 = \frac{2 \times 4 + 1 \times(-2)}{2+1}$ …Here m = 2 and n = 1

$\therefore x_2 = \frac{-12}{3}$, $y_2 = \frac{6}{3}$

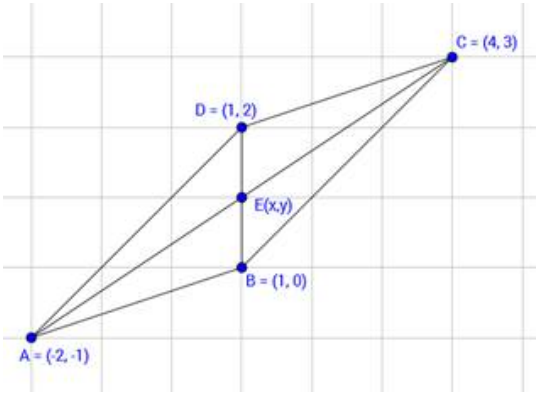
$\therefore$ D $(x_2 , y_2) \equiv$ (-4, 2)

3. Question

Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.

Answer

Let our points of parallelogram be A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) and mid point of diagonals be E(x,y)



We know that diagonals of parallelogram bisect each other.

Hence, we find mid point of AC.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For point E(x, y)

$x_1 = \frac{-2+4}{2}$, $y_1 = \frac{-1+3}{2}$

$\therefore x_1 = \frac{2}{2}$, $y_1 = \frac{2}{2}$

$\therefore$ E (x, y) $\equiv (1, 1)$

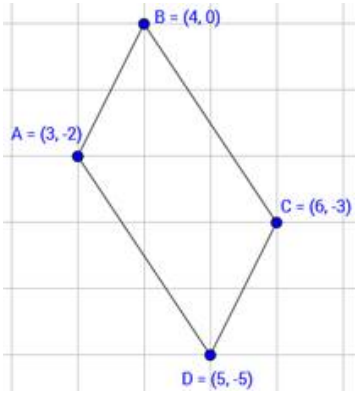
4. Question

Prove that the points (3, -2), (4, 0), (6, -3) and (5, -5) are the vertices of a parallelogram.

Answer

We know if the quadrilateral is parallelogram if opposite sides are equal.

Let our points be A(3, -2), B(4, 0), C(6, -3) and D(5, -5).



By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For AB,

$AB = \sqrt{(4-3)^2 - (0-(-2))^2}$

= $\sqrt{1+4}$

= $\sqrt{5}$ units

For BC,

BC = $\sqrt{(6-4)^2-\left((-3)-0\right)^2}$

= $\sqrt{4+9}$

= $\sqrt{13}$ units

For CD,

CD = $\sqrt{(5-6)^2-\left((-5)-(-3)\right)^2}$

= $\sqrt{1+4}$

= $\sqrt{5}$ units

For AD,

AD = $\sqrt{(5-3)^2-\left((-5)-(-2)\right)^2}$

= $\sqrt{4+9}$

= $\sqrt{13}$ units

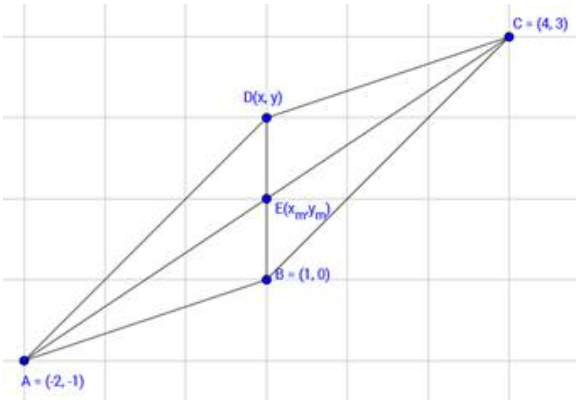
Here, we observe that AB = CD and AD = BC, which means that the quadrilateral formed by lines joining by points, is parallelogram.

5. Question

Three consecutive vertices of a parallelogram are (-2, -1), (1, 0) and (4, 3). Find the fourth vertex

Answer

Let three vertices be A(-2, -1), B(1, 0) and C(4, 3) and fourth vertex be D(x, y)



It is given that quadrilateral joining these four vertices is parallelogram.

∴□ABCD is parallelogram

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.

Let E(x_m , y_m) be the midpoint of diagonals AC and BD.

By midpoint formula,

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For diagonal AC,

$x_m = \frac{-2+4}{2}$, $y_m = \frac{-1+3}{2}$

$\therefore x_m = \frac{2}{2}$, $y_m = \frac{2}{2}$

$\therefore E(x_m , y_m) \equiv (1, 1)$

For diagonal BD,

$1 = \frac{1+x}{2}$, $1 = \frac{0+y}{2}$

$\therefore x = 2 - 1$, $y = 2 - 0$

$\therefore x = 1$ and $y = 2$

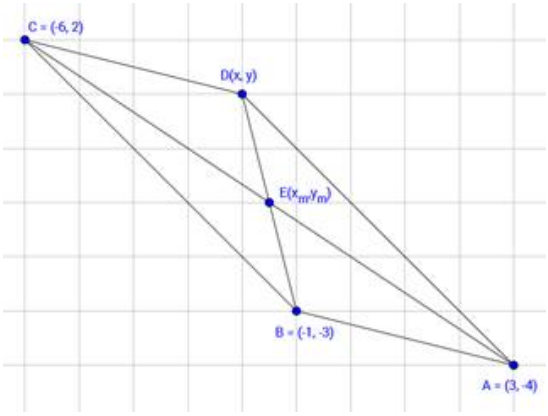
Hence, our fourth vertex is D(1, 2)

6. Question

The points (3, -4) and (-6, 2) are the extremities of a diagonal of a parallelogram. If the third vertex is (-1, -3). Find the coordinates of the fourth vertex.

Answer

Let three vertices be A(3, -4), B(-1, -3) and C(-6, 2) and fourth vertex be D(x, y)



It is given that quadrilateral joining these four vertices is parallelogram.

$\therefore$□ABCD is parallelogram

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.

Let E(x_m , y_m) be the midpoint of diagonals AC and BD.

By midpoint formula,

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For diagonal AC,

$x_m = \frac{3+(-6)}{2}$, $y_m = \frac{-4+2}{2}$

$\therefore x_m = \frac{-3}{2}$, $y_m = \frac{-2}{2}$

$\therefore E(x_m , y_m) \equiv (\frac{-3}{2}, -1)$

For diagonal BD,

$\frac{-3}{2} = \frac{-1+x}{2}$, $-1 = \frac{-3+y}{2}$

$\therefore x = -3 + 1$, $y = -2 + 3$

$\therefore x = -2$ and $y = 1$

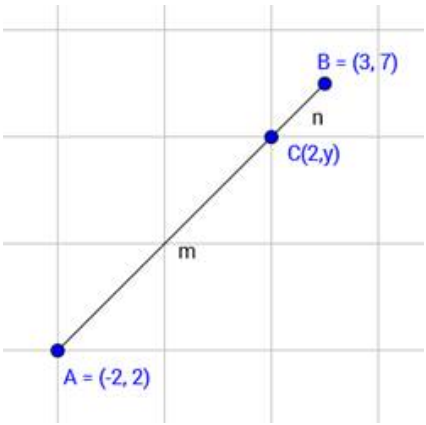
Hence, our fourth vertex is D(-2, 1)

7. Question

Find the ratio in which the point (2,y) divides the line segment joining the points A (-2, 2) and B (3, 7). Also, find the value of y.

Answer

Here, given points are A (-2, 2) and B (3, 7) and let the point dividing the line joining two points be C(2,y).



Let the ratio be m:n

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(2,y),

$2 = \frac{m \times 3+n \times(-2)}{m+n}$ ...(1)

And $y = \frac{m \times 7+n \times 2}{m+n}$ ...(2)

Solving 1 for finding ratio between m and n,

$2 = \frac{m \times 3+n \times(-2)}{m+n}$

$2(m + n) = 3m - 2n$

$2m + 2n = 3m - 2n$

$\therefore m = 4n$

$\therefore \frac{m}{n} = \frac{4}{1}$

∴ m : n = 4 : 1

Now solving for equation 2, where m = 4 and n = 1

$y = \frac{m \times 7 + n \times 2}{m+n}$

$y = \frac{4 \times 7 + 1 \times 2}{4+1}$

$\therefore y = \frac{28+2}{5}$

$\therefore y = \frac{30}{5}$

∴ y =6

Hence, our point is (2, 6)

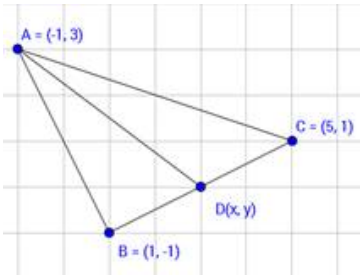
8. Question

If A (-1, 3), B (1, -1) and C (5, 1) are the vertices of a triangle ABC, find the length of the median through A.

Answer

Here given vertices of triangle are A (-1, 3), B (1, -1) and C (5, 1).

Let D, E and F be the midpoints of the sides BC, CA and AB respectively.



We need to find length of median passing through A, ie distance between AD.

Let point D ≡ (x, y)

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of side BC,

$x = \frac{1+5}{2}$, $y = \frac{-1+1}{2}$

$\therefore x = \frac{6}{2}$, $y = \frac{0}{2}$

∴D(x , y) ≡ (3, 0)

Now, by distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For AD,

$AD = \sqrt{(3-(-1))^2 + (0-3)^2}$

∴ AD = $\sqrt{16+9}$

AD = $\sqrt{25}$

∴AD = 5 units

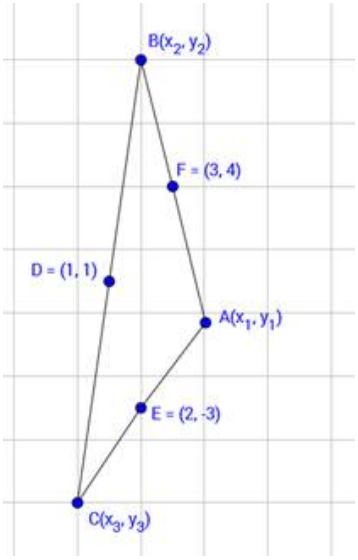
Hence, the length of the median through A is 5 units

9. Question

If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4), find the vertices of the triangle.

Answer

Let A(x_1 , y_1), B(x_2 , y_2) and C(x_3 , y_3) be the vertices of triangle.



Let D(1, 1), E(2, -3) and F(3, 4) be the midpoints of sides BC, CA and AB respectively.

By midpoint formula.

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For midpoint D(1, 1) of side BC,

1 = $\frac{x_2+x_3}{2}$, 1 = $\frac{y_2+y_3}{2}$

∴ $x_2 + x_3$ = 2 and $y_2 + y_3$ = 2 …(1)

For midpoint E(2, -3) of side CA,

2 = $\frac{x_1+x_3}{2}$, -3 = $\frac{y_1+y_3}{2}$

∴ $x_1 + x_3$ = 4 and $y_1 + y_3$ = -6 …(2)

For midpoint F(3, 4) of side AB,

3 = $\frac{x_1+x_2}{2}$, 4 = $\frac{y_1+y_2}{2}$

∴ $x_1 + x_2$ = 6 and $y_1 + y_2$ = 8 …(3)

Adding 1,2 and 3, we get,

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2$ = 2 + 4 + 6

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2$ = 2 – 6 +8

∴ $2(x_1 + x_2 + x_3)$ = 12 and $2(y_1 + y_2 + y_3)$ = 2

∴ $x_1 + x_2 + x_3$ = 6 and $y_1 + y_2 + y_3$ = 1

x_1 + 2 = 6 and y_1 + 2 = 2 …from 1

∴ x_1 = 4 and y_1 = 0

Substituting above values in 3,

4 + $x_2 = 6$ and 0 + y_2 = 8

∴ x_2 = 2 and y_2 = 89

Similarly for equation 2,

4 + x_3 = 6 and 0 + y_3 = -6

∴ x_3 = 2 and y_3 = -6

Hence the vertices of triangle are A(4 , 0), B(2 ,8) and C(0 ,-6)

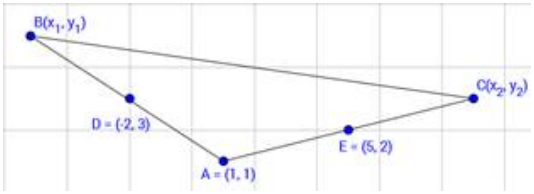
10. Question

If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (-2, 3) and (5, 2), find the other vertices.

Answer

Let in ΔABC, A(1,1), B(x_1, y_1) and C(x_2, y_2).

Let D(-2, 3) and E(5, 2) be the midpoints of sides AB and AC respectively.



By midpoint formula.

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For mid point D (-2, 3) of side AB,

-2 = $\frac{1+x_1}{2}$, 3 = $\frac{1+y_1}{2}$

1 + x_1 = -4 and 1 + y_1 = 6

x_1 = -5 and y_1 = 5

∴ B (x_1, y_1) ≡ (-5, 5)

For midpoint E(5, 2) of side AC,

5 = $\frac{1+x_2}{2}$, 2 = $\frac{1+y_2}{2}$

$1 + x_2 = 10$ and $1 + y_2 = 4$

$\therefore x_2 = 9$ and $y_2 = 3$

$\therefore C(x_2, y_2) \equiv (9, 3)$

Hence other two vertices are B (-5, 5) and C(9, 3)

11 A. Question

In what ratio is the line segment joining the points (-2, -3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

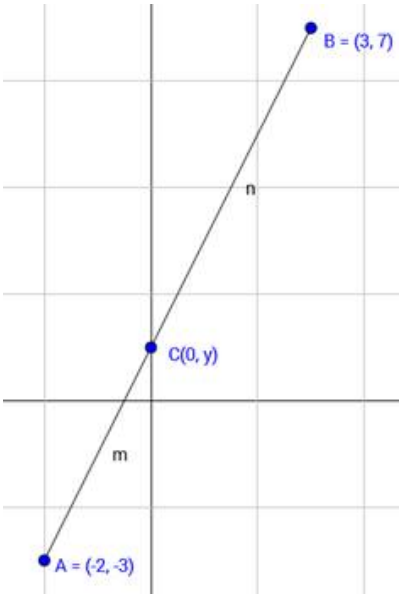
Answer

Here y axis divides our line joined by the points (say) A (-2, -3) and B (3, 7).

Let coordinate of the point be C(0, y).

Here our x- coordinate is zero, as point C lie on x-axis.

Let y axis divide AB in ratio of m:n.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(0, y) on line joined by the points A and B,

$0 = \frac{m \times 3 + n \times (-2)}{m+n}$ ...(1)

And, $y = \frac{m \times 7 + n \times (-3)}{m+n}$ ...(2)

Solving 1,

$0(m + n) = 3m - 2n$

$\therefore 3m = 2n$

$\therefore m:n = 2:3$

Now solving for 2, for values m = 2 and n = 3,

$y = \frac{2 \times 7 + 3 \times (-3)}{2+3}$

$y = \frac{14-6}{5}$

$\therefore y = \frac{5}{5} = 1$

$\therefore C(0, y) \equiv (0, 1)$

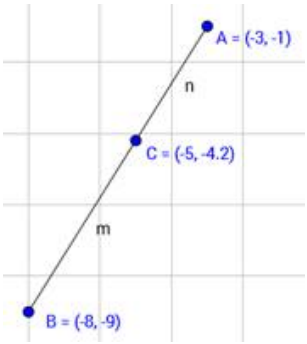
11 B. Question

In what ratio is the line segment joining (-3, -1) and (-8, -9) divided at the point (-5, -21 /5)?

Answer

Let given points be A (-3, -1) and B (-8, -9).

Let the point C(-5, -21 /5) divide AB in ratio m:n.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(-5, -21 /5) on the line joined by the points A and B.

$-5 = \frac{m \times (-8) + n \times (-3)}{m+n}$ ...(1)

And, $-\frac{21}{5} = \frac{m \times (-9) + n \times (-1)}{m+n}$ ...(2)

Solving 1,

-5(m + n) = -8m - 3n

∴5m + 5n = 8m + 3n

∴ 2n = 3m

$\therefore \frac{m}{n} = \frac{2}{3}$

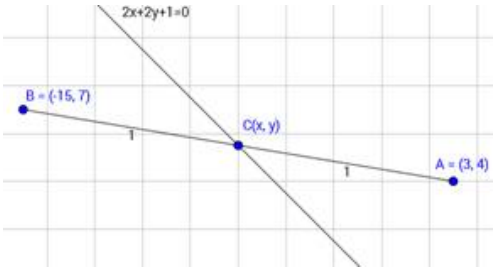
Hence, ratio is 2:3.

12. Question

If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and 2x + 2y + 1 = 0, find the value of k.

Answer

Let A(3, 4) and B(k, 7) and midpoint be C(x, y) which lies on the line 2x+2y+1 = 0



By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For point C(x, y),

$x = \frac{3+k}{2}$, $y = \frac{4+7}{2}$ …(1)

Here, $y = \frac{11}{2}$,

Hence, substituting value of y in given equation of line,

$2x + 2 \times \frac{11}{2} + 1 = 0$

∴ 2x = -12

∴ x = -6

Now substituting value of x in equation(1), we get.

$x = \frac{3+k}{2}$

$-6 = \frac{3+k}{2}$

∴ -12 = 3 + k

∴ k = -15

Hence, the value of k is -15.

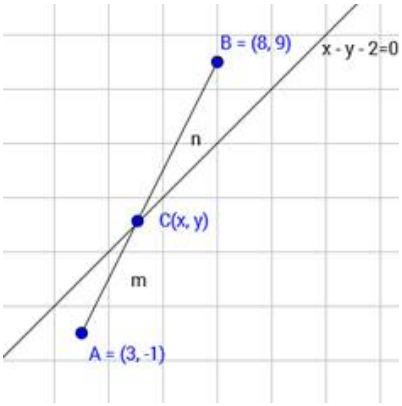
13. Question

Determine the ratio in which the straight line x - y - 2 = 0 divides the line segment joining (3, -1) and (8, 9).

Answer

Let point be A(3, -1) and B(8, 9).

Let the line divide the line joining the points A and B in the ratio m:n at any point C(x, y)



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x, y),

$x = \frac{m\times8+n\times3}{m+n}$, $y = \frac{m\times9+n\times(-1)}{m+n}$

$\therefore x = \frac{8m+3n}{m+n}$, $y = \frac{9m-n}{m+n}$

Now, substituting value of x and y in equation x - y - 2 = 0,

$\frac{8m+3n}{m+n} - \frac{9m-n}{m+n} - 2 = 0$

$\frac{8m+3n-9m+n-2m-2n}{m+n} = 0$

$\therefore$ -3m +2n =0

$\therefore \frac{m}{n} = \frac{2}{3}$

$\therefore$ m:n = 2:3

Hence, the line divides the line segment joining A and B in the ratio 2:3 internally.

14. Question

Find the ratio in which the line segment joining (-2, -3) and (5, 6) is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case.

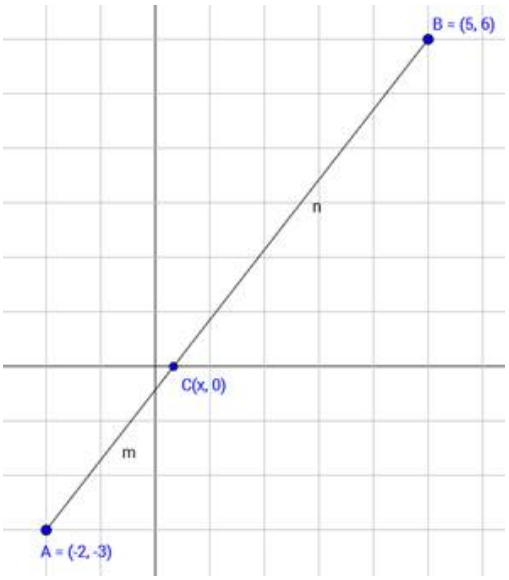
(i) x-axis

Answer

(i) x-axis

Let our points be A(-2, -3) and B(5, 6).

Let point C(x, 0) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x, 0)

$x = \frac{m\times5+n\times(-2)}{m+n}$, $0 = \frac{m\times6+n\times(-3)}{m+n}$

Solving for y coordinate,

$0 = \frac{m\times6+n\times(-3)}{m+n}$

$\therefore$ 6m -3n = 0

$\therefore$ 2m = n

$\therefore \frac{m}{n} = \frac{1}{2}$

$\therefore$ m : n = 1 : 2

Now solving for x coordinate, with m = 1 and n = 2,

$x = \frac{1\times5+2\times(-2)}{1+2}$

$\therefore x = \frac{5-4}{3}$

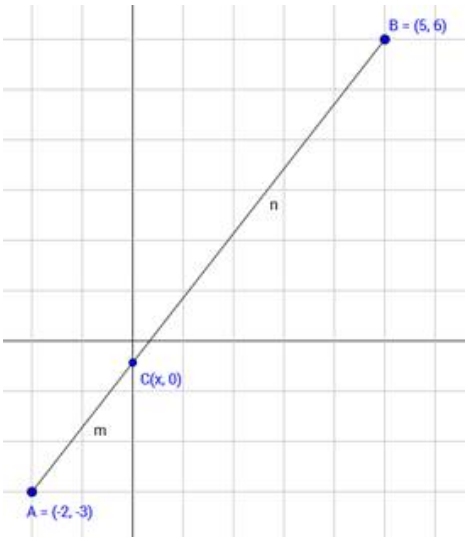
$\therefore x = \frac{1}{3}$

Hence, the coordinates of required point is $C(\frac{1}{3}, 0)$

(ii) y-axis.

Let our points be A(-2, -3) and B(5, 6).

Let point C(0, y) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(0, y)

$0 = \frac{m\times5+n\times(-2)}{m+n}$, $y = \frac{m\times6+n\times(-3)}{m+n}$

Solving for x coordinate,

$0 = \frac{m\times5+n\times(-2)}{m+n}$

$\therefore 5m - 2n = 0$

$\therefore \frac{m}{n} = \frac{2}{5}$

$\therefore m : n = 2 : 5$

Now solving for y coordinate, with m = 2 and n = 5,

$y = \frac{2\times6+5\times(-3)}{2+5}$

$y = \frac{12-15}{7}$

$\therefore y = \frac{-3}{7}$

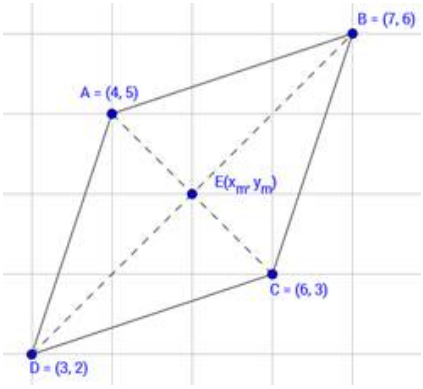
Hence, the coordinates of required point is C(0 , $\frac{-3}{7}$)

15. Question

Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle.

Answer

Let given points be A(4, 5), B(7, 6), C(6, 3), D(3, 2) and let the intersection of diagonals be E(x_m , y_m)



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint of diagonal AC,

$X_1 = \frac{4+6}{2}$, $y_1 = \frac{5+3}{2}$

$\therefore x_1 = \frac{10}{2} = 5$, $y_1 = \frac{8}{2} = 4$

∴ midpoint of diagonal AC is $(x_1, y_1) \equiv (5, 4)$ …(1)

For midpoint of diagonal BD,

$X_2 = \frac{7+3}{2}$, $y_2 = \frac{6+2}{2}$

$\therefore x_2 = \frac{10}{2} = 5$, $y_2 = \frac{8}{2} = 4$

∴ midpoint of diagonal BD is $(x_2, y_2) \equiv (5, 4)$ …(2)

Here, from 1 and 2 we say that midpoint of both the diagonals intersect at same point, ie (5, 4)

But our intersection of diagonals is at E, which means that midpoint of diagonals intersect at single point, ie E(5, 4)

We know that if midpoints of diagonals intersect at single point, then quadrilateral formed by joining the points is parallelogram.

Hence, our □ABCD is parallelogram.

Now, we shall check whether □ABCD is rectangle.

If the lengths of diagonals are same, then given quadrilateral is rectangle.

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For diagonal AC,

$AC = \sqrt{(6-4)^2 + (3-5)^2}$

$= \sqrt{4+4}$

$= 2\sqrt{2}$ units

For diagonal BD,

AC = $\sqrt{(7-3)^2+(6-2)^2}$

= $\sqrt{16+16}$

= $4\sqrt{2}$ units.

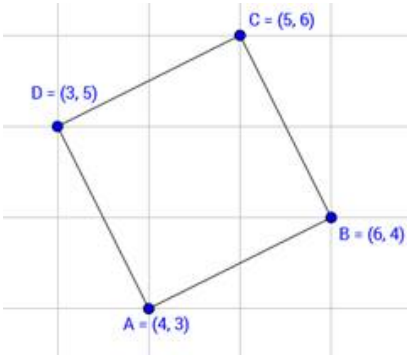
Here, AC ≠ BD, hence □ABCD is not rectangle.

16. Question

Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.

Answer

Let given points be A(4, 3), B(6, 4), C(5, 6) and D(3, 5).



By distance formula,

XY = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

For AB,

AB = $\sqrt{(6-4)^2+(4-3)^2}$

= $\sqrt{4+1}$

= $\sqrt{5}$ units.

For BC,

BC = $\sqrt{(5-6)^2+(6-4)^2}$

= $\sqrt{1+4}$

=$\sqrt{5}$ units.

For CD,

CD =$\sqrt{(3-5)^2+(5-6)^2}$

= $\sqrt{4+1}$

= $\sqrt{5}$ units.

For AD,

AD = $\sqrt{(3-4)^2+(5-3)^2}$

= $\sqrt{1+4}$

=$\sqrt{5}$ units.

Here, we can observe that □ABCD is a parallelogram.

Now,

For diagonal AC,

AC = $\sqrt{(5-4)^2+(6-3)^2}$

= $\sqrt{1+9}$

= $\sqrt{10}$ units.

For diagonal BD,

BD =$\sqrt{(3-6)^2+(5-4)^2}$

= $\sqrt{9+1}$

= $\sqrt{10}$ units.

∴ AC = BD, which means diagonals are equal.

We know that quadrilateral in which all sides are equal and diagonals are equal, is a square.

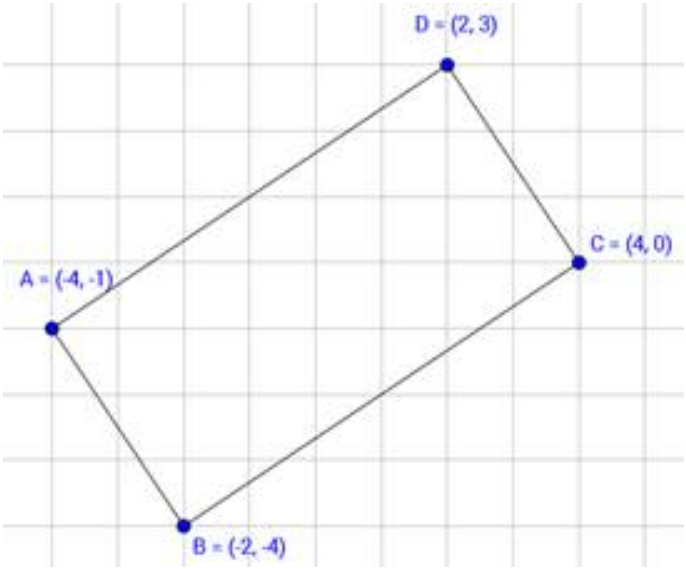
∴□ABCD is a square.

17. Question

Prove that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices of a rectangle.

Answer

Solution : Let the given points be A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3).



Use distance formula, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

For AB,

AB = $\sqrt{(-2-(-4))^2+(-4-(-1))^2}$

$=\sqrt{(-2+4)^2+(-4+1)^2}$

= $\sqrt{(2)^2+(-3)^2}$

= $\sqrt{4+9}$

=$\sqrt{13}$ units

For BC,

BC = $\sqrt{(4-(-2))^2+(0-(-4))^2}=\sqrt{(4+2)^2+(0+4)^2}\quad=\sqrt{6^2+4^2}$

= $\sqrt{36+16}$

= $\sqrt{52}$ units

For CD,

$CD=\sqrt{(2-4)^2+(3-0)^2}\quad=\sqrt{(-2)^2+(3)^2}$

= $\sqrt{4+9}$

=$\sqrt{13}$ units

For AD,

AD = $\sqrt{(2-(-4))^2+(3-(-1))^2}=\sqrt{(2+4)^2+(3+1)^2}\quad=\sqrt{(6)^2+(4)^2}$

= $\sqrt{36+16}$

= $\sqrt{52}$ units

Also, for diagonal AC,

AC = $\sqrt{(4-(-4))^2+(0-(-1))^2}=\sqrt{(4+4)^2+(0+1)^2}\quad=\sqrt{(8)^2+(1)^2}$

$=\sqrt{64+1}$

= $\sqrt{65}$ units

For diagonal BD,

BD = $\sqrt{(2-(-2))^2+(3-(-4))^2}=\sqrt{(2+2)^2+(3+4)^2}\quad=\sqrt{(4)^2+(7)^2}$

= $\sqrt{16+49}$

= $\sqrt{65}$ units

We can observe that AB = CD and BC = AD and also diagonal AC = BD.

We know that a quadrilateral whose opposite sides are equal and the diagonal are equal is rectangle.

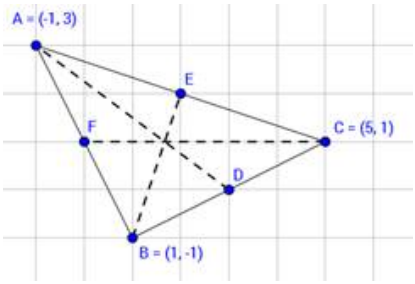
∴ ABCD is a rectangle.

18. Question

Find the lengths of the medians of a triangle whose vertices are A (-1,3), B (1,-1) and C(5,1).

Answer

Here given vertices are A(-1,3), B (1,-1) and C(5,1) and let midpoints of BC, CA and AB be D,E and F respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of side BC,

$x = \frac{1+5}{2}$, $y = \frac{-1+1}{2}$

$x = \frac{6}{2}$, $y = \frac{0}{2}$

∴midpoint of side BC is D(3, 0)

For midpoint E of side AB,

$x = \frac{-1+5}{2}$, $y = \frac{3+1}{2}$

$x = \frac{4}{2}$, $y = \frac{4}{2}$

∴midpoint of side AB is E(2, 2)

For midpoint F of side CA,

$x = \frac{-1+1}{2}$, $y = \frac{3-1}{2}$

$x = \frac{0}{2}$, $y = \frac{2}{2}$

∴midpoint of side CA is F(0, 1)

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For median AD,

$AD = \sqrt{(3-(-1))^2 + (0-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

= 5 units

For median BE,

BE = $\sqrt{(2-1)^2+(2-(-1))^2}$

= $\sqrt{1+9}$

= $\sqrt{10}$ units.

For median CF,

CF = $\sqrt{(0-5)^2+(1-1)^2}$

= $\sqrt{25}$

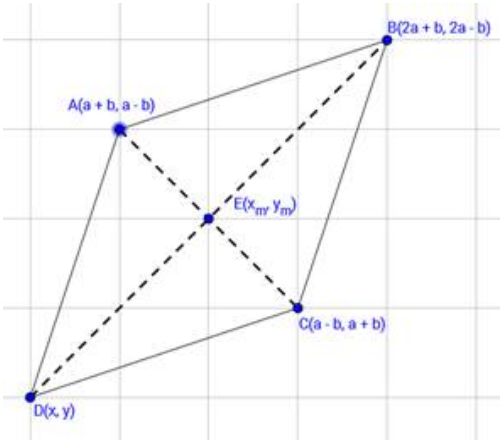
= 5 units

19. Question

Three vertices of a parallelogram are (a + b, a - b), (2a + b, 2a - b), (a - b, a + b). Find the fourth vertex.

Answer

Let A(a + b, a - b), B(2a + b, 2a - b), C(a - b, a + b) and fourth vertex be D(x, y).



It is given that □ABCD is parallelogram.

We know that diagonals of parallelogram bisect each other.

Let intersection of diagonals be E(x_m, y_m)

By midpoint formula.

$x_m = \frac{x_1+x_2}{2}$, $y_m = \frac{y_1+y_2}{2}$

For midpoint E of diagonal AC,

$x_m = \frac{a+b+a-b}{2}$, $y_m = \frac{a-b+a+b}{2}$

∴ x_m = a , y_m = a

∴E(x_m, y_m) ≡ (a, a)

For diagonal BD,

$a = \frac{2a+b+x}{2}$, $a = \frac{2a-b+y}{2}$

∴ 2a = 2a + b +x , 2a = 2a – b +y

∴ x = -b and y = b

Hence, the fourth vertex is D(-b, b)

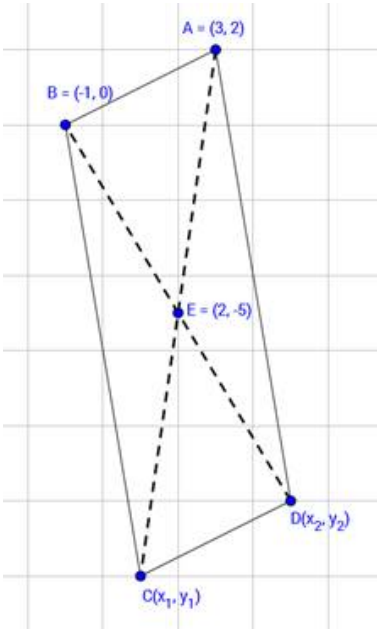
20. Question

If two vertices of a parallelogram are (3, 2), (-1, 0) and the diagonals cut at (2, -5), find the other vertices of the parallelogram.

Answer

Let the vertices be A(3, 2), B(-1, 0), C(x_1, y_1) and D(x_2, y_2).

Let diagonals cut at E(2, - 5).



We know that mid points of diagonals of parallelogram coincide.

By midpoint formula.

$x_m = \frac{x_1+x_2}{2}$, $y_m = \frac{y_1+y_2}{2}$

For midpoint E of diagonal AC,

$2 = \frac{3+x_1}{2}$, $-5 = \frac{2+y_1}{2}$

∴x_1 = 1 and y_1 = -12

∴ coordinates of C are (1, -12)

For midpoint E of diagonal BD,

$2 = \frac{-1+x_2}{2}$, $-5 = \frac{0+y_2}{2}$

∴ x_2 = 5 and y_2 = -10

∴ coordinates of D are (5, -10)

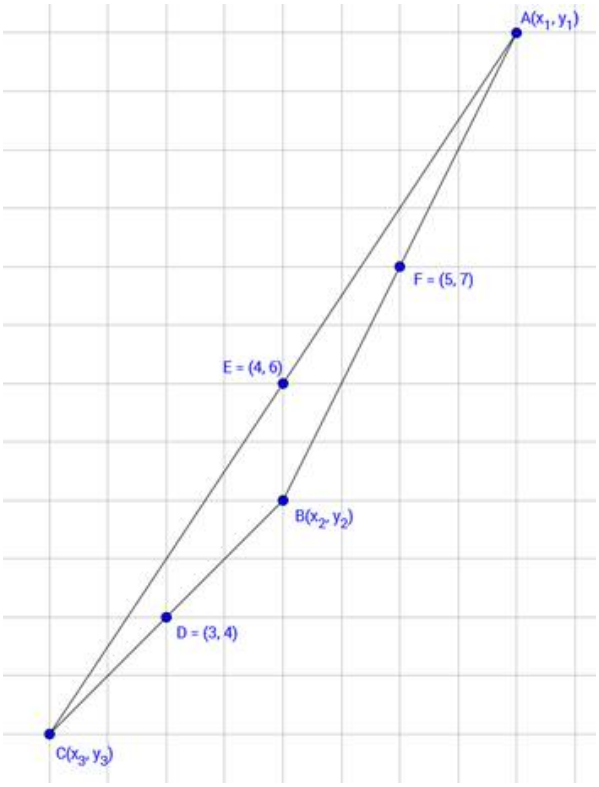
21. Question

If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7), find its vertices.

Answer

Let A(x_1 , y_1), B(x_2 , y_2) and C(x_3 , y_3) be the vertices of triangle.

Let D(3, 4), E(4, 6) and F(5, 7) be the midpoints of sides BC, CA and AB respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D(3, 4) of side BC,

$3 = \frac{x_2 + x_3}{2}$, $4 = \frac{y_2 + y_3}{2}$

$\therefore x_2 + x_3 = 6$ and $y_2 + y_3 = 8$ …(1)

For midpoint E(4, 6) of side CA,

$4 = \frac{x_1 + x_3}{2}$, $6 = \frac{y_1 + y_3}{2}$

$\therefore x_1 + x_3 = 8$ and $y_1 + y_3 = 12$ …(2)

For midpoint F(5, 7) of side AB,

$5 = \frac{x_1 + x_2}{2}$, $7 = \frac{y_1 + y_2}{2}$

$\therefore x_1 + x_2 = 10$ and $y_1 + y_2 = 14$ …(3)

Adding 1,2 and 3, we get,

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + 8 + 10$

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 8 + 12 + 14$

$\therefore 2(x_1 + x_2 + x_3) = 24$ and $2(y_1 + y_2 + y_3) = 34$

$\therefore x_1 + x_2 + x_3 = 12$ and $y_1 + y_2 + y_3 = 17$

$x_1 + 6 = 12$ and $y_1 + 8 = 17$ …from 1

$\therefore x_1 = 6$ and $y_1 = 9$

Substituting above values in 3,

$6 + x_2 = 10$ and $9 + y_2 = 14$

$\therefore x_2 = 4$ and $y_2 = 5$

Similarly for equation 2,

$6 + x_3 = 8$ and $9 + y_3 = 12$

$\therefore x_3 = 2$ and $y_3 = 3$

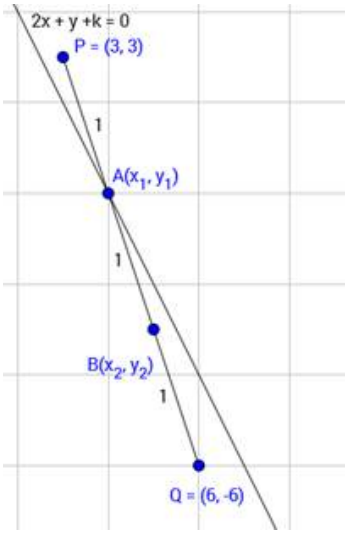
Hence the vertices of triangle are A(6 , 9), B(4 ,5) and C(2 ,3)

22. Question

The line segment joining the points P (3, 3) and Q (6, - 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

Answer

Here, given points are P (3, 3) and Q (6, - 6) which is trisected at the points(say) A(x_1 , y_1) and B(x_2 , y_2)such that A is nearer to P.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point A(x_1 , y_1) of PQ, where m = 2 and n = 1,

$x_1 = \frac{2\times3+1\times6}{2+1}$, $y_1 = \frac{2\times3+1\times(-6)}{2+1}$

$\therefore x_1 = 4$, $y_1 = 0$

∴Coordinates of A is (4,0)

It is given that point A lies on the line 2x + y + k = 0.

So, substituting value of x and y as coordinates of A,

2 × 4 + 0 + k = 0

∴ k = - 8

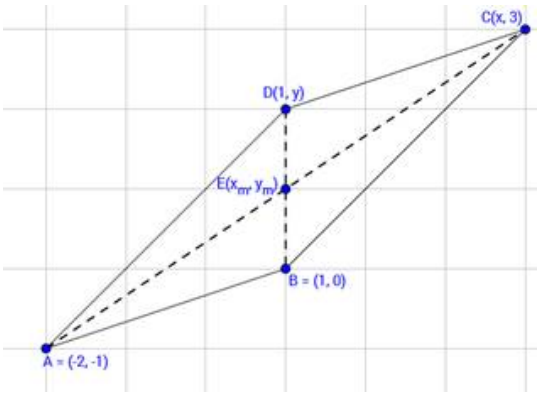
23. Question

If the points (-2 , -1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Answer

Let given points be A(-2, -1), B(1, 0), C(x, 3), D(1, y) and let the intersection of diagonals be E(x_m , y_m)

It is given that □ABCD is a parallelogram.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

We know that midpoint of parallelogram coincide.

∴ Midpoint of AC = Midpoint of BD

$\therefore \left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$

$\therefore \frac{x-2}{2} = \frac{1+1}{2}$ and $\frac{3-1}{2} = \frac{y+0}{2}$

∴x = 4 and y = 2

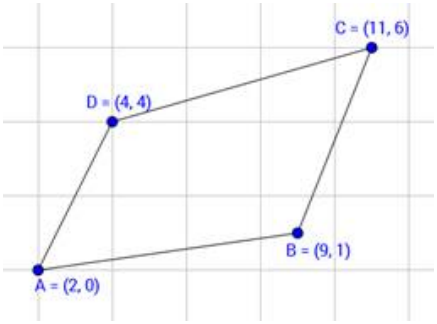
24. Question

The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Answer

Here given points are A (2, 0), B (9, 1), C (11, 6) and D (4, 4).

For a quadrilateral to be rhombus, all sides must be equal.



By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For side AB,

AB = $\sqrt{(9-2)^2 + (1-0)^2}$

= $\sqrt{49+1}$

= $\sqrt{50}$ units.

For BC,

BC = $\sqrt{(11-9)^2 + (6-1)^2}$

= $\sqrt{4+25}$

= $\sqrt{29}$ units

CD = $\sqrt{(11-4)^2 + (6-4)^2}$

= $\sqrt{49+4}$

= $\sqrt{53}$ units.

AD = $\sqrt{(4-2)^2 + (4-0)^2}$

= $\sqrt{4+16}$

= $\sqrt{20}$ units.

Here all sides are unequal.

Hence □ABCD is not a rhombus.

25. Question

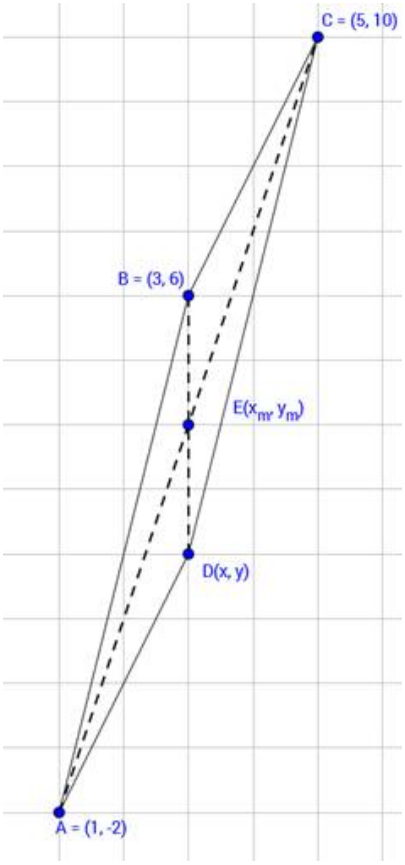
If three consecutive vertices of a parallelogram are (1, -2), (3, 6) and (5, 10), find its fourth vertex.

Answer

Let three vertices be A(1, -2), B(3, 6) and C(5, 10) and fourth vertex be D(x, y)

It is given that quadrilateral joining these four vertices is parallelogram, ie □ABCD is parallelogram.

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.



Let E(x_m , y_m) be the midpoint of diagonals AC and BD.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For diagonal AC,

$x_m = \frac{1+5}{2}$, $y_m = \frac{-2+10}{2}$

$\therefore x_m = \frac{6}{2}$, $y_m = \frac{8}{2}$

$\therefore$ E(x_m , y_m) $\equiv$ (3, 4)

For diagonal BD,

$3 = \frac{3 + x}{2}$, $4 = \frac{6 + y}{2}$

$\therefore$ x = 6 – 3 , y = 8 – 6

$\therefore$ x = 3 and y = 2

Hence, our fourth vertex is D(3 , 2)

26. Question

If the points A(a, -11), B(5, b), C(2, 15) and D(1, 1) are the vertices of a parallelogram ABCD, find the values of a and b.

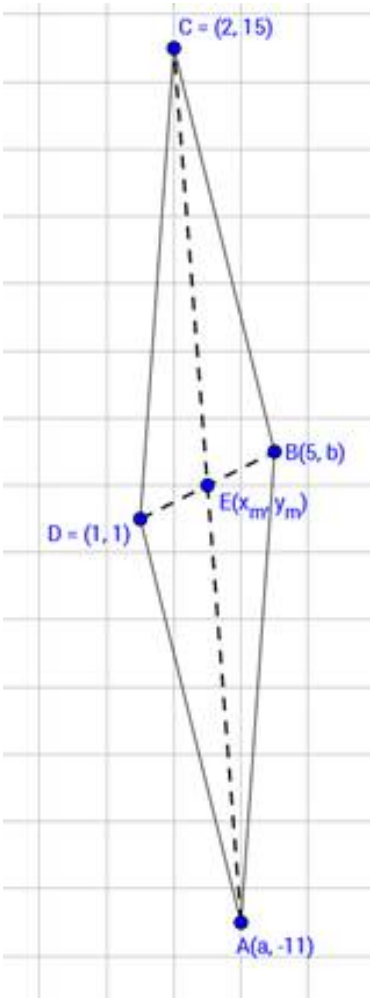
Answer

Given: the points A(a, -11), B(5, b), C(2, 15) and D(1, 1) are the vertices of a parallelogram ABCD.

To find: the values of a and b.

Solution: Given points are A(a, -11), B(5, b), C(2, 15) and D(1, 1) and let the intersection of diagonals be E(x_m , y_m)

It is given that □ABCD is a parallelogram.



By midpoint formula.

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

We know that midpoint of parallelogram coincide.

∴ Midpoint of AC = Midpoint of BD

∴ ($\frac{a+2}{2}$, $\frac{15-11}{2}$) = ($\frac{5+1}{2}$, $\frac{b+1}{2}$)

∴ $\frac{a+2}{2}$ = $\frac{5+1}{2}$ and $\frac{15-11}{2}$ = $\frac{b+1}{2}$

$$\Rightarrow \frac{a+2}{2}=\frac{6}{2}\ and\ \frac{15-11}{2}=\frac{b+1}{2}$$

$$\Rightarrow \frac{a+2}{2}=3 and\ \frac{4}{2}=\frac{b+1}{2}$$

$$\Rightarrow \frac{a+2}{2} = 3 \ and \ \ 2 = \frac{b+1}{2}$$

⇒ a + 2 = 6 and 4 = b + 1⇒ a = 6 - 2 and 4 - 1 = b

⇒ a = 4 and 3 = b

∴a = 4 and b = 3

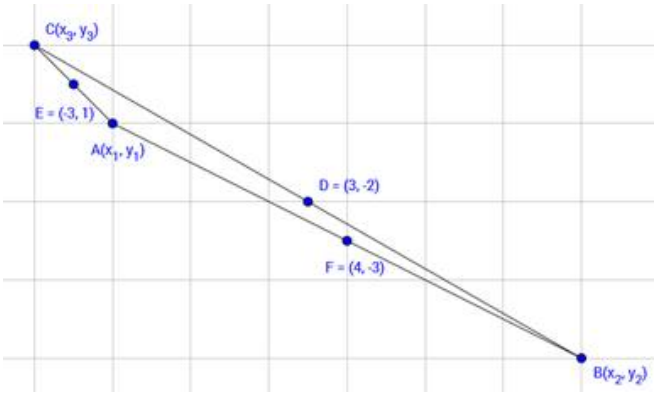
27. Question

If the coordinates of the mid-points of the sides of a triangle be (3, -2), (-3, 1) and (4, -3), then find the coordinates of its vertices.

Answer

Let A(x_1 , y_1), B(x_2 , y_2) and C(x_3 , y_3) be the vertices of triangle.

Let D(3, -2), E(-3, 1) and F(4, -3) be the midpoints of sides BC, CA and AB respectively.



By midpoint formula.

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For midpoint D(3, -2) of side BC,

3 = $\frac{x_2+x_3}{2}$, -2 = $\frac{y_2+y_3}{2}$

∴ $x_2 + x_3$ = 6 and $y_2 + y_3$ = -4 …(1)

For midpoint E(-3, 1) of side CA,

-3 = $\frac{x_1+x_3}{2}$, 1 = $\frac{y_1+y_3}{2}$

∴ $x_1 + x_3$ = -6 and $y_1 + y_3$ = 2 …(2)

For midpoint F(4, -3) of side AB,

4 = $\frac{x_1+x_2}{2}$, -3 = $\frac{y_1+y_2}{2}$

∴ $x_1 + x_2$ = 8 and $y_1 + y_2$ =-6 …(3)

Adding 1,2 and 3, we get,

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2$ =6 -6 + 8

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2$ = -4 + 2 -6

$\therefore 2(x_1 + x_2 + x_3) = 8$ and $2(y_1 + y_2 + y_3) = -8$

$\therefore x_1 + x_2 + x_3 = 4$ and $y_1 + y_2 + y_3 = -4$

$x_1 + 6 = 4$ and $y_1 - 4 = -4$ …from 1

$\therefore x_1 = -2$ and $y_1 = 0$

Substituting above values in 3,

$-2 + x_2 = 8$ and $0 + y_2 = -6$

$\therefore x_2 = 10$ and $y_2 = -6$

Similarly for equation 2,

$-2 + x_3 = -6$ and $0 + y_3 = 2$

$\therefore x_3 = -4$ and $y_3 = 2$

Hence the vertices of triangle are A(-2 , 0), B(10 ,-6) and C(-4 ,2)

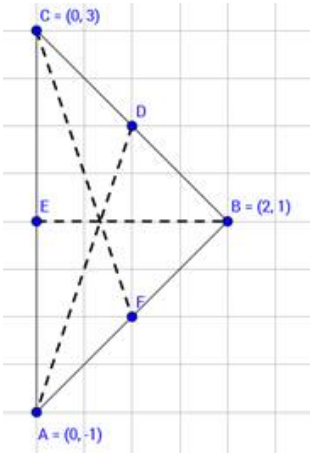
28. Question

Find the lengths of the medians of a $\triangle$ABC having vertices at A (0,-1), B (2, 1) and C (0, 3).

Answer

Here given vertices are A (0,-1), B (2, 1) and C (0, 3) and let midpoints of BC, CA and AB be D,E and F respectively.

By midpoint formula.



$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of side BC,

$x = \frac{2+0}{2}$, $y = \frac{1+3}{2}$

$x = \frac{2}{2}$, $y = \frac{4}{2}$

∴midpoint of side BC is D(1, 2)

For midpoint E of side AB,

$x = \frac{0+0}{2}$, $y = \frac{-1+3}{2}$

$x = \frac{0}{2}$, $y = \frac{2}{2}$

∴midpoint of side AB is E(0, 1)

For midpoint F of side CA,

$x = \frac{2+0}{2}$, $y = \frac{1-1}{2}$

$x = \frac{2}{2}$, $y = \frac{0}{2}$

∴midpoint of side CA is F(1, 0)

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For median AD,

AD = $\sqrt{(1-0)^2 + (2-(-1))^2}$

= $\sqrt{1+9}$

= $\sqrt{10}$ units

For median BE,

BE = $\sqrt{(0-2)^2 + (1-1)^2}$

= $\sqrt{4}$

= 2 units.

For median CF,

CF = $\sqrt{(1-0)^2 + (0-3)^2}$

= $\sqrt{1+9}$

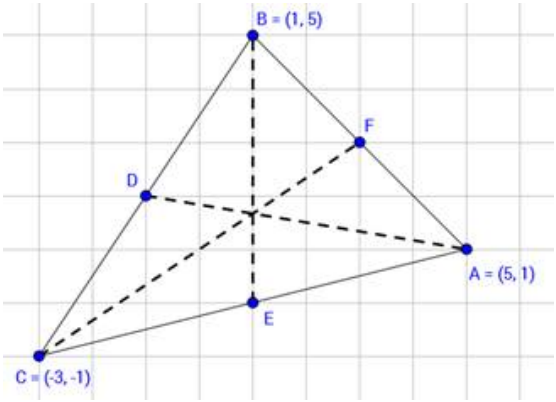
= $\sqrt{10}$ units

29. Question

Find the lengths of the medians of a $\triangle$ABC having vertices at A (5, 1), B (1, 5), and C(-3, -1).

Answer

Here given vertices are A (0,-1), B (2, 1) and C (0, 3) and let midpoints of BC, CA and AB be D,E and F respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of side BC,

$x = \frac{-3+1}{2}$, $y = \frac{-1+5}{2}$

$x = \frac{-2}{2}$, $y = \frac{4}{2}$

∴midpoint of side BC is D(-1, 2)

For midpoint E of side AB,

$x = \frac{-3+5}{2}$, $y = \frac{-1+1}{2}$

$x = \frac{2}{2}$, $y = \frac{0}{2}$

∴midpoint of side AB is E(1, 0)

For midpoint F of side CA,

$x = \frac{1+5}{2}$, $y = \frac{1+5}{2}$

$x = \frac{6}{2}$, $y = \frac{6}{2}$

∴midpoint of side CA is F(3, 3)

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For median AD,

AD = $\sqrt{(-1-5)^2 + (2-1)^2}$

= $\sqrt{36+1}$

= $\sqrt{37}$ units

For median BE,

BE = $\sqrt{(1-1)^2 + (0-5)^2}$

$= \sqrt{25}$

= 5 units.

For median CF,

CF = $\sqrt{(-3-3)^2+(-1-3)^2}$

$= \sqrt{36+16}$

$= 2\sqrt{13}$ units

30. Question

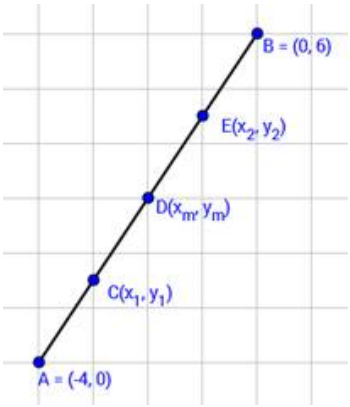
Find the coordinates of the points which divide the line segment joining the points (-4, 0) and (0, 6) in four equal parts.

Answer

Let given coordinates be A(-4, 0) and B (0, 6).

We need to divide AB into 4 equal parts, ie first we need to find midpoint of AB, which will be D and then find out midpoints of AD and DB respectively.

Let required points be C(x_1 , y_1), D(x_m , y_m) and E(x , y_2)



By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint D of AB,

$x_m = \frac{-4+0}{2}$, $y_m = \frac{0+6}{2}$

∴ x_m = -2 and y_m = 3

∴ D(x_m , y_m) ≡ (-2 , 3)

Now, for midpoint C of AD,

$X_1 = \frac{-4-2}{2}$, $y_2 = \frac{0+3}{2}$

x_1 = -3 and y_1 = 1.5

∴C(x_1 , y_1) ≡ (-3, 1.5)

For midpoint E of DB,

$X_2 = \frac{0-2}{2}$, $y_2 = \frac{6+3}{2}$

∴ x_2 = -1 and y_2 = 4.5

∴ D(x_2 , y_2) ≡ (-1 , 4.5)

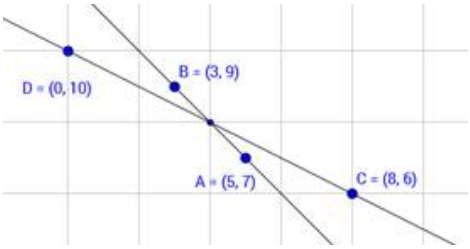
Hence the co-ordinates of the points are (-3, 1.5), (-2 , 3) and (-1 , 4.5)

31. Question

Show that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).

Answer

Let given points be A(5, 7) and B(3, 9) and the points of other segment line be C(8, 6) and D(0, 10)



By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint of AB,

$x = \frac{5+3}{2}$, $y = \frac{7+9}{2}$

x = 4 and y = 8 …(1)

Now for midpoint of CD,

$e = \frac{8+0}{2}$, $d = \frac{6+10}{2}$ …(say)

∴ e = 4 and d = 8 …(2)

Here from 1 and 2 we say that midpoints of AB and CD are same, ie they coincide.

32. Question

Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).

Answer

Let D(x, y) be the midpoints of A(6, 8) and B(2, 4). Let our third given point be C(1, 2).

By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint D of AB,

$x = \frac{6+2}{2}$ and $y = \frac{8+4}{2}$

∴ x =4 and y = 6

∴D(x, y) ≡ (4, 6)

Now to find distance between C and D,

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For CD,

CD = $\sqrt{(4-1)^2 + (6-2)^2}$

= $\sqrt{9+16}$

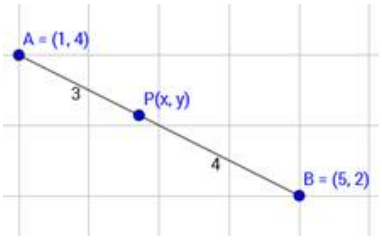
= 5 units

33. Question

If A and B are (1, 4) and (5, 2) respectively, find the coordinates of P when AP/BP = 3/4.

Answer

Given points are A(1, 4) and B(5, 2). Let P be (x, y) and given ratio is 3:4.



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$

For point P on AB,

x = $\frac{5 \times 3+1 \times 4}{3+4}$, y = $\frac{3 \times 2+4 \times 4}{3+4}$

x = $\frac{19}{7}$ and y = $\frac{22}{7}$

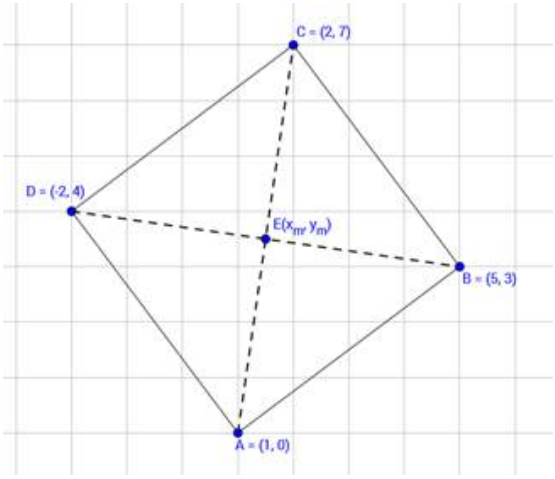
Hence, required coordinates is P($\frac{19}{7}$, $\frac{22}{7}$)

34. Question

Show that the points A (1, 0), B (5, 3), C (2, 7) and D (-2, 4) are the vertices of a parallelogram.

Answer

Let given points be A (1, 0), B (5, 3), C (2, 7) and D (-2, 4) and let the intersection of diagonals be E(x_m , y_m)



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint of diagonal AC,

$X_1 = \frac{1+2}{2}$, $y_1 = \frac{0+7}{2}$

$\therefore x_1 = \frac{3}{2}$, $y_1 = \frac{7}{2}$

∴ midpoint of diagonal AC is $(x_1, y_1) \equiv (\frac{3}{2}, \frac{7}{2})$ …(1)

For midpoint of diagonal BD,

$X_2 = \frac{5-2}{2}$, $y_2 = \frac{3+4}{2}$

$\therefore x_2 = \frac{3}{2}$, $y_2 = \frac{7}{2}$

∴ midpoint of diagonal BD is $(x_2, y_2) \equiv (\frac{3}{2}, \frac{7}{2})$ …(2)

Here, from 1 and 2 we say that midpoint of both the diagonals intersect at same point, ie $(\frac{3}{2}, \frac{7}{2})$

But our intersection of diagonals is at E, which means that midpoint of diagonals intersect at single point, ie $E(\frac{3}{2}, \frac{7}{2})$

We know that if midpoints of diagonals intersect at single point, then quadrilateral formed by joining the points is parallelogram.

Hence, our □ABCD is parallelogram.

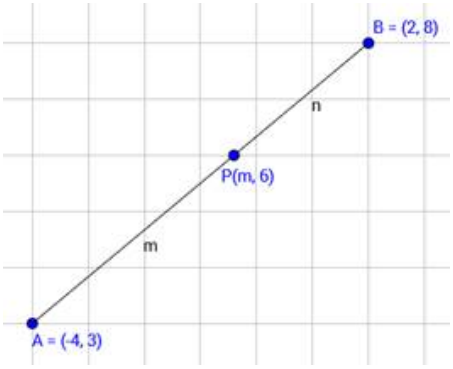
35. Question

Determine the ratio in which the point P (m, 6) divides the join of A(-4, 3) and B(2, 8). Also, find the value of m.

Answer

Here, given points are A (-4, 3) and B (2, 8) and let the point dividing the line joining two points be P(m,6).

Let the ratio be m:n



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P(m,6),

$m = \frac{m\times2+n\times(-4)}{m+n}$ …(1)

And $6 = \frac{m\times8+n\times3}{m+n}$ …(2)

Solving 2 for finding ratio between m and n,

$6 = \frac{m\times8+n\times3}{m+n}$

6(m + n) = 8m +3n

6m + 6n = 8m +3n

∴ 2m = 3n

$\therefore \frac{m}{n} = \frac{3}{2}$

∴ m : n = 3 : 2

Now solving for equation 1, where m = 3 and n =2

$m = \frac{m\times2+n\times(-4)}{m+n}$

$\therefore m = \frac{6-8}{5}$

$\therefore m = \frac{-2}{5}$

Hence, our point is $(\frac{-2}{5}, 6)$

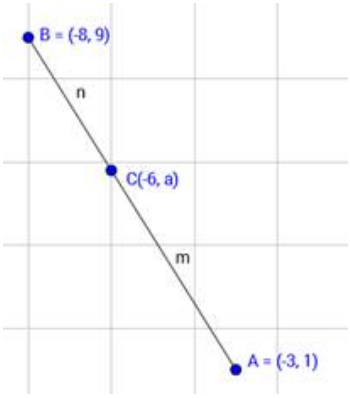
36. Question

Determine the ratio in which the point (-6, a) divides the join of A(-3, 1) and B(-8, 9). Also find the value of a.

Answer

Here, given points are A(-3, 1) and B(-8, 9) and let the point dividing the line joining two points be C(-6,a).

Let the ratio be m:n



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(-6,a),

$-6 = \frac{m\times(-8)+n\times(-3)}{m+n}$ ...(1)

And $a = \frac{m\times9+n\times1}{m+n}$ ...(2)

Solving 1 for finding ratio between m and n,

$-6 = \frac{m\times(-8)+n\times(-3)}{m+n}$

-6 (m + n) = -8m -3n

6m + 6n = 8m + 3n

∴ 2m = 3n

$\therefore \frac{m}{n} = \frac{3}{2}$

∴ m : n = 3 : 2

Now solving for equation 2, where m = 3 and n = 2

$a = \frac{m\times9+n\times1}{m+n}$

$a = \frac{3\times9+2\times1}{3+2}$

$\therefore a = \frac{27+2}{5}$

$\therefore a = \frac{29}{5}$

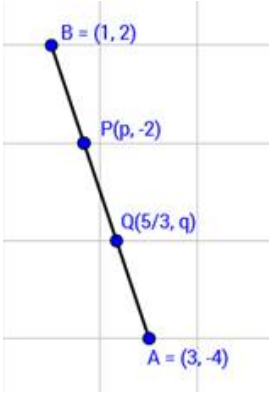
∴ value of a is $\frac{29}{5}$

37. Question

The line segment joining the points (3, -4) and (1, 2) is trisected at the points P and Q . If the coordinates of P and Q are (p, -2) and (5/3, q) respectively. Find the values of p and q.

Answer

Let given points be A(3, -4) and B(1, 2) , which is trisected at points P(p, -2) and Q(5/3, q).



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$As point P divides the line in 1:2 and Q divides the line in 2:1.

For point P(p, -2)of AB, where m = 1 and n = 2,

$p= \frac{1\times1+3\times2}{1+2}$, $-2 = \frac{2\times1+2\times(-4)}{1+2}$

Solving for p,

$P = \frac{7}{3}$

For point Q(5/3, q) of AB, where m = 2 and n = 1,

$\frac{5}{3} = \frac{2\times1+1\times3}{2+1}$,$q = \frac{2\times2+1\times(-4)}{2+1}$

Solving for q,

$q = \frac{4-4}{3}$

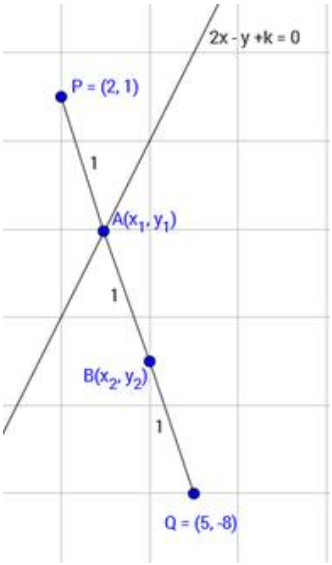
$\therefore q = 0$Hence, the value of p and q are $\frac{7}{3}$ and 0 respectively.

38. Question

The line joining the points (2,1) and (5,-8) is trisected at the points P and Q. If point P lies on the line 2x - y + k = 0. Find the value of k.

Answer

Here, given points are P (2, 1) and Q (5, - 8) which is trisected at the points(say) A(x_1 , y_1) and B(x_2 , y_2)such that A is nearer to P.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point A(x_1 , y_1) of PQ, where m = 1 and n = 2,

$x_1 = \frac{1\times5+2\times2}{1+2}$, $y_1 = \frac{1\times(-8)+2\times1}{1+2}$

∴ x_1 = 3 , y_1 = -2

∴Coordinates of A is (3,-2)

It is given that point A lies on the line 2x - y + k = 0.

So, substituting value of x and y as coordinates of A,

2 × 3 – (-2) + k = 0

∴ k = -8

39. Question

If A and B are two points having coordinates (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$AB.

Answer

Given points are A(-2, -2) and B(2, -4). Let P be (x, y)

Here given that AP = $\frac{3}{7}$AB.

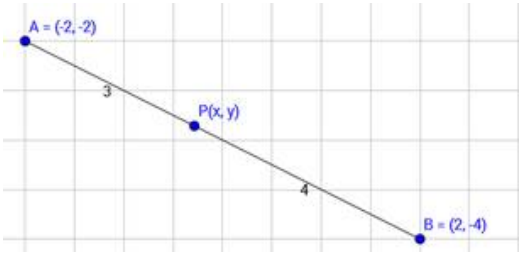
But AB = AP + BP

∴7AP = 3AB

7AP = 3(AP + BP)

∴4AP = 3BP

$\therefore \frac{AP}{BP} = \frac{3}{4}$



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P on AB, where m = 3 and n = 4

$x = \frac{3 \times 2+4 \times(-2)}{3+4}$, $y = \frac{3 \times(-4)+4 \times(-2)}{3+4}$

$x = \frac{-2}{7}$ and $y = \frac{-20}{7}$

Hence, required coordinates is $P(\frac{-2}{7}, \frac{-20}{7})$

40. Question

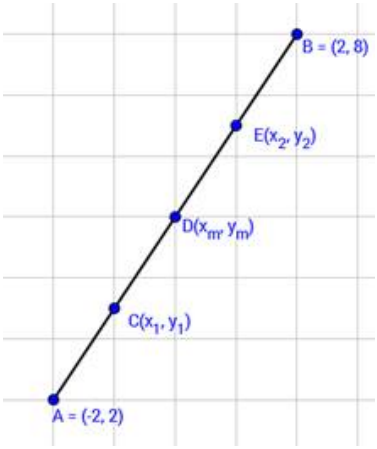
Find the coordinates of the points which divide the line segment joining A (-2, 2) and B (2, 8) into four equal parts.

Answer

Let given coordinates be A(-2, 2) and B (2, 8).

We need to divide AB into 4 equal parts, ie first we need to find midpoint of AB, which will be D and then find out midpoints of AD and DB respectively.

Let required points be $C(x_1 , y_1)$, $D(x_m , y_m)$ and $E(x , y_2)$



By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint D of AB,

$x_m = \frac{-2+2}{2}$, $y_m = \frac{2+8}{2}$

$\therefore x_m = 0$ and $y_m = 5$

$\therefore D(x_m , y_m) \equiv (0 , 5)$

Now, for midpoint C of AD,

$X_1 = \frac{-2+0}{2}$, $y_2 = \frac{2+5}{2}$

x_1 = -1 and $y_1 = \frac{7}{2}$

$\therefore C(x_1 , y_1) \equiv (-1, \frac{7}{2})$

For midpoint E of DB,

$X_2 = \frac{2+0}{2}$, $y_2 = \frac{8+5}{2}$

$\therefore x_2 = 1$ and $y_2 = \frac{13}{2}$

$\therefore E(x_2 , y_2) \equiv (1 , \frac{13}{2})$

Hence the co-ordinates of the points are $(-1, \frac{7}{2})$, (0 , 5) and $(1 , \frac{13}{2})$

41. Question

A (4, 2), B (6, 5) and C (1, 4) are the vertices of $\triangle$ABC.

(i) The median from A meets BC in D. Find the coordinates of the point D.

(ii) Find the coordinates of point P on AD such that AP : PD = 2 :1.

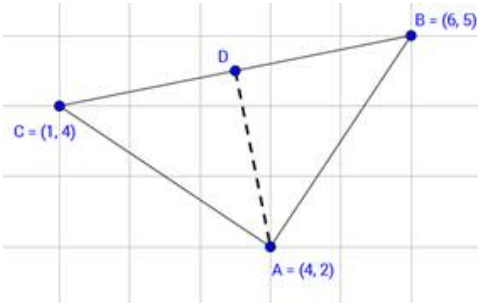
(iii) Find the coordinates of the points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

(iv) What do you observe?

Answer

(i) The median from A meets BC in D. Find the coordinates of the point D.

Here given vertices are A (4, 2), B (6, 5) and C (1, 4).



By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

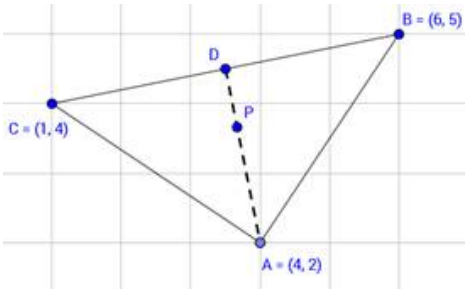
For midpoint D of side BC,

$x = \frac{6+1}{2}$, $y = \frac{5+4}{2}$

$x = \frac{7}{2}$, $y = \frac{9}{2}$

Hence, the coordinates of D are $(\frac{7}{2}, \frac{9}{2})$

(ii) Find the coordinates of point P on AD such that AP : PD = 2 :1.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P on AD, where m = 2 and n = 1

$x = \frac{2\times\frac{7}{2}+1\times4}{2+1}$, $y = \frac{2\times\frac{9}{2}+1\times2}{2+1}$

$\therefore x = \frac{11}{3}$ and $y = \frac{11}{3}$

(iii) Find the coordinates of the points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint E of side AC,

$x = \frac{1+4}{2}$, $y = \frac{4+2}{2}$

$x = \frac{5}{2}$, $y = \frac{6}{2}$

Hence, the coordinates of E are $(\frac{5}{2}, 3)$

For midpoint F of side AB,

$x = \frac{6+4}{2}$, $y = \frac{5+2}{2}$

$x = \frac{10}{2}$, $y = \frac{7}{2}$

Hence, the coordinates of F are $(5, \frac{7}{2})$

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point Q on BE, where m = 2 and n = 1

$x = \frac{2\times\frac{5}{2}+1\times6}{2+1}$, $y = \frac{2\times3+1\times5}{2+1}$

$\therefore x = \frac{11}{3}$ and $y = \frac{11}{3}$

For point R on CF, where m = 2 and n = 1

$x = \frac{2 \times 5 + 1 \times 1}{2+1}$, $y = \frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}$

$\therefore x = \frac{11}{3}$ and $y = \frac{11}{3}$

(iv) What do you observe?

We observe that the point P,Q and R coincides with the centroid.

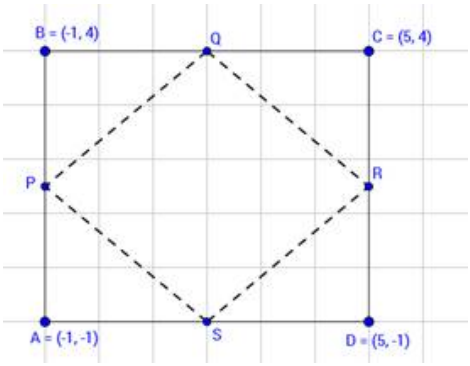
This also shows that centroid divides the median in the ratio 2:1

42. Question

ABCD is a rectangle formed by joining the points A (-1, -1), B (-1, 4), C (5, 4) and D (5,-1). P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Answer

Here given that A (-1, -1), B (-1, 4), C (5, 4) and D (5,-1).Also P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint P of side AB,

$x = \frac{-1-1}{2}$, $y = \frac{-1+4}{2}$

$x = -1$, $y = \frac{3}{2}$

Hence, the coordinates of P are $(-1, \frac{3}{2})$

For midpoint Q of side BC,

$x = \frac{-1+5}{2}$, $y = \frac{4+4}{2}$

$x = 2$, $y = 4$

Hence, the coordinates of Q are (2 ,4)

For midpoint R of side CD,

$x = \frac{5+5}{2}$, $y = \frac{-1+4}{2}$

$x = 5$, $y = \frac{3}{2}$

Hence, the coordinates of R are $(5 , \frac{3}{2})$

For midpoint S of side AD,

$x = \frac{-1+5}{2}$, $y = \frac{-1-1}{2}$

$x = 2$, $y = -1$

Hence, the coordinates of S are (2 ,-1)

Now we find length of the length of the □PQRS,

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For PQ,

PQ = $\sqrt{(2-(-1))^2 + \left(4 - \frac{3}{2}\right)^2}$

= $\sqrt{9 + \frac{25}{2}}$

= $\sqrt{\frac{61}{2}}$ units

For QR,

QR = $\sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2}$

= $\sqrt{9 + \frac{25}{2}}$

= $\sqrt{\frac{61}{2}}$ units

For RS,

RS = $\sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2}$

= $\sqrt{9 + \frac{25}{2}}$

= $\sqrt{\frac{61}{2}}$ units

For PS,

PS = $\sqrt{(2-(-1))^2 + \left(-1 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{2}}$

$= \sqrt{\frac{61}{2}}$ units

Here we can observe that all lengths of □PQRS are equal.

Now for diagonal PR,

PR = $\sqrt{(5-(-1))^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$

$= \sqrt{36 + 0}$

$= 6$ units

Now for diagonal QS,

QS = $\sqrt{(2-2)^2 + (-1 - 4)^2}$

$= \sqrt{0 + 25}$

= 5 units

Here in □PQRS, diagonals are unequal.

We know that a quadrilateral whose all sides are equal and diagonals are unequal, it is a rhombus.

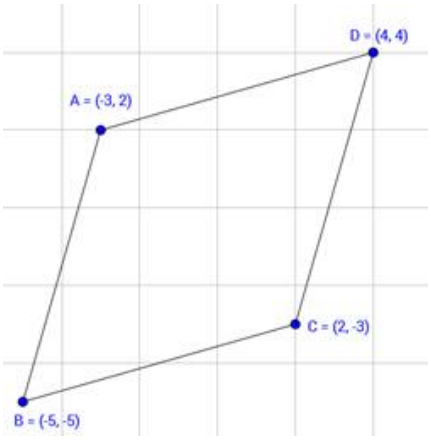
Hence, our □PQRS is rhombus .

43. Question

Show that A(-3, 2), B (-5, -5), C (2, -3) and D(4, 4) are the vertices of a rhombus.

Answer

solution: Given points are A(-3, 2), B (-5, -5), C (2, -3) and D(4, 4)



Use distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For AB ,

AB = $\sqrt{(-5-(-3))^2 + (-5-2)^2}$

$$= \sqrt{(-5+3)^2 + (-5-2)^2}$$

$$= \sqrt{(-2)^2 + (-7)^2}$$

= $\sqrt{4+49}$

= $\sqrt{53}$ units

For BC ,

BC = $\sqrt{(2-(-5))^2 + (-3-(-5))^2}$

$$= \sqrt{(2+5)^2 + (-3+5)^2}$$

$$= \sqrt{7^2 + 2^2}$$

= $\sqrt{49+4}$

= $\sqrt{53}$ units

For CD,

CD = $\sqrt{(4-2)^2 + (4-(-3))^2}$

$$= \sqrt{(4-2)^2 + (4+3)^2}$$

$$= \sqrt{(2)^2 + (7)^2}$$

= $\sqrt{4+49}$

= $\sqrt{53}$ units

For AD ,

AD= $\sqrt{(4-(-3))^2 + (4-2)^2}$

$$= \sqrt{(4+3)^2 + (4-2)^2}$$

$$= \sqrt{(7)^2 + (2)^2}$$

= $\sqrt{49+4}$

= $\sqrt{53}$ units

Here we can observe that all lengths of □PQRS are equal.

Now for diagonal AC,

AC = $\sqrt{(2-(-3))^2 + (-3-2)^2}$

$$= \sqrt{(2+3)^2 + (-3-2)^2}$$

$$= \sqrt{(5)^2 + (-5)^2}$$

= $\sqrt{25+25}$

= $\sqrt{50}$units

Now for diagonal BD,

BD = $\sqrt{(4-(-5))^2 + (4-(-5))^2}$

$$= \sqrt{(4+5)^2 + (4+5)^2}$$

$$= \sqrt{(9)^2 + (9)^2}$$

= $\sqrt{81+81}$

= $\sqrt{162}$ unitsAB = BC = CD = AD

And AC ≠ BD

Here in ABCD, diagonals are unequal.

We know that a quadrilateral whose all sides are equal and diagonals are unequal, it is a rhombus.

Hence, ABCD is rhombus .

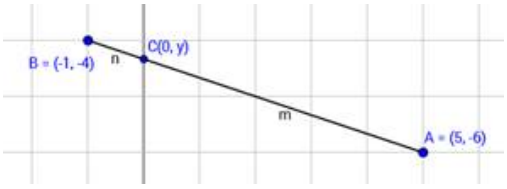
44. Question

Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also, find the coordinates of the point of division.

Answer

Let our points be A(5, -6) and B(-1, -4).

Let point C(0, y) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$

For point C(0, y)

0 = $\frac{m\times(-1)+n\times5}{m+n}$, y = $\frac{m\times(-4)+n\times(-6)}{m+n}$

Solving for x coordinate,

0 = $\frac{m\times(-1)+n\times5}{m+n}$

∴ m = 5n

∴ $\frac{m}{n} = \frac{5}{1}$

∴ m : n = 5 : 1

Now solving for y coordinate, with m = 5 and n = 1,

y = $\frac{my_2+ny_1}{m+n}$

y = $\frac{-4\times5-6}{6}$

∴ y = $\frac{-13}{6}$

There is no need to solve for x, as our point lies on y-axis

Hence, the coordinates of required point is C(0 , $\frac{-13}{6}$)

45. Question

If the points A (6, 1), B (8, 2), C (9, 4) and D (k, p) are the vertices of a parallelogram taken in order, then find the values of k and p.

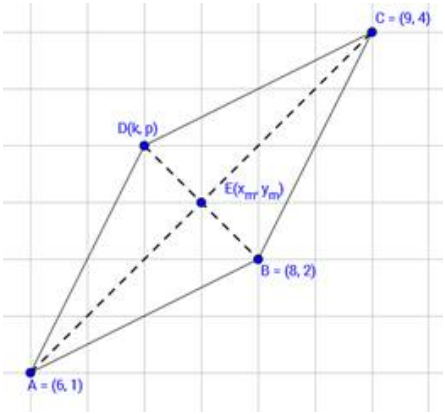
Answer

Our given vertices are A(1, -2), B(3, 6) and C(5, 10) and fourth vertex be D(k, p)

It is given that quadrilateral joining these four vertices is parallelogram, ie □ABCD is parallelogram.

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.

Let E(x_m , y_m) be the midpoint of diagonals AC and BD.



By midpoint formula,

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For diagonal AC,

x_m = $\frac{6+9}{2}$, y_m = $\frac{4+1}{2}$

∴ x_m = $\frac{15}{2}$, y_m = $\frac{5}{2}$

∴ E(x_m , y_m) ≡ ($\frac{15}{2}$, $\frac{5}{2}$)

For diagonal BD,

$\frac{15}{2} = \frac{8+k}{2}, \frac{5}{2} = \frac{p+2}{2}$

∴ k = 15 – 8 , y = 5 – 2

∴ k = 7 and p = 3

Hence, our fourth vertex is D(7 , 3)

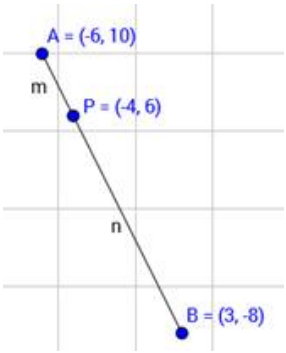
46. Question

In what ratio does the point (-4, 6) divide the line segment joining the points A (-6, 10) and B(3, -8)?

Answer

Given points are A (-6, 10) and B(3, -8)

Let the point C(-4, 6) divide AB in ratio m:n.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(-4, 6) on the line joined by the points A and B.

$-4 = \frac{m\times3+n\times(-6)}{m+n}$ …(1)

And, $6 = \frac{m\times(-8)+n\times10}{m+n}$ …(2)

Solving 1,

-4(m + n) = 3m - 6n

∴4m + 4n = -3m + 6n

∴ 7m = 2n

$\therefore \frac{m}{n} = \frac{2}{7}$

Hence, ratio is 2:7.

47. Question

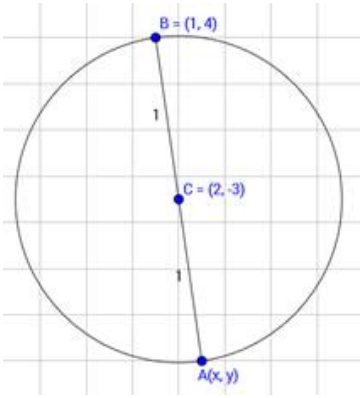
Find the coordinates of a point A, where AB is a diameter of the circle whose centre is (2, -3) and B is (1, 4).

Answer

Here given that AB is a diameter of the circle whose centre is (say) C(2, -3) and B is (1, 4)

Let A be (x, y)

We know that as C is center, AC = CB or C is midpoint of AB.



By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For Center C,

$2 = \frac{x + 1}{2}$ and $-3 = \frac{y + 4}{2}$

∴ x = 4 – 1 and y = -6 – 4

∴ x = 3 and y = -10

Hence, coordinates of A are (3, -10)

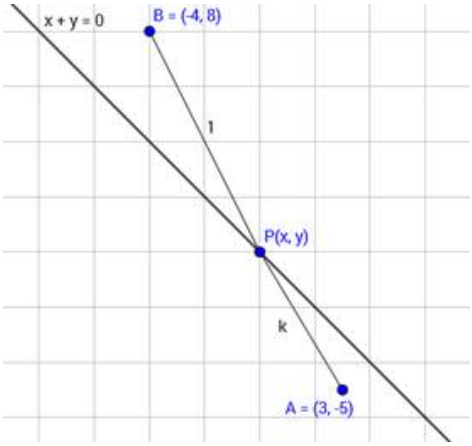
48. Question

A point P divides the line segment joining the points A (3, -5) and B (-4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of k.

Answer

Here given points are A (3, -5) and B (-4, 8) .

Let point P be (x, y) which divides AB in ratio of k:1, also point P lies on line x + y = 0



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point P on the line joined by the points A and B.

$x = \frac{k \times (-4) + 1 \times 3}{k+1}$, $y = \frac{k \times 8 + 1 \times (-5)}{k+1}$

Putting in given equation,

$\frac{k \times (-4) + 1 \times 3}{k+1}, \frac{k \times 8 + 1 \times (-5)}{k+1} = (x, y)$

$$x = \frac{-4k + 3}{k + 1} \text{ and } y = \frac{8k - 8}{k + 1}$$

$$\text{Now } (x, y) \text{ lies on the line } x + y = 0$$

$$\text{Therefore, the points will satisfy the equation.}$$

$$\text{Hence,}$$

$$\frac{-4k + 3}{k + 1} + \frac{8k - 8}{k + 1} = 0$$

- 4 k + 3 + 8 k - 8 = 04k - 5 = 0

∴ 4k = 5

∴ k = 5/2

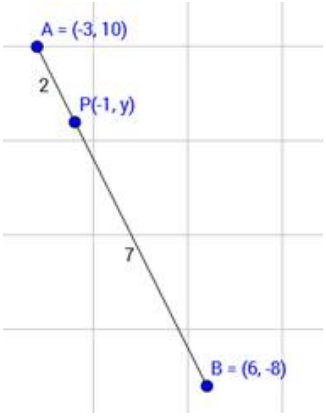
49. Question

Find the ratio in which the point P(-1, y) line segment joining A (-3,10) and B(6, -8) divides it. Also find the value of y.

Answer

Here, given points are A (-3,10) and B(6, -8) and the point dividing the line joining two points is P(-1,y).

Let the ratio be m:n



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point P(-1,a),

$-1 = \frac{m \times 6 + n \times (-3)}{m+n}$ ...(1)

And $y = \frac{m \times (-8) + n \times 10}{m+n}$ …(2)

Solving 1 for finding ratio between m and n,

$-1 = \frac{m \times 6 + n \times (-3)}{m+n}$

$-(m + n) = 6m - 3n$

$m + n = -6m + 3n$

$\therefore 7m = 2n$

$\therefore \frac{m}{n} = \frac{2}{7}$

$\therefore m : n = 2 : 7$

Now solving for equation 2, where m = 2 and n = 7

$y = \frac{m \times (-8) + n \times 10}{m+n}$

$y = \frac{2 \times (-8) + 7 \times 10}{2+7}$

$\therefore y = \frac{-16+70}{9}$

$\therefore y = \frac{54}{9}$

$\therefore$ value of y is 6

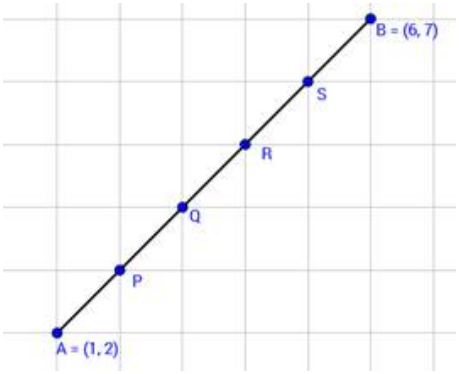
50. Question

Points p, Q, R and S divide the segment joining the points A (1, 2) and B (6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

Answer

Here given points are A (1, 2) and B (6, 7) which is divided into 5 equal parts by points P, Q, R and S

$\therefore$ AP = PQ= QR = RS = SB



The point P divides the line segment AB in the ratio 1:4.

By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point P,

$x = \frac{1 \times 6+4 \times 1}{1+4}$, $y = \frac{1 \times 7+4 \times 1}{1+4}$

x = 2 and y = 3

∴ Coordinate of P is (2 ,3)

The point Q divides the line segment AB in the ratio of 2:3.

For point Q,

$x = \frac{2 \times 6+3 \times 1}{2+3}$, $y = \frac{2 \times 7+3 \times 1}{2+3}$

x = 3 and y = 4

∴ Coordinate of Q is (3 ,4)

The point R divides the line segment AB in the ratio of 3:2.

For point R,

$x = \frac{3 \times 6+2 \times 1}{3+2}$, $y = \frac{3 \times 7+2 \times 1}{3+2}$

x = 4 and y = 5

∴ Coordinate of R is (4 , 5)

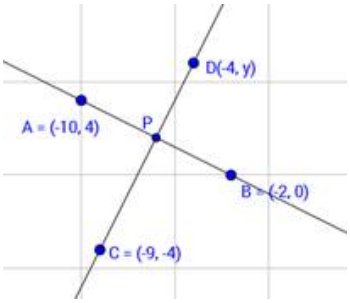
51. Question

The mid-point P of the line segment joining the points A (- 10, 4) and B (- 2, 0) lies on the line segment joining the points C (- 9,-4) and D (- 4, y). Find the ratio in which P divides CD. Also, find the value of y.

Answer

Here given points are A (- 10, 4) and B (- 2, 0)and the points of other segment line are C (- 9,-4) and D (- 4, y)

Let the point of intersection between AB and CD be P



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint of AB,

$e = \frac{-10-2}{2}$, $d = \frac{4+0}{2}$ …(say)

e = -6 and d = 2 …(1)

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P on CD, where ratio is m:n,

$-6 = \frac{m \times(-4)+n\times(-9)}{m+n}$ and $2 = \frac{m \times y+n\times(-4)}{m+n}$

Solving for m and n,

$-6 = \frac{m \times(-4)+n\times(-9)}{m+n}$

$\therefore -6(m + n) = -4m - 9n$

$6m + 6n = 4m + 9n$

$2m = 3n$

$\therefore \frac{m}{n} = \frac{3}{2}$

∴ Ratio is 3:2

Now solving for y, where m = 3 and n = 2,

$2 = \frac{3 \times y+2\times(-4)}{3+2}$

$\therefore 3y - 8 = 10$

$\therefore 3y = 18$

$\therefore y = 6$

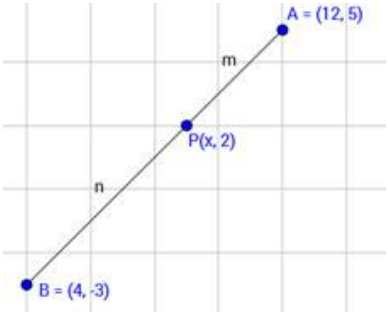
52. Question

Find the ratio in which the point P (x, 2) divides the line segment joining the points A (12,5) and B (4, -3). Also, find the value of x.

Answer

Here, given points are A (12,5) and B (4, -3) and let the point dividing the line joining two points be P(x,2)

Let the ratio be m:n



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P(x,2),

$x = \frac{m \times 4+n \times 12}{m+n}$ …(1)

And $2 = \frac{m \times(-3)+n \times 5}{m+n}$ …(2)

Solving 2 for finding ratio between m and n,

$2 = \frac{m \times(-3)+n \times 5}{m+n}$

2(m + n) = -3m +5n

2m + 2n = -3m +5n

∴ 5m = 3n

$\therefore \frac{m}{n} = \frac{3}{5}$

∴ m : n = 3 : 5

Now solving for equation 1, where m = 3 and n =5

$x = \frac{3 \times 4 + 5 \times 12}{3+5}$

$\therefore x = \frac{12+60}{8}$

∴ x = 9

Hence, our point is (9, 2)

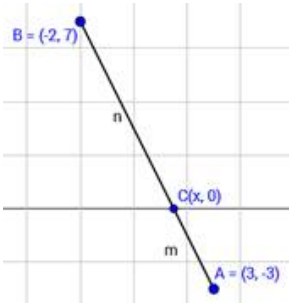
53. Question

Find the ratio in which the line segment joining the points A (3,-3) and B (-2, 7) is divided by x-axis. Also, find the coordinates of the point of division.

Answer

Our points are A (3,-3) and B (-2, 7)

Let point C(x, 0) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(x, 0)

$x = \frac{m \times (-2) + n \times 3}{m+n}$, $0 = \frac{m \times 7 + n \times (-3)}{m+n}$

Solving for y coordinate,

$0 = \frac{m \times 7 + n \times (-3)}{m+n}$

∴ 7m -3n = 0

∴ 7m = 3n

$\therefore \frac{m}{n} = \frac{3}{7}$

∴ m : n = 3 : 7

Now solving for x coordinate, with m = 3 and n = 7,

$x = \frac{3 \times(-2)+7 \times 3}{3+7}$

$\therefore x = \frac{-6+21}{10}$

$\therefore x = \frac{15}{10} = \frac{3}{2}$

Hence, the coordinates of required point is C($\frac{3}{2}$, 0)

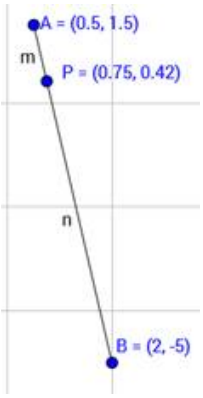
54. Question

Find the ratio in which the points P (3/4, 5/12) divides the line segments joining the points A(1/2 , 3/2) and B(2, -5).

Answer

Given points are A(1/2 , 3/2) and B(2, -5)

Let the point P(3/4, 5/12) divide AB in ratio m:n.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P on the line joined by the points A and B.

$\frac{3}{4} = \frac{m \times 2+n \times \frac{1}{2}}{m+n}$ ...(1)

And, $\frac{5}{12} = \frac{m \times(-5)+n \times \frac{3}{2}}{m+n}$ ...(2)

Solving 1,

3(m + n) = 8m + 2n

∴3m + 3n = 8m + 2n

∴ 5m = n

$\therefore \frac{m}{n} = \frac{1}{5}$

Hence, ratio is 1:5.

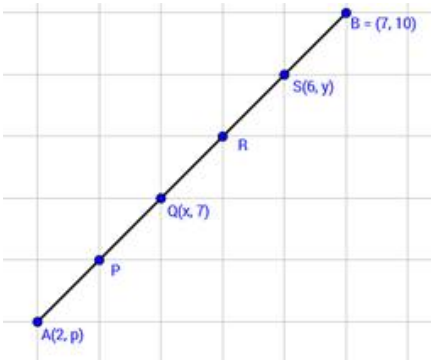
55. Question

If the points P, Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B (7, 10) in 5 equal parts, find x, y and p.

Answer

Here given points are A (2, p) and B (7, 10) which is divided into 5 equal parts by points P, Q(x, 7), R and S(6, y)

∴ AP = PQ= QR = RS = SB



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

The point Q divides the line segment AB in the ratio of 2:3.

For point Q,

$x = \frac{2\times7+3\times2}{2+3}$, $7 = \frac{2\times10+3\times p}{2+3}$

Solving above equations, we get,

x = 4 and p = 5

For point P, divides the line segment AB in the ratio 4:1.

$6 = \frac{4\times7+1\times2}{4+1}$, $y = \frac{4\times10+1\times p}{4+1}$

Solving for y and substituting value of p,

$y = \frac{40+5}{5}$

∴ y = 9

Hence, values are x = 4, y = 9 and p = 5

Exercise 14.4

1. Question

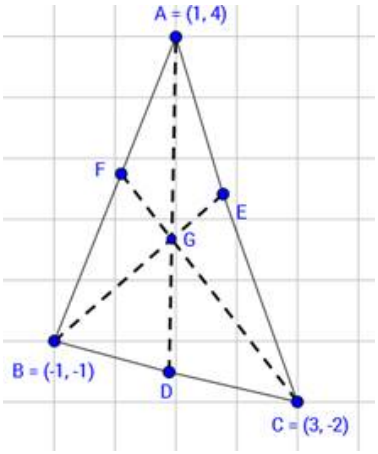
Find the centroid of the triangle whose vertices are:

(i) (1, 4), (-1, -1), (3, -2)

(ii) (- 2, 3), (2, -1), (4, 0)

Answer

(i) (1, 4), (-1, -1), (3, -2)



We know that centroid of a triangle for the vertices (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) is

$G(x, y) = \left(\frac{x_1+x_2+x_3}{3} , \frac{y_1+y_2+y_3}{3} \right)$

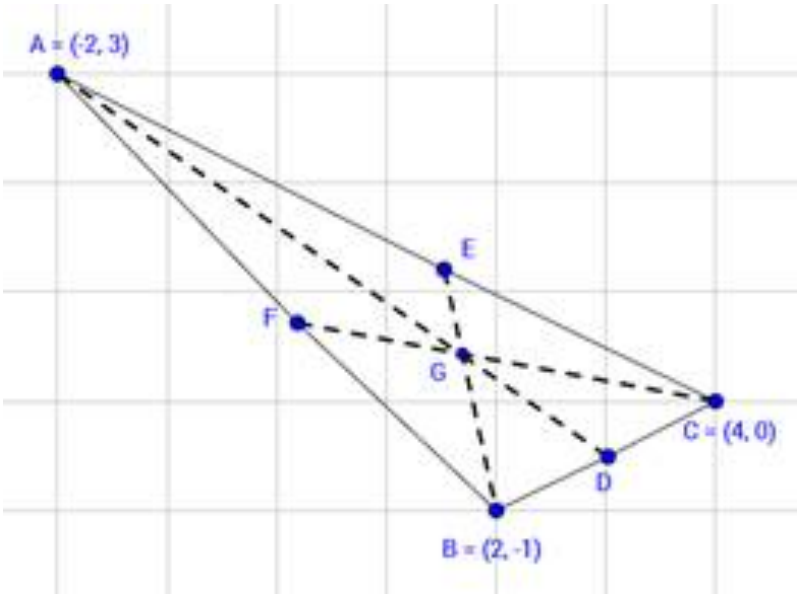
∴For coordinates (1, 4), (-1, -1), (3, -2),

Centroid of triangle $= \left(\frac{1-1+3}{3} , \frac{4-1-2}{3} \right)$

$= \left(1, \frac{1}{3} \right)$

Hence, centroid of triangle is $\left(1, \frac{1}{3} \right)$

(ii) (- 2, 3), (2, -1), (4, 0)



We know that centroid of a triangle for (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) is

$G(x, y) = \left(\frac{x_1+x_2+x_3}{3} , \frac{y_1+y_2+y_3}{3} \right)$

∴For coordinates (- 2, 3), (2, -1), (4, 0)

Centroid of triangle $= \left(\frac{-2+2+4}{3} , \frac{3-1+0}{3} \right)$

$= \left(\frac{4}{3}, \frac{2}{3} \right)$

5. Trigonometric Ratios

Exercise 5.1

1. Question

In each of the following, one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.

(i) $\sin A = \frac{2}{3}$ (ii) $\cos A = \frac{4}{5}$

(iii) $\tan\theta = 11$ (iv) $\sin\theta = \frac{11}{15}$

(v) $\tan\alpha = \frac{5}{12}$ (vi) $\sin\theta = \frac{\sqrt{3}}{2}$

(vii) $\cos\theta = \frac{7}{25}$ (viii) $\tan\theta = \frac{8}{15}$

(ix) $\cot\theta = \frac{12}{5}$ (x) $\sec\theta = \frac{13}{5}$

(xi) $\operatorname{cosec}\theta = \sqrt{10}$ (xii) $\cos\theta = \frac{12}{15}$

Answer

(i)

$$\sin A = \frac{2}{3} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = (3)^2 - (2)^2 = 9 - 4 = 5$$

$$base = \sqrt{5}$$

$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{\sqrt{5}}{3}, \ \tan A = \frac{perpendicular}{base} = \frac{2}{\sqrt{5}},$$

$$\cot A = \frac{base}{perpendicular} = \frac{\sqrt{5}}{2}, \sec A = \frac{hypotenuse}{base} = \frac{3}{\sqrt{5}},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{3}{2}$$

(ii)

$$\cos A = \frac{2}{5} = \frac{base}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$$

$$(perpendicular)^2 = (5)^2 - (4)^2$$

$$(perpendicular)^2 = 25 - 16 = 9$$

$$perpendicular = 3$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{3}{5}, \ \tan A = \frac{perpendicular}{base} = \frac{3}{4},$$

$$\cot A = \frac{base}{perpendicular} = \frac{4}{3}, \sec A = \frac{hypotenuse}{base} = \frac{5}{4},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{5}{3}$$

(iii)

$$\tan\theta = \frac{11}{1} = \frac{Perpendicular}{base}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(hypotenuse)^2 = (1)^2 + (11)^2 = 1 + 121 = 122$$

$$hypotenuse = \sqrt{122}$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{11}{\sqrt{122}}, \ \cos A = \frac{base}{hypotenuse} = \frac{1}{\sqrt{122}},$$

$$\cot A = \frac{base}{perpendicular} = \frac{1}{11}, \sec A = \frac{hypotenuse}{base} = \sqrt{122},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{\sqrt{122}}{11}$$

(iv)

$$\sin\theta = \frac{11}{15} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = (15)^2 - (11)^2 = 225 - 121 = 104$$

$$base = \sqrt{104} = 2\sqrt{26}$$

$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{2\sqrt{26}}{15}, \ \tan A = \frac{perpendicular}{base} = \frac{11}{2\sqrt{26}},$$

$$\cot A = \frac{base}{perpendicular} = \frac{2\sqrt{26}}{11}, \sec A = \frac{hypotenuse}{base} = \frac{15}{2\sqrt{26}},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{15}{11}$$

(v)

$$\tan\theta = \frac{5}{12} = \frac{Perpendicular}{base}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(hypotenuse)^2 = (12)^2 + (5)^2 = 144 + 25 = 169$$

$$hypotenuse = \sqrt{169} = 13$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{5}{13}, \ \cos A = \frac{base}{hypotenuse} = \frac{12}{13},$$

$$\cot A = \frac{base}{perpendicular} = \frac{12}{5}, \sec A = \frac{hypotenuse}{base} = \frac{13}{12},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{13}{5}$$

(vi)

$$\sin A = \frac{\sqrt{3}}{2} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = (2)^2 - \left(\sqrt{3}\right)^2 = 4 - 3 = 1$$

$$base = 1$$

$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{1}{2}, \ \tan A = \frac{perpendicular}{base} = \frac{\sqrt{3}}{1} = \sqrt{3},$$

$$\cot A = \frac{base}{perpendicular} = \frac{1}{\sqrt{3}}, \sec A = \frac{hypotenuse}{base} = \frac{2}{1} = 2,$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{2}{\sqrt{3}}$$

(vii)

$$\cos A = \frac{7}{25} = \frac{base}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$$

$$(perpendicular)^2 = (25)^2 - (7)^2$$

$$(perpendicular)^2 = 625 - 49 = 576$$

$$perpendicular = 24$$

$$\therefore \ \sin A = \frac{perpendicular}{hypotenuse} = \frac{24}{25}, \ \tan A = \frac{perpendicular}{base} = \frac{24}{7},$$

$$\cot A = \frac{base}{perpendicular} = \frac{7}{24}, \sec A = \frac{hypotenuse}{base} = \frac{25}{7},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{25}{24}$$

(viii)

$$\tan\theta = \frac{8}{15} = \frac{Perpendicular}{base}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(hypotenuse)^2 = (15)^2 + (8)^2 = 225 + 64 = 289$$

$$hypotenuse = \sqrt{289} = 17$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{8}{17}, \quad \cos A = \frac{base}{hypotenuse} = \frac{15}{17},$$

$$\cot A = \frac{base}{perpendicular} = \frac{15}{8}, \sec A = \frac{hypotenuse}{base} = \frac{17}{15},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{17}{8}$$

(ix)

$$\cot A = \frac{base}{perpendicular} = \frac{12}{5}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(hypotenuse)^2 = (12)^2 + (5)^2 = 144 + 25 = 169$$

$$hypotenuse = \sqrt{169} = 13$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{5}{13}, \quad \cos A = \frac{base}{hypotenuse} = \frac{12}{13},$$

$$\tan A = \frac{perpendicular}{base} = \frac{5}{12}, \sec A = \frac{hypotenuse}{base} = \frac{13}{12},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{13}{5}$$

(x)

$$\sec A = \frac{hypotenuse}{base} = \frac{13}{5}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$$

$$(perpendicular)^2 = (13)^2 - (5)^2$$

$$(perpendicular)^2 = 169 - 25 = 144$$

$$perpendicular = 12$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{12}{13}, \quad \cos A = \frac{base}{hypotenuse} = \frac{5}{13}$$

$$\tan A = \frac{perpendicular}{base} = \frac{12}{5}, \cot A = \frac{base}{perpendicular} = \frac{5}{12},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{13}{12}$$

(xi)

$$\operatorname{cosec}\theta = \frac{hypotenuse}{Perpendicular} = \frac{\sqrt{10}}{1}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = \left(\sqrt{10}\right)^2 - (1)^2 = 10 - 1 = 9$$

$$base = 3$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{1}{\sqrt{10}}, \cos A = \frac{base}{hypotenuse} = \frac{3}{\sqrt{10}},$$

$$\tan A = \frac{perpendicular}{base} = \frac{1}{3}, \cot A = \frac{base}{perpendicular} = \frac{3}{1} = 3,$$

$$\sec A = \frac{hypotenuse}{base} = \frac{\sqrt{10}}{3},$$

(xii)

$$\cos A = \frac{12}{15} = \frac{base}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$$

$$(perpendicular)^2 = (15)^2 - (12)^2$$

$$(perpendicular)^2 = 225 - 144 = 81$$

$$perpendicular = 9$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{9}{15}, \quad \tan A = \frac{perpendicular}{base} = \frac{9}{12},$$

$$\cot A = \frac{base}{perpendicular} = \frac{12}{9}, \sec A = \frac{hypotenuse}{base} = \frac{15}{12},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{15}{9}$$

2. Question

In a ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determinei-sinA,cosAii-sinC, cosC

Answer

In a ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Therefore, By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 7^2$
$AC^2 = 576 + 49$
$AC^2 = 625$
$AC = 25$

Therefore,

$$\sin A = \frac{BC}{AC} = \frac{7}{25}, \ \cos A = \frac{AB}{AC} = \frac{24}{25}$$

$$\sin C = \frac{AB}{AC} = \frac{24}{25}, \ \cos C = \frac{BC}{AC} = \frac{7}{25}$$

3. Question

In Fig. 5.37, find tan P and cot R. Is tan P = cot R?

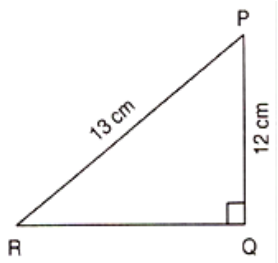


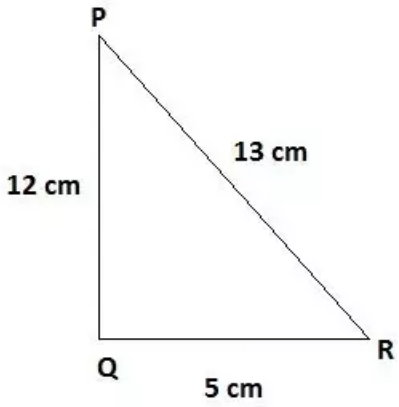
Fig. 5.37

Answer

By Pythagoras theorem we know that,(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

$$PR^2 = PQ^2 + RQ^2$$
$$RQ^2 = PR^2 - PQ^2$$
$$RQ^2 = (13)^2 - (12)^2$$
$$RQ^2 = 169 - 144$$
$$RQ^2 = 25$$
$$RQ = 5$$

Now we have the figure as



we also know that, $\tan\ \theta\ =\ \frac{Perpendicular}{Base}\quad \cot\ \theta\ =\ \frac{Base}{Perpendicular}$

Note: When finding any trigonometric ratio, the main part is to decide perpendicular and base for that angle. Perpendicular is the side opposite to the angle for which we are calculating. For example from above figure if we are calculating sin R, then side opposite to R is PQ, So PQ will be perpendicular, PR is Hypotenuse and the side left out will be base.

$$\tan\ P\ =\ \frac{5}{12}$$

$$\cot\ R\ =\ \frac{5}{12}$$

4. Question

If $\sin A = \frac{9}{41}$, compute cos A and tan A.

Answer

$$\sin A = \frac{9}{41} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (41)^2 - (9)^2$$

$$(base)^2 = 1681 - 81 = 1600$$

$$base = 40$$

$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{40}{41},$$

$$\tan A = \frac{perpendicular}{base} = \frac{9}{40}$$

5. Question

Given 15 cot A =8, find sin A and sec A.

Answer

Given

$$15 \cot A = 8$$

$$\Rightarrow \quad \cot A = \frac{8}{15} = \frac{\text{base}}{\text{perpendicular}}$$

By Pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(hypotenuse)^2 = (8)^2 + (15)^2$$

$$(hypotenuse)^2 = 64 + 225 = 289$$

$$hypotenuse = 17$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenous} = \frac{15}{17}$$

$$\cos A = \frac{base}{hypotenous} = \frac{8}{17}$$

6. Question

In ΔPQR, right angled at Q, PQ = 4 cm and RQ = 3 cm. Find the values of sin P, sin R, sec P and sec R.

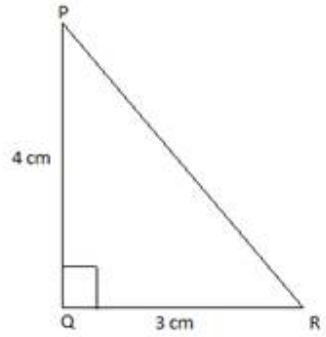
Answer

Given: In ΔPQR, right angled at Q, PQ = 4 cm and RQ = 3 cm.

To find: the values of sin P, sin R, sec P and sec R.

Solution: In triangle PQR, $\angle Q = 90^\circ$, PQ = 4 cm and RQ = 3 cm

By Pythagoras theorem,



$PR^2 = PQ^2 + RQ^2$
$RQ^2 = PR^2 - PQ^2$
$RQ^2 = (13)^2 - (12)^2$ Use the formula sinθ = perpendicular / hypotenuseSecθ = hypotenuse / base
$RQ^2 = 169 - 144$
$RQ^2 = 25$
$RQ = 5$

$\sin P = \frac{QR}{PR} = \frac{3}{5}, \sec P = \frac{PR}{PQ} = \frac{5}{4}, \sec R = \frac{PR}{PQ} = \frac{5}{3}$

$\sin R = \frac{PQ}{PR} = \frac{4}{5}$

NOTE: Always check on which angle you are asked to find any trignometricvalue and take perpendicular,base,hypotenuse accordingly.

7. Question

If $\cot\theta = \frac{7}{2}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Answer

(i)

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \cot^2\theta = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

(ii) $\cot^2\theta = \frac{49}{4}$

8. Question

If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer

$$\cot A = \frac{4}{3} \Rightarrow \tan A = \frac{3}{4}$$

$$\therefore \quad \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$= \frac{7/16}{25/16} = \frac{7}{16} \times \frac{16}{25} = \frac{7}{25}$$

And, $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Thus, it is true.

9. Question

If $\tan\theta = \frac{a}{b}$, find the value of $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

Answer

Given: $\tan\theta = \frac{a}{b}$

To find: the value of $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

Solution: Take cosΘ common,And use the formula: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Solve,

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{\cos\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right)}{\cos\theta\left(1 - \frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{(1+\tan\theta)}{(1-\tan\theta)}$$

$$= \frac{\left(1+\frac{a}{b}\right)}{\left(1-\frac{a}{b}\right)} = \frac{b+a}{b-a}$$

Thus, it is true.

10. Question

If 3 tanθ = 4, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Answer

Given:3 tanθ = 4

To find: the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Solution: $\tan\theta = \frac{P}{B}$

Here,3 tanθ = 4 $tan\theta = \frac{4}{3}$

Use the formula, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Solve,

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{4\cos\theta\left(1 - \frac{1}{4}\frac{\sin\theta}{\cos\theta}\right)}{2\cos\theta\left(1 + \frac{1}{2}\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{2\left(1 - \frac{1}{4}\tan\theta\right)}{\left(1 + \frac{1}{2}\tan\theta\right)}$$

$$= \frac{2\left(1 - \frac{4}{12}\right)}{\left(1 + \frac{4}{6}\right)} = 2 \times \frac{2/3}{5/3} = 2 \times \frac{2}{5} = \frac{4}{5}$$

11. Question

If 3 cot $_\theta$ = 2, find the value of $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$

Answer

$$\frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\cos\theta}=\frac{4\sin\theta\left(1-\frac{3}{4}\frac{\cos\theta}{\sin\theta}\right)}{2\sin\theta\left(1+3\frac{\cos\theta}{\sin\theta}\right)}$$

$$=2\times\frac{\left(1-\frac{3}{4}\cot\theta\right)}{(1+3\cot\theta)}$$

$$=2\times\frac{\left(1-\frac{3}{4}\times\frac{2}{3}\right)}{\left(1+3\times\frac{2}{3}\right)}=2\times\frac{\left(1-\frac{1}{2}\right)}{(1+2)}=2\times\frac{\frac{1}{2}}{3}=2\times\frac{1}{6}=\frac{1}{3}$$

12. Question

If $\tan\theta=\frac{a}{b}$, prove that $\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}=\frac{a^2-b^2}{a^2+b^2}$

Answer

Given: $\tan\theta=\frac{a}{b}$

To prove: $\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}=\frac{a^2-b^2}{a^2+b^2}$ ..... (1)

Solution: Consider LHS of eq. (1)Take b cosθ common from both numerator and denominator.

$$\frac{a\ \sin\theta\ -\ b\ \cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(\frac{a\ sin\theta}{b\ cos\theta}-1\right)b\ cos\theta}{\left(\frac{a\ sin\theta}{b\ cos\theta}+1\right)b\ cos\theta}$$

Solve using the formula:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(\frac{a}{b}\tan\theta-1\right)}{\left(\frac{a}{b}tan\theta+1\right)}$$

Put the value of tanθ to get,

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta} = \frac{\left(\frac{a}{b}\times\frac{a}{b}-1\right)}{\left(\frac{a}{b}\times\frac{a}{b}+1\right)}$$

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta} = \frac{\left(\frac{a^2}{b^2}-1\right)}{\left(\frac{a^2}{b^2}+1\right)}$$

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta} = \frac{\left(a^2-b^2\right)}{\left(a^2+b^2\right)}$$

hence proved

13. Question

If $\sec\theta = \frac{13}{5}$, show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

Answer

$$\sec\theta = \frac{13}{5} \qquad \Rightarrow \cos\theta = \frac{5}{13}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\therefore \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} = \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{9/13}{3/13} = 3$$

14. Question

If $\cos\theta = \frac{12}{13}$, show that $\sin\theta\ (1-\tan\theta) = \frac{35}{156}$

Answer

Given: $\cos\theta = \frac{12}{13}$

To prove: $\sin\theta\ (1-\tan\theta) = \frac{35}{156}$

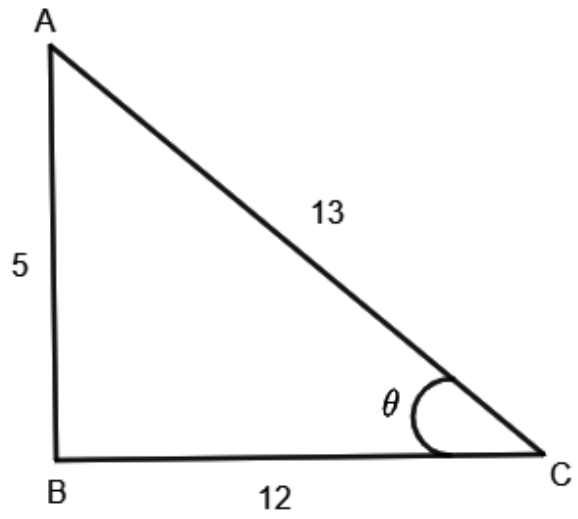
Proof: we know, $\cos\theta = \frac{B}{H}$

Where B is base and H is hypotenuse of the right angled triangle.We construct a right triangle ABC right angled at B such that $\angle ACB = \theta$

Perpendicular is AB, Base is BC = 12 and hypotenuse is AC = 13.In the triangle ABC,By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

13^2 = AB^2+$12^2$169 = AB^2+144169-144=$AB^2$25=AB^2 $AB = \sqrt{25} = 5$



$$\sin\theta = \frac{P}{H} = \frac{5}{13}$$

So,

$$\tan\theta = \frac{P}{B} = \frac{5}{12}$$

Put the values in sin θ (1-tan θ) to find its value,

$$\sin\theta(1-\tan\theta) = \frac{5}{13}\left(1-\frac{5}{12}\right) = \frac{5}{13}\times\frac{7}{12} = \frac{35}{156}$$ Hence Proved.

15. Question

If cot $\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Answer

$$\frac{1-\cos^2\theta}{2-\sin^2\theta}=\frac{\sin^2\theta}{1+\cos^2\theta}$$
$$=\frac{1}{\frac{1+\cos^2\theta}{\sin^2\theta}}$$
$$=\frac{1}{\operatorname{cosec}^2\theta+\cot^2\theta}$$
$$=\frac{1}{1+\cot^2\theta+\cot^2\theta}$$
$$=\frac{1}{1+2\cot^2\theta}=\frac{1}{1+2\times\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{1}{1+2\times\frac{1}{3}}=\frac{1}{1+\frac{2}{3}}=\frac{3}{5}$$

Hence Proved.

16. Question

If $\tan\theta=\frac{1}{\sqrt{7}}$, show that $\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}=\frac{3}{4}$

Answer

$$\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}=\frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$
$$=\frac{1+\cot^2\theta-1-\tan^2\theta}{1+\cot^2\theta+1+\tan^2\theta}$$
$$=\frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$
$$=\frac{\left(\sqrt{7}\right)^2-\left(\frac{1}{\sqrt{7}}\right)^2}{2+\left(\sqrt{7}\right)^2+\left(\frac{1}{\sqrt{7}}\right)^2}$$
$$=\frac{7-\frac{1}{7}}{2+7+\frac{1}{7}}=\frac{48/7}{64/7}=\frac{48}{64}=\frac{3}{4}$$

Hence Proved.

17. Question

If $\sin\theta=\frac{12}{13}$, find the value of $\frac{sin^2\theta-cos^2\theta}{2\,sin\theta\,cos\theta}\times\frac{1}{tan^2\theta}$

Answer

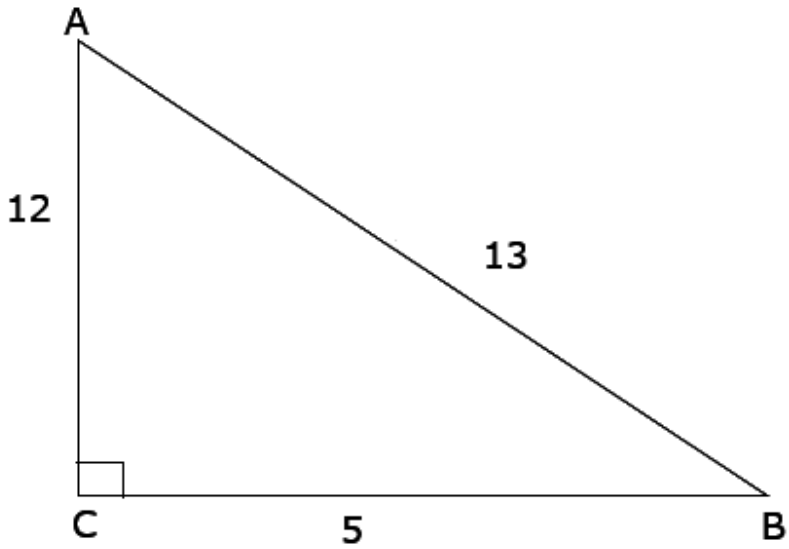
Given: $\sin\theta=\frac{12}{13}$

To find: the value of $\frac{sin^2\theta-cos^2\theta}{2\,sin\theta\,cos\theta}\times\frac{1}{tan^2\theta}$

Solution: Since $\sin\theta=\frac{perpendicular}{hypotenuse}$

So $\sin\theta = \frac{12}{13}$ implies:

Perpendicular = AC = 12, Hypotenuse = BC = 13Draw a right angled triangle at C,



By Pythagoras theorem,$AB^2 = AC^2 + BC^2$

$\Rightarrow (13)^2 = (12)^2 + BC^2$

$\Rightarrow BC^2 = (13)^2 - (12)^2$

$\Rightarrow BC^2 = 169 - 144$

$\Rightarrow BC^2 = 25$

$$\Rightarrow BC = \sqrt{25}$$

$\Rightarrow$ BC = 5Since cosθ = Base/Hypotenuse and tanθ = Perpendicular/Base

$$\Rightarrow \cos\theta = \frac{5}{13} \; and \; \tan\theta = \frac{12}{5}$$

Substitute the known values in $\frac{\sin^2\theta - \cos^2\theta}{2\sin^2\theta\cos\theta} \times \frac{1}{\tan^2\theta}$,

$$\Rightarrow \frac{\sin^2\theta - \cos^2\theta}{2\ \sin\theta\ \cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{25}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{119}{120} \times \frac{25}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{119}{\cancel{120}_{24}} \times \frac{\cancel{25}^{5}}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{119 \times 5}{24 \times 144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{595}{3456}$$

18. Question

If $\sec\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Answer

$\sec\theta = \frac{5}{4} \quad \Rightarrow \quad \cos\theta = \frac{4}{5}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$ *and* $\cot\theta = \frac{4}{3}$

$\therefore \quad \frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\times\frac{4}{5}}{\frac{3}{4} - \frac{4}{3}} = \frac{\frac{3}{5} - \frac{8}{5}}{\frac{3}{4} - \frac{4}{3}} = \frac{-1}{-\frac{7}{12}} = \frac{12}{7}$

19. Question

If $\cos\theta = \frac{5}{13}$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$

Answer

$$\cos\theta = \frac{5}{13} \Rightarrow \cos^2\theta = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos\theta} = \frac{144/169}{25/169} = \frac{144}{25}$$

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin^2\theta\cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{144}{169} \times \frac{25}{169}} \times \frac{25}{144} = \frac{595}{3456}$$

20. Question

If $\tan\theta = \frac{12}{13}$, find the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta\sin^2\theta}$

Answer

$$\tan\theta = \frac{12}{13} \quad \Rightarrow \quad \sin\theta = \frac{12}{\sqrt{313}} \text{ and } \cos\theta = \frac{13}{\sqrt{313}}$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta\sin^2\theta} = \frac{2}{\cos\theta\sin\theta} = \frac{2}{\frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}} = \frac{2 \times 313}{60} = \frac{313}{30}$$

21. Question

If $\cos\theta = \frac{3}{5}$, find the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2tan\theta}$

Answer

Given: $\cos\theta = \frac{3}{5}$

To find: the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2tan\theta}$

Solution: We know:

$$\cos\theta = \frac{Base}{Hypotenuse}$$

By applying Pythagoras theorem,we have(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

$\Rightarrow BC^2 = AB^2 + AC^2$

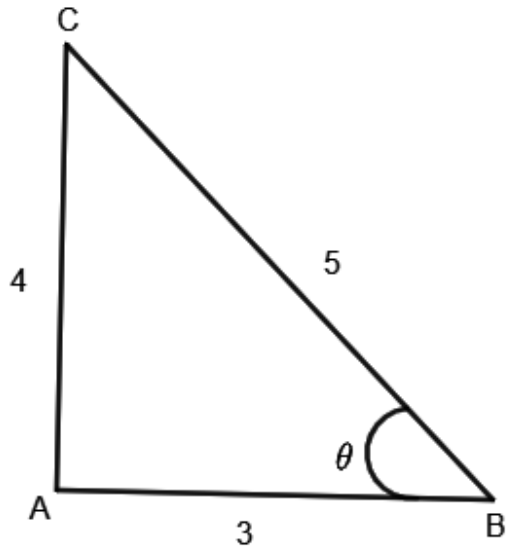
$\Rightarrow BC^2 = 3^2 + 4^2$

$\Rightarrow BC^2 = 9 + 16$

$\Rightarrow BC^2 = 25$

⇒ BC = $\sqrt{25}$

⇒ BC = 5



Use: $Sin\theta = \frac{Perpendicular}{Hypotenuse}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\Rightarrow \sin\theta = \frac{4}{5}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2tan\theta} = \frac{\frac{4}{5} - \frac{\cos\theta}{\sin\theta}}{2\frac{sin\theta}{cos\theta}}$$

Substitute the known values,

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{\frac{4}{5} - \frac{\frac{3}{5}}{\frac{4}{5}}}{2 \times \frac{\frac{4}{5}}{\frac{3}{5}}}$$

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{\frac{4}{5} - \frac{3}{4}}{2 \times \frac{4}{3}}$$

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{\frac{16-15}{20}}{\frac{8}{3}}$$

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{\frac{1}{20}}{\frac{8}{3}}$$

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{3}{20 \times 8}$$

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{3}{160}$$

22. Question

If $\sin\theta = \frac{3}{5}$, find the value of $\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$

Answer

$\sin\theta = \frac{3}{5}$

$$\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta} = \frac{\cos\theta - \cot\theta}{2\cot\theta}$$

$$= \frac{1}{2}\left(\frac{\cos\theta}{\cot\theta} - 1\right)$$

$$= \frac{1}{2}(\sin\theta - 1)$$

$$= \frac{1}{2}\left(\frac{3}{5} - 1\right) = \frac{1}{2}\left(-\frac{2}{5}\right) = -\frac{1}{5}$$

23. Question

If $\sec A = \frac{5}{4}$, verify that $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Answer

$\sec A = \frac{5}{4} \quad \Rightarrow \cos A = \frac{4}{5}$ *and* $\sin A = \frac{3}{5}$ $\tan A = \frac{3}{4}$

$$\therefore \frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\times\frac{3}{5} - 4\times\frac{27}{125}}{4\times\frac{64}{125} - 3\times\frac{4}{5}}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{216}{125} - \frac{12}{5}} = \frac{\frac{225-108}{125}}{\frac{216-300}{125}} = -\frac{117}{84}$$

$$\textit{and} \quad \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = \frac{3\times\frac{3}{4} - \frac{27}{64}}{1 - 3\times\frac{9}{16}} = \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}} = \frac{\frac{144-27}{64}}{\frac{16-27}{64}} = -\frac{117}{84}$$

24. Question

If $\sin\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$

Answer

$$\sin\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \frac{\sqrt{7}}{4}$$

$$\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{1 + \cot^2\theta - \cot^2\theta}{1 + \tan^2\theta - 1}}$$

$$= \sqrt{\frac{1}{\tan^2\theta}} = \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{7}}{4}\times\frac{4}{3} = \frac{\sqrt{7}}{3}$$

25. Question

If sec A = $\frac{17}{8}$, verify that $\frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$

Answer

$$\sec A = \frac{17}{8} \quad \Rightarrow \cos A = \frac{8}{17} \quad \Rightarrow \sin A = \frac{15}{17} \textit{ and } \tan A = \frac{15}{8}$$

$$\therefore \frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - 4\times\frac{225}{289}}{4\times\frac{64}{289} - 3} = \frac{3 - \frac{900}{289}}{\frac{216}{289} - 3} = \frac{-\frac{33}{289}}{-\frac{651}{289}} = \frac{33}{651} = \frac{11}{217}$$

$$\frac{3 - \tan^2 A}{1 - 3\tan^2 A} = \frac{3 - \frac{225}{64}}{1 - 3\times\frac{225}{64}} = \frac{11}{217}$$

26. Question

If $\cot\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta - \text{cosec}\theta}{\sec\theta + \text{cosec}\theta}} = \frac{1}{\sqrt{7}}$

Answer

$$\cot\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{4}{5} \text{ or } \operatorname{cosec}\theta = \frac{5}{4}$$

$$\text{and } \cos\theta = \frac{3}{5} \text{ or } \sec\theta = \frac{5}{3}$$

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}} = \sqrt{\frac{\frac{5}{12}}{\frac{35}{12}}} = \frac{1}{\sqrt{7}}$$

27. Question

If $\tan\theta = \frac{24}{7}$, find that $\sin\theta + \cos\theta$.

Answer

$$\tan\theta = \frac{24}{7} \Rightarrow \sin\theta = \frac{24}{25} \text{ and } \cos\theta = \frac{7}{25}$$

$$\therefore \quad \sin\theta + \cos\theta = \frac{24}{25} + \frac{7}{25} = \frac{31}{25}$$

28. Question

If $\sin\theta = \frac{a}{b}$, find $\sec\theta + \tan\theta$ in terms of a and b.

Answer

Given: $\sin\theta = \frac{a}{b}$

To find: $\sec\theta + \tan\theta$ in terms of a and b.

Solution:

$$\sin\theta = \frac{a}{b} \sin\theta = \frac{P}{H}$$

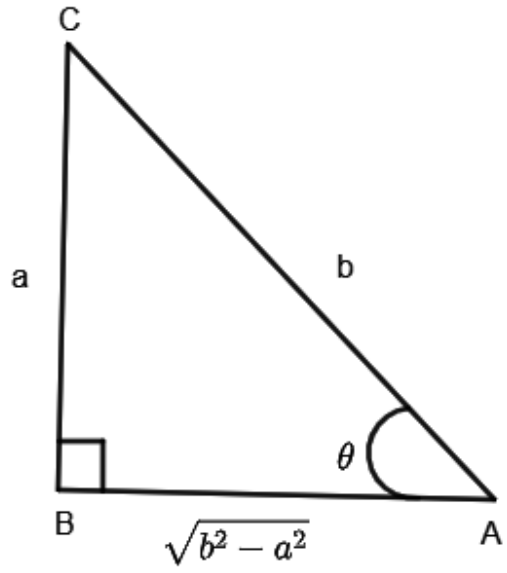
We will construct a right angled triangle, right angled at B such that, $\angle BAC = \theta$

Perpendicular=BC=a and hypotenuse=AC=b$AC^2 = AB^2 + BC^2$

$b^2 = AB^2 + a^2$

$AB^2 = b^2 - a^2$

$$AB = \sqrt{b^2 - a^2}$$



Use the formula: $\cos\theta = \frac{B}{H},\ \tan\theta = \frac{P}{B}, \sec\theta = \frac{H}{B}$

Solve,

$\sin\theta = \frac{a}{b} \quad \Rightarrow \cos\theta = \frac{\sqrt{b^2-a^2}}{b} \quad or \quad \sec\theta = \frac{b}{\sqrt{b^2-a^2}}$

$and\ \tan\theta = \frac{a}{\sqrt{b^2-a^2}}$

$\therefore \quad \sec\theta + \tan\theta = \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$

29. Question

If 8 tan A = 15, find $\sin A - \cos A$

Answer

$\tan A = \frac{15}{8} \quad \Rightarrow \sin A = \frac{15}{17}\ and\ \cos A = \frac{8}{17}$

$\therefore \sin A - \cos A = \frac{15}{17} - \frac{8}{17} = \frac{7}{17}$

30. Question

If $3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$, find $\tan\theta$

Answer

$3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$

$3\cos\theta - 2\cos\theta = 4\sin\theta + \sin\theta$

$\cos\theta = 5\sin\theta$

$\therefore \quad \tan\theta = \frac{1}{5}$

31. Question

If $\tan\theta = \frac{20}{21}$, show that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

Answer

Given: $\tan\theta = \frac{20}{21}$

To show: $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

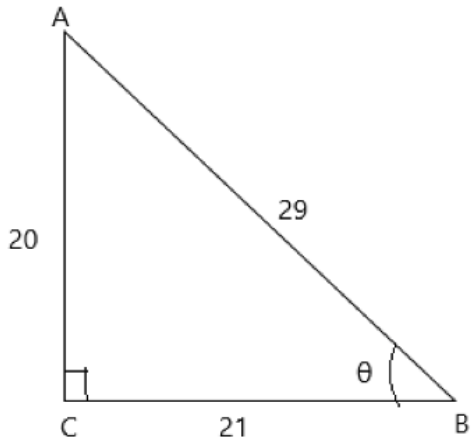
Solution: Since tanθ = perpendicular / baseSo we construct a right triangle ABC right angled at C such that∠ABC=θ and AC = Perpendicular = 20BC = base = 21By Pythagoras theorem,$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (20)^2 + (21)^2$

$\Rightarrow AB^2 = 400 + 441$

$\Rightarrow AB^2 = 841$

$\Rightarrow AB = \sqrt{841} \Rightarrow AB = 29$



As sinθ = perpendicular / hypotenusecosθ = base / hypotenuseSo,

$$\tan\theta = \frac{20}{21} \Rightarrow \sin\theta = \frac{20}{29} \text{ and } \cos\theta = \frac{21}{29}$$

$$\therefore \frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}} = \frac{\frac{30}{29}}{\frac{70}{29}} = \frac{3}{7}$$

Hence proved

32. Question

If $\operatorname{cosec} A = 2$, find the value of $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

Answer

$$\text{cosec}A = \frac{2}{1} \Rightarrow \sin A = \frac{1}{2}, \cos A = \frac{\sqrt{3}}{2}, \tan A = \sqrt{3}$$

$$\therefore \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\sqrt{3}} + \frac{1/2}{1+\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{2} \times \frac{2}{2+\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{2+\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3}+1\right)}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3}+1\right)\left(2-\sqrt{3}\right)}{4-3} = 2$$

33. Question

If $\angle A \text{ and } \angle B$ are acute angles such that cos A = cos B, then show that $\angle A = \angle B$.

Answer

In a right angled triangle ABC,

$$\cos A = \frac{AC}{AB} \text{ and } \cos B = \frac{BC}{AB}$$

$$\because \cos A = \cos B$$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$\therefore \quad AC = BC$$

We have, opposite sides of equal angles are equal. Therefore, In a right angled triangle ABC

$$\angle A = \angle B = 45^\circ$$

34. Question

If $\angle A \text{ and } \angle P$ are acute angles such that tan A = tan P, then show that $\angle A = \angle P$.

Answer

In a right angled triangle APQ,

$$\tan A = \frac{PQ}{AQ} \text{ and } \tan P = \frac{AQ}{PQ}$$

$$\because \tan A = \tan P$$

$$\frac{PQ}{AQ} = \frac{AQ}{PQ}$$

$$\therefore \quad PQ = AQ$$

$$\angle P = \angle A = 45^\circ$$

35. Question

In a ΔABC, right angled at A, if tan C = √3, find the value of sin B cos C + cos B sin C.

Answer

In a ΔABC, right angled at A,

tan C = √3

i.e ∠C = 60° and ∠B = 90 - 60 = 30°

Sin C = Sin 60° = √3/2

Cos C = Cos 60° = 1/2

Sin B = Sin 30° = 1/2

Cos B = Cos 30° = √3/2

According to the question,

sin B cos C + cos B sin C

= (1/2) (1/2) + (√3/2) (√3/2)

= 1/4 + 3/4

= 1

36. Question

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than (ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Answer

(i) The value of tan 90° is greater than 1. Therefore, given statement is false.

(ii) $\sec A = \frac{12}{5} \Rightarrow \cos A = \frac{5}{12}$ as 12 is the hypotenuse largest side. Therefore, given statement is true.

(iii) cos A is the abbreviation used for cosine of angle A Therefore, given statement is true.

(iv) cot A is not the product of cot and A. Therefore, given statement is false.

(v) Since, the hypotenuse is the longest side whereas in sin A = $\frac{4}{3}$, 3 which is the denominator and cannot be hypotenous.

Exercise 5.2

1. Question

Evaluate each of the following:

$\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

Answer

Given: $\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

To find: The value of above equation.

Solution: Use the values:

$$\sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, sin45^\circ = \frac{1}{\sqrt{2}}, \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Solve, $\sin 45^\circ sin30^\circ + \cos 45^\circ \cos 30^\circ = \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right)$

$$\Rightarrow sin45^\circ sin30^\circ + cos45^\circ cos30^\circ = \left(\frac{1}{2\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$$

$$\Rightarrow sin45^\circ sin30^\circ + cos45^\circ cos30^\circ = \left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)$$

2. Question

Evaluate each of the following:

$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Answer

Given : $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

To find : The value of $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Solution : Use the values:

$$sin30^\circ = \frac{1}{2}, \sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2} and\ cos60^\circ = \frac{1}{2}$$

Solve, $\begin{aligned} &\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = 1 \end{aligned}$

+Hence the value of $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$ is 1.

3. Question

Evaluate each of the following:

cos60° cos45° - sin60° sin 45°

Answer

$$\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

4. Question

Evaluate each of the following:

$$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$$

Answer

$$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 190^\circ$$
$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$$
$$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 = \frac{5}{2}$$

5. Question

Evaluate each of the following:

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$$

Answer

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2$$
$$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$$

6. Question

Evaluate each of the following:

$$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ$$

Answer

$$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ$$
$$= \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + (1)^2$$
$$= \frac{1}{3} + 3 + 1 = \frac{13}{3}$$

7. Question

Evaluate each of the following:

$$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$$

Answer

$$2\sin^2 30^o - 3\cos^2 45^o + \tan^2 60^o$$
$$= 2\times\left(\frac{1}{2}\right)^2 - 3\times\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2 3$$
$$= 2\times\frac{1}{4} - 3\times\frac{1}{2} + 3$$
$$= \frac{1}{2} - \frac{3}{2} + 3 = 2$$

8. Question

Evaluate each of the following:

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ \ -2\cos^2 90^\circ + \frac{1}{24}\cos^2 0^\circ$$

Answer

$$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin^2 90^o - 2\cos^2 90^o + \frac{1}{24}\cos^2 0^o$$
$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\times\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}\times(1)^2 - 2\times(0)^2 + \frac{1}{24}\times(1)^2$$
$$= \frac{1}{4}\times\frac{1}{2} + \frac{4}{3} + \frac{1}{2} - 0 + \frac{1}{24}$$
$$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = 2$$

9. Question

Evaluate each of the following:

$$4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - tan^2 45^\circ) \ +5\cos^2 45^\circ$$

Answer

Given: 4 ($\sin^4 60°$ + $\cos^4 30°$)-3 ($\tan^2 60°$ - $\tan^2 45°$) + 5$\cos^2 45°$

To find: The value of 4 ($\sin^4 60°$ + $\cos^4 30°$)-3 ($\tan^2 60°$ - $\tan^2 45°$) + 5$\cos^2 45°$.

Solution:

We know,

$$\sin\ 60^\circ = \frac{\sqrt{3}}{2}, \cos\ 30^\circ = \frac{\sqrt{3}}{2}, \cos\ 45^\circ = \frac{1}{\sqrt{2}}, \tan\ 45^\circ = 1, tan\ 60^\circ = \sqrt{3}$$

Substitute the above values in 4 ($\sin^4 60°$ + $\cos^4 30°$)-3 ($\tan^2 60°$ - $\tan^2 45°$) + 5$\cos^2 45°$,

Solve,

$$4(\sin^4 60^o + \cos^4 30^o) - 3(\tan^2 60^o - tan^2 45^o) + 5\cos^2 45^o$$
$$= \left[4\left\{\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4\right\} - 3\left\{\left(\sqrt{3}\right)^2 - (1)^2\right\} + 5\times\left(\frac{1}{\sqrt{2}}\right)^2\right]$$
$$= \left[4\left\{\frac{9}{16} + \frac{9}{16}\right\} - 3\{3-1\} + 5\times\frac{1}{2}\right]$$
$$= \frac{18}{4} - 6 + \frac{5}{2} = 1$$

10. Question

Evaluate each of the following:

$(\cos ec^2 45^\circ \sec^2 30^\circ)\ (\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$

Answer

$$\left(\cos ec^2 45^\circ \sec^2 30^\circ\right)\left(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ\right)$$
$$= \left(2\times \frac{2}{3}\right)\left(\frac{1}{4} + 4\times 1 - 4\right)$$
$$= \frac{4}{3}\left(\frac{1}{4} + 4\times 1 - 4\right) = \frac{1}{3}$$

11. Question

Evaluate each of the following:

$\cos ec^3 30^\circ \cos 60^\circ \tan^3 45^\circ\ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

Answer

$$\cos ec^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$$
$$= (2)^3 \times \frac{1}{2} \times (1)^3 \times (1)^2 \times \left(\sqrt{2}\right)^2 - \sqrt{3}$$
$$= 8\times \frac{1}{2} \times 1 \times 1 \times 2 - \sqrt{3}$$
$$= 8\sqrt{3}$$

12. Question

Evaluate each of the following:

$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$

Answer

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$$
$$= \left(\sqrt{3}\right)^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}\times\left(\sqrt{2}\right)^2 - 4\left(\frac{2}{\sqrt{3}}\right)^2$$
$$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3} = -\frac{13}{3}$$

13. Question

Evaluate each of the following:(cos 0°+sin 45°+sin 30°) (sin 90° - cos 45° + cos 60°)

Answer

Given: (cos 0°+sin 45°+sin 30°) (sin 90° - cos 45° + cos 60°)

To find: The value of the above.

Solution: Use the formulas:

$\sin 30^\circ = \frac{1}{2}, sin45^\circ = \frac{1}{\sqrt{2}}, sin90^\circ = 1$

$cos0^\circ = 1, cos45^\circ = \frac{1}{\sqrt{2}}, cos60^\circ = \frac{1}{2}$

Solve,(cos 0°+sin 45°+sin 30°) (sin 90° - cos 45° + cos 60°)

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(\frac{2\sqrt{2} + 2 + \sqrt{2}}{2\sqrt{2}}\right)\left(\frac{2\sqrt{2} - 2 + \sqrt{2}}{2\sqrt{2}}\right)$$

$$= \left(\frac{3\sqrt{2} + 2}{2\sqrt{2}}\right)\left(\frac{3\sqrt{2} - 2}{2\sqrt{2}}\right)$$

$$= \frac{(3\sqrt{2} + 2)(3\sqrt{2} - 2)}{(2\sqrt{2})^2}$$

Use the identity: $(a-b)(a+b)=a^2-b^2$

$$= \frac{(3\sqrt{2})^2 - (2)^2}{(2\sqrt{2})^2}$$

$$= \frac{(9 \times 2) - 4}{(2\sqrt{2})^2}$$

$$= \frac{18 - 4}{8}$$

$$= \frac{14}{8}$$

$$= \frac{7}{2}$$

14. Question

Evaluate each of the following:

$$\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

Answer

$$\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ} = \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{3}{2}$$

15. Question

Evaluate each of the following:

$$\frac{4}{\cot^2 30^\circ}+\frac{1}{\sin^2 60^\circ}-\cos^2 45^\circ$$

Answer

$$\frac{4}{\cot^2 30^\circ}+\frac{1}{\sin^2 60^\circ}-\cos^2 45^\circ=\frac{4}{\left(\sqrt{3}\right)^2}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}-\left(\frac{1}{\sqrt{2}}\right)^2$$

$$=\frac{4}{3}+\frac{4}{3}-\frac{1}{2}=\frac{13}{6}$$

16. Question

Evaluate each of the following:

$$4(\sin^4 30^\circ+\cos^2 60^\circ)\ -3(\cos^2 45^\circ\ -\sin^2 90^\circ)\ -\sin^2 60^\circ$$

Answer

$$4(\sin^4 30^\circ+\cos^2 60^\circ)-3(\cos^2 45^\circ-\sin^2 90^\circ)-\sin^2 60^\circ$$

$$=4\left(\frac{1}{16}+\frac{1}{4}\right)-3\left(\frac{1}{2}-1\right)-\frac{3}{4}$$

$$=\frac{20}{16}+\frac{3}{2}-\frac{3}{4}$$

$$=2$$

17. Question

Evaluate each of the following:

$$\frac{\tan^2 60^\circ+4\cos^2 45^\circ+3\sec^2 30^\circ+5\cos^2 90^\circ}{\operatorname{cosec}30^\circ+\sec 60^\circ-\cot^2 30^\circ}$$

Answer

$$\frac{\tan^2 60^\circ+4\cos^2 45^\circ+3\sec^2 30^\circ+5\cos^2 90^\circ}{\operatorname{cosec}30^\circ+\sec 60^\circ-\cot^2 30^\circ}$$

$$=\frac{\left(\sqrt{3}\right)^2+4\left(\frac{1}{\sqrt{2}}\right)^2+3\left(\frac{2}{\sqrt{3}}\right)^2+5\times(0)^2}{2+2-\left(\sqrt{3}\right)^2}$$

$$=\frac{3+2+4}{2+2-3}=9$$

18. Question

Evaluate each of the following:

$$\frac{\sin 30^\circ}{\sin 45^\circ}+\frac{\tan 45^\circ}{\sec 60^\circ}-\frac{\sin 60^\circ}{\cot 45^\circ}-\frac{\cos 30^\circ}{\sin 90^\circ}$$

Answer

$$\frac{\sin 30^\circ}{\sin 45^\circ}+\frac{\tan 45^\circ}{\sec 60^\circ}-\frac{\sin 60^\circ}{\cot 45^\circ}-\frac{\cos 30^\circ}{\sin 90^\circ}$$
$$=\frac{1/2}{1/\sqrt{2}}+\frac{1}{2}-\frac{\sqrt{3}/2}{1}-\frac{\sqrt{3}/2}{1}$$
$$=\frac{1}{\sqrt{2}}+\frac{1}{2}-\frac{2\sqrt{3}}{2}=\frac{\sqrt{2}+1-2\sqrt{3}}{2}$$

19. Question

Evaluate each of the following:

$$\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ}+\frac{\sec 60^\circ}{\cot 45^\circ}-\frac{5\sin 90^\circ}{2\cos 0^\circ}$$

Answer

$$\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ}+\frac{\sec 60^\circ}{\cot 45^\circ}-\frac{5\sin 90^\circ}{2\cos 0^\circ}$$
$$=\frac{1}{2}+\frac{2}{1}-\frac{5\times 1}{2\times 1}$$
$$=\frac{1}{2}+2-\frac{5}{2}=0$$

20. Question

Find the value of x in each of the following:

$2\sin 3x=\sqrt{3}$

Answer

$$2\sin 3x=\sqrt{3}$$
$$\Rightarrow \quad \sin 3x=\frac{\sqrt{3}}{2}$$
$$\Rightarrow \quad \sin 3x=\sin 60^\circ$$
$$\Rightarrow \quad 3x=60^\circ$$
$$\Rightarrow \quad x=20^\circ$$

21. Question

Find the value of x in each of the following:

$$2\sin\frac{x}{2}=1$$

Answer

$$2\sin\frac{x}{2}=1$$
$$\Rightarrow \quad \sin\frac{x}{2}=\frac{1}{2}$$
$$\Rightarrow \quad \sin\frac{x}{2}=\sin 30^\circ$$
$$\Rightarrow \quad \frac{x}{2}=30^\circ$$
$$\Rightarrow \quad x=60^\circ$$

22. Question

Find the value of x in each of the following:

$\sqrt{3}\sin x = \cos x$

Answer

Given: $\sqrt{3}\sin x = \cos x$

To find: The value of $\sqrt{3}\sin x = \cos x$

Solution: Apply cross multiplication in the given expression,

$$\begin{aligned} & \sqrt{3}\sin x = \cos x \\ \Rightarrow \quad & \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}} \\ \Rightarrow \quad & \tan x = \frac{1}{\sqrt{3}} \\ \Rightarrow \quad & \tan x = \tan 30^\circ \\ \Rightarrow \quad & x = 30^\circ \end{aligned}$$

23. Question

Find the value of x in each of the following:

$\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

Answer

Given : $\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

To find : The value of x.

Solution : Put the values:

$$sin30^\circ = \frac{1}{2}, \sin 45^\circ = \frac{1}{\sqrt{2}} \; and \; \cos 45^\circ = \frac{1}{\sqrt{2}}$$

solve,

$$\begin{aligned} & \tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ \\ \Rightarrow \quad & \tan x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \\ \Rightarrow \quad & \tan x = \frac{1}{2} + \frac{1}{2} \\ \Rightarrow \quad & \tan x = 1 \\ \Rightarrow \quad & \tan x = \tan 45^\circ \\ \Rightarrow \quad & x = 45^\circ \end{aligned}$$

24. Question

Find the value of x in each of the following:

$\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

Answer

$$\sqrt{3}\tan 2x = \cos 60^o + \sin 45^o \cos 45^o$$

$$\Rightarrow \quad \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \quad \sqrt{3}\tan 2x = 1$$

$$\Rightarrow \quad \tan 2x = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan 2x = \tan 30^\circ$$

$$\Rightarrow \quad 2x = 30^\circ$$

$$\Rightarrow \quad x = 15^\circ$$

25. Question

Find the value of x in each of the following:

$\cos 2x = \cos 60^\circ \cos 30^\circ \ + \sin 60^\circ \sin 30^\circ$

Answer

$$\cos 2x = \cos 60^o \cos 30^o + \sin 60^o \sin 30^o$$

$$\Rightarrow \quad \cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \cos 2x = \frac{2\sqrt{3}}{4}$$

$$\Rightarrow \quad \cos 2x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos 2x = \cos 30^\circ$$

$$\Rightarrow \quad 2x = 30^\circ$$

$$\Rightarrow \quad x = 15^\circ$$

26. Question

If $\theta = 30^\circ$, verify that:

(i) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

(ii) $\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$

(iii) $\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

(iv) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Answer

(i) Use the values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, tan 60^\circ = \sqrt{3}$$

L.H.S.

$\tan 2\theta = \tan(2\times 30^\circ) = \tan 60^\circ = \sqrt{3}$

R.H.S

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}\times\frac{3}{2} = \sqrt{3}$$

Hence proved

(ii) Use the formula:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, tan 30^\circ = \frac{1}{\sqrt{3}}$$

L.H.S.

$\sin 2\theta = \sin(2\times 30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

R.H.S

$$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2/\sqrt{3}}{4/3} = \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{\sqrt{3}}{2}$$

Hence proved

(iii) Use the formula,

$$\cos 60^\circ = \frac{1}{2}, tan 30^\circ = \frac{1}{\sqrt{3}}$$

L.H.S.

$\cos 2\theta = \cos(2\times 30^\circ) = \cos 60^\circ = \frac{1}{2}$

R.H.S

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2/3}{4/3} = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$$

Hence proved

(iv) Use the formula, $cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 90^\circ = 1$

L.H.S.

$\cos 3\theta = \cos(3\times 30^\circ) = \cos 90^\circ = 0$

R.H.S

$4\cos^3\theta - 3\cos\theta = 4\cos^3 30^\circ - 3\cos 30^\circ$

$= 4\times\left(\frac{\sqrt{3}}{2}\right)^3 - 3\times\frac{\sqrt{3}}{2}$

$= 4\times\frac{3\sqrt{3}}{8} - 3\times\frac{\sqrt{3}}{2} = 0$

Hence proved

27. Question

If $A = B = 60^\circ$, verify that

(i) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(ii) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(iii) $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$

Answer

(i)

$\cos(A-B) = \cos(60^\circ - 60^\circ) = \cos 0^\circ = 1$

$\cos A\cos B + \sin A\sin B = \cos 60^\circ\cos 60^\circ + \sin 60^\circ\sin 60^\circ$

$= \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = 1$

(ii)

$\sin(A-B) = \sin(60^\circ - 60^\circ) = \sin 0^\circ = 0$

$\sin A\cos B - \cos A\sin B = \sin 60^\circ\cos 60^\circ - \cos 60^\circ\sin 60^\circ$

$= \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2}$

$= 0$

(iii)

$\tan(A-B) = \tan(60^\circ - 60^\circ) = \tan 0^\circ = 0$

$\frac{\tan A - \tan B}{1+\tan A\tan B} = \frac{\tan 60^\circ - \tan 60^\circ}{1+\tan 60^\circ\tan 60^\circ} = \frac{\sqrt{3}-\sqrt{3}}{1-\sqrt{3}\sqrt{3}} = \frac{0}{1-3} = 0$

28. Question

If A = 30° and B = 60°, verify that

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

Answer

(i) Use the formula,

$$\sin 30^\circ = \frac{1}{2}, sin60^\circ = \frac{\sqrt{3}}{2}, sin90^\circ = 1, \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}$$

$\sin(A+B) = \sin(30^\circ + 60^\circ) = \sin 90^\circ = 1$

$\sin A \cos B + \cos A \sin B = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

Hence proved

(ii) Use the formula,

$$sin30^\circ = \frac{1}{2}, sin60^\circ = \frac{\sqrt{3}}{2}, \cos 90^\circ = 0, cos30^\circ = \frac{\sqrt{3}}{2}, cos60^\circ = \frac{1}{2}$$

$\cos(A+B) = \cos(30^\circ + 60^\circ) = \cos 90^\circ = 0$

$\cos A \cos B - \sin A \sin B = \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= 0$$

29. Question

If sin (A - B) = sin A cos B – cos A sin B and cos (A - B) = cos A cos B + sin A sin B, find the values of sin 15° and cos 15°.

Answer

For finding the values of sin 15° and cos 15°, we can split 15° into two angles such that,15° = 45° - 30° or we also can split 15° as 15° = 60° - 45°You can use either way, answer won't change.Formula to use for calculating this value is already given in the question,

sin(A - B) = sin A cos B - cos A sin B (1)

cos(A - B) = cos A cos B + sin A sin B (2)

Put A = 60° and B = 45°A - B = 15°Now put the values in formula 1 and 2,

$$\sin 15^\circ = \sin(60^\circ - 45^\circ)$$

$$= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

And,

$$\cos 15^\circ = \cos(60^\circ - 45^\circ)$$
$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Thus we have,

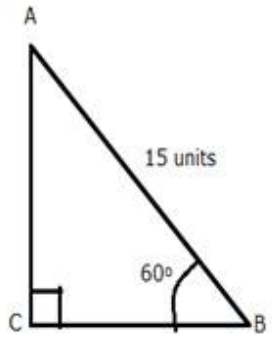
$$\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$\cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

30. Question

In a right triangle ABC, right angled at C, if $\angle B$ = 60° and AB = 15 units. Find the remaining angles and sides.

Answer



In a right triangle ABC, right angled at C, if $\angle B$ = 60° and AB = 15 units. Therefore,

$$\sin 60^\circ = \frac{AC}{AB}$$
$$\frac{\sqrt{3}}{2} = \frac{AC}{15}$$
$$AC = \frac{15}{2}\sqrt{3} \text{ units}$$

And,

$$\cos 60^\circ = \frac{BC}{AB}$$
$$\frac{1}{2} = \frac{BC}{15}$$
$$BC = \frac{15}{2} = 7.5 \text{units}$$

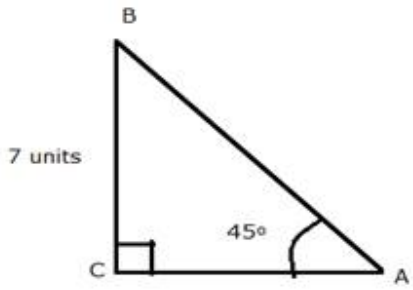
$$\angle A = 180^\circ - (90^\circ + 60^\circ)$$
$$= 180^\circ - 150^\circ$$
$$= 30^\circ$$

31. Question

If ΔABC is a right triangle such that $\angle C$ = 90°, $\angle A$ = 45° and BC = 7 units. Find $\angle B$, AB and AC.

Answer

If ΔABC is a right triangle such that $\angle C$ = 90°, $\angle A$ = 45° and BC = 7 units. Therefore,



$$\sin A = \frac{BC}{AB}$$
$$\sin 45^\circ = \frac{AC}{AB}$$
$$\frac{1}{\sqrt{2}} = \frac{7}{AB}$$
$$AB = 7\sqrt{2} \text{ units}$$

And,

$$\cos A^\circ = \frac{AC}{AB}$$
$$\cos 45^\circ = \frac{AC}{AB}$$
$$\frac{1}{\sqrt{2}} = \frac{AC}{7\sqrt{2}}$$
$$AC = 7 \text{ units}$$

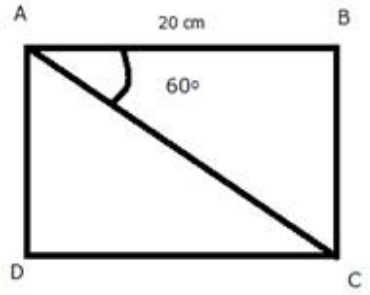
$$\angle B = 180^\circ - (90^\circ + 45^\circ)$$
$$= 180^\circ - 135^\circ$$
$$= 45^\circ$$

32. Question

In a rectangle ABCD, AB = 20 cm, $\angle BAC$ = 60°, calculate side BC and diagonals AC and BD.

Answer

If rectangle ABCD AB = 20 cm, $\angle BAC$ = 60°. Therefore,



$$\tan A = \frac{BC}{AB}$$

$$\tan 60^\circ = \frac{BC}{20}$$

$$\sqrt{3} = \frac{BC}{20}$$

$$BC = 20\sqrt{3} \text{ cm}$$

And,

$$\cos 60^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{20}{AC}$$

$$AC = 40 \text{ cm}$$

Therefore, AC = BD = 40 cm

33. Question

If sin (A + B) = 1 and cos (A – B) = 1, 0° < A + B ≤90°, A ≥ B find A and B.

Answer

Given: sin (A + B) = 1 and cos (A – B) = 1

sin(A+B) = 1

Also, we know, sin 90^0 = 1

$\Rightarrow$ sin(A+B) = sin 90^0

or (A+B) = 90^0 .....(1)

Now, cos (A – B) = 1

And, we know, cos 0^0 = 1

$\Rightarrow$ (A - B) = 0^0 .....(2)

On solving both equations (1) and (2) , we get

2A = 90^0

or A = $90^0/2$

or **A = 45°**

Similarly, **B = 45°**

34. Question

If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, 0° < A + B ≤90°, A > B find A and B.

Answer

$$\tan(A-B)=\frac{1}{\sqrt{3}}$$
$$\tan(A-B)=\tan 30^\circ$$
$$A-B=30^\circ \ldots\ldots (1)$$
$$\tan(A+B)=\sqrt{3}$$
$$\tan(A+B)=\tan 60^\circ$$
$$A+B=60^\circ \ldots\ldots (2)$$

On solving both equations, we get

A = 45° and B = 15°

35. Question

If sin (A – B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, 0° < A + B ≤90°, A < B find A and B.

Answer

$$\sin(A-B)=\frac{1}{2}$$
$$\sin(A-B)=\sin 30^\circ$$
$$A-B=30^\circ \ldots\ldots (1)$$
$$\cos(A+B)=\frac{1}{2}$$
$$\cos(A+B)=\cos 60^\circ$$
$$A+B=60^\circ \ldots\ldots (2)$$

On solving both equations, we get

A = 45°, B = 15°

36. Question

In a ΔABC right angled at B, $\angle A = \angle C$. Find the values of

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\sin A \sin B + \cos A \cos B$

Answer

In a ΔABC right angled at B, $\angle A = \angle C$, therefore,

$\angle A = \angle C = 45^\circ$

(i)

$$\sin A\cos C + \cos A\sin C = \sin(45° + 45°)$$
$$= \sin 90° = 1$$

(ii)

$$\sin A\sin B + \cos A\cos B = \cos(A + B)$$
$$= \cos(90° + 45°)$$
$$= \sin(45°)$$
$$= \frac{1}{\sqrt{2}}$$

37. Question

Find acute angles A and B, if sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0, A > B.

Answer

Given: sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0

To find: The values of acute angles A and B.

Solution: We know,

$\sin 60^\circ = \frac{\sqrt{3}}{2}$ *and* $\cos 90^\circ = 0$ So,

$$\sin(A + 2B) = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \quad \sin(A + 2B) = \sin 60°$$
$$\Rightarrow \quad A + 2B = 60° \quad \ldots.(1)$$
$$\text{and} \quad \cos(A + 4B) = \cos 90°$$
$$\Rightarrow \quad A + 4B = 90° \quad \ldots.(2)$$

Solve eq. (1) and eq. (2) to get the values of a and b.

Subtract eq. (1) from eq. (2),⇒A+4B-(A+2B)=90°-60°

⇒A+4B-A-2B=90°-60°

⇒2B=30° ⇒B=15° Substitute the value of B in eq. (1) to get,⇒A + 2B =60° ⇒A + 30°=60°⇒A =60°-30°⇒A =30°

Hence the values of A and B are A = 30°, B = 15°

38. Question

If A and B ae acute angles such that tan A = $\frac{1}{2}$, tan B = $\frac{1}{3}$ and tan (A + B) = $\frac{\tan A + \tan B}{1 - \tan A\tan B}$, find A + B.

Answer

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A+B)=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$$

$$\tan(A+B)=\frac{\frac{5}{6}}{1-\frac{1}{6}}=\frac{\frac{5}{6}}{\frac{5}{6}}=1$$

$$\therefore \quad \tan(A+B)=\tan 45^\circ$$

$$\Rightarrow \quad A+B=45^\circ$$

39. Question

In ΔPQR, right-angled at Q, PQ = 3 cm and PR = 6 cm. Determine $\angle P$ and $\angle R$.

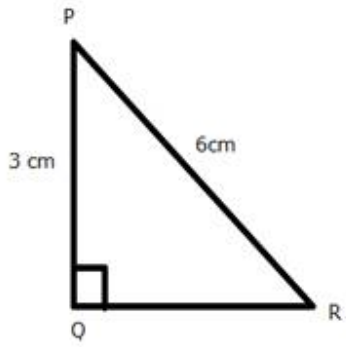
Answer

Given: In ΔPQR, right-angled at Q, PQ = 3 cm and PR = 6 cm.

To find: $\angle P$ and $\angle R$

Solution:

Take right angle ΔPQR right angled at Q.



Since,

$$\sin\theta=\frac{P}{H}$$

Here,

$$\sin R=\frac{PQ}{PR}$$

$$sinR=\frac{3}{6}=\frac{1}{2}$$

since, $\sin 30^\circ=\frac{1}{2}$

So, sin R = sin 30°

⇒ R = 30°

Now, in ΔPQRAs sum of all angles of a triangle is 180°.∠P = 180° - (90° + 30°)∠P = 180° - 120° ∠P = 60°

Hence the value of ∠R is 30° and ∠P is 60°.

Exercise 5.3

1. Question

Evaluate the following:

(i) $\frac{\sin 20^\circ}{\cos 70^\circ}$ (ii) $\frac{\cos 19^\circ}{\sin 71^\circ}$

(iii) $\frac{\sin 21^\circ}{\cos 69^\circ}$ (iv) $\frac{\tan 10^\circ}{\cot 80^\circ}$

(v) $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ}$

Answer

Use:

$$\sin\left(90^\circ - \theta\right) = \cos\theta$$

$$cos\left(90^\circ - \theta\right) = \sin\theta$$

$$\tan\left(90^\circ - \theta\right) = \cot\theta$$

$$cot\left(90^\circ - \theta\right) = tan\theta$$

$$\sec\left(90^\circ - \theta\right) = \operatorname{cosec}\theta$$

$$cosec\left(90^\circ - \theta\right) = sec\theta$$

(i) $\frac{\sin 20^\circ}{\cos 70^\circ} = \frac{\sin(90^\circ - 70^\circ)}{\cos 70^\circ} = \frac{\cos 70^\circ}{\cos 70^\circ} = 1$

(ii) $\frac{\cos 19^\circ}{\sin 71^\circ} = \frac{\cos(90^\circ - 71^\circ)}{\sin 71^\circ} = \frac{\sin 71^\circ}{\sin 71^\circ} = 1$

(iii) $\frac{\sin 21^o}{\cos 69^o} = \frac{\sin(90^o - 69^o)}{\cos 69^o} = \frac{\cos 69^o}{\cos 69^o} = 1$

(iv) $\frac{\tan 10^o}{\cot 80^o} = \frac{\tan(90^\circ - 80^o)}{\cot 80^o} = \frac{\cot 80^\circ}{\cot 80^o} = 1$

(v) $\frac{\sec 11^o}{\operatorname{cosec} 79^o} = \frac{\sec(90^\circ - 79^o)}{\operatorname{cosec} 79^o} = \frac{\operatorname{cosec} 79^o}{\operatorname{cosec} 79^o} = 1$

2. Question

Evaluate the following:

(i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

(ii) $\cos 48^\circ - \sin 42^\circ$

(iii) $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$

(iv) $\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$

(v) $\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$

(vi) $\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$

(vii) cosec 31° - sec 59°

(viii) $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$

(ix) $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

(x) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

(xi) $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Answer

(i)

$$\left(\frac{\sin 49^o}{\cos 41^o}\right)^2 + \left(\frac{\cos 41^o}{\sin 49^o}\right)^2$$
$$= \left(\frac{\sin(90^o - 41^o)}{\cos 41^o}\right)^2 + \left(\frac{\cos(90^o - 49^o)}{\sin 49^o}\right)^2$$
$$= \left(\frac{\cos 41^\circ}{\cos 41^o}\right)^2 + \left(\frac{\sin 49^\circ}{\sin 49^o}\right)^2$$
$$= (1)^2 + (1)^2 = 2$$

(ii)

$$\cos 48^\circ - \sin 42^\circ = \cos 48^\circ - \sin(90^\circ - 48^\circ)$$
$$= \cos 48^\circ - \cos 48^\circ$$
$$= 0$$

(iii)

$$\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$$
$$= \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$$
$$= \frac{\cot(90^\circ - 40^\circ)}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos(90^\circ - 35^\circ)}{\sin 55^\circ}\right)$$
$$= \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\sin 55^\circ}{\sin 55^\circ}\right)$$
$$= 1 - \frac{1}{2} \times 1 = \frac{1}{2}$$

(iv)

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2 = \left(\frac{\sin(90^\circ - 63^\circ)}{\cos 63^\circ}\right)^2 - \left(\frac{\cos(90^\circ - 27^\circ)}{\sin 27^\circ}\right)^2$$
$$= \left(\frac{\cos 63^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\sin 27^\circ}{\sin 27^\circ}\right)^2$$
$$= (1)^2 - (1)^2 = 0$$

(v)

$$\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$
$$= \frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ} + \frac{\cot(90^\circ - 12^\circ)}{\tan 12^\circ} - 1$$
$$= \frac{\cot 55^\circ}{\cot 55^\circ} + \frac{\tan 12^\circ}{\tan 12^\circ} - 1$$
$$= 1 + 1 - 1 = 0$$

(vi)

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ} = \frac{\sec(90^\circ - 20^\circ)}{\operatorname{cosec} 20^\circ} + \frac{\sin(90^\circ - 31^\circ)}{\cos 31^\circ}$$
$$= \frac{\operatorname{cosec} 20^\circ}{\operatorname{cosec} 20^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$$
$$= 1 + 1 = 2$$

(vii)

$$\operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$$
$$= \sec 59^\circ - \sec 59^\circ$$
$$= 0$$

(viii)

$$
\begin{aligned}
&(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ) \\
&= \{\sin(90^\circ - 72^\circ) + \cos 18^\circ\}\{\sin(90^\circ - 72^\circ) - \cos 18^\circ\} \\
&= \{\cos 18^\circ + \cos 18^\circ\}\{\cos 18^\circ - \cos 18^\circ\} \\
&= 2 \times 0 \\
&= 0
\end{aligned}
$$

(ix)

$$
\begin{aligned}
&\sin 35^\circ \sin(90^\circ - 35^\circ) - \cos 35^\circ \cos(90^\circ - 55^\circ) \\
&= \sin 35^\circ \cos 35^\circ - \cos 35^\circ \sin 35^\circ \\
&= \sin(35^\circ - 35^\circ) \\
&= \sin 0^\circ = 0
\end{aligned}
$$

(x)

$$
\begin{aligned}
&\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \\
&= \tan 48^\circ \tan 23^\circ \tan(90^\circ - 48^\circ) \tan(90^\circ - 23^\circ) \\
&= \tan 48^\circ \tan 23^\circ \cot 48^\circ \cot 23^\circ \\
&= (\tan 48^\circ \cot 48^\circ)(\tan 23^\circ \cot 23^\circ) \\
&= 1 \times 1 = 1
\end{aligned}
$$

(xi)

$$
\begin{aligned}
&\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \\
&= \sec 50^\circ \sin(90^\circ - 50^\circ) + \cos(90^\circ - 50^\circ) \operatorname{cosec} 50^\circ \\
&= \frac{1}{\cos 50^\circ} \cos 50^\circ + \sin 50^\circ \frac{1}{\sin 50^\circ} \\
&= 1 + 1 = 2
\end{aligned}
$$

3. Question

Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) $\sin 59^\circ + \cos 56^\circ$

(ii) $\tan 65^\circ + \cot 49^\circ$

(iii) $\sec 76^\circ + \operatorname{cosec} 52^\circ$

(iv) $\cos 78^\circ + \sec 78^\circ$

(v) $\operatorname{cosec} 54^\circ + \sin 72^\circ$

(vi) $\cot 85^\circ + \cos 75^\circ$

(vii) $\sin 67^\circ + \cos 75^\circ$

Answer

Use the formula:

$$\sin\left(90^\circ - \theta\right) = cos\theta$$

$$cos\left(90^{\circ}-\theta\right)=\sin\theta$$

$$\tan\left(90^{\circ}-\theta\right)=\cot\theta$$

$$cot\left(90^{\circ}-\theta\right)=\tan\theta$$

$$\sec\left(90^{\circ}-\theta\right)=\operatorname{cosec}\theta$$

$$cosec\left(90^{\circ}-\theta\right)=sec\theta$$

(i)

$$\begin{aligned}\sin 59^{\circ}+\cos 56^{\circ} &= \sin(90^{\circ}-31^{\circ})+\cos(90^{\circ}-34^{\circ})\\ &= \cos 31^{\circ}+\sin 34^{\circ}\end{aligned}$$

(ii)

$$\begin{aligned}&\tan 65^{\circ}+\cot 49^{\circ}\\ &= \tan(90^{\circ}-25^{\circ})+\cot(90^{\circ}-41^{\circ})\\ &= \cot 25^{\circ}+\tan 41^{\circ}\end{aligned}$$

(iii)

$$\begin{aligned}&\sec 76^{\circ}+\operatorname{cosec}52^{\circ}\\ &= \sec(90^{\circ}-14^{\circ})+\operatorname{cosec}(90^{\circ}-52^{\circ})\\ &= \operatorname{cosec}14^{\circ}+\sec 38^{\circ}\end{aligned}$$

(iv)

$$\begin{aligned}&\cos 78^{\circ}+\sec 78^{\circ}\\ &= \cos(90^{\circ}-78^{\circ})+\sec(90^{\circ}-78^{\circ})\\ &= \sin 12^{\circ}+\operatorname{cosec}12^{\circ}\end{aligned}$$

(v)

$$\begin{aligned}&\operatorname{cosec}54^{\circ}+\sin 54^{\circ}\\ &= \operatorname{cosec}(90^{\circ}-54^{\circ})+\sin(90^{\circ}-72^{\circ})\\ &= \sec 36^{\circ}+\cos 18^{\circ}\end{aligned}$$

(vi)

$$\begin{aligned}&\cot 85^{\circ}+\cos 75^{\circ}\\ &= \cot(90^{\circ}-85^{\circ})+\cos(90^{\circ}-75^{\circ})\\ &= \tan 5^{\circ}+\sin 15^{\circ}\end{aligned}$$

(vii)

$$\begin{aligned}&\sin 77^\circ + \cos 75^\circ \\ &= \sin(90^\circ - 77^\circ) + \cos(90^\circ - 75) \\ &= \cos 23^\circ + \sin 15^\circ\end{aligned}$$

4. Question

Express $\cos 75^\circ + \cot 75^\circ$ in terms of angles between 0° and 30°.

Answer

Given : $\cos 75^\circ + \cot 75^\circ$

To find : Expression in terms of angles between 0° and 30°.

Solution : Use the values:

$$\cos\left(90^\circ - \theta\right) = \sin\theta$$

$$\cot\left(90^\circ - \theta\right) = \tan\theta$$

Solve,

$$\begin{aligned}\cos 75^\circ + \cot 75^\circ &= \cos(90^\circ - 15^\circ) + \cot(90^\circ - 15^\circ) \\ &= \sin 15^\circ + \tan 15^\circ\end{aligned}$$

5. Question

If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

Answer

$$\begin{aligned}\sin 3A &= \cos(A - 26^\circ) \\ \cos(90^\circ - 3A) &= \cos(A - 26^\circ) \\ 90^\circ - 3A &= A - 26^\circ \\ 4A &= 116^\circ \\ A &= \frac{116^\circ}{4} = 29^\circ\end{aligned}$$

6. Question

If A, B, C, are the interior angles of a triangle ABC, prove that

(i) $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

(ii) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Answer

(i) Since, A, B, C, are the interior angles of a triangle ABC.

Therefore,

$$A+B+C=180^\circ$$
$$\Rightarrow \quad A+C=180^\circ-B$$
$$\Rightarrow \quad \frac{A+C}{2}=\frac{180^\circ-B}{2}$$
$$\Rightarrow \quad \tan\left(\frac{A+C}{2}\right)=\tan\left(90^\circ-\frac{B}{2}\right)$$
$$\Rightarrow \quad \tan\left(\frac{A+C}{2}\right)=\cot\left(\frac{B}{2}\right)$$

Hence proved.

(ii)

$$A+B+C=180^\circ$$
$$\Rightarrow \quad A+C=180^\circ-B$$
$$\Rightarrow \quad \frac{A+C}{2}=\frac{180^\circ-B}{2}$$
$$\Rightarrow \quad \sin\left(\frac{A+C}{2}\right)=\sin\left(90^\circ-\frac{B}{2}\right)$$
$$\Rightarrow \quad \sin\left(\frac{A+C}{2}\right)=\cos\left(\frac{B}{2}\right)$$

Hence proved.

7. Question

Prove that:

(i) $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$

(ii) $\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ = 2$

(iii) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \operatorname{cosec} 20^\circ = 0$

(iv) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$

Answer

(i)

we know,

tan(90 - A) = cot Aand cotA tanA = 1By using above concepts, we can solve the question as:Consider

LHS,$\tan20^\circ\tan35^\circ\tan45^\circ\tan55^\circ\tan70^\circ=\tan(90^\circ-70^\circ)\tan(90^\circ-55^\circ)\tan45^\circ\tan55^\circ\tan70^\circ$

$\Rightarrow \tan20^\circ\tan35^\circ\tan45^\circ\tan55^\circ\tan70^\circ=\cot70^\circ\cot55^\circ\tan45^\circ\tan55^\circ\tan70^\circ$

$\Rightarrow \tan20^\circ\tan35^\circ\tan45^\circ\tan55^\circ\tan70^\circ=(\cot70^\circ\tan70^\circ)(\cot55^\circ\tan55^\circ)\tan45^\circ$Since $\tan45^\circ=1$,

$\Rightarrow \tan20^\circ\tan35^\circ\tan45^\circ\tan55^\circ\tan70^\circ=1\times1\times1$

Which is equal to RHS.

Hence Proved

(ii)

we know, sec(90 - A) = cosec Aand sinA.cosecA = 1By using above concepts, we can solve the question as

$$\begin{aligned}
&\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ \\
&= \sin 48^\circ \sec(90^\circ - 48^\circ) + \cos 48^\circ \operatorname{cosec}(90^\circ - 48^\circ) \\
&= \sin 48^\circ \operatorname{cosec} 48^\circ + \cos 48^\circ \sec 48^\circ \\
&= \sin 48^\circ \frac{1}{\sin 48^\circ} + \cos 48^\circ \frac{1}{\cos 48^\circ} \\
&= 1 + 1 = 2
\end{aligned}$$

Proved

(iii)

we know, sin(90 - A) = cos A andcosec(90 - A) = sec A, By using this information we can solve our question as

$$\begin{aligned}
&\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \operatorname{cosec} 20^\circ \\
&= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\operatorname{cosec}(90^\circ - 70^\circ)}{\sec 70^\circ} - 2\cos 70^\circ \operatorname{cosec}(90^\circ - 70^\circ) \\
&= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2\cos 70^\circ \sec 70^\circ \\
&= 1 + 1 - 2 \times 1 \\
&= 0
\end{aligned}$$

Proved

(iv)

we know, sin(90 - A) = cos A andcosec(90 - A) = sec A, By using this information we can solve our question as

$$\begin{aligned}
&\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ \\
&= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec}(90^\circ - 59^\circ) \\
&= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \sec 59^\circ \\
&= 1 + 1 = 2
\end{aligned}$$

Proved

8. Question

Prove the following:

(i) $\sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta) = 0$

(ii) $\dfrac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\operatorname{cosec}(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \dfrac{\tan(90^\circ - \theta)}{\cot\theta} = 2$

(iii) $\dfrac{\tan(90^\circ - A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

(iv) $\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

(v) $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ = 1$

Answer

In the given parts use:

$$\sin\left(90^\circ - \theta\right) = \cos\theta,\ cos\left(90^\circ - \theta\right) = \sin\theta,$$

$$\sec\left(90^\circ - \theta\right) = \operatorname{cosec}\theta,\ cosec\left(90^\circ - \theta\right) = \sec\theta$$

$$\tan\left(90^\circ - \theta\right) = \cot\theta,\ cot\left(90^\circ - \theta\right) = tan\theta$$

(i)

$\sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta) = 0$

solve LHS,

$$\begin{aligned}&\sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta)\\ &= \sin\theta\cos\theta - \cos\theta\sin\theta\\ &= 0\end{aligned}$$

Which is equal to RHS.

(ii)

$$\frac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\operatorname{cosec}(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta} = 2$$

solve LHS,Use: $\sin\theta = \frac{1}{\operatorname{cosec}\theta},\ \sec\theta = \frac{1}{\cos\theta}$

Solve,

$$\begin{aligned}&\frac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\operatorname{cosec}(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta}\\ &= \frac{\sin\theta\ \operatorname{cosec}\theta\tan\theta}{\sec\theta\ \cos\theta\ \tan\theta} + \frac{\cot\theta}{\cot\theta}\\ &= 1 + 1 = 2\end{aligned}$$

Which is equal to RHS.

(iii) $\frac{\tan(90^\circ - A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

solve LHS,Use:

$$\tan\theta = \frac{sin\theta}{cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin\theta = \frac{1}{\text{cosec}\theta},\ \sec\theta = \frac{1}{\cos\theta}$$

Solve,

$$\begin{aligned}
&\frac{\tan(90^\circ - A)\cot A}{\text{cosec}^2 A} - \cos^2 A \\
&= \frac{\cot A \cot A}{\text{cosec}^2 A} - \cos^2 A \\
&= \frac{\cot^2 A}{\text{cosec}^2 A} - \cos^2 A \\
&= \frac{\cos^2 A}{\sin^2 A \times \text{cosec}^2 A} - \cos^2 A \\
&= \frac{\cos^2 A}{1} - \cos^2 A \\
&= \cos^2 A - \cos^2 A = 0
\end{aligned}$$

Which is equal to RHS.

(iv) $\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

solve LHS,Use: $\tan\theta = \frac{sin\theta}{cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\begin{aligned}
&\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} \\
&= \frac{\sin A \cos A}{\cot A} \\
&= \frac{\sin A \cos A}{\frac{\cos A}{\sin A}} \\
&= \frac{\sin^2 A \cos A}{\cos A} = \sin^2 A
\end{aligned}$$

Which is equal to RHS.

(v) $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ = 1$

solve LHS,

$$\begin{aligned}
&\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ \\
&= \sin(50^\circ + \theta) - \sin\{90^\circ - (40^\circ - \theta)\} + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan(90^\circ - 20^\circ)\tan(90^\circ - 10^\circ)\tan(90^\circ - 1^\circ) \\
&= \sin(50^\circ + \theta) - \sin(50^\circ + \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \cot 20^\circ \cot 10^\circ \cot 1^\circ \\
&= (\tan 1^\circ \cot 1^\circ)(\tan 10^\circ \cot 10^\circ)(\tan 20^\circ \cot 20^\circ) \\
&= 1 \times 1 \times 1 = 1
\end{aligned}$$

Which is equal to RHS.

9. Question

Evaluate:

(i) $\frac{2}{3}(\cos^4 30^\circ - \sin^4 45^\circ) - 3(\sin^2 60^\circ - \sec^2 45^\circ) + \frac{1}{4}\cot^2 30^\circ$

(ii) $4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(sin^2 60^\circ - cos^2 45^\circ) + \frac{1}{2}\tan^2 60^\circ$

(iii) $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\cos ec 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ \cos ec 40^\circ$

(iv) $\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$

(v) $\cos ec(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cos(35^\circ + \theta)$

(vi) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

(vii) $\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$

(viii) $\frac{3\cos 55^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \cos ec 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$

(ix) $\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$

(x) $\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ \cos ec 52^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ)}$

Answer

(i)

$$\frac{2}{3}\left(\cos^4 30^0 - \sin^4 45^0\right) - 3\left(\sin^2 60^0 - \sec^2 45^0\right) + \frac{1}{4}\cot^2 30^0$$

$$= \frac{2}{3}\left\{\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right\} - 3\left\{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\sqrt{2}\right)^2\right\} + \frac{1}{4}\left(\sqrt{3}\right)^2$$

$$= \frac{2}{3}\left\{\frac{9}{16} - \frac{1}{4}\right\} - 3\left\{\frac{3}{4} - 2\right\} + \frac{3}{4}$$

$$= \frac{2}{3} \times \frac{5}{16} + 3 \times \frac{5}{4} + \frac{3}{4}$$

$$= \frac{5}{24} + \frac{15}{4} + \frac{3}{4} = \frac{113}{24}$$

(ii) $4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(sin^2 60^\circ - cos^2 45^\circ) + \frac{1}{2}\tan^2 60^\circ$

$$= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - \frac{2}{3}\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right) + \frac{1}{2}(\sqrt{3})^2$$

$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - \frac{2}{3}\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{1+1}{16}\right) - \frac{2}{3}\left(\frac{3-2}{4}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{2}{16}\right) - \frac{2}{3}\left(\frac{1}{4}\right) + \frac{1}{2}(3)$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{3}{2}$$

$$= \frac{3-1+9}{6}$$

$$= \frac{11}{6}$$

(iii)

$$= \frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ \operatorname{cosec} 40^\circ$$

$$= \frac{\sin(90^\circ - 40^\circ)}{\cos 40^\circ} + \frac{\operatorname{cosec}(90^\circ - 50^\circ)}{\sec 50^\circ} - 4\cos 50^\circ \operatorname{cosec}(90^\circ - 50^\circ)$$

$$= \frac{\cos 40^\circ}{\cos 40^\circ} + \frac{\sec 50^\circ}{\sec 50^\circ} - 4\cos 50^\circ \sec 50^\circ$$

$$= 1 + 1 - 4 = -2$$

(iv)

$$\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$$

$$= \tan 35^\circ \tan 40^\circ \tan 45^\circ \tan(90^\circ - 40^\circ) \tan(90^\circ - 35^\circ)$$

$$= \tan 35^\circ \tan 40^\circ \tan 45^\circ \cot 40^\circ \cot 35^\circ$$

$$= (\tan 35^\circ \cot 35^\circ)(\tan 40^\circ \cot 40^\circ) \tan 45^\circ$$

$$= 1 \times 1 \times 1 = 1$$

(v)

$$\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$$

$$= \sec\{90^\circ - (65^\circ + \theta)\} - \sec(25^\circ - \theta) - \cot\{90^\circ - (55^\circ - \theta)\} + \cot(35^\circ + \theta)$$

$$= \sec(25^\circ - \theta) - \sec(25^\circ - \theta) - \cot(35^\circ + \theta) + \cot(35^\circ + \theta)$$

$$= 0$$

(vi)

$$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \tan(90^\circ - 23^\circ) \tan(90^\circ - 7^\circ)$$

$$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \cot 23^\circ \cot 7^\circ$$

$$= (\tan 7^\circ \cot 7^\circ)(\tan 23^\circ \cot 23^\circ) \tan 60^\circ$$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

(vii)

$$\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$$
$$= \frac{2\sin(90^\circ - 22^\circ)}{\cos 22^\circ} - \frac{2\cot(90^\circ - 75^\circ)}{5\tan 75^\circ} - \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan(90^\circ - 40^\circ)\tan(90^\circ - 20^\circ)}{5}$$
$$= \frac{2\cos 22^\circ}{\cos 22^\circ} - \frac{2\tan 75^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ(\tan 20^\circ \cot 20^\circ)(\tan 40^\circ \cot 40^\circ)}{5}$$
$$= 2 - \frac{2}{5} - \frac{3}{5} = 1$$

(viii)

$$\frac{3\cos 55^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$$
$$= \frac{3\cos(90^\circ - 35^\circ)}{7\sin 35^\circ} - \frac{4\cos 70^\circ \operatorname{cosec}(90^\circ - 70^\circ)}{7\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan(90^\circ - 25^\circ)\tan(90^\circ - 5^\circ)}$$
$$= \frac{3\sin 35^\circ}{7\sin 35^\circ} - \frac{4\cos 70^\circ \sec 70^\circ}{7(\tan 5^\circ \cot 5^\circ)(\tan 25^\circ \cot 25^\circ)\tan 45^\circ}$$
$$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$$

(ix)

$$\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$$
$$= \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} + \sqrt{3}\{\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan(90^\circ - 40^\circ)\tan(90^\circ - 10^\circ)\}$$
$$= \frac{\cos 72^\circ}{\cos 72^\circ} + \sqrt{3}\{(\tan 10^\circ \cot 10^\circ)\tan 30^\circ(\tan 40^\circ \cot 40^\circ)\}$$
$$= 1 + \sqrt{3}\left\{1 \times \frac{1}{\sqrt{3}} \times 1\right\} = 1 + 1 = 2$$

(x)

$$\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ \operatorname{cosec} 52^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ)}$$
$$= \frac{\cos(90^\circ - 32^\circ)}{\sin 32^\circ} + \frac{\sin(90^\circ - 68^\circ)}{\cos 68^\circ} - \frac{\cos 38^\circ \operatorname{cosec}(90^\circ - 38^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan(90^\circ - 18^\circ)\tan(90^\circ - 35^\circ)}$$
$$= \frac{\sin 32^\circ}{\sin 32^\circ} + \frac{\cos 68^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ \sec 38^\circ}{(\tan 18^\circ \cot 18^\circ)(\tan 35^\circ \cot 35^\circ)\tan 60^\circ}$$
$$= 1 + 1 - \frac{1}{1 \times 1 \times \sqrt{3}}$$
$$= 2 - \frac{1}{\sqrt{3}} = \frac{6 - \sqrt{3}}{3}$$

10. Question

In $\sin\theta = \cos(\theta - 45^\circ)$, where θ *and* $\theta - 45^\circ$ are acute angles, find the degree measure of θ.

Answer

$$\sin\theta = \cos(\theta - 45^\circ)$$
$$\Rightarrow \cos(90^\circ - \theta) = \cos(\theta - 45^\circ)$$
$$\Rightarrow 90^\circ - \theta = \theta - 45^\circ$$
$$\Rightarrow 2\theta = 135^\circ$$
$$\Rightarrow \theta = \frac{135^\circ}{2} = 67\frac{1}{2}^\circ$$

11. Question

If A, B, C are the interior angles of a ΔABC, show that:

(i) $\sin\frac{B+C}{2}=\cos\frac{A}{2}$ (ii) $\cos\frac{B+C}{2}=\sin\frac{A}{2}$

Answer

(i) Since, A, B, C are the interior angles of a ΔABC

$$\begin{aligned} & A+B+C=180^\circ \\ \Rightarrow\quad & B+C=180^\circ-A \\ \Rightarrow\quad & \frac{B+C}{2}=\frac{180^\circ-A}{2} \\ \Rightarrow\quad & \sin\left(\frac{B+C}{2}\right)=\sin\left(90^\circ-\frac{A}{2}\right) \\ \Rightarrow\quad & \sin\left(\frac{B+C}{2}\right)=\cos\left(\frac{A}{2}\right) \end{aligned}$$

(ii)

$$\begin{aligned} & A+B+C=180^\circ \\ \Rightarrow\quad & B+C=180^\circ-A \\ \Rightarrow\quad & \frac{B+C}{2}=\frac{180^\circ-A}{2} \\ \Rightarrow\quad & \cos\left(\frac{B+C}{2}\right)=\cos\left(90^\circ-\frac{A}{2}\right) \\ \Rightarrow\quad & \cos\left(\frac{B+C}{2}\right)=\sin\left(\frac{A}{2}\right) \end{aligned}$$

12. Question

If $2\theta+45^\circ$ *and* $30^\circ-\theta$ are acute angles, find the degree measure of θ satisfying

Answer

$$\begin{aligned} & \sin(2\theta+45^0)=\cos(30^0-\theta) \\ \Rightarrow\quad & \sin(2\theta+45^0)=\sin\{90^\circ-(30^0-\theta)\} \\ \Rightarrow\quad & 2\theta+45^0=90^\circ-30^0+\theta \\ \Rightarrow\quad & \theta=15^\circ \end{aligned}$$

13. Question

If θ is a positive acute angle such that $\sec\theta=\operatorname{cosec}60^\circ$, find the value of $2\cos^2\theta-1$.

Answer

$$\begin{aligned} & \sec\theta=\operatorname{cosec}60^0 \\ \Rightarrow\quad & \operatorname{cosec}(90^\circ-\theta)=\operatorname{cosec}60^0 \\ \Rightarrow\quad & 90^\circ-\theta=60^0 \\ \Rightarrow\quad & \theta=30^\circ \\ \textit{Now,}\quad & 2\cos^2\theta-1=2\cos^2 30^\circ-1 \\ & =2\times\left(\frac{\sqrt{3}}{2}\right)^2-1 \\ & =2\times\frac{3}{4}-1=\frac{1}{2} \end{aligned}$$

14. Question

If $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles, find the value of θ.

Answer

Given: $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles.

To find: The value of θ.

Solution:since, $\sin(90^\circ - \theta) = \cos\theta$ So,

$$\begin{aligned}
&\cos 2\theta = \sin 4\theta \\
\Rightarrow &\sin(90^\circ - 2\theta) = \sin 4\theta \\
\Rightarrow &\quad 90^\circ - 2\theta = 4\theta \\
\Rightarrow &\quad 6\theta = 90^\circ \\
\Rightarrow &\quad \theta = 15^\circ
\end{aligned}$$

15. Question

If $\sin 3\theta = \cos(\theta - 6^\circ)$, where 3θ and $\theta - 6^\circ$ are acute angles, find the value of θ.

Answer

$$\begin{aligned}
&\sin 3\theta = \cos(\theta - 6^\circ) \\
\Rightarrow &\cos(90^\circ - 3\theta) = \cos(\theta - 6^\circ) \\
\Rightarrow &\quad 90^\circ - 3\theta = \theta - 6^\circ \\
\Rightarrow &\quad 4\theta = 96^\circ \\
\Rightarrow &\quad \theta = \frac{96^\circ}{4} = 24^\circ
\end{aligned}$$

16. Question

If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A.

Answer

$$\begin{aligned}
&\sec 4A = \operatorname{cosec}(A - 20^\circ) \\
\Rightarrow &\operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ) \\
\Rightarrow &\quad 90^\circ - 4A = A - 20^\circ \\
\Rightarrow &\quad 5A = 110^\circ \\
\Rightarrow &\quad A = \frac{110^\circ}{5} = 22^\circ
\end{aligned}$$

17. Question

If $\sec 2A = \operatorname{cosec}(A - 42^\circ)$, where $2A$ is an acute angle, find the value of A.

Answer

$$\begin{aligned}
&\sec 2A = \operatorname{cosec}(A - 42^\circ) \\
\Rightarrow &\operatorname{cosec}(90^\circ - 2A) = \operatorname{cosec}(A - 42^\circ) \\
\Rightarrow &\quad 90^\circ - 2A = A - 42^\circ \\
\Rightarrow &\quad 3A = 132^\circ \\
\Rightarrow &\quad A = \frac{132^\circ}{3} = 44^\circ
\end{aligned}$$

CCE - Formative Assessment

1. Question

Write the maximum and minimum values of sin θ.

Answer

With the help of Minimum-Maximum Value Table we can find the Value of sin θ

Therefore,

Minimum Value of sin θ = - 1 and

Maximum Value of sin θ = 1

2. Question

Write the maximum and minimum values of cos θ.

Answer

With the help of Minimum-Maximum Value Table we can find the Value of cos θ

Therefore,

Minimum Value of cos θ = - 1 and

Maximum Value of cos θ = 1

3. Question

What is the maximum value of $\frac{1}{\sec\theta}$?

Answer

As we know,

$$\frac{1}{\sec\theta} = \frac{1}{\frac{1}{\cos\theta}} = \cos\theta$$

And,

Maximum value of cos θ = 1

So,

The maximum value of 1/sec θ is 1

4. Question

What is the maximum value of $\frac{1}{\text{cosec}\theta}$?

Answer

As we know,

$$\frac{1}{\text{cosec}\,\theta} = \sin\theta$$

And,

Maximum value of sin θ = 1

So,

The maximum value of 1/cosec θ is 1.

5. Question

If $\tan\theta = \frac{4}{5}$, find the value of $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

Answer

Given,

tan θ = 4/5

As we know,

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{4}{5}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{16}{25}}} = \frac{5}{\sqrt{41}}$$

And,

$$\sin\theta = \sqrt{1 - \left(\frac{5}{\sqrt{41}}\right)^2}$$

$$= \sqrt{1 - \frac{25}{41}}$$

$$= \sqrt{\frac{41 - 25}{41}} = \frac{4}{\sqrt{41}}$$

Now,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\frac{5}{\sqrt{41}} - \frac{4}{\sqrt{41}}}{\frac{5}{\sqrt{41}} + \frac{4}{\sqrt{41}}}$$

$$= \frac{\frac{1}{\sqrt{41}}}{\frac{9}{\sqrt{41}}} = \frac{1}{9}$$

Therefore,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1}{9}$$

6. Question

If $\cos\theta = \frac{2}{3}$, find the value of $\frac{\sec\theta - 1}{\sec\theta + 1}$

Answer

Given,

cos θ = 2/3

We know,

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

So,

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{3}{2} - 1}{\frac{3}{2} + 1}$$

$$= \frac{\frac{1}{2}}{\frac{5}{2}}$$

$$= \frac{2}{10} = \frac{1}{5}$$

Therefore,

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1}{5}$$

7. Question

If 3cot θ = 4, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Answer

Given,

3 cot θ = 4

So,

cot θ = 4/3

As we know,

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{4}{5}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{16}{25}}} = \frac{5}{\sqrt{41}}$$

And,

$$\sin\theta = \sqrt{1 - \left(\frac{5}{\sqrt{41}}\right)^2}$$

$$= \sqrt{1 - \frac{25}{41}}$$

$$= \sqrt{\frac{41 - 25}{41}} = \frac{4}{\sqrt{41}}$$

Now,

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{(4)\frac{5}{\sqrt{41}} - \frac{4}{\sqrt{41}}}{(2)\frac{5}{\sqrt{41}} + \frac{4}{\sqrt{41}}} =$$

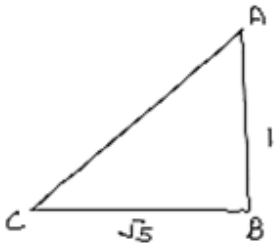
8. Question

Given $\tan\theta = \frac{1}{\sqrt{5}}$, what is the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$?

Answer

Given,

tan θ = 1/√5



In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + (\sqrt{5})^2$

$AC^2 = 1 + 5 = 6$

AC = √6

We have,

cosec θ = AC/AB = √6/1

sec θ = AC/BC = √6/√5

Now,

$$\frac{\text{cosec}^2\,\theta - \sec^2\theta}{\text{cosec}^2\,\theta + \sec^2\theta} = \frac{(\sqrt{6})^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{(\sqrt{6})^2 + \left(\frac{\sqrt{6}}{\sqrt{6}}\right)^2}$$

$$= \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}}$$

$$= \frac{\frac{30 - 6}{5}}{\frac{30 + 6}{5}}$$

$$= \frac{24}{36} = \frac{2}{3}$$

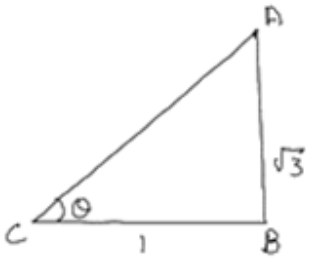
9. Question

If $\cot\theta = \frac{1}{\sqrt{3}}$, rite the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Answer

Given,

cot θ = 1/√3



In ΔABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (\sqrt{3})^2 + 1^2$

$AC^2 = 3 + 1 = 4$

AC= 2

We get;

cos θ = BC/AC = 1/2

sin θ = AB/AC = √3/2

Now,

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

$$= \frac{\frac{4-1}{4}}{\frac{8-3}{4}} = \frac{3}{5}$$

10. Question

If tan A = 3/4 and A + B = 90°, then what is the value of cot B ?

Answer

Given,

tan A = 3/4 and

A + b = 90°

So we have,

A = 90° - B

So,

tan A = tan (90° - B) = 3/4

∵ cot B = tan A

Therefore,

cot B = 3/4

11. Question

If A + B = 90° and cos B = 3/5, what is the value of sin A?

Answer

Given,

cos B = 3/5 and

A + B = 90°

So we have,

B = 90° - A

So,

cos B = cos (90° - A) = 3/5

∵ cos B = sin A

Therefore,

sin A = 3/5

12. Question

Write the acute angle θ satisfying √3 sin θ = cos θ

Answer

Given,

√3 sin θ = cos θ

sin θ / cos θ = 1/√3tan θ = 1/ √3 …(the value of tan 30° is 1/√3)

Therefore,tan θ = tan 30°θ = 30°

13. Question

Write the value of cos 1° cos 2° cos 3°…. cos 179° cos 180°.

Answer

cos 1° cos 2° cos 3°…. cos 179° cos 180°= cos 1° × cos 2° × cos 3° × .....× cos 90° × ..... × cos 179°

As we know,

cos 90° = 0

So,= cos 1° × cos 2° × cos 3° × .....× 0 × ..... × cos 179°= 0

As, cos 90° has the value 0 that's why the whole answer will be zero.

14. Question

Write the value of tan 10° tan 15° tan 75° tan 80°.

Answer

tan 10° × tan 15° × tan 75° × tan 80°

= tan 10 × tan 15 × tan(90-75) × tan (90-80)= tan10 × tan15 × cot 15 × cot10As we know,

tan θ × cot θ = 1So,tan 10 × cot 10 × tan 15 × cot 15= 1

15. Question

If A + B = 90° and tan A = 3/4, what is cot B?

Answer

Given,

tan A = 3/4

A + B = 90°B = 90 − A

So,cot B = cot (90-A)cot B = tan A

Therefore,cot B = 3/4

16. Question

If tan A = 5/12, find the value of (sin A + cos A) sec A.

Answer

Given,

tan A = 5/12

We get,

$$\cot A = \frac{1}{\tan A} = \frac{1}{\frac{5}{12}} = \frac{5}{12}$$

Now,

(sin A + cos A) sec A = (sin A + cos A)× 1/cos A

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A}$$

= tan A + 1

$$= \frac{5}{12} + 1 = \frac{5 + 12}{12} = \frac{17}{12}$$

Therefore,

cot A = 17/12

1. Question

If θ is an acute angle such that cos θ = 3/5, then $\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta}$ =

A. $\frac{160}{3}$

B. $\frac{1}{36}$

C. $\frac{3}{160}$

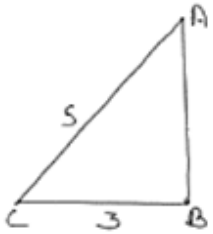
D. $\frac{160}{3}$

Answer

Given,

cos θ = 3/5

In Δ ABC,



$AC^2 = AB^2 + BC^2$

$(5)^2 = AB^2 + (3)^2$

$25 = AB^2 + 9$

$25 - 9 = AB^2$

$16 = AB^2$

$4 = AB$

Therefore,

AB = 4

As,

$\sin \theta = AB/AC = 4/5$

$\tan \theta = AB/BC = 4/3$

Now,

$$\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta} = \frac{\frac{4}{5}\times\frac{4}{3} - 1}{2\times\left(\frac{4}{3}\right)^2}$$

$$= \frac{\frac{16-15}{15}}{2\times\frac{16}{9}}$$

$$= \frac{\frac{1}{15}}{\frac{32}{9}}$$

$$= \frac{1}{15}\times\frac{9}{32}$$

$$= \frac{9}{480} = \frac{3}{160}$$

2. Question

If $\tan\theta = a/b$ then $\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta}$ is equal to

A. $\frac{a+b}{a-b}$

B. $\frac{a^2 - b^2}{a^2 + b^2}$

C. $\frac{a + b}{a - b}$

D. $\frac{a - b}{a + b}$

Answer

Given,

tan θ = a/b

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = a^2 + b^2$

$AC = \sqrt{a^2 + b^2}$

Therefore,

$$\sin\theta = \frac{AB}{AC} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\cos\theta = \frac{BC}{AC} = \frac{b}{\sqrt{a^2 + b^2}}$$

Now,

$$\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta} = \frac{a \times \frac{a}{\sqrt{a^2 + b^2}} + b \times \frac{b}{\sqrt{a^2 + b^2}}}{a \times \frac{a}{\sqrt{a^2 + b^2}} - b \times \frac{b}{\sqrt{a^2 + b^2}}}$$

$$= \frac{\frac{a^2 + b^2}{\sqrt{a^2 + b^2}}}{\frac{a^2 - b^2}{\sqrt{a^2 + b^2}}} = \frac{a^2 + b^2}{a^2 - b^2}$$

3. Question

If 5 tan θ – 4 = 0, then the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is

A. 5/3

B. 5/6

C. 0

D. 1/6

Answer

Given,

5 tan θ – 4 = 0

So,

5 tan θ = 4

tan θ = 4/5

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (4)^2 + (5)^2$

$AC^2 = 16 + 25$

$AC^2 = 41$

AC = √41

Therefore,

sin θ = AB/AC = 4/√41

cos θ = BC/AC = 5/√41

Now,

$$\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta} = \frac{5 \times \frac{4}{\sqrt{41}} - 4 \times \frac{5}{\sqrt{41}}}{5 \times \frac{4}{\sqrt{41}} + 4 \times \frac{5}{\sqrt{41}}} = 0$$

4. Question

If 16 cot x = 12, then $\frac{\sin x - \cos x}{\sin x + \cos x}$ equals

A. 1/7

B. 3/7

C. 2/7

D. 0

Answer

Given:16 cot x = 12

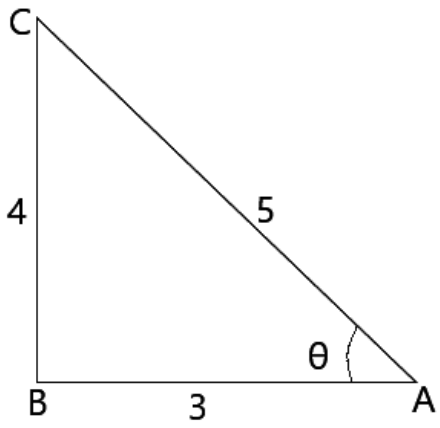
To find: The value of $\frac{\sin x - \cos x}{\sin x + \cos x}$.

Solution:

cot x = 12/16

cot x = 3/4Also cotθ = base/perpendicularSo we now construct a right triangle ABC, right angled at B such that∠BAC = θ, Base = 3 and perpendicular = 4

In Δ ABC,



$AC^2 = AB^2 + BC^2$

$AC^2 = (3)^2 + (4)^2$

$AC^2 = 9 + 16 = 25$

AC = 5

As we know sinθ = perpendicular / hypotenuse cosθ = base / hypotenuse

Therefore,

sin x = AB/AC = 4/5

cos x = BC/AC = 3/5

Now,

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{\frac{4}{5} - \frac{3}{5}}{\frac{4}{5} + \frac{3}{5}}$$

$$= \frac{\frac{4-3}{5}}{\frac{4+3}{5}} = \frac{1}{7}$$

5. Question

If 8 tan x = 15, then sin x - cos x is equal to

A. $\frac{7}{17}$

B. $\frac{17}{7}$

C. $\frac{1}{17}$

D. $\frac{7}{17}$

Answer

Given,

8 tan x = 15

So,

tan x = 15/8

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (15)^2 + (8)^2$

$AC^2 = 225 + 64 = 289$

AC = 17

Therefore,

sin x = AB/AC = 15/17

cos x = BC/AC = 8/17

Now,

$$\sin x - \cos x = \frac{15}{17} - \frac{8}{17}$$

$$= \frac{15-8}{17} = \frac{7}{17}$$

6. Question

If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2 + \sec^2\theta}$ =

A. 5/7

B. 3/7

C. 1/12

D. 3/4

Answer

Given,

tan θ = 1/√7

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (1)^2 + (\sqrt{7})^2$

$AC^2 = 8$

AC = 2√2

Therefore,

$$\operatorname{cosec}\theta = \frac{2\sqrt{2}}{1} \text{ and } \sec\theta = \frac{2\sqrt{2}}{\sqrt{7}}$$

Now,

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{\frac{56 - 8}{7}}{\frac{56 + 8}{7}}$$

$$= \frac{48}{64} = \frac{3}{4}$$

7. Question

If tan θ = 3/4, then $\cos^2\theta - \sin^2\theta$ =

A. $\frac{7}{25}$

B. 1

C. $\frac{4}{25}$

D. $\frac{4}{25}$

Answer

Given,

tan θ = 3/4

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (3)^2 + (4)^2$

$AC^2 = 9 + 16 = 25$

AC = 5

Therefore,

sin θ = p/h = 3/5 and

cos θ = b/h = 4/5

Now putting these values in the given equation we get,

$\cos^2 \theta - \sin^2 \theta = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{16 - 9}{25}$

$= \frac{7}{25}$

8. Question

If θ is an acute angle such that $\tan^2 \theta = 8/7$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is

A. 7/8

B. 8/7

C. 7/4

D. 64/49

Answer

Given,

$\tan^2 \theta = 8/7$

Now,

$$\frac{(1 + \sin\theta)(1-\sin\theta)}{(1 + \cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

$$= \frac{1}{\tan^2\theta}$$

$$= \frac{1}{\frac{8}{7}} = \frac{7}{8}$$

9. Question

If $3\cos\theta = 5\sin\theta$, then the value of $\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$ is

A. $\frac{542}{2937}$

B. $\frac{316}{2937}$

C. $\frac{542}{2937}$

D. None of these

Answer

Given,

$3\cos\theta = 5\sin\theta$

$$\frac{\cos\theta}{\sin\theta} = \frac{5}{3}$$

$$\cot\theta = \frac{5}{3}$$

In ΔABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (3)^2 + (5)^2$

$AC^2 = 9 + 25 = 34$

$AC = \sqrt{34}$

Therefore,

$\sin\theta = 3/\sqrt{34}$

$\cos\theta = 5/\sqrt{34}$

$\sec\theta = \sqrt{34}/5$

Now,

$$\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta} = \frac{5\times\frac{3}{\sqrt{34}} - 2\left(\frac{\sqrt{34}}{5}\right)^3 + 2\times\frac{5}{\sqrt{34}}}{5\times\frac{3}{\sqrt{34}} + 2\left(\frac{\sqrt{34}}{5}\right)^3 - 2\times\frac{5}{\sqrt{34}}}$$

$$= \frac{\frac{125\times 15 - 2\times 34\times 34 + 10\times 125}{125\sqrt{34}}}{\frac{125\times 15 + 2\times 34\times 34 - 10\times 125}{125\sqrt{34}}}$$

$$= \frac{1875 - 2312 + 1250}{1875 + 2312 - 1250}$$

$$= \frac{813}{2937}$$

$$= \frac{271}{979}$$

10. Question

If $\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$, then x =

A. 2

B. - 2

C. –1/2

D. 1/2

Answer

Given,

$\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$ …….(i)

put the values in equation (i),

$$(1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = x \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$1 - \frac{3}{4} = x \times \frac{1}{2}$$

$$\frac{1}{4} = \frac{x}{2}$$

$$x = \frac{1}{2}$$

11. Question

The value of $\cos^2 17° - \sin^2 73°$ is

A. 1

B. $\frac{1}{3}$

C. 0

D. -1

Answer

$\cos^2 17° - \sin^2 73° = \cos^2 17° - \sin^2(90° - 17°)$

$= \cos^2 17° - \cos^2 17° = 0$

12. Question

The value of $\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°}$ is

A. 1/2

B. 1/√2

C. 1

D. 2

Answer

$$\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°} = \frac{\cos^3 20° - \cos^3 (90° - 20°)}{\sin^3 (90° - 20°) - \sin^3 20°}$$

$$= \frac{\cos^3 20° - \sin^3 20°}{\cos^3 20° - \sin^3 20°} = 1$$

13. Question

If $\frac{x \operatorname{cosec}^2 30° \sec^2 45°}{8 \cos^2 45° \sin^2 60°}$ = $\tan^2 60° - \tan^2 30°$ then

A. 1

B. - 1

C. 2

D. 0

Answer

Given,

$$\frac{x \operatorname{cosec}^2 30° \sec^2 45°}{8 \cos^2 45° \sin^2 60°} = \tan^2 60° - \tan^2 30°$$

$$\frac{x \times (2)^2}{8\left(\frac{1}{\sqrt{2}}\right)^2} = \left(\sqrt{3}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\frac{4x \times 2}{\frac{8}{2} \times \frac{3}{4}} = 3 - \frac{1}{3}$$

$$\frac{8x}{3} = \frac{9 - 1}{3}$$

8x = 8

x = 8/8 = 1

So, the value of x = 1

14. Question

If A and B are complementary angles, then

A. sin A = sin B

B. cos A = cos B

C. tan A = tan B

D. sec A = cosec B

Answer

Given,

A + B = 90°

B = 90° - A

sin B = sin (90° - A)

sin B = cos A

Taking the reciprocal,

cosec B = sec A

Or

sec A = cosec B

15. Question

If x sin (90° - 0) cot (90° - θ) = cos (90° - θ), then x =?

A. 0

B. 1

C. - 1

D. 2

Answer

Given: x sin (90° - 0) cot (90° - θ) = cos (90° - θ)

To find: The value of x.

Solution:

x sin (90° - 0) cot (90° - θ) = cos (90° - θ)

Since, sin (90° - θ)= cos θcot (90° - θ) = tan θ

⇒ x cos θ.tan θ = sin θ

We know $\tan\ \theta\ = \frac{\sin\ \theta}{\cos\ \theta}$ ..... (1)Put this value in (1)

$\Rightarrow x\ \cos\ \theta\ .\frac{\sin\theta}{cos\ \theta} = sin\ \theta$

$\Rightarrow x = \frac{\sin\ \theta .\ \cos\ \theta}{\cos\ \theta . \sin\ \theta}$

⇒ x = 1

Hence, the value is x = 1

16. Question

If x tan 45° cos 60° = sin 60° cot 60°, then x is equal to

A. 1

B. √3

C. 1/2

D. 1/√2

Answer

Given,

x tan 45° cos 60° = sin 60° cot 60°

(x)×(1)× 1/2 = √3/2 × 1/√3

x/2 = 1/2

x = 2/2 = 1

So the value of x is 1.

17. Question

If angles A, B, C of a ΔABC form an increasing AP, then sin B =

A. 1/2

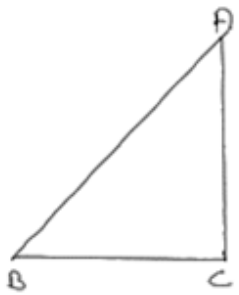
B. √3/2

C. 1

D. 1/√2

Answer

Let suppose A, B and C are the angles of a triangle ABC,



In ΔABC,

∠ A = (a − d)

∠ B = a

∠ C = a + d

Now, form an increasing A.P

As we know Sum of all the angle of a triangle is 180°,

Therefore,

∠A + ∠B + ∠C = 180°

(a – d) + a + (a + d) = 180°

3a = 180°

a = 180/3 = 60°

From the table,

sin B = sin A = sin 60°

= √3/2

18. Question

If θ is an acute angle such that $\sec^2\theta = 3$, then the value of $\frac{\tan^2\theta - \operatorname{cosec}^2\theta}{\tan^2\theta + \operatorname{cosec}^2\theta}$ is

A. 4/7

B. 3/7

C. 2/7

D. 1/7

Answer

Given,

$\sec^2\theta = 3$

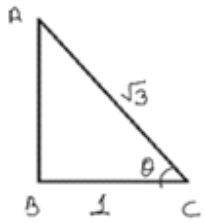
So,

Sec θ = √3 = h/b = k

Therefore,

h = √3k, b = k

In ΔABC,



$h^2 = p^2 + b^2$

$(\sqrt{3}k)^2 = p^2 + (k)^2$

$3k^2 = p^2 + k^2$

$3k^2 - k^2 = p^2$

$2k^2 = p^2$

$\sqrt{2}\ K = p$

We know,

$\tan\theta = p/b = \sqrt{2}k/k = \sqrt{2}$

$\text{cosec}\ \theta = h/p = \sqrt{3}k/\sqrt{2}k = \sqrt{3}/\sqrt{2}$

Put these value in,

$$\frac{\tan^2 - \text{cosec}^2}{\tan^2 + \text{cosec}^2} = \frac{(\sqrt{2})^2 - \left(\sqrt{\frac{3}{2}}\right)^2}{(\sqrt{2})^2 + \left(\sqrt{\frac{3}{2}}\right)^2}$$

$$= \frac{2 - \frac{3}{2}}{2 + \frac{3}{2}}$$

$$= \frac{\frac{4-3}{2}}{\frac{4+3}{2}} = \frac{1}{7}$$

19. Question

The value of tan 1° tan 2° tan 3° ……tan 89° is

A. 1

B. -1

C. 0

D. None of these

Answer

tan 1° × tan 2° × tan 3° × ……. × tan 89°

As we know tan (90 – θ) = cot θ

So here we get,

tan (90 – 89) × tan (90 - 88) × tan (90 - 87) × …….× tan 87 × tan 88 × tan 89°

cot 89° × cot 88° × cot 87°…….. tan 45°× tan 46° ……..× tan 87°× tan 88°× tan 89°

∵ cot θ = 1/tan θ

Therefore,

(cot 89° × tan89°)(cot 88° × tan88°)(cot 87° × tan87°)…… (cot 46° × tan46°)(tan 45°)

$$\left(\frac{1}{\tan 89°} \times \tan 89°\right)\left(\frac{1}{\tan 88°} \times \tan 88°\right)\left(\frac{1}{\tan 87°} \times \tan 87°\right) \ldots . \left(\frac{1}{\tan 46°} \times \tan 46°\right)(\tan 45°$$

= tan 45°

= 1

20. Question

The value of cos 1° cos 2° cos 3°…… cos 180° is

A. 1

B. 0

C. -1

D. None of these

Answer

Given,

cos 1° cos 2° cos 3° ……..cos 180°

cos 1° × cos 2° × cos 3° …… cos 89° × cos 90° × cos 91° …….cos 180°

As we know from the table,

cos 90° = 0

Therefore,

cos 1° × cos 2° × cos 3° …… cos 89° × 0 × cos 91° …….cos 180°

= 0

21. Question

6. Trigonometric Identities

Exercise 6.1

1. Question

Prove the following trigonometric identities:

$(1-\cos^2 A)\cos ec^2 A = 1$

Answer

$To\ Prove: (1\ -\ \cos^2 A)\ \text{cosec}^2\ A\ =\ 1$

$Proof:$

$1\ -\ \cos^2\ A\ =\ \sin^2 A$

$Therefore,\ L.H.S\ =\ \sin^2\ A\ .\ \text{cosec}^2\ A$

$Now,\ \text{cosec}^2\ A\ =\ \frac{1}{\sin^2\ A}$

$Therefore,\ L.H.S\ =\ \sin^2\ A\ .\ \frac{1}{\sin^2\ A}\ =\ 1$

$R.H.S\ =\ 1$

$L.H.S\ =\ R.H.S$

$Hence,\ Proved$

2. Question

Prove the following trigonometric identities:

$(1+\cot^2 A)\sin^2 A = 1$

Answer

Consider,

$(1+\cot^2 A)\sin^2 A$

As we know $1+\cot^2 A = \text{cosec}^2 A$

Putting the values we get,

$(\text{cosec}^2 A)\sin^2 A$

As we know,cosec A = 1/sinA

So,

$$\Rightarrow \frac{1}{\sin^2 A} \times sin^2 A = 1$$

hence proved

3. Question

Prove the following trigonometric identities:

$\tan^2\theta\cos^2\theta = 1-\cos^2\theta$

Answer

$$\tan^2\theta\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta$$
$$= \sin^2\theta$$
$$= 1-\cos^2\theta$$

Hence Proved.

4. Question

Prove the following trigonometric identities:

$$\operatorname{cosec}\theta\sqrt{1-\cos^2\theta} = 1$$

Answer

$$\operatorname{cosec}\theta\sqrt{1-\cos^2\theta} = \frac{1}{\sin\theta}\sqrt{\sin^2\theta}$$
$$= \frac{1}{\sin\theta}\times\sin\theta$$
$$= 1$$

Hence Proved.

5. Question

Prove the following trigonometric identities:

$$(\sec^2\theta-1)(\operatorname{cosec}^2\theta-1) = 1$$

Answer

$$(\sec^2\theta-1)(\operatorname{cosec}^2\theta-1) = \tan^2\theta\times\cot^2\theta$$
$$= 1$$

Hence Proved.

6. Question

Prove the following trigonometric identities:

$$\tan\theta+\frac{1}{\tan\theta} = \sec\theta\operatorname{cosec}\theta$$

Answer

$$\tan\theta+\frac{1}{\tan\theta} = \frac{\tan^2\theta+1}{\tan\theta}$$
$$= \sec^2\theta\times\frac{\cos\theta}{\sin\theta}$$
$$= \frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}$$
$$= \frac{1}{\cos\theta}\times\frac{1}{\sin\theta}$$
$$= \sec\theta\operatorname{cosec}\theta$$

Hence Proved.

7. Question

Prove the following trigonometric identities:

$$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$$

Answer

$$\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$$
$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$
$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$
$$= \frac{1+\sin\theta}{\cos\theta}$$

Hence Proved.

8. Question

Prove the following trigonometric identities:

$$\frac{\cos\theta}{1+\sin\theta}=\frac{1-\sin\theta}{\cos\theta}$$

Answer

$$\begin{aligned}\frac{\cos\theta}{1+\sin\theta}&=\frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}\\&=\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}\\&=\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}\\&=\frac{1-\sin\theta}{\cos\theta}\end{aligned}$$

Hence Proved.

9. Question

Prove the following trigonometric identities:

$$\cos^2 A+\frac{1}{1+\cot^2 A}=1$$

Answer

$$\begin{aligned}\cos^2 A+\frac{1}{1+\cot^2 A}&=\cos^2 A+\frac{1}{\operatorname{cosec}^2 A}\\&=\cos^2 A+\sin^2 A\\&=1\end{aligned}$$

Hence Proved.

10. Question

Prove the following trigonometric identities:

$$\sin^2 A+\frac{1}{1+\tan^2 A}=1$$

Answer

$$\begin{aligned}\sin^2 A+\frac{1}{1+\tan^2 A}&=\sin^2 A+\frac{1}{\sec^2 A}\\&=\sin^2 A+\cos^2 A\\&=1\end{aligned}$$

Hence Proved.

11. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\operatorname{cosec}\theta-\cot\theta a$$

Answer

$$\begin{aligned}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}&=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}\\&=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}\\&=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}\\&=\frac{1-\cos\theta}{\sin\theta}\\&=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\\&=\operatorname{cosec}\theta-\cot\theta\end{aligned}$$

Hence Proved.

12. Question

Prove the following trigonometric identities:

$$\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}$$

Answer

$$\begin{aligned}\frac{1-\cos\theta}{\sin\theta}&=\frac{1-\cos\theta}{\sin\theta}\times\frac{1+\cos\theta}{1+\cos\theta}\\&=\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}\\&=\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}\\&=\frac{\sin\theta}{1+\cos\theta}\end{aligned}$$

Hence Proved.

13. Question

Prove the following trigonometric identities:

$$\frac{\sin\theta}{1-\cos\theta}=\operatorname{cosec}\theta+\cot\theta$$

Answer

$$\begin{aligned}\frac{\sin\theta}{1-\cos\theta}&=\frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}\\&=\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}\\&=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}\\&=\frac{1+\cos\theta}{\sin\theta}\\&=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\\&=\operatorname{cosec}\theta+\cot\theta\end{aligned}$$

$$As,\ \frac{1}{\sin\ \theta}=\operatorname{cosec}\ \theta$$

$$and\ \frac{\cos\ \theta}{\sin\ \theta}=\cot\ \theta$$

Hence Proved.

14. Question

Prove the following trigonometric identities:

$$\frac{1-\sin\theta}{1+\sin\theta}=(\sec\theta-\tan\theta)^2$$

Answer

$$\begin{aligned}\frac{1-\sin\theta}{1+\sin\theta}&=\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}\\&=\frac{(1-\sin\theta)^2}{1-\sin^2\theta}\\&=\frac{(1-\sin\theta)^2}{\cos^2\theta}\\&=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2\\&=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2\\&=(\sec\theta-\tan\theta)^2\end{aligned}$$

Hence Proved.

15. Question

Prove the following trigonometric identities:

$(\cos ec\theta + \sin\theta)(\cos ec\theta - \sin\theta) = \cot^2\theta + \cos^2\theta$

Answer

Consider,

(cosecθ + sinθ)(cosecθ - sinθ)

Apply the formula $(a^2 - b^2) = (a+b)(a-b)$

we get,

$(\text{cosec}\theta + \sin\theta)(\text{cosec}\theta - \sin\theta) = \text{cosec}^2\theta - \sin^2\theta$

As we know $1+\cot^2 A = \text{cosec}^2 A$

and $1-\cos^2 A = \sin^2 A$

So,

$(\text{cosec}\theta + \sin\theta)(\text{cosec}\theta - \sin\theta) = (1+\cot^2 A) - (1-\cos^2 A)$

$= 1+\cot^2 A - 1+\cos^2 A$

$=\cot^2 A + \cos^2 A$

Hence Proved.

16. Question

Prove the following trigonometric identities:

$\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$

Answer

To prove: $\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$

Proof: Use the identity $\text{cosec}^2\theta = 1 + \cot^2\theta$and the formula $\cos\theta=1/\sec\theta$ and $\text{cosec}\theta=1/\sin\theta$, $\tan\theta= 1 / \cot\theta\cot\theta = \cos\theta / \sin\theta$

$$\begin{aligned}\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} &= \frac{\cos ec^2\theta \times \tan\theta}{\sec^2\theta} \\ &= \frac{\cos^2\theta \times \tan\theta}{\sin^2\theta} \\ &= \frac{\cos^2\theta}{\sin^2\theta} \times \tan\theta \\ &= \cot^2\theta \times \tan\theta \\ &= \cot^2\theta \times \frac{1}{\cot\theta} \\ &= \cot\theta\end{aligned}$$

Hence Proved.

17. Question

Prove the following trigonometric identities:

$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

Answer

To Prove: $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

Proof: Use the formula:$(a + b)(a - b) = a^2 - b^2$ on $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

Where $a = \sec\theta$ and $b = \cos\theta$

so,

$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \sec^2\theta - \cos^2\theta$ ..... (1)

We know,$\sec^2\theta = \tan^2\theta + 1$

$\sin^2\theta + \cos^2\theta = 1$

Use the identities in the eq. (1)$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \sec^2\theta - \cos^2\theta$

$= (\tan^2\theta + 1) - (1 - \sin^2\theta)$

$= \tan^2\theta + 1 - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$ Hence proved.

18. Question

Prove the following trigonometric identities:

$\sec A(1-\sin A)(\sec A+\tan A)=1$

Answer

$$\begin{aligned}&\sec A(1-\sin A)(\sec A+\tan A)\\&=\left(\sec A-\frac{1}{\cos A}\times\sin A\right)(\sec A+\tan A)\\&=(\sec A-\tan A)(\sec A+\tan A)\\&=\left(\sec^2 A-\tan^2 A\right)\\&=\left(\tan^2 A+1-\tan^2 A\right)\\&=1\end{aligned}$$

Hence Proved.

19. Question

Prove the following trigonometric identities:

$(\operatorname{cosec}A-\sin A)(\sec A-\cos A)(\tan A+\cot A)=1$

Answer

taking LHS

(cosecA-sinA)(secA-cosA)(tanA+cotA)As we know,cosec A = 1/sinAsec A = 1/cos Atan A = sin / cos ASo,

$$=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=\left(\frac{1-\sin^2 A}{sinA}\right)\left(\frac{1-\cos^2 A}{cosA}\right)\left(\frac{sin^2 A+\cos^2 A}{cosA}\right)$$

As we know,$\sin^2 A + \cos^2 A = 1$

$$=\left(\frac{\cos^2 A}{sinA}\right)\left(\frac{\sin^2 A}{cosA}\right)\left(\frac{1}{cosA}\right) \quad =1$$

Hence Proved.

20. Question

Prove the following trigonometric identities:

$\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$

Answer

$$LHS.: \tan^2\theta - \sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$
$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$
$$= \frac{\sin^2\theta - \sin^2\theta(1-\sin^2\theta)}{\cos^2\theta}$$
$$= \frac{\sin^2\theta - \sin^2\theta + \sin^4\theta}{\cos^2\theta}$$
$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$$
$$= \tan^2\theta \sin^2\theta$$
$$= R.H.S$$

Hence Proved.

21. Question

Prove the following trigonometric identities:

$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) = 1$

Answer

$$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$$
$$= \sec^2\theta(1-\sin^2\theta)$$
$$= \sec^2\theta\cos^2\theta$$
$$= 1$$

Hence Proved.

22. Question

Prove the following trigonometric identities:

$\sin^2 A\cot^2 A + \cos^2 A\tan^2 A = 1$

Answer

given : $\sin^2 A\cot^2 A + \cos^2 A\tan^2 A = 1$

To prove : Above equality holds.

Proof: Consider LHS,we know,

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \; and \; \tan\theta = \frac{\sin\theta}{\cos\theta}$$

using these

$$\sin^2 A\cot^2 A + \cos^2 A\tan^2 A$$
$$= \sin^2 A \times \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \times \frac{\sin^2 A}{\cos^2 A}$$
$$= \cos^2 A + \sin^2 A$$
$$= 1$$

Which is equal to RHS.

Hence Proved.

23 A. Question

Prove the following trigonometric identities:

$$\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$$

Answer

$$L.H.S: \cot\theta - \tan\theta = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta.\cos\theta}$$
$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta.\cos\theta}$$
$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta.\cos\theta}$$
$$= \frac{2\cos^2\theta - 1}{\sin\theta.\cos\theta} = R.H.S$$

Hence Proved.

23 B. Question

Prove the following trigonometric identities:

$$\tan\theta - \cot\theta = \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

Answer

$$L.H.S: \quad \tan\theta - \cot\theta = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

Hence Proved.

24. Question

Prove the following trigonometric identities:

$$\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta = 0$$

Answer

$$\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta = \frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta} + \sin\theta$$
$$= \frac{\cos^2\theta - 1 + \sin^2\theta}{\sin\theta}$$
$$= \frac{(\cos^2\theta + \sin^2\theta) - 1}{\sin\theta}$$
$$= \frac{1 - 1}{\sin\theta} = 0$$

Hence Proved.

25. Question

Prove the following trigonometric identities:

$$\frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} = 2\sec^2 A$$

Answer

$$\frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} = \frac{1 - \sin A + 1 + \sin A}{1 - \sin^2 A}$$
$$= \frac{2}{\cos^2 A}$$
$$= 2\sec^2 A$$

Hence Proved.

26. Question

Prove the following trigonometric identities:

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

Answer

$$\begin{aligned}=\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}&=\frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}\\&=\frac{1+1+2\sin\theta}{\cos\theta(1+\sin\theta)}\\&=\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}\\&=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}\\&=\frac{2}{\cos\theta}\\&=2\sec\theta\end{aligned}$$

Hence Proved.

27. Question

Prove the following trigonometric identities:

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

Answer

$$\begin{aligned}\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}&=\frac{1+\sin^2\theta}{1-\sin^2\theta}\\&=\frac{1+\sin^2\theta+2\sin\theta+1+\sin^2\theta-2\sin\theta}{2\cos^2\theta}\\&=\frac{2+2\sin^2\theta}{2\cos^2\theta}\\&=\frac{2(1+\sin^2\theta)}{2\cos^2\theta}\\&=\frac{(1+\sin^2\theta)}{\cos^2\theta}\\&=\frac{1+\sin^2\theta}{1-\sin^2\theta}\end{aligned}$$

Hence Proved.

28. Question

Prove the following trigonometric identities:

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}=\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2=\tan^2\theta$$

Answer

Use the formula:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\ and\ \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}} = \frac{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

Now,

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \left(\frac{1-\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}\right) = \left(\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}\right)^2$$

$$= \left(\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}\right)^2 = \left(\frac{(\cos\theta-\sin\theta)}{\cos\theta}\times\frac{\sin\theta}{-(\cos\theta-\sin\theta)}\right)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \tan^2\theta$$

Hence Proved.

29. Question

Prove the following trigonometric identities:

$$\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$$

Answer

$$\frac{1+\sec\theta}{\sec\theta} = \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{1+\cos\theta}{1}$$

$$= \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

Hence Proved.

30. Question

Prove the following trigonometric identities:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$$

Answer

Given : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$

To prove: Above equality.

Taking LHS Use $\tan\theta = \frac{1}{\cot\theta}$, $cot\theta = \frac{1}{tan\theta}$

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta\,(1-\tan\theta)}$$

$$=\frac{1}{(\tan\theta-1)}\left(\tan^2\theta-\frac{1}{\tan\theta}\right)$$

$$=\frac{\tan^3\theta-1}{\tan\theta\,(\tan\theta-1)}$$

$$=\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta\,(\tan\theta-1)}$$

$[\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$=\frac{\tan^2\theta+\tan\,\theta+1}{\tan\,\theta}$$

$$=\tan\,\theta+1+\cot\,\theta$$

= RHS**Hence Proved.**

31. Question

Prove the following trigonometric identities:

$\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Answer

Taking RHS

$\tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

$= (\sec^2\theta - 1)^3 + 3(\sec^2\theta - 1)\sec^2\theta + 1$

[As, $\tan^2\theta = \sec^2\theta - 1$]

$= (\sec^6\theta - 1 - 3\sec^4\theta + 3\sec^2\theta) + (3\sec^4\theta - 3\sec^2\theta) + 1$

$[(a + b)^3 = a^3 - b^3 - 3a^2b + 3ab^2]$

$= \sec^6\theta$= LHSHence Proved.

32. Question

Prove the following trigonometric identities:

$\operatorname{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\operatorname{cosec}^2\theta + 1$

Answer

$\text{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\text{cosec}^2\theta + 1$

$\text{cosec}^6\theta - \cot^6\theta - 3\cot^2\theta\,\text{cosec}^2\theta = 1 \quad \ldots(i)$

since we know that

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

so we can write LHS of eq. (i) as

$\left(\text{cosec}^2\theta\right)^3 - \left(\cot^2\theta\right)^3 - 3\cot^2\theta\,\text{cosec}^2\theta\left(\text{cosec}^2\theta - \cot^2\theta\right) \quad \ldots\ldots(\because \text{cosec}^2\theta - \cot^2\theta = 1)$

$= \left(\text{cosec}^2\theta - \cot^2\theta\right)^3$

$= 1 = RHS$

Hence Proved

33. Question

Prove the following trigonometric identities:

$$\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \tan\theta$$

Answer

$$\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \frac{\sec^2\theta \times \cot\theta}{\text{cosec}^2\theta}$$
$$= \frac{\sin^2\theta}{\cos^2\theta} \times \cot\theta$$
$$= \tan^2\theta \times \cot\theta$$
$$= \tan^2\theta \times \frac{1}{\tan\theta}$$
$$= \tan\theta$$

Hence Proved.

34. Question

Prove the following trigonometric identities:

$$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$

Answer

$$\frac{1+\cos A}{\sin^2 A} = \frac{1+\cos A}{\sin^2 A} \times \frac{1-\cos A}{1-\cos A}$$
$$= \frac{1-\cos^2 A}{\sin^2 A(1-\cos A)}$$
$$= \frac{\sin^2 A}{\sin^2 A(1-\cos A)}$$
$$= \frac{1}{1-\cos A}$$

Hence Proved.

35. Question

Prove the following trigonometric identities:

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$$

Answer

$$R.H.S: \frac{\cos^2 A}{(1+\sin A)^2} = \frac{1-\sin^2 A}{(1+\sin A)^2}$$
$$= \frac{(1-\sin A)(1+\sin A)}{(1+\sin A)^2}$$
$$= \frac{(1-\sin A)/\cos A}{(1+\sin A)/\cos A}$$
$$= \frac{\left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)}$$
$$= \frac{\sec A - \tan A}{\sec A + \tan A} = LH.S$$

Hence Proved.

36. Question

Prove the following trigonometric identities:

$$\frac{1+\cos A}{\sin A}=\frac{\sin A}{1-\cos A}$$

Answer

$$\begin{aligned}\frac{1+\cos A}{\sin A}&=\frac{1+\cos A}{\sin A}\times\frac{1-\cos A}{1-\cos A}\\&=\frac{1-\cos^2 A}{\sin A(1-\cos A)}\\&=\frac{\sin^2 A}{\sin A(1-\cos A)}\\&=\frac{\sin A}{1-\cos A}\end{aligned}$$

Hence Proved.

37. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$$

Answer

$$\begin{aligned}\sqrt{\frac{1+\sin A}{1-\sin A}}&=\sqrt{\frac{1+\sin A}{1-\sin A}\times\frac{1+\sin A}{1+\sin A}}\\&=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}\\&=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}\\&=\frac{1+\sin A}{\cos A}\\&=\frac{1}{\cos A}+\frac{\sin A}{\cos A}\\&=\sec A+\tan A\end{aligned}$$

Hence Proved.

38. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1-\cos A}{1+\cos A}}+\sqrt{\frac{1+\cos A}{1-\cos A}}=2\,\mathrm{cosec}A$$

Answer

$$\begin{aligned}\sqrt{\frac{1-\cos A}{1+\cos A}}+\sqrt{\frac{1+\cos A}{1-\cos A}}&=\sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}+\sqrt{\frac{1+\cos A}{1+\cos A}\times\frac{1+\cos A}{1+\cos A}}\\&=\sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}+\sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}\\&=\sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}+\sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}\\&=\frac{(1-\cos A)}{\sin A}+\frac{(1+\cos A)}{\sin A}\\&=\frac{1}{\sin A}-\frac{\cos A}{\sin A}+\frac{1}{\sin A}+\frac{\cos A}{\sin A}\\&=\frac{2}{\sin A}\\&=2\,\mathrm{cosec}A\end{aligned}$$

Hence Proved.

39. Question

Prove the following trigonometric identities:

$$(\sec A-\tan A)^2=\frac{1-\sin A}{1+\sin A}$$

Answer

$$(\sec A - \tan A)^2 = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$
$$= \frac{(1-\sin A)^2}{\cos^2 A}$$
$$= \frac{(1-\sin A)^2}{1-\sin^2 A}$$
$$= \frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)}$$
$$= \frac{1-\sin A}{1+\sin A}$$

Hence Proved.

40. Question

Prove the following trigonometric identities:

$$\frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)^2$$

Answer

Given: $\frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)^2$

To prove: Above equality

Proof: Rationalize the LHS,Use $\sin^2 x + \cos^2 x = 1$Solve,

$$\frac{1-\cos A}{1+\cos A} = \frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$
$$= \frac{(1-\cos A)^2}{1-\cos^2 A}$$
$$= \frac{(1-\cos A)^2}{\sin^2 A}$$
$$= \left(\frac{1-\cos A}{\sin A}\right)^2$$
$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$
$$= (\cot A - \operatorname{cosec} A)^2$$

Hence proved

41. Question

Prove the following trigonometric identities:

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$$

Answer

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = \frac{\sec A + 1 + \sec A - 1}{\sec^2 A - 1}$$
$$= \frac{2 \sec A}{\tan^2 A}$$
$$= \frac{2}{\cos A} \times \frac{\cos A}{\sin^2 A}$$
$$= 2 \operatorname{cosec} A \cot A$$

Hence Proved.

42. Question

Prove the following trigonometric identities:

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

Answer

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$
$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}$$
$$=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$
$$=\frac{\cos^2 A}{\cos A-\sin A}-\frac{\sin^2 A}{\cos A-\sin A}$$
$$=\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$
$$=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$$
$$=\sin A+\cos A$$

Hence Proved.

43. Question

Prove the following trigonometric identities:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2\sec^2 A$$

Answer

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=\frac{\operatorname{cosec} A(\operatorname{cosec} A+1)+\operatorname{cosec} A(\operatorname{cosec} A-1)}{\operatorname{cosec}^2 A-1}$$
$$=\frac{\operatorname{cosec}^2 A+\operatorname{cosec} A+\operatorname{cosec}^2 A-\operatorname{cosec} A}{\operatorname{cosec}^2 A-1}$$
$$=\frac{2\operatorname{cosec}^2 A}{\cot^2 A}=\frac{2}{\sin^2 A}\times\frac{\sin^2 A}{\cos^2 A}=2\sec^2 A$$

Hence Proved.

44. Question

Prove the following trigonometric identities:

$$\left(1+\tan^2 A\right)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$$

Answer

$$\left(1+\tan^2 A\right)+\left(1+\frac{1}{\tan^2 A}\right)=\sec^2 A+\frac{1+\tan^2 A}{\tan^2 A}$$
$$=\sec^2 A+\frac{\sec^2 A}{\tan^2 A}$$
$$=\frac{1}{\cos^2 A}+\frac{1}{\cos^2 A}\times\frac{\cos^2 A}{\sin^2 A}$$
$$=\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}$$
$$=\frac{1}{1-\sin^2 A}+\frac{1}{\sin^2 A}$$
$$=\frac{\sin^2 A+1-\sin^2 A}{(1-\sin^2 A)\sin^2 A}$$
$$=\frac{1}{\sin^2 A-\sin^4 A}$$

Hence Proved.

45. Question

Prove the following trigonometric identities:

$$\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}=1$$

Answer

$$\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}=\frac{\tan^2 A}{\sec^2 A}+\frac{\cot^2 A}{\operatorname{cosec}^2 A}$$
$$=\frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}}+\frac{\frac{\cos^2 A}{\sin^2 A}}{\frac{1}{\sin^2 A}}$$
$$=\sin^2 A+\cos^2 A$$
$$=1$$

Hence Proved.

46. Question

Prove the following trigonometric identities:

$$\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$$

Answer

$$\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A}$$
$$=\frac{\cos A\left(\frac{1}{\sin A}-1\right)}{\cos A\left(\frac{1}{\sin A}+1\right)}$$
$$=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$$

Hence Proved.

47 A. Question

Prove the following trigonometric identities:

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$$

Answer

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)-\sin\theta\}}\times\frac{\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)+\sin\theta\}}$$
$$=\frac{\{(1+\cos\theta)+\sin\theta\}^2}{(1+\cos\theta)^2-\sin^2\theta}$$
$$=\frac{(1+\cos\theta)^2+\sin^2\theta+2(1+\cos\theta)\sin\theta}{(1+\cos\theta)^2-\sin^2\theta}$$
$$=\frac{1+\cos^2\theta+2\cos\theta+\sin^2\theta+2\sin\theta+2\sin\theta\cos\theta}{1+\cos^2\theta+2\cos\theta-\sin^2\theta}$$
$$=\frac{1+\cos^2\theta+\sin^2\theta+2\cos\theta+2\sin\theta+2\sin\theta\cos\theta}{1+\cos^2\theta+2\cos\theta-1+\cos^2\theta}$$
$$=\frac{1+1+2\sin\theta+2\cos\theta(1+\sin\theta)}{2\cos^2\theta+2\cos\theta}$$
$$=\frac{2(1+\sin\theta)+2\cos\theta(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$$
$$=\frac{2(1+\sin\theta)(1+\cos\theta)}{2\cos\theta(1+\cos\theta)}$$
$$=\frac{1+\sin\theta}{\cos\theta}$$

Hence Proved.

47 B. Question

Prove the following trigonometric identities:

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$$

Answer

To Prove: $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$

$$\mathrm{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing the numerator and denominator by cosθ, we get,

$$= \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

Now, we know that, $\sec^2\theta - \tan^2\theta = 1$

Therefore, replacing 1 by $\sec^2\theta - \tan^2\theta$ in the numerator only, we get,

$$= \frac{\left(\tan\theta + \sec\theta\right) - \left(\sec^2\theta - \tan^2\theta\right)}{\tan\theta - \sec\theta + 1}$$

As we know, $a^2 - b^2 = (a-b)(a+b)$

$$= \frac{\left(\tan\theta + \sec\theta\right) - \left[\left(\sec\theta + \tan\theta\right)\left(\sec\theta - \tan\theta\right)\right]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\left(\tan\theta + \sec\theta\right)\left[1 - \left(\sec\theta - \tan\theta\right)\right]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\left(\tan\theta + \sec\theta\right)\left[1 - \sec\theta + \tan\theta\right]}{\tan\theta - \sec\theta + 1}$$

= secθ + tanθNow, multiplying and dividing by secθ - tanθ, we get,

$$= \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} \times (\sec\theta - \tan\theta)$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

As we know, $\sec^2\theta - \tan^2\theta = 1$

$$= \frac{1}{\sec\theta - \tan\theta}$$

= R.H.SHence, proved.

47 C. Question

Prove the following trigonometric identities:

$\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Answer

$$\text{L.H.S} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosec A + cot A

Hence Proved.

48. Question

Prove the following trigonometric identities:

$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Answer

To prove: $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Proof: Consider LHS,

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A}$$

Use the formula:secθ = 1/cos θ and tanθ=sinθ/cosθ

$$= \frac{1}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}} - \frac{1}{cos A}$$

$$= \frac{\cos A}{1 + \sin A} - \frac{1}{cos A}$$

Do rationalization,

$$= \left(\frac{cos A}{1 + sin A} \times \frac{1 - \sin A}{1 - \sin A}\right) - \frac{1}{cos A}$$

$$= \frac{cosA(1-sinA)}{(1+sinA)(1-sinA)} - \frac{1}{cosA}$$

$$= \frac{cosA(1-sinA)}{(1-sin^2A)} - \frac{1}{cosA}$$

Use the formula $\cos^2\theta + \sin^2\theta = 1$

$$= \frac{\cos A(1-\sin A)}{\cos^2 A} - \frac{1}{\cos A}$$

$$= \frac{(1-sinA)}{cos\ \ A} - \frac{1}{cosA}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} - \frac{1}{\cos A}$$

= - tan AConsider RHS, $\frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Use the formula:$\sec\theta = 1/\cos\theta$ and $\tan\theta = \sin\theta/\cos\theta$

$$= \frac{1}{cosA} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{1}{cosA} - \frac{\cos A}{1-\sin A}$$

Do rationalization,

$$= \frac{1}{cosA} - \left(\frac{cosA}{1-sinA} \times \frac{1+\sin A}{1+\sin A}\right)$$

$$= \frac{1}{cosA} - \frac{cosA(1+sinA)}{(1-sinA)(1+sinA)}$$

$$= \frac{1}{cosA} - \frac{cosA(1+\sin A)}{(1-sin^2A)}$$

Use the formula $\cos^2\theta + \sin^2\theta = 1$

$$= \frac{1}{\cos A} - \frac{\cos A(1+\sin A)}{\cos^2 A}$$

$$= \frac{1}{cosA} - \frac{(1+sinA)}{cosA}$$

$$= \frac{1}{cosA} - \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

= - tan ALHS = RHSHence proved.

49. Question

Prove the following trigonometric identities:

$\tan^2 A + \cot^2 A = \sec^2 A \operatorname{cosec}^2 A - 2$

Answer

$$\begin{aligned} RHS &= \sec^2 A \operatorname{cosec}^2 A - 2 \\ &= (1 + \tan^2 A)(1 + \cot^2 A) - 2 \\ &= (\tan^2 A + \cot^2 A + \tan^2 A \cot^2 A + 1) - 2 \\ &= (\tan^2 A + \cot^2 A + 1 + 1) - 2 \\ &= \tan^2 A + \cot^2 A \\ &= LHS \end{aligned}$$

Hence Proved.

50. Question

Prove the following trigonometric identities:

$\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Answer

To prove:

$\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Use the formula

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; and \; \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\begin{aligned} \frac{1 - \tan^2 A}{\cot^2 A - 1} &= \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1} = \\ &= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\cos^2 A} \\ &= \tan^2 A \end{aligned}$$

Hence Proved.

51. Question

Prove the following trigonometric identities:

$1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec}\theta} = \operatorname{cosec}\theta$

Answer

$$\begin{aligned} 1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec}\theta} &= 1 + \frac{\operatorname{cosec}^2\theta - 1}{1 + \operatorname{cosec}\theta} \\ &= 1 + \frac{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}{(1 + \operatorname{cosec}\theta)} \\ &= 1 + \operatorname{cosec}\theta - 1 \\ &= \operatorname{cosec}\theta \end{aligned}$$

Hence Proved.

52. Question

Prove the following trigonometric identities:

$\frac{\cos\theta}{\operatorname{cosec}\theta + 1} + \frac{\cos\theta}{\operatorname{cosec}\theta - 1} = 2\tan\theta$

Answer

$$\frac{\cos\theta}{\operatorname{cosec}\theta+1}+\frac{\cos\theta}{\operatorname{cosec}\theta-1}=\frac{\cos\theta(\operatorname{cosec}\theta-1)+\cos\theta(\operatorname{cosec}\theta+1)}{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}$$
$$=\frac{2\cot\theta}{\cot^2\theta}$$
$$=2\tan\theta$$

Hence Proved.

53. Question

Prove the following trigonometric identities:

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$$

Answer

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\frac{(1+\cos\theta)-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$$
$$=\frac{(1+\cos\theta)-(1+\cos\theta)(1-\cos\theta)}{\sin\theta(1+\cos\theta)}$$
$$=\frac{(1+\cos\theta)\{1-(1-\cos\theta)\}}{\sin\theta(1+\cos\theta)}$$
$$=\frac{\cos\theta}{\sin\theta}$$
$$=\cot\theta$$

Hence Proved.

54. Question

Prove the following trigonometric identities:

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

Answer

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\frac{\tan^3\theta}{\sec^2\theta}+\frac{\cot^3\theta}{\operatorname{cosec}^2\theta}$$
$$=\tan^3\theta\cos^2\theta+\cot^3\theta\sin^2\theta$$
$$=\frac{\sin^3\theta}{\cos^3\theta}\cos^2\theta+\frac{\cos^3\theta}{\sin^3\theta}\sin^2\theta$$
$$=\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}$$
$$=\frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$
$$=\frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$
$$=\frac{1}{\sin\theta\cos\theta}-2\sin\theta\cos\theta$$
$$=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

Hence Proved.

55. Question

If $T_n=\sin^n\theta+\cos^n\theta$, prove that $\frac{T_3-T_5}{T_1}=\frac{T_5-T_7}{T_3}$

Answer

$T_n = \sin^n\theta + \cos^n\theta$

$T_1 = \sin^1\theta + \cos^1\theta = \sin\theta + \cos\theta$

$T_3 = \sin^3\theta + \cos^3\theta$, $T_5 = \sin^5\theta + \cos^5\theta$ *and* $T_7 = \sin^7\theta + \cos^7\theta$

Now, we have,

$$\frac{T_3 - T_5}{T_1} = \frac{\sin^3\theta + \cos^3\theta - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta(\sin\theta + \cos\theta)}{\sin\theta + \cos\theta}$$

$$= \sin^2\theta\cos^2\theta \quad \ldots. (1)$$

$$\frac{T_5 - T_7}{T_3} = \frac{\sin^5\theta + \cos^5\theta - (\sin^7\theta + \cos^7\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta(1 - \sin^2\theta) + \cos^5\theta(1 - \cos^2\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta\cos^2\theta + \cos^5\theta\sin^2\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta(\sin^3\theta + \cos^3\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \sin^2\theta\cos^2\theta \quad \ldots. (2)$$

From(1) *and* (2), *we get*

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

Hence Proved.

56. Question

Prove the following trigonometric identities:

$$\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1 + \sin^2\theta}{1 - \sin^2\theta}\right)$$

Answer

$$\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2$$

$$= (\tan\theta + \sec\theta)^2 + (\tan\theta - \sec\theta)^2$$

$$= (\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta) + (\tan^2\theta + \sec^2\theta - 2\tan\theta\sec\theta)$$

$$= \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + \tan^2\theta + \sec^2\theta - 2\tan\theta\sec\theta$$

$$= 2(\tan^2\theta + \sec^2\theta)$$

$$= 2\left(\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}\right)$$

$$= 2\left(\frac{1 + \sin^2\theta}{\cos^2\theta}\right) = 2\left(\frac{1 + \sin^2\theta}{1 - \sin^2\theta}\right)$$

Hence Proved.

57. Question

Prove the following trigonometric identities:

$$\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta = \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$$

Answer

$$\left(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\operatorname{cosec}^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta}+\frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}}+\frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta-\cos^4\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\cos^2\theta\left(1-\cos^2\theta\right)+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta\left(1-\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta\cos^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\sin^2\theta\left(\cos^2\theta+1\right)}+\frac{\sin^2\theta}{\cos^2\theta\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^4\theta\left(1+\sin^2\theta\right)+\sin^4\theta\left(\cos^2\theta+1\right)}{\sin^2\theta\cos^2\theta\left(\cos^2\theta+1\right)\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$=\frac{\cos^4\theta\left(1+\sin^2\theta\right)+\sin^4\theta\left(\cos^2\theta+1\right)}{\left(\cos^2\theta+1\right)\left(1+\sin^2\theta\right)}$$

$$=\frac{\left(\cos^4\theta+\sin^2\theta\cos^4\theta\right)+\left(\sin^4\theta\cos^2\theta+\sin^4\theta\right)}{1+\cos^2\theta+\sin^2\theta+\cos^2\theta\sin^2\theta}$$

$$=\frac{\left(\cos^4\theta+\sin^4\theta\right)+\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{1+1+\cos^2\theta\sin^2\theta}$$

$$=\frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta\times1}{2+\cos^2\theta\sin^2\theta}$$

$$=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

Hence Proved.

58. Question

Prove the following trigonometric identities:

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Answer

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\times\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2$$

$$= \left[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2-\cos^2\theta}\right]^2$$

$$= \left[\frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta}\right]^2$$

$$= \left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2$$

$$= \left[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2$$

$$= \left[\frac{2(1+\sin\theta)-2\cos\theta(1+\sin\theta)}{2\sin^2\theta+2\sin\theta}\right]^2$$

$$= \left[\frac{2(1+\sin\theta)(1-\cos\theta)}{2\sin\theta(1+\sin\theta)}\right]^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

Hence Proved.

59. Question

Prove the following trigonometric identities:

$(\sec A+\tan A-1)(\sec A-\tan A+1)=2\tan A$

Answer

$$(\sec A+\tan A-1)(\sec A-\tan A+1)$$
$$=\left[\sec A+\tan A-\left(\sec^2 A-\tan^2 A\right)\right]\left[\sec A-\tan A+\left(\sec^2 A-\tan^2 A\right)\right]$$
$$=\left[\sec A+\tan A-(\sec A-\tan A)(\sec A+\tan A)\right]\left[\sec A-\tan A+(\sec A-\tan A)(\sec A+\tan A)\right]$$
$$=(\sec A+\tan A)\left[1-(\sec A-\tan A)\right](\sec A-\tan A)\left[1+(\sec A+\tan A)\right]$$
$$=(\sec A+\tan A)(\sec A-\tan A)\left[1-\sec A+\tan A\right]\left[1+\sec A+\tan A\right]$$
$$=\left(\sec^2 A-\tan^2 A\right)\left[1-\sec A+\tan A\right]\left[1+\sec A+\tan A\right]$$
$$=1\times\left[1-\sec A+\tan A\right]\left[1+\sec A+\tan A\right]$$
$$=\left[1-\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]\left[1+\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]$$
$$=\left[\frac{\cos A+\sin A-1}{\cos A}\right]\left[\frac{\cos A+\sin A+1}{\cos A}\right]$$
$$=\left[\frac{(\cos A+\sin A)^2-1}{\cos^2 A}\right]$$
$$=\left[\frac{\cos^2 A+\sin^2 A+2\sin A\cos A-1}{\cos^2 A}\right]$$
$$=\left[\frac{1+2\sin A\cos A-1}{\cos^2 A}\right]$$
$$=\left[\frac{2\sin A}{\cos A}\right]$$
$$=2\tan A$$

Hence Proved.

60. Question

Prove the following trigonometric identities:

$(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)=2$

Answer

$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$

$= \frac{[(\sin\theta + \cos\theta) - 1][(\sin\theta + \cos\theta) + 1]}{\sin\theta \cdot \cos\theta}$

$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta \cdot \cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta \cdot \cos\theta}$

$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta \cdot \cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta \cdot \cos\theta} = 2 = \text{R.H.S.}$

Hence Proved.

61. Question

Prove the following trigonometric identities:

$(\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta) = (\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

Answer

$LHS = (\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta)$

$= \left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)$

$= \left(\frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}\right)$

$= \left(\frac{(\cos\theta - \sin\theta)^2(\cos\theta + \sin\theta)}{\sin^2\theta\cos^2\theta}\right)$

$RHS = (\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

$= \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta} \times \frac{1}{\cos\theta} - 2\right)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{1 - 2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{(\cos\theta - \sin\theta)^2}{\sin\theta\cos\theta}\right)$

$= \left(\frac{(\cos\theta - \sin\theta)^2(\cos\theta + \sin\theta)}{\sin^2\theta\cos^2\theta}\right)$

Therefore, LHS = RHS

Hence Proved.

62. Question

Prove the following trigonometric identities:

$(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A\sec A - \cot A\operatorname{cosec} A$

Answer

To prove: $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A\sec A - \cot A\operatorname{cosec} A$

Proof: Consider LHS,(secA-cosecA)(1+tanA+cotA)We know,cosecA=1/sinA, secA=1/cosA, tanA=sinA/cosA,cotA=cosA/sinASo,

$$(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{\sin A - \cos A}{cosA\ \sin A}\right)\left(\frac{sinA\ \cos A + \sin^2 A + \cos^2 A}{cosA\ \sin A}\right)$$

Using the formula $a^3 - b^3 = (a-b)(a^2+b^2+ab)$ we get,

$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \ \cos^2 A}$$

$$RHS = \tan A \sec A - \cot A \operatorname{cosec} A$$
$$= \frac{\sin A}{\cos A} \times \frac{1}{\cos A} - \frac{\cos A}{\sin A} \times \frac{1}{\sin A}$$
$$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A}$$
$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$
$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$

LHS = RHS

Hence Proved.

63. Question

Prove the following trigonometric identities:

$$\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$$

Answer

$$\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A}$$
$$= \frac{\cos A \times \frac{1}{\sin A} - \sin A \frac{1}{\cos A}}{\cos A + \sin A}$$
$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A}$$
$$= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A}$$
$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\sin A \cos A (\cos A + \sin A)}$$
$$= \frac{(\cos A - \sin A)}{\sin A \cos A}$$
$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$
$$= \operatorname{cosec} A - \sec A$$

Hence Proved.

64. Question

Prove the following trigonometric identities:

$$\frac{\mathbf{sinA}}{\mathbf{secA + tanA - 1}} + \frac{\mathbf{cotA}}{\mathbf{cosecA + cotA - 1}} = 1$$

Answer

$$\frac{\sin A}{\sec A+\tan A-1}+\frac{\cot A}{\operatorname{cosec} A+\cot A-1}$$

$$= \frac{\sin A}{\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1}+\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1}$$

$$= \frac{\sin A\cos A}{1+\sin A-\cos A}+\frac{\sin A\cos A}{1+\cos A-\sin A}$$

$$= \sin A\cos A\left(\frac{1}{1+\sin A-\cos A}+\frac{1}{1+\cos A-\sin A}\right)$$

$$= \sin A\cos A\left(\frac{1+\cos A-\sin A+1+\sin A-\cos A}{(1+\sin A-\cos A)(1+\cos A-\sin A)}\right)$$

$$= \sin A\cos A\left(\frac{2}{1-\sin^2 A-\cos^2 A+2\sin A\cos A}\right)$$

$$= \sin A\cos A\left(\frac{2}{1-\left(\sin^2 A+\cos^2 A\right)+2\sin A\cos A}\right)$$

$$= \sin A\cos A\left(\frac{2}{1-1+2\sin A\cos A}\right)$$

$$= \sin A\cos A\left(\frac{1}{\sin A\cos A}\right)$$

$$= 1$$

Hence Proved.

65. Question

Prove the following trigonometric identities:

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\sin A\cos A$$

Answer

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\frac{\tan A}{(\sec^2 A)^2}+\frac{\cot A}{(\operatorname{cosec}^2 A)^2}$$

$$=\frac{\sin A}{\cos A}\times\cos^4 A+\frac{\cos A}{\sin A}\times\sin^4 A$$

$$=\sin A\cos^3 A+\cos A\sin^3 A$$

$$=\sin A\cos A\left(\cos^2 A+\sin^2 A\right)$$

$$=\sin A\cos A\times 1$$

$$=\sin A\cos A$$

Hence Proved.

66. Question

Prove the following trigonometric identities:

$$\sec^4 A(1-\sin^4 A)-2\tan^2 A=1$$

Answer

$$\sec^4 A(1-\sin^4 A)-2\tan^2 A$$

$$=\sec^4 A-\sec^4 A\sin^4 A-2\tan^2 A$$

$$=\sec^4 A-\frac{\sin^4 A}{\cos^4 A}-2\tan^2 A$$

$$=\left(\sec^2 A\right)^2-\tan^4 A-2\tan^2 A$$

$$=\left(1+\tan^2 A\right)^2-\tan^4 A-2\tan^2 A$$

$$=1+\tan^4 A+2\tan^2 A-\tan^4 A-2\tan^2 A$$

$$=1$$

Hence Proved.

67. Question

Prove the following trigonometric identities:

$$\frac{\cot^2 A(\sec A-1)}{1+\sin A}=\sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$$

Answer

$$\frac{\cot^2 A(\sec A-1)}{1+\sin A}=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A}-1\right)}{1+\sin A}$$
$$=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$$
$$=\frac{\frac{\cos A}{1-\cos^2 A}(1-\cos A)}{1+\sin A}$$
$$=\frac{\frac{\cos A}{(1-\cos A)(1+\cos A)}(1-\cos A)}{1+\sin A}$$
$$=\frac{\cos A}{(1+\cos A)(1+\sin A)} \quad .....(1)$$

$$RHS=\frac{1}{\cos^2 A}\left(\frac{1-\sin A}{1+\frac{1}{\cos A}}\right)$$
$$=\frac{1}{\cos^2 A}\frac{(1-\sin A)\cos A}{(1+\cos A)}$$
$$=\frac{1}{\cos A}\frac{(1-\sin A)}{(1+\cos A)}\times\frac{(1+\sin A)}{(1+\sin A)}$$
$$=\frac{1}{\cos A}\frac{(1-\sin^2 A)}{(1+\cos A)(1+\sin A)}$$
$$=\frac{1}{\cos A}\frac{\cos^2 A}{(1+\cos A)(1+\sin A)}$$

$$=\frac{\cos A}{(1+\cos A)(1+\sin A)} \quad ....(2)$$

From (1) and (2) we get

LHS = RHS

Hence Proved.

68. Question

Prove the following trigonometric identities:

$$(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}=\sin A\tan A-\cot A\cos A$$

Answer

To Prove: $(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}=\sin A\tan A-\cot A\cos A$

Proof: Consider the LHS,

$$\Rightarrow \frac{\sec A}{\operatorname{cosec}^2 A}-\frac{cosec A}{\sec^2 A}=\left(\frac{1}{\cos A}\times\sin^2 A\right)-\left(\frac{1}{\sin A}\times\cos^2 A\right)$$

= sinA - cosA+ cotA sinA - cotA cosA + tanA sinA - tanA cosA

Use the formula:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\ and\ \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$=sinA-cosA+\left(\frac{cosA}{sinA}\times sinA\right)-\left(\frac{cosA}{sinA}\times cosA\right)+\left(\frac{sinA}{cosA}\times sinA\right)-\left(\frac{sinA}{cosA}\times cosA\right)$$

$$=sinA-cosA+\cos A-\frac{\cos^2 A}{\sin A}+\frac{\sin^2 A}{\cos A}-\sin A$$

$$= \frac{sin^2A}{cosA} - \frac{cos^2A}{sinA}$$

We know:

$$\sin\theta = \frac{1}{\text{cosec}\theta} \; and \; \cos\theta \; = \frac{1}{\sec\theta}$$

So,

$$(\; 1 \; + \; \cot A \; + \; \tan A)(\sin A \; - \; \cos A) \; = \; \frac{\sec A}{\text{cosec}^2 A} \; - \; \frac{\text{cosec} A}{sec^2 A}$$

Again use the formula:

$$\sin\theta = \frac{1}{\text{cosec}\theta} \; and \; \cos\theta \; = \frac{1}{\sec\theta}$$

So,

$$\Rightarrow \frac{secA}{cosec^2A} - \frac{cosecA}{sec^2A} = \left(\frac{1}{cosA} \times sin^2A\right) - \left(\frac{1}{sinA} \times cos^2A\right)$$

$$\Rightarrow \frac{secA}{cosec^2A} - \frac{cosecA}{sec^2A} = \frac{sinA \; \times \; \sin \; A}{cosA} - \frac{cosA \; \cos A}{sinA}$$

Use the formula:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; and \; \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{secA}{cosec^2A} - \frac{cosecA}{sec^2A} = \sin A \; \tan A - \cot A \; \cos A$$

Therefore, $(\; 1 + cotA + tanA)(sinA - cosA) \; = \frac{secA}{cosec^2A} - \frac{cosecA}{sec^2A} \; = sinA \; tanA - cotA \; cosA$

Hence Proved.

69. Question

Prove the following trigonometric identities:

$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$

Answer

To prove: $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$ **Proof:** Take LHS,Use the identity $\sin^2\theta+\cos^2\theta=1$

$\sin^2A \cos^2B - \cos^2A \sin^2B = \sin^2A(1 - \sin^2B) - (1 - \sin^2A)\sin^2B$ $= \sin^2A - \sin^2A \sin^2B - \sin^2B + \sin^2A \sin^2B$

$= \sin^2A - \sin^2B$ = RHSHence Proved

70. Question

Prove the following trigonometric identities:

$\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

Answer

Use the formula $\tan\theta = \frac{\sin\theta}{\cos\theta}\ and\ \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$
\begin{aligned}
L.H.S &= \frac{\cot A + \tan B}{\cot B + \tan A} \\
&= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\cos A}{\cot A}} \\
&= \frac{\frac{\cos A \cot B \sin A \sin B}{\sin A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \sin B}} \\
&= \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \times \frac{\cos A \sin B}{\cos A \cos B + \sin A \sin B} \\
&= \frac{\cos A \sin B}{\sin A \cos B} \\
&= \cot A \tan B \\
&= R.H.S
\end{aligned}
$$

Hence Proved.

71. Question

Prove the following trigonometric identities:

$$\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$$

Answer

$$
\begin{aligned}
\frac{\tan A + \tan B}{\cot A + \cot B} &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} \\
&= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin B \cos A + \cos B \sin A}{\sin A \sin B}} \\
&= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin B \cos A + \cos B \sin A}{\sin A \sin B}} \\
&= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \times \frac{\sin A \sin B}{\sin B \cos A + \cos B \sin A} \\
&= \frac{\sin A \sin B}{\cos A \cos B} \\
&= \tan A \tan B
\end{aligned}
$$

Hence Proved.

72. Question

Prove the following trigonometric identities:

$$\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$$

Answer

$$
\begin{aligned}
&\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A \\
&= \cot^2 A \left(1 + \cot^2 B\right) - \cot^2 B \left(1 + \cot^2 A\right) \\
&= \cot^2 A + \cot^2 A \cot^2 B - \cot^2 B - \cot^2 A \cot^2 B \\
&= \cot^2 A - \cot^2 B
\end{aligned}
$$

Hence Proved.

73. Question

Prove the following trigonometric identities:

$$\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$$

Answer

$$\tan^2 A \sec^2 B - \sec^2 A \tan^2 B$$
$$= \tan^2 A(1 + \tan^2 B) - (1 + \tan^2 A)\tan^2 B$$
$$= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 A \tan^2 B$$
$$= \tan^2 A - \tan^2 B$$

Hence Proved.

74. Question

If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$

$x^2 - y^2 = a^2 - b^2$

Answer

$$x = a\sec\theta + b\tan\theta$$
$$\Rightarrow x^2 = (a\sec\theta + b\tan\theta)^2 = a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta$$
$$y = a\tan\theta + b\sec\theta$$
$$\Rightarrow y^2 = (a\tan\theta + b\sec\theta)^2 = a^2\tan^2\theta + b^2\sec^2\theta + 2ab\tan\theta\sec\theta$$

Now,

$$x^2 - y^2 = a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta - b^2\sec^2\theta - 2ab\tan\theta\sec\theta$$
$$x^2 - y^2 = a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$$
$$x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$$
$$x^2 - y^2 = (a^2 - b^2)(\sec^2\theta - \tan^2\theta)$$
$$x^2 - y^2 = (a^2 - b^2) \times 1$$
$$x^2 - y^2 = a^2 - b^2$$

Hence Proved.

75. Question

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

Answer

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = 2$$
$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\cos\theta\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{xy}{ab}\sin\theta\cos\theta = 2$$
$$\frac{x^2}{a^2}\cos^2\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta = 2$$
$$\frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$
$$\frac{x^2}{a^2} \times 1 + \frac{y^2}{b^2} \times 1 = 2$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

Hence Proved.

76. Question

If $\text{cosec}\theta - \sin\theta = a^3$, $\sec\theta - \cos\theta = b^3$, prove that $a^2b^2(a^2 + b^2)$.

Answer

$$\frac{1}{\sin\theta} - \sin\theta = a^3$$
$$\frac{1 - \sin^2\theta}{\sin\theta} = a^3$$
$$\frac{\cos^2\theta}{\sin\theta} = a^3$$
$$a^3 = \frac{\cos^2\theta}{\sin\theta}$$
$$a = \frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$
$$\Rightarrow \quad a^2 = \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \quad \ldots.(1)$$

Similarly we can see that,

$$\sec\theta - \cos\theta = b^3$$

$$\frac{1}{\cos\theta} - \cos\theta = b^3$$

$$\frac{1 - \cos^2\theta}{\cos\theta} = b^3$$

$$b^3 = \frac{\sin^2\theta}{\cos\theta}$$

$$b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

$$b^2 = \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta} \ldots\ldots (2)$$

From (1) and (2), we get

$$a^2b^2\left(a^2+b^2\right) = \cos^{\frac{4}{3}-\frac{2}{3}}\theta \sin^{\frac{4}{3}-\frac{2}{3}}\theta\left(\frac{\cos^2\theta+\sin^2\theta}{\sin^{2/3}\theta\cos^{2/3}\theta}\right)$$
$$= \cos^{\frac{2}{3}}\theta \sin^{\frac{2}{3}}\theta\left(\frac{1}{\sin^{2/3}\theta\cos^{2/3}\theta}\right)$$
$$= 1$$

Hence Proved.

77. Question

If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = m$, $a\sin^3\theta + 3a\cos^2\theta\sin\theta = n$, prove that

$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$.

Answer

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$$
$$= \left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{2/3}$$
$$+ \left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{2/3}$$
$$= a^{2/3}\left(\cos^3\theta + 3\cos\theta\sin^2\theta + \sin^3\theta + 3\cos^2\theta\sin\theta\right)^{2/3}$$
$$+ a^{2/3}\left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{2/3}$$
$$= a^{2/3}\left[\left(\cos\theta+\sin\theta\right)^3\right]^{2/3} + a^{2/3}\left[\left(\cos\theta-\sin\theta\right)^3\right]^{2/3}$$
$$= a^{2/3}\left(\cos\theta+\sin\theta\right)^2 + a^{2/3}\left(\cos\theta-\sin\theta\right)^2$$
$$= a^{2/3}\left(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta\right) + a^{2/3}\left(\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta\right)$$
$$= a^{2/3}\left[\left(1+2\sin\theta\cos\theta\right)+\left(1-2\sin\theta\cos\theta\right)\right]$$
$$= a^{2/3}\times 2$$
$$= a^{2/3}$$

Hence Proved.

78. Question

If $x = a\cos^3\theta$, $y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

Answer

$$x = a\cos^3\theta \quad \Rightarrow \quad \frac{x}{a} = \cos^3\theta$$
$$y = b\sin^3\theta \quad \Rightarrow \quad \frac{y}{b} = \sin^3\theta$$
$$\text{Now, } \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = \left(\cos^3\theta\right)^{2/3} + \left(\sin^3\theta\right)^{2/3}$$
$$= \cos^2\theta + \sin^2\theta$$
$$= 1$$
$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

79. Question

If $3\sin\theta + 5\cos\theta = 5$, prove that $5\sin\theta - 3\cos\theta = \pm 3$.

Answer

$$
\begin{aligned}
& 3\sin\theta + 5\cos\theta = 5 \\
\Rightarrow \quad & 3\sin\theta = 5 - 5\cos\theta \\
\Rightarrow \quad & 3\sin\theta = 5(1 - \cos\theta) \\
\Rightarrow \quad & 3\sin\theta = \frac{5(1-\cos\theta)\times(1+\cos\theta)}{(1+\cos\theta)} \\
\Rightarrow \quad & 3\sin\theta = \frac{5(1-\cos^2\theta)}{(1+\cos\theta)} \\
\Rightarrow \quad & 3\sin\theta = \frac{5\sin^2\theta}{(1+\cos\theta)} \\
\Rightarrow \quad & 3 = \frac{5\sin\theta}{(1+\cos\theta)} \\
\Rightarrow \quad & 3 + 3\cos\theta = 5\sin\theta \\
\Rightarrow \quad & 3 = 5\sin\theta - 3\cos\theta
\end{aligned}
$$

Hence Proved.

80. Question

If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

Answer

$$
\begin{aligned}
m^2 + n^2 &= (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 \\
&= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta \\
&= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) \\
&= a^2 \times 1 + b^2 \times 1 \\
&= a^2 + b^2
\end{aligned}
$$

Hence Proved.

81. Question

If $\operatorname{cosec}\theta + \cot\theta = m$ and $\operatorname{cosec}\theta - \cot\theta = n$, prove that $mn = 1$.

Answer

$$
\begin{aligned}
& \operatorname{cosec}\theta + \cot\theta = m \\
& \operatorname{cosec}\theta - \cot\theta = n \\
& mn = (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) \\
& mn = (\operatorname{cosec}^2\theta - \cot^2\theta) \\
& mn = 1
\end{aligned}
$$

Hence Proved.

82. Question

If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$

Answer

Consider,cos A+ $\cos^2$A = 1⇒cosA = 1 - $\cos^2$A As we know 1-$\cos^2$A = $\sin^2$A⇒ cosA = $\sin^2$A .... (1)Now $\sin^2$A + $\sin^4$A = $\sin^2$A + ($\sin^2$A)^2 From 1$\sin^2$A + $\sin^4$A = $\sin^2$A + (cosA)^2 = $\sin^2$A + $\cos^2$A = 1

Hence Proved.

83. Question

Prove that:

(i) $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

(iii) $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2\operatorname{cosec}\theta$

(iv) $\frac{\sec\theta - 1}{\sec\theta + 1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

Answer

(i)

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\sqrt{\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}}+\sqrt{\frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}}$$
$$=\sqrt{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}+\sqrt{\frac{\frac{1+\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}}$$
$$=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$
$$=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1-\cos\theta}{1+\cos\theta}}$$
$$=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$
$$=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$
$$=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$
$$=\frac{1-\cos\theta}{\sin\theta}+\frac{1+\cos\theta}{\sin\theta}$$
$$=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$$
$$=\frac{1}{\sin\theta}+\frac{1}{\sin\theta}$$
$$=\frac{2}{\sin\theta}$$
$$=2\operatorname{cosec}\theta$$

Hence Proved.

(ii)

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$
$$=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$
$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$
$$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$$
$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$$
$$=\frac{2}{\cos\theta}$$
$$=2\sec\theta$$

Hence Proved.

(iii)

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$$
$$=\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$
$$=\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$
$$=\frac{1+\cos\theta}{\sin\theta}+\frac{1-\cos\theta}{\sin\theta}$$
$$=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$$
$$=\frac{2}{\sin\theta}$$
$$=2\operatorname{cosec}\theta$$

Hence Proved.

(iv)

$$\frac{\sec\theta-1}{\sec\theta+1}=\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}$$

$$=\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$$

$$=\frac{1-\cos^2\theta}{(1+\cos\theta)^2}$$

$$=\frac{\sin^2\theta}{(1+\cos\theta)^2}$$

$$=\left(\frac{\sin\theta}{1+\cos\theta}\right)^2$$

Hence Proved.

84. Question

If $\cos\theta+\cos^2\theta=1$, prove that

Answer

$\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2=1$

$\cos\theta+\cos^2\theta=1$

$\cos=1-\cos^2\theta$

$\cos=\sin^2\theta-(i)$

Now, $\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2$

$=(\sin^4\theta)^3+\sin^4\theta-\sin^2\theta[\sin^4\theta+\sin^2\theta]$

$+(\sin^2\theta)^3+2(\sin^2\theta)^2+2\sin^2\theta-2$

Using $(a+b)^3=a^3+b^3+3(a+b)$ and

Also from (i) $\sin^2\theta\cos^2$

$(\sin^4\theta+\sin^2\theta)^3+2(\cos\theta)^2+2\cos\theta-2.$

$((\sin^2\theta)^2+\sin^2\theta)+2\cos^2\theta+2\cos\theta-2$

$(\cos^2+\sin^2\theta)^3+2\cos^2\theta+2\cos\theta-2$

$(\cos)^3+2\cos^2\theta+2\sin^2\theta-2$

$[\because\sin^2\theta+\cos^2\theta=1]$

$1+2(\sin^2\theta+\cos^2\theta)-2$

$1+2(1)-2=1$

Hence Proved.

85. Question

Given that:

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$$

Show that one of the values of each member of this equality is $\sin\alpha\sin\beta\sin\gamma$

Answer

L.H.S

we know that $1+\cos\theta = 1+\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$

$= 2\cos^2\frac{\theta}{2}$.

$\therefore \Rightarrow 2\cos^2\frac{\alpha}{2}-2\cos^2\frac{\beta}{2}.2\cos^2\frac{\gamma}{2}$ ...(i)

Multiply (i) with $\sin\alpha\sin\beta\sin\chi$ and divide with same

we get $\frac{8\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}{\sin\alpha\sin\beta\sin\chi}\times\sin\alpha\sin\beta\sin\chi$

$\Rightarrow \frac{8\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}\times\sin\alpha\sin\beta\sin\chi}{2.2\theta\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\sin\frac{\beta}{2}\sin\frac{\beta}{2}\cos\frac{\gamma}{2}}$

$\Rightarrow \sin\alpha\sin\beta\sin\chi\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$

R.H.S $(1-\cos\alpha)(1-\cos\beta)(1\cos\chi)$

we know that $1-\cos\theta = 1-\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$

$\Rightarrow 2\sin^2\frac{\alpha}{2},2\sin^2\frac{\beta}{2}2\sin^2\frac{\gamma}{2}$

Multiply and divide by $\sin\alpha\sin\beta\sin\chi$ we get

$\frac{2\sin^2\frac{\alpha}{2},2\sin^2\frac{\beta}{2}2\sin^2\frac{\gamma}{2}.\sin\alpha\sin\beta\sin\chi}{\sin\alpha\sin\beta\sin\chi}$

$\Rightarrow \frac{2\sin^2\frac{\alpha}{2},2\sin^2\frac{\beta}{2}2\sin^2\frac{\gamma}{2}.\sin\alpha\sin\beta\sin\chi}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}2\sin\frac{\beta}{2}\cos\frac{\beta}{2}2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

$\Rightarrow \text{Tan}\frac{\alpha}{2}\text{Tan}\frac{\beta}{2}\text{Tan}\frac{\gamma}{2}\sin\alpha\sin\beta\sin\chi$

hence $\sin\alpha\sin\beta\sin\chi$ is the member of equality

Hence Proved.

86. Question

If $\sin\theta+\cos\theta = x$, prove that $\sin^6\theta+\cos^6\theta = \frac{4-3(x^2-1)^2}{4}$

Answer

$\text{Sin}\theta+\text{Cos}\theta = x$

Squaring on both sides

$(\text{Sin}\theta+\text{Cos})^2 = x^2$

$\Rightarrow \text{Sin}^2\theta+\text{Cos}^2\theta+2\text{Sin}\theta\text{Cos}\theta = x^2$

$\therefore \text{Sin}\theta\text{Cos}\theta = \frac{x^2-1}{2}$ ....(1)

We know $\text{Sin}^2\theta+\text{Cos}^2\theta = 1$

cobing on both since

$(\sin^2\theta+\cos^2\theta)^3 = (1)^3$

$\text{Sin}^6\theta+\text{Cos}^6\theta+3\text{Sin}\theta\text{Cos}^2(\text{Sin}^2\theta+\text{Cos}^2\theta) = 1$

$\Rightarrow \text{Sin}^6\theta+\text{Cos}^2 = 1-3\text{Sin}^2\theta\text{Cos}^2\theta$

$= 1-\frac{3(x^2-1)^2}{4}$ form -(1)

$\therefore \text{Sin}^6\theta+\text{Cos}^6\theta = \frac{4-3(x^2-1)^2}{4}$

Hence proved

Hence Proved.

87. Question

If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\cos\phi$, and $z = c\tan\theta$, show that $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2} = 1$

Answer

$x = a\sec\theta\cos\phi \quad \Rightarrow x^2 = a^2\sec^2\theta\cos^2\phi$

$\therefore \quad \frac{x^2}{a^2} = \sec^2\theta\cos^2\phi \quad \ldots.(1)$

$y = b\sec\theta\sin\phi \quad \Rightarrow y^2 = b^2\sec^2\theta\sin^2\phi$

$\therefore \quad \frac{y^2}{a^2} = \sec^2\theta\sin^2\phi \quad \ldots.(2)$

$z = c\tan\theta \quad \Rightarrow \; z^2 = c^2\tan^2\theta$

$\therefore \quad \frac{z^2}{c^2} = \tan^2\theta \quad \ldots.(3)$

$$\text{Now, } \frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2} = \sec^2\theta\cos^2\phi+\sec^2\theta\sin^2\phi-\tan^2\theta$$
$$= \sec^2\theta\left(\cos^2\phi+\sin^2\phi\right)-\tan^2\theta$$
$$= \sec^2\theta\times 1-\tan^2\theta$$
$$= \sec^2\theta-\tan^2\theta = 1$$

$$\text{hence, } \frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2} = 1$$

Hence Proved.

Exercise 6.2

1. Question

If $\cos\theta = \frac{4}{5}$, find all other trigonometric ratios of angle θ.

Answer

$\sin\theta = 3/5, \tan\theta = 3/4$, $\sec\theta = 5/4,$

$\cos\theta = \frac{4}{5} \quad \Rightarrow \sin\theta = \frac{3}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{4/5} = \frac{5}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4}$

Hence Proved.

2. Question

If $\sin\theta = \frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle θ

Answer

$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$

3. Question

If $\tan\theta = \frac{1}{\sqrt{2}}$, find the value of $\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\cot^2\theta}$

Answer

$$Given,\ \tan\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{P}{B} = \frac{1}{\sqrt{2}}$$

$$Let\ P = k\ and\ B = \sqrt{2}k$$

$$Then,$$

$$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{k^2 + 2k^2}$$

$$= \sqrt{3k^2}$$

$$= \sqrt{3}k$$

$$Hence,$$

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

$$\frac{\mathrm{cosec}^2\theta - \sec^2\theta}{\mathrm{cosec}^2\theta + \cot^2\theta} = \frac{\mathrm{cosec}^2\theta\left(1 - \frac{\sec^2\theta}{\mathrm{cosec}^2\theta}\right)}{\mathrm{cosec}^2\theta\left(1 + \frac{\cot^2\theta}{\mathrm{cosec}^2\theta}\right)}$$

$$= \frac{\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right)}{\left(1 + \sin^2\theta \times \cot^2\theta\right)}$$

$$= \frac{\left(1 - \tan^2\theta\right)}{\left(1 + \sin^2\theta \times \frac{1}{\tan^2\theta}\right)}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{2}}\right)^2}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)} = \frac{1 - \frac{1}{2}}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)}$$

$$= \frac{1/2}{1 + \frac{2}{3}} = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

4. Question

If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1 - \cos\theta}{1 + \cos\theta}$

Answer

$$\tan\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$Now,\ \frac{1 - \cos\theta}{1 + \cos\theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{1/5}{9/5} = \frac{1}{9}\ \frac{1 - \cos\theta}{1 + \cos\theta}$$

5. Question

If $\tan\theta = \frac{12}{5}$, find the value of $\frac{1+\sin\theta}{1-\sin\theta}$

Answer

$$\frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$
$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$
$$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$$
$$= \sec^2\theta + \tan^2\theta$$
$$= \tan^2\theta + 1 + \tan^2\theta$$
$$= 2\tan^2\theta + 1$$
$$= 2\times\left(\frac{12}{5}\right)^2 + 1 \qquad \left(\because \tan\theta = \frac{12}{5}\right)$$
$$= \frac{288}{25} + 1$$
$$= \frac{288+25}{25} = \frac{313}{25}$$

6. Question

If $\cot\theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Answer

Given, $\cot\theta = \frac{1}{\sqrt{3}}$

Now,

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$$
$$= \frac{1}{\frac{1+\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{1}{\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{1}{\operatorname{cosec}^2\theta + \cot^2\theta}$$
$$= \frac{1}{1+\cot^2\theta+\cot^2\theta}$$
$$= \frac{1}{1+2\cot^2\theta}$$
$$= \frac{1}{1+2\times\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{1+2\times\frac{1}{3}} = \frac{1}{1+\frac{2}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

7. Question

If $\operatorname{cosec}A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$

Answer

Given, $\operatorname{cosec}A = \sqrt{2} \quad \Rightarrow \cot^2 A = \operatorname{cosec}^2 A - 1 = \left(\sqrt{2}\right)^2 - 1 = 2 - 1 = 1$

and, $\tan^2 A = \frac{1}{\cot^2 A} = \frac{1}{1} = 1$

$\Rightarrow \quad \sin^2 A = \frac{1}{\operatorname{cosec}^2 A} = \frac{1}{\left(\sqrt{2}\right)^2} = \frac{1}{2}$

$\Rightarrow \quad \cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$

Now,

$$\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)} = \frac{2\times\frac{1}{2} + 3\times 1}{4\left(1 - \frac{3}{4}\right)} = \frac{1+3}{4\times\frac{1}{4}} = 4$$

8. Question

If $\cot\theta = \sqrt{3}$, find the value of $\frac{\operatorname{cosec}^2\theta + \cot^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}$

Answer

Given, $\cot\theta = \sqrt{3}$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4$$

and $\cot\theta = \sqrt{3} \quad \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1 + \frac{1}{3} = \frac{4}{3}$$

Now,

$$\frac{\operatorname{cosec}^2\theta + \cot^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta} = \frac{4+3}{4-\frac{4}{3}} = \frac{7}{8/3} = \frac{21}{8}$$

9. Question

If $3\cos\theta = 1$, find the value of $\frac{6\sin^2\theta + \tan^2\theta}{4\cos\theta}$

Answer

$$3\cos\theta = 1 \quad \Rightarrow \cos\theta = \frac{1}{3}$$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}/3}{1/3} = 2\sqrt{2}$$

$$\frac{6\sin^2\theta + \tan^2\theta}{4\cos\theta} = \frac{6\times\frac{8}{9} + 8}{4\times\frac{1}{3}} = \frac{\frac{16}{3} + 8}{\frac{4}{3}} = \frac{\frac{30}{3}}{\frac{4}{3}} = \frac{30}{4} = 7\frac{1}{2}$$

10. Question

If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$

Answer

$$\sqrt{3}\tan\theta = 3\sin\theta$$

$$\sqrt{3}\frac{\sin\theta}{\cos\theta} = \sqrt{3}\times\sqrt{3}\sin\theta$$

$$\frac{1}{\cos\theta} = \sqrt{3}$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

Now, $\sin^2\theta - \cos^2\theta = 1 - \cos^2\theta - \cos^2\theta$

$$= 1 - 2\cos^2\theta$$

$$= 1 - 2\times\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 1 - 2\times\frac{1}{3} = \frac{1}{3}$$

11. Question

If $\operatorname{cosec}\theta = \frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Answer

$$\text{cosec}\theta = \frac{13}{12} \quad \Rightarrow \sin\theta = \frac{12}{13} \text{ and } \cos\theta = \frac{5}{13}$$

$$\text{now, } \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

$$= \frac{\frac{24-15}{13}}{\frac{48-45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13}\times\frac{13}{3} = 3$$

$$\text{cosec}\theta = \frac{13}{12}$$

12. Question

If $\sin\theta + \cos\theta = \sqrt{2}\cos(90^\circ - \theta)$, find $\cot\theta$.

Answer

$$\sin\theta + \cos\theta = \sqrt{2}\cos(90^\circ - \theta)$$

$$\cos\theta = \sqrt{2}\sin\theta - \sin\theta$$

$$\cos\theta = \left(\sqrt{2} - 1\right)\sin\theta$$

$$\frac{\cos\theta}{\sin\theta} = \sqrt{2} - 1$$

CCE - Formative Assessment

1. Question

Define an identity.

Answer

An equation that is true for all values of the variables involved is said to be an identity. For example:

$a^2 - b^2 = (a - b)(a + b)$

$\sin^2\theta + \cos^2\theta = 1$

2. Question

What is the value of $(1 - \cos^2\theta)\,\text{cosec}^2\theta$?

Answer

To find: $(1 - \cos^2\theta)\,\text{cosec}^2\theta$

$\because \text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\therefore \text{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$\Rightarrow (1 - \cos^2\theta)\,\text{cosec}^2\theta = (1 - \cos^2\theta)\frac{1}{\sin^2\theta}$ .....(i)

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow$ from (i), we have

$$(1 - \cos^2\theta)\,\text{cosec}^2\theta = \sin^2\theta\frac{1}{\sin^2\theta} = 1$$

3. Question

What is the value of $(1 + \cot^2\theta)\sin^2\theta$?

Answer

To find: $(1 + \cot^2\theta) \sin^2\theta$

$\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$\therefore (1 + \cot^2\theta) \sin^2\theta = \operatorname{cosec}^2 \theta \sin^2 \theta$

Also, $\operatorname{cosec} \theta = \frac{1}{\sin\theta}$

$$\Rightarrow \operatorname{cosec}^2\theta = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow (1 + \cot^2 \theta) \sin^2 \theta = \operatorname{cosec}^2\theta \sin^2 \theta = \frac{1}{\sin^2 \theta} \sin^2 \theta = 1$$

4. Question

What is the value of $\sin^2 \theta + \frac{1}{1+\tan^2 \theta}$?

Answer

To find: $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$

$\because 1 + \tan^2 \theta = \sec^2 \theta$

$$\therefore \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta}$$

Also, we know that $\cos \theta = \frac{1}{\sec \theta}$

$$\Rightarrow \cos^2 \theta = \frac{1}{\sec^2 \theta}$$

$$\Rightarrow \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta} = \sin^2 \theta + \cos^2 \theta$$

Also,

$\because \sin^2 \theta + \cos^2 \theta = 1$

$$\therefore \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta} = \sin^2 \theta + \cos^2 \theta = 1$$

5. Question

If $\sec \theta + \tan\theta = x$, write the value of $\sec \theta - \tan \theta$ in terms of x.

Answer

Given: $\sec \theta + \tan \theta = x$ ……(i)

To find: $\sec \theta - \tan \theta$

We know that $1 + \tan^2 \theta = \sec^2 \theta$

$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta$

Now, $\because a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta)$

⇒ From (i), we have

$1 = (\sec \theta - \tan \theta) x$

$$\Rightarrow \sec\theta - \tan \theta = \frac{1}{x}$$

6. Question

If $\operatorname{cosec} \theta - \cot \theta = \alpha$, write the value of $\operatorname{cosec} \theta + \cot \alpha$.

Answer

Given: cosec θ − cot θ = α ……(i)

To find: cosec θ + cot θ

We know that $1 + \cot^2 \theta = \text{cosec}^2 \theta$

$\Rightarrow 1 = \text{cosec}^2 \theta - \cot^2 \theta$

Now, ∵ $a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow 1 = \text{cosec}^2 \theta - \cot^2 \theta = (\text{cosec}\ \theta - \cot \theta)(\text{cosec}\ \theta + \cot \theta)$

⇒ From (i), we have

$1 = \alpha (\text{cosec}\ \theta + \cot \theta)$

$$\Rightarrow \text{cosec}\,\theta + \cot\theta = \frac{1}{\alpha}$$

7. Question

Write the value of $\text{cosec}^2 (90° - \theta) - \tan^2\theta$.

Answer

To find: $\text{cosec}^2 (90° - \theta) - \tan^2\theta$

∵ cosec (90° − θ) = sec θ

∴ $\text{cosec}^2 (90° - \theta) = \sec^2 \theta$

$\Rightarrow \text{cosec}^2 (90° \theta) - \tan^2 \theta = \sec^2 \theta - \tan^2 \theta$

Now, ∵ $1 + \tan^2 \theta = \sec^2 \theta$

∴ $\text{cosec}^2 (90° - \theta) - \tan^2 \theta = \sec^2 \theta - \tan^2 \theta$

$= 1 + \tan^2 \theta - \tan^2 \theta = 1$

8. Question

Write the value of sin A cos (90° − A) + cos A sin (90° − A).

Answer

To find: sin A cos (90° − A) + cos A sin (90° − A)

∵ cos (90° − A) = sin A and sin (90° − A) = cos A ……(i)

∴ sin A cos (90° − A) + cos A sin (90° − A)

= sin A sin A + cos A cos A [Using (i)]

$= \sin^2 A + \cos^2 A$

Now, ∵ $\sin^2 \theta + \cos^2 \theta = 1$

∴ sin A cos (90° − A) + cos A sin (90° − A)

$= \sin^2 A + \cos^2 A = 1$

9. Question

Write the value of $\cot^2 \theta - \frac{1}{\sin^2 \theta}$.

Answer

To find: $\cot^2 \theta - \frac{1}{\sin^2 \theta}$

∵ $\text{cosec}\ \theta = \frac{1}{\sin\theta}$

$\Rightarrow \text{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$\Rightarrow \cot^2\theta - \frac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta$

Also, we know that $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow \cot^2\theta - \text{cosec}^2\theta = -1$

$\Rightarrow \cot^2\theta - \frac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta = -1$

10. Question

If $x = a \sin\theta$ and $y = b \cos\theta$, what is the value of $b^2x^2 + a^2y^2$?

Answer

Given: $x = a \sin\theta$ and $y = b \cos\theta$

$\Rightarrow x^2 = a^2 \sin^2\theta$ and $y^2 = b^2 \cos^2\theta$ .....(i)

To find: $b^2x^2 + a^2y^2$

Consider $b^2x^2 + a^2y^2 = b^2 a^2 \sin^2\theta + a^2 b^2 \cos^2\theta$

$= a^2 b^2 (\sin^2\theta + \cos^2\theta)$

$= a^2 b^2 (1)$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= a^2 b^2$

11. Question

If $\sin\theta = \frac{4}{5}$, what is the value of cot θ + cosec θ?

Answer

Given: $\sin\theta = \frac{4}{5}$

To find: cot θ + cosec θ

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$

$\Rightarrow \cos\theta = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Now, as $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{3/5}{4/5} = \frac{3}{4}$

Also, $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{4/5} = \frac{5}{4}$

$\Rightarrow \cot\theta + \text{cosec}\,\theta = \frac{3}{4} + \frac{5}{4} = \frac{3 + 5}{4} = \frac{8}{4} = 2$

12. Question

What is the value of $9 \cot^2\theta - 9 \text{cosec}^2\theta$?

Answer

To find: $9 \cot^2\theta - 9 \text{cosec}^2\theta$

Consider 9 $\cot^2\theta - 9\,\text{cosec}^2\theta = 9(\cot^2\theta - \text{cosec}^2\theta)$

Now ∵ $1 + \cot^2\theta = \text{cosec}^2\theta$

⇒ $\cot^2\theta - \text{cosec}^2\theta = -1$

⇒ $9\cot^2\theta - 9\,\text{cosec}^2\theta = 9(\cot^2\theta - \text{cosec}^2\theta) = 9(-1) = -9$

13. Question

What is the value of $6\tan^2\theta - \frac{6}{\cos^2\theta}$?

Answer

To find: $6\tan^2\theta - \frac{6}{\cos^2\theta}$

∵ $\sec\theta = \frac{1}{\cos\theta}$

$$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$$

$$\Rightarrow 6\tan^2\theta - \frac{6}{\cos^2\theta} = 6\tan^2\theta - 6\sec^2\theta = 6(\tan^2\theta - \sec^2\theta)$$

Now, as $1 + \tan^2\theta = \sec^2\theta$

⇒ $\tan^2\theta - \sec^2\theta = -1$

$$\Rightarrow 6\tan^2\theta - \frac{6}{\cos^2\theta} = 6(\tan^2\theta - \sec^2\theta) = 6(-1) = -6$$

14. Question

What is the value of $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \text{cosec}^2\theta}$?

Answer

To find: $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \text{cosec}^2\theta}$

We know that $1 + \tan^2\theta = \sec^2\theta$

And $1 + \cot^2\theta = \text{cosec}^2\theta$

⇒ $\tan^2\theta - \sec^2\theta = -1$

And $\cot^2\theta - \text{cosec}^2\theta = -1$

$$\Rightarrow \frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \text{cosec}^2\theta} = \frac{-1}{-1} = 1$$

15. Question

What is the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$?

Answer

To find: $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

∵ $(a - b)(a + b) = a^2 - b^2$

∴ $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (1 + \tan^2\theta)(1 - \sin^2\theta)$

Now, as $\sin^2\theta + \cos^2\theta = 1$

⇒ $1 - \sin^2\theta = \cos^2\theta$ ……(i)

Also, we know that $1 + \tan^2 \theta = \sec^2 \theta$ .....(ii)

Using (i) and (ii), we have

$(1 + \tan^2\theta)(1 - \sin \theta)(1 + \sin \theta)$

$= (1 + \tan^2 \theta)(1 - \sin^2 \theta)$

$= \sec^2 \theta \cos^2 \theta$

$\because \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$

$\Rightarrow (1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$

$= \sec^2 \theta \cos^2 \theta$

$= \frac{1}{\cos^2\theta}\cos^2\theta = 1$

16. Question

If $\cos A = \frac{7}{25}$, find the value of tan A + cot A.

Answer

Given: $\cos A = \frac{7}{25}$

To find: tan A + cot A

$\because \sin^2 A + \cos^2 A = 1$

$\Rightarrow \sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin A = \sqrt{1 - \cos^2 A}$

$$= \sqrt{1 - \left(\frac{7}{25}\right)^2} = \sqrt{1 - \frac{49}{625}} = \sqrt{\frac{625 - 49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

Now, as $\tan A = \frac{\sin A}{\cos A} = \frac{24/25}{7/25} = \frac{24}{7}$

And $\cot A = \frac{1}{\tan A} = \frac{7}{24}$

$$\Rightarrow \tan A + \cot A = \frac{24}{7} + \frac{7}{24} = \frac{576 + 49}{168} = \frac{625}{168}$$

17. Question

If $\sin \theta = \frac{1}{3}$, then find the value of $2 \cot^2\theta + 2$.

Answer

Given: $\sin \theta = \frac{1}{3}$

To find: The value of $2 \cot^2 \theta + 2$.

Solution: $\sin \theta = \frac{1}{3}$

$\because \operatorname{cosec} \theta = \frac{1}{\sin\theta} = \frac{1}{1/3} = 3$

$\Rightarrow \operatorname{cosec}^2 \theta = 3^2 = 9$

Also, $1 + \cot^2 \theta = \text{cosec}^2 \theta$

$\Rightarrow \cot^2 \theta = \text{cosec}^2 \theta - 1 = 9 - 1 = 8$

$\Rightarrow 2 \cot^2 \theta + 2 = 2 (8) + 2 = 16 + 2 = 18$Hence, the value of $2 \cot^2 \theta + 2$ is 18.

18. Question

If $\cos\theta = \frac{3}{4}$, then find the value of $9 \tan^2\theta + 9$.

Answer

Given: $\cos\theta = \frac{3}{4}$

To find: $9 \tan^2 \theta + 9$

$\because \sec\theta = \frac{1}{\cos\theta} = \frac{4}{3}$

$\therefore \sec^2\theta = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

Also, we know that $1 + \tan^2 \theta = \sec^2 \theta$

$$\Rightarrow \tan^2\theta = \sec^2\theta - 1 = \frac{16}{9} - 1 = \frac{16-9}{9} = \frac{7}{9}$$

$$\Rightarrow 9\tan^2\theta + 9 = 9\left(\frac{7}{9}\right) + 9 = 7 + 9 = 16$$

19. Question

If $\sec^2\theta (1 + \sin \theta) (1 - \sin \theta) = k$, then find the value of k.

Answer

Given: $\sec^2\theta (1 + \sin \theta) (1 - \sin \theta) = k$

To find: k

Consider $\sec^2\theta (1 + \sin \theta) (1 - \sin \theta)$

$\because (a - b) (a + b) = a^2 - b^2$

$\therefore \sec^2\theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$

Now, as $\sin^2 \theta + \cos^2 \theta = 1$

$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$

$\Rightarrow \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$

$= \sec^2 \theta \cos^2 \theta$

Now, $\because \sec\theta = \frac{1}{\cos\theta}$

$$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$$

$\Rightarrow \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$

$= \sec^2 \theta \cos^2 \theta$

$$= \frac{1}{\cos^2\theta}\cos^2\theta = 1$$

$\Rightarrow k = 1$

20. Question

If $\text{cosec}^2\theta (1 + \cos \theta) (1 - \cos \theta) = \lambda$, then find the value of λ.

Answer

Given: $\text{cosec}^2\theta\ (1 + \cos\theta)\ (1 - \cos\theta) = \lambda$

To find: λ

Consider $\text{cosec}^2\theta\ (1 + \cos\theta)\ (1 - \cos\theta)$

$\because (a - b)(a + b) = a^2 - b^2$

$\therefore \text{cosec}^2\theta\ (1 + \cos\theta)\ (1 - \cos\theta) = \text{cosec}^2\ \theta\ (1 - \cos^2\theta)$

Now, as $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \text{cosec}^2\theta\ (1 + \cos\theta)\ (1 - \cos\theta) = \text{cosec}^2\ \theta\ (1 - \cos^2\theta)$

$= \text{cosec}^2\ \theta \sin^2\theta$

Now, $\because \text{cosec}\,\theta = \frac{1}{\sin\theta}$

$$\Rightarrow \text{cosec}^2\theta = \frac{1}{\sin^2\theta}$$

$\Rightarrow \text{cosec}^2\ \theta\ (1 + \cos\theta)\ (1 - \cos\theta) = \text{cosec}^2\ \theta\ (1 - \cos^2\theta)$

$= \text{cosec}^2\ \theta \sin^2\theta$

$$= \frac{1}{\sin^2\theta}\sin^2\theta = 1$$

21. Question

If $\sin^2\theta \cos^2\theta\ (1 + \tan^2\theta)\ (1 + \cot^2\theta) = \lambda$, then find the value of λ.

Answer

Given: $\sin^2\theta \cos^2\theta\ (1 + \tan^2\theta)\ (1 + \cot^2\theta) = \lambda$

To find: λ

We know that $1 + \tan^2\ \theta = \sec^2\ \theta$

And $1 + \cot^2\ \theta = \text{cosec}^2\ \theta$

$\Rightarrow \sin^2\theta \cos^2\theta\ (1 + \tan^2\theta)\ (1 + \cot^2\theta)$

$= \sin^2\ \theta \cos^2\ \theta \sec^2\ \theta\ \text{cosec}^2\ \theta$

Now, $\because \text{cosec}\,\theta = \frac{1}{\sin\theta}$

$$\Rightarrow \text{cosec}^2\theta = \frac{1}{\sin^2\theta}$$

And $\because \sec\theta = \frac{1}{\cos\theta}$

$$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$$

$\Rightarrow \sin^2\ \theta \cos^2\ \theta\ (1 + \tan^2\ \theta)\ (1 + \cot^2\ \theta)$

$= \sin^2\ \theta \cos^2\ \theta \sec^2\ \theta\ \text{cosec}^2\ \theta$

$$= \sin^2\theta \cos^2\theta\ \frac{1}{\cos^2\theta}\ \frac{1}{\sin^2\theta} = 1$$

$\Rightarrow \lambda = 1$

22. Question

If 5x = sec θ and $\frac{5}{x} = \tan\theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

Answer

Given: 5x = sec θ

$\Rightarrow x = \frac{\sec\theta}{5}$

$\Rightarrow x^2 = \frac{\sec^2\theta}{25}$ .....(i)

And $\frac{5}{x} = \tan\theta$

$\Rightarrow x = \frac{5}{\tan\theta}$

$\Rightarrow x^2 = \frac{25}{\tan^2\theta}$

$\Rightarrow \frac{1}{x^2} = \frac{\tan^2\theta}{25}$ .....(ii)

To find: $5\left(x^2 - \frac{1}{x^2}\right)$

Consider $5\left(x^2 - \frac{1}{x^2}\right) = 5\left(\frac{\sec^2\theta}{25} - \frac{1}{x^2}\right)$ [Using (i)]

$= 5\left(\frac{\sec^2\theta}{25} - \frac{\tan^2\theta}{25}\right)$ [Using (ii)]

$= 5\left(\frac{\sec^2\theta - \tan^2\theta}{25}\right) = \frac{1}{5}(\sec^2\theta - \tan^2\theta)$

Now, as $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow 1 = \sec^2\theta - \tan^2\theta$

$\Rightarrow 5\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{5}(\sec^2\theta - \tan^2\theta) = \frac{1}{5}$

23. Question

If cosec θ = 2x and $\cot\theta = \frac{2}{x}$, find the value of $2\left(x^2 - \frac{1}{x^2}\right)$

Answer

Given: cosec θ = 2x

$\Rightarrow x = \frac{\text{cosec}\,\theta}{2}$

$\Rightarrow x^2 = \frac{\text{cosec}^2\theta}{4}$ .....(i)

And $\cot\theta = \frac{2}{x}$

$\Rightarrow x = \frac{2}{\cot\theta}$

$\Rightarrow x^2 = \frac{4}{\cot^2\theta}$

$\Rightarrow \frac{1}{x^2} = \frac{\cot^2\theta}{4}$ .....(ii)

To find: $2\left(x^2 - \frac{1}{x^2}\right)$

Consider $2\left(x^2-\frac{1}{x^2}\right)=2\left(\frac{\text{cosec}^2\theta}{4}-\frac{1}{x^2}\right)$ [Using (i)]

$=2\left(\frac{\text{cosec}^2\theta}{4}-\frac{\cot^2\theta}{4}\right)$ [Using (ii)]

$=2\left(\frac{\text{cosec}^2\theta-\cot^2\theta}{4}\right)=\frac{1}{2}\left(\text{cosec}^2\theta-\cot^2\theta\right)$

Now, as $1+\cot^2\theta=\text{cosec}^2\theta$

$\Rightarrow 1=\text{cosec}^2\theta-\cot^2\theta$

$\Rightarrow 2\left(x^2-\frac{1}{x^2}\right)=\frac{1}{2}\left(\text{cosec}^2\theta-\cot^2\theta\right)=\frac{1}{2}$

1. Question

If $\sec\theta+\tan\theta=x$, then $\sec\theta=$

A. $\frac{x^2+1}{x}$

B. $\frac{x^2+1}{2x}$

C. $\frac{x^2-1}{2x}$

D. $\frac{x^2-1}{x}$

Answer

Given: $\sec\theta+\tan\theta=x$ ……(i)

To find: $\sec\theta$

We know that $1+\tan^2\theta=\sec^2\theta$

$\Rightarrow \sec^2\theta-\tan^2\theta=1$

$\because a^2-b^2=(a-b)(a+b)$

$\therefore \sec^2\theta-\tan^2\theta=(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)=1$

$\Rightarrow$ From (i), we have

$\Rightarrow (\sec\theta-\tan\theta)x=1$

$\Rightarrow \sec\theta-\tan\theta=\frac{1}{x}$ ……(ii)

Adding (i) and (ii), we get

$\sec\theta+\sec\theta=x+\frac{1}{x}$

$\Rightarrow \sec\theta=\frac{x^2+1}{2x}$

$\Rightarrow \sec\theta=\frac{x^2+1}{2x}$

2. Question

If $\sec\theta+\tan\theta=x$, then $\tan\theta=$

A. $\frac{x^2+1}{x}$

B. $\frac{x^2-1}{x}$

C. $\frac{x^2+1}{2x}$

D. $\frac{x^2-1}{2x}$

Answer

Given: sec θ + tan θ = x ……(i)

To find: tan θ

We know that $1 + \tan^2 \theta = \sec^2 \theta$

$\Rightarrow \sec^2 \theta - \tan^2 \theta = 1$

$\because a^2 - b^2 = (a - b)(a + b)$

$\therefore \sec^2 \theta - \tan^2 \theta = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$

⇒ From (i), we have

$\Rightarrow (\sec \theta - \tan \theta) x = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x}$ ……(ii)

Subtracting (ii) from (i), we get

$$\tan\theta + \tan\theta = x - \frac{1}{x}$$

$$\Rightarrow 2\tan\theta = \frac{x^2-1}{x}$$

$$\Rightarrow \tan\theta = \frac{x^2-1}{2x}$$

3. Question

$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

A. sec θ + tan θ

B. sec θ − tan θ

C. $\sec^2\theta + \tan^2\theta$

D. $\sec^2\theta - \tan^2\theta$

Answer

Note: Since all the options involve the trigonometric ratios sec θ and tan θ, so we divide the whole term (numerator as well as denominator) by cos θ.

To find: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

Consider $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

Dividing numerator and denominator by cos θ, we get

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}} = \sqrt{\frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}} \left[\begin{matrix} \because \sec\theta = \frac{1}{\cos\theta} \\ \text{and } \tan\theta = \frac{\sin\theta}{\cos\theta} \end{matrix}\right]$$

Rationalizing the term by multiplying it by $\sqrt{\sec\theta+\tan\theta}$,

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}} = \sqrt{\frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}} = \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}}$$

$$= \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}} \times \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}}$$

$$= \sqrt{\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}}$$

Now, as $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}} = \sqrt{(\sec\theta+\tan\theta)^2} = \sec\theta+\tan\theta$$

4. Question

The value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is

A. $\cot\theta - \operatorname{cosec}\theta$

B. $\operatorname{cosec}\theta + \cot\theta$

C. $\operatorname{cosec}^2\theta + \cot^2\theta$

D. $(\cot\theta + \operatorname{cosec}\theta)^2$

Answer

Note: Since all the options involve the trigonometric ratios cosec θ and cot θ, so we divide the whole term (numerator as well as denominator) by sin θ.

To find: $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

Consider $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

Dividing numerator and denominator by sin θ, we get

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}} = \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}} \left[\begin{matrix} \because \operatorname{cosec}\theta = \frac{1}{\sin\theta} \\ \text{and } \cot\theta = \frac{\cos\theta}{\sin\theta} \end{matrix}\right]$$

Rationalizing the term by multiplying it by $\sqrt{\operatorname{cosec}\theta+\cot\theta}$,

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}} = \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}}$$

$$= \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}} \times \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta+\cot\theta}}$$

$$= \sqrt{\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{\operatorname{cosec}^2\theta-\cot^2\theta}}$$

Now, as $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$\Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{\operatorname{cosec}^2\theta-\cot^2\theta}} = \sqrt{(\operatorname{cosec}\theta+\cot\theta)^2}$$

$$= \operatorname{cosec}\theta + \cot\theta$$

5. Question

$\sec^4 A - \sec^2 A$ is equal to

A. $\tan^2 A - \tan^4 A$

B. $\tan^4 A - \tan^2 A$

C. $\tan^4 A + \tan^2 A$

D. $\tan^2 A + \tan^4 A$

Answer

Note: Since all the options involve the trigonometric ratio $\tan \theta$, so we use the identity $1 + \tan^2 \theta = \sec^2 \theta$.

To find: $\sec^4 A - \sec^2 A$

Consider $\sec^4 A - \sec^2 A = (\sec^2 A)^2 - \sec^2 A$

Now, as $\sec^2 A = 1 + \tan^2 A$

$\Rightarrow \sec^4 A - \sec^2 A = (\sec^2 A)^2 - \sec^2 A$

$= (1 + \tan^2 A)^2 - (1 + \tan^2 A)$

$= 1 + \tan^4 A + 2 \tan^2 A - 1 - \tan^2 A$

$= \tan^4 A + \tan^2 A$

6. Question

$\cos^4 A - \sin^4 A$ is equal to

A. $2 \cos^2 A + 1$

B. $2 \cos^2 A - 1$

C. $2 \sin^2 A - 1$

D. $2 \sin^2 A + 1$

Answer

To find: $\cos^4 A - \sin^4 A$

Consider $\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$

$\because a^2 - b^2 = (a - b)(a + b)$

∴ $\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A)$ [∵ $\cos^2 A + \sin^2 A = 1$]

$= \cos^2 A - (1 - \cos^2 A)$ [∵ $\sin^2 A = 1 - \cos^2 A$]

$= \cos^2 A - 1 + \cos^2 A = 2\cos^2 A - 1$

7. Question

$\frac{\sin\theta}{1+\cos\theta}$ is equal to

A. $\frac{1+\cos\theta}{\sin\theta}$

B. $\frac{1-\cos\theta}{\cos\theta}$

C. $\frac{1-\cos\theta}{\sin\theta}$

D. $\frac{1-\sin\theta}{\cos\theta}$

Answer

To find: $\frac{\sin\theta}{1+\cos\theta}$

Consider $\frac{\sin\theta}{1+\cos\theta}$

Rationalizing the above fraction by $(1 - \cos\theta)$,

$$\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$= \frac{\sin\theta(1-\cos\theta)}{(1-\cos^2\theta)}$ [∵ $(a - b)(a + b) = a^2 - b^2$]

∵ $\sin^2\theta + \cos^2\theta = 1$

⇒ $\sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow \frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{(1-\cos^2\theta)} = \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}$$

8. Question

$\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$ is equal to

A. 0

B. 1

C. $\sin\theta + \cos\theta$

D. $\sin\theta - \cos\theta$

Answer

Given: $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$

To find: The value of $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}$

Solution:

Use:

$$\tan\theta=\frac{sin\theta}{\cos\theta},\cot\theta=\frac{cos\theta}{sin\theta}$$

So,

$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$$

Using the identity,

$a^2 - b^2 = (a - b)(a + b)$

$$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}=\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$$
$$=\sin\theta+\cos\theta$$

9. Question

The value of $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$ is

A. 1

B. 2

C. 4

D. 0

Answer

To find: $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

Consider $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$$=\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$\left[\because\cot\theta=\frac{\cos\theta}{\sin\theta},\operatorname{cosec}\theta=\frac{1}{\sin\theta},\tan\theta=\frac{\sin\theta}{\cos\theta},\sec\theta=\frac{1}{\cos\theta}\right]$$

$$=\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$

$$=\left(\frac{(\sin\theta+\cos\theta)-1}{\sin\theta}\right)\left(\frac{(\sin\theta+\cos\theta)+1}{\cos\theta}\right)$$

$=\frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta}$ [$\because (a - b)(a + b) = a^2 - b^2$]

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$=\frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

10. Question

$\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$ is equal to

A. 2 tan θ

B. 2 sec θ

C. 2 cosec θ

D. 2 tan θ sec θ

Answer

To find: $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$

Consider $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$

$\because \tan\theta=\frac{\sin\theta}{\cos\theta}, \sec\theta=\frac{1}{\cos\theta}$

$\therefore \frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-1}+\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+1}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}+\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$=\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}$

$=\frac{\sin\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$=\frac{\sin\theta+\sin\theta\cos\theta+\sin\theta-\sin\theta\cos\theta}{(1-\cos^2\theta)}$

$=\frac{2\sin\theta}{\sin^2\theta}$ [$\because \sin^2\theta = 1 - \cos^2\theta$]

$=\frac{2}{\sin\theta}$

$=2\operatorname{cosec}\theta\left[\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}\right]$

11. Question

(cosec θ − sin θ) (sec θ − cos θ) (tan θ + cot θ) is equal

A. 0

B. 1

C. −1

D. None of these

Answer

To find: (cosec θ − sin θ) (sec θ − cos θ) (tan θ + cot θ)

$\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}, \sec\theta=\frac{1}{\cos\theta}, \tan\theta=\frac{\sin\theta}{\cos\theta}, \cot\theta=\frac{\cos\theta}{\sin\theta}$

∴ (cosec θ − sin θ) (sec θ − cos θ) (tan θ + cot θ)

$=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$

$=\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$

Now, as $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

And $\cos^2\theta = 1 - \sin^2\theta$

$$\Rightarrow \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$
$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta\cos\theta}\right) = 1$$

Hence the answer is 'B'

12. Question

If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 =$

A. a^2b^2

B. ab

C. a^4b^4

D. $a^2 + b^2$

Answer

Given: $x = a\sin\theta$ and $y = b\cos\theta$

$\Rightarrow x^2 = a^2\sin^2\theta$ and $y^2 = b^2\cos^2\theta$ .....(i)

To find: $b^2x^2 + a^2y^2$

Consider $b^2x^2 + a^2y^2 = b^2a^2\sin^2\theta + a^2b^2\cos^2\theta$

$= a^2b^2(\sin^2\theta + \cos^2\theta)$

$= a^2b^2(1)$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= a^2b^2$

13. Question

If $x = a\sec\theta$ and $y = b\tan\theta$, then $b^2x^2 - a^2y^2 =$

A. ab

B. $a^2 - b^2$

C. $a^2 + b^2$

D. a^2b^2

Answer

Given: $x = a\sec\theta$ and $y = b\tan\theta$

$\Rightarrow x^2 = a^2\sec^2\theta$ and $y^2 = b^2\tan^2\theta$ .....(i)

To find: $b^2x^2 - a^2y^2$

Consider $b^2x^2 - a^2y^2 = b^2a^2\sec^2\theta - a^2b^2\tan^2\theta$

$= a^2b^2(\sec^2\theta - \tan^2\theta)$

$= a^2b^2(1)$ [$\because \sec^2\theta - \tan^2\theta = 1$]

$= a^2b^2$

14. Question

$$\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}$$ is equal to

A. 0

B. 1

C. −1

D. 2

Answer

To find: $\frac{\cot\theta}{\cot\theta-\cot3\theta}+\frac{\tan\theta}{\tan\theta-\tan3\theta}$

Consider $\frac{\cot\theta}{\cot\theta-\cot3\theta}+\frac{\tan\theta}{\tan\theta-\tan3\theta}$

$$=\frac{\cot\theta(\tan\theta-\tan3\theta)\ +\ \tan\theta\,(\cot\theta-\cot3\theta)}{(\cot\theta-\cot3\theta)(\tan\theta-\tan3\theta)}$$

$$=\frac{\cot\theta\tan\theta-\cot\theta\tan3\theta\ +\ \tan\theta\cot\theta-\tan\theta\cot3\theta}{\cot\theta\tan\theta-\cot\theta\tan3\theta-\cot3\theta\tan\theta\ +\ \cot3\theta\tan3\theta}$$

$$=\frac{1-\cot\theta\tan3\theta\ +\ 1-\tan\theta\cot3\theta}{1-\cot\theta\tan3\theta-\cot3\theta\tan\theta\ +\ 1}\left[\because\cot\theta=\frac{1}{\tan\theta}\Rightarrow\cot\theta\tan\theta=1\right]$$

$$=\frac{2-\cot\theta\tan3\theta-\tan\theta\cot3\theta}{2-\cot\theta\tan3\theta-\tan\theta\cot3\theta}=1$$

15. Question

$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)$ is equal to

A. 0

B. 1

C. −1

D. None of these

Answer

To find: $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)$

First, we consider

$\sin^6\theta+\cos^6\theta=(\sin^2\theta)^3+(\cos^2\theta)^3$

Now, as $(a+b)^3=a^3+b^3+3a^2b+3ab^2$

$\Rightarrow a^3+b^3=(a+b)^3-3a^2b-3ab^2$

$\Rightarrow \sin^6\theta+\cos^6\theta$

$=(\sin^2\theta)^3+(\cos^2\theta)^3$

$=(\sin^2\theta+\cos^2\theta)^3-3(\sin^2\theta)^2\cos^2\theta-3\sin^2\theta(\cos^2\theta)^2$

$=1-3\sin^4\theta\cos^2\theta-3\sin^2\theta\cos^4\theta\ [\because \sin^2\theta+\cos^2\theta=1]$

$=1-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$=1-3\sin^2\theta\cos^2\theta\ [\because \sin^2\theta+\cos^2\theta=1]$ .....(i)

Next, we consider

$\sin^4\theta+\cos^4\theta=(\sin^2\theta)^2+(\cos^2\theta)^2$

Now, as $(a+b)^2=a^2+b^2+2ab$

$\Rightarrow a^2+b^2=(a+b)^2-2ab$

$\Rightarrow\sin^4\theta+\cos^4\theta$

$=(\sin^2\theta)^2+(\cos^2\theta)^2$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$] ……(ii)

Now, using (i) and (ii), we have

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$

$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta)$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta$

$= 2 - 3 = -1$

16. Question

If $a\cos\theta + b\sin\theta$ and $a\sin\theta - b\cos\theta = 3$, then $a^2 + b^2 =$

A. 7

B. 12

C. 25

D. None of these

Answer

Given: $a\cos\theta + b\sin\theta = 4$

Squaring both sides, we get

$(a\cos\theta + b\sin\theta)^2 = 4^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = 16$ ……(i)

and $a\sin\theta - b\cos\theta = 3$

Squaring both sides, we get

$(a\sin\theta - b\cos\theta)^2 = 3^2$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = 9$ ……(ii)

To find: $a^2 + b^2$

Adding (i) and (ii), we get

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta$

$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = 16 + 9$

$\Rightarrow a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = 25$

$\Rightarrow a^2 + b^2 = 25$ [$\because \sin^2\theta + \cos^2\theta = 1$]

17. Question

If $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$, then $p^2 - q^2 =$

A. $a^2 - b^2$

B. $b^2 - a^2$

C. $a^2 + b^2$

D. $b - a$

Answer

Given: $a\cot\theta + b\operatorname{cosec}\theta = p$

Squaring both sides, we get

$(a\cot\theta + b\operatorname{cosec}\theta)^2 = p^2$

$\Rightarrow a^2 \cot^2 \theta + b^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta = p^2$ ……(i)

and $b \cot \theta + a \operatorname{cosec} \theta = q$

Squaring both sides, we get

$(b \cot \theta + a \operatorname{cosec} \theta)^2 = q^2$

$\Rightarrow b^2 \cot^2 \theta + a^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta = q^2$ ……(ii)

To find: $p^2 - q^2$

Subtracting (ii) from (i), we get

$a^2 \cot^2 \theta + b^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta - b^2 \cot^2 \theta - a^2 \operatorname{cosec}^2 \theta - 2ab \cot \theta \operatorname{cosec} \theta = p^2 - q^2$

$\Rightarrow P^2 - q^2 = a^2 (\cot^2 \theta - \operatorname{cosec}^2 \theta) + b^2 (\operatorname{cosec}^2 \theta - \cot^2 \theta)$

$= a^2 (- 1) + b^2 (1)$ [$\because 1 = \operatorname{cosec}^2 \theta - \cot^2 \theta$]

$= b^2 - a^2$

18. Question

The value of $\sin^2 29° + \sin^2 61°$ is

A. 1

B. 0

C. $2 \sin^2 29°$

D. $2 \cos^2 61$

Answer

To find: $\sin^2 29° + \sin^2 61°$

Consider $\sin^2 29° + \sin^2 61°$

$\because 29 = 90 - 61$

$\therefore \sin^2 29° + \sin^2 61° = \sin^2 (90° - 61°) + \sin^2 61°$

Now, as $\sin (90° - \theta) = \cos \theta$

$\Rightarrow \sin^2 29° + \sin^2 61° = \sin^2 (90° - 61°) + \sin^2 61°$

$= \cos^2 61° + \sin^2 61°$

$= 1$ [$\sin^2 \theta + \cos^2 \theta = 1$]

19. Question

If $x = r \sin \theta \cos \varphi$, $y = r \sin \theta \sin \varphi$ and $z = r \cos \theta$, then

A. $x^2 + y^2 + z^2 = r^2$

B. $x^2 + y^2 - z^2 = r^2$

C. $x^2 - y^2 + z^2 = r^2$

D. $z^2 + y^2 - x_2 = r^2$

Answer

Given: $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \varphi$ and $z = r \cos \theta$,

Solution:$x = r \sin \theta \cos \phi$

Squaring both sides, we get

$x^2 = r^2 \sin^2 \theta \cos^2 \phi$ …….(i)

and $y = r \sin \theta \sin \phi$

Squaring both sides, we get

$\Rightarrow y^2 = r^2 \sin^2\theta \sin^2\phi$ …….(ii)

z = r cos θSquaring both sides, we get

$\Rightarrow z^2 = r^2 \cos^2\theta$ …….(iii)

Adding (i), (ii) and (iii), we get

$x^2 + y^2 + z^2 = r^2 \sin^2\theta \cos^2\phi + r^2 \sin^2\theta \sin^2\phi + r^2 \cos^2\theta$

$= r^2 (\sin^2\theta \cos^2\phi + \sin^2\theta \sin^2\phi + \cos^2\theta)$

$= r^2 [\sin^2\theta (\cos^2\phi + \sin^2\phi) + \cos^2\theta]$

$\because \sin^2\theta + \cos^2\theta = 1$

$= r^2 [\sin^2\theta + \cos^2\theta]$

Again apply the identity $\sin^2\theta + \cos^2\theta = 1$

$= r^2$ Hence $x^2 + y^2 + z^2 = r^2$

20. Question

If $\sin\theta + \sin^2 = 1$, then $\cos^2\theta + \cos^4\theta =$

A. −1

B. 1

C. 0

D. None of these

Answer

Given: $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta = \cos^2\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$]……(i)

$\Rightarrow \text{Sin}^2\theta = (\cos^2\theta)^2 = \cos^4\theta$ ……(ii)

To find: $\cos^2\theta + \cos^4\theta$

Consider $\cos^2\theta + \cos^4\theta = \sin\theta + \sin^2\theta$ [Using (i) and (ii)]

= 1 [Given]

21. Question

If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then $a^2 + b^2 =$

A. $m^2 - n^2$

B. m^2n^2

C. $n^2 - m^2$

D. $m^2 + n^2$

Answer

Given: $a\cos\theta + b\sin\theta = m$

Squaring both sides, we get

$(a\cos\theta + b\sin\theta)^2 = m^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = m^2$ ……(i)

And $a\sin\theta - b\cos\theta = n$

Squaring both sides, we get

$(a \sin \theta - b \cos \theta)^2 = n^2$

$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = n^2$ ……(ii)

To find: $a^2 + b^2$

Adding (i) and (ii), we get

$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = m^2 + n^2$

$\Rightarrow a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$

$\Rightarrow a^2 + b^2 = m^2 + n^2$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

22. Question

If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$

A. −1

B. 0

C. 1

D. None of these

Answer

Given: $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$ [$\because \sin^2 A + \cos^2 A = 1$]……(i)

Squaring both sides, we get

$\Rightarrow \cos^2 A = (\sin^2 A)^2 = \sin^4 A$ ……(ii)

To find: $\sin^2 A + \sin^4 A$

Consider $\sin^2 A + \sin^4 A = \cos A + \cos^2 A$ [From (i) and (ii)]

= 1

23. Question

If $x = a \sec \theta \cos \varphi$, $y = b \sec \theta \sin \varphi$ and $z = c \tan \theta$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

A. $\frac{z^2}{c^2}$

B. $1 - \frac{z^2}{c^2}$

C. $\frac{z^2}{c^2} - 1$

D. $1 + \frac{z^2}{c^2}$

Answer

Given: x a $\sec \theta \cos \phi$

Squaring both sides, we get

$x^2 = a^2 \sec^2 \theta \cos^2 \phi$

and $y = b \sec \theta \sin \phi$

Squaring both sides, we get

$y^2 = b^2 \sec^2 \theta \sin^2\phi$

And $z = c \tan \theta$

$\Rightarrow z^2 = c^2 \tan^2 \theta$

$\Rightarrow \tan^2 \theta = \frac{z^2}{c^2}$ .....(i)

To find: $\frac{x^2}{a^2} + \frac{y^2}{b^2}$

Consider $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2 \sec^2 \theta \cos^2 \phi}{a^2} + \frac{b^2 \sec^2 \theta \sin^2 \phi}{b^2}$

$= \sec^2 \theta \cos^2\phi + \sec^2 \theta \sin^2\phi$

$= \sec^2 \theta (\cos^2\phi + \sin^2\phi)$

$= \sec^2 \theta$ [$\because \sin^2\phi + \cos^2\phi = 1$]

$= 1 + \tan^2 \theta$ [$\because 1 + \tan^2 \theta = \sec^2 \theta$]

$= 1 + \frac{z^2}{c^2}$

24. Question

If $a \cos \theta - b \sin \theta = c$, then $a \sin \theta + b \cos \theta =$

A. $\pm\sqrt{a^2 + b^2 + c^2}$

B. $\pm\sqrt{a^2 + b^2 - c^2}$

C. $\pm\sqrt{c^2 - a^2 + b^2}$

D. None of these

Answer

Given: $a \cos \theta - b \sin \theta = c$

To find: $a \sin \theta + b \cos \theta$

Consider $a \cos \theta - b \sin \theta = c$

Squaring both sides, we get

$(a \cos \theta - b \sin \theta)^2 = c^2$

$\because (a - b)^2 = a^2 + b^2 - 2ab$

$\therefore a \cos \theta - b \sin \theta = c$

$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin\theta \cos \theta = c^2$ .....(i)

Now, $\because \sin^2 \theta + \cos^2 \theta = 1$

$\therefore \sin^2 \theta = 1 - \cos^2 \theta$ and $\cos^2 \theta = 1 - \sin^2 \theta$

$\Rightarrow$ From (i), we have

$\Rightarrow a^2 (1 - \sin^2 \theta) + b^2 (1 - \cos^2 \theta) - 2ab \sin \theta \cos \theta = c^2$

$\Rightarrow a^2 - a^2 \sin^2 \theta + b^2 - b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = c^2$

$\Rightarrow a^2 + b^2 - (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2$

$\Rightarrow - (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2 - a^2 - b^2$

$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = a^2 + b^2 - c^2$

⇒ $(a \sin \theta)^2 + (b \cos \theta)^2 + 2 (a \sin \theta) (b \cos \theta) = a^2 + b^2 - c^2$

⇒ $(a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$

⇒ $a \sin \theta + b \cos \theta = \pm\sqrt{a^2 + b^2 - c^2}$

25. Question

$9 \sec^2 A - 9 \tan^2 A$ is equal to

A. 1

B. 9

C. 8

D. 0

Answer

To find: $9 \sec^2 A - 9 \tan^2 A$

Consider $9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$\because 1 + \tan^2 A = \sec^2 A$

$\therefore 9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$= 9 (1 + \tan^2 A - \tan^2 A) = 9$

26. Question

$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta) =$

A. 0

B. 1

C. 1

D. −1

Answer

To find: $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

Consider $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right)\left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$\left[\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta}, \sec \theta = \frac{1}{\cos \theta}\right]$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right)\left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

$$= \left(\frac{(\sin \theta + \cos \theta) + 1}{\cos \theta}\right)\left(\frac{(\sin \theta + \cos \theta) - 1}{\sin \theta}\right)$$

$= \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta}$ [$\because (a - b) (a + b) = a^2 - b^2$]

$$= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

27. Question

$(\sec A + \tan A) (1 - \sin A) =$

A. sec A

B. sin A

C. cosec A

D. cos A

Answer

To find: (sec A + tan A) (1 − sin A)

Consider (sec A + tan A) (1 − sin A)

We know that $\sec A = \frac{1}{\cos A}$ and $\tan A = \frac{\sin A}{\cos A}$

$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$

∵ (a + b) (a − b) = $a^2 - b^2$

$\therefore (\sec A + \tan A)(1 - \sin A) = \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A}$

Also, $\sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A$

$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$

28. Question

$\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is equal to

A. $\sec^2 A$

B. −1

C. $\cot^2 A$

D. $\tan^2 A$

Answer

To find: $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

Consider $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

∵ $1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \text{cosec}^2 A$

$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A}$ $\left[\because \sec A = \frac{1}{\cos A} \text{ and } \text{cosec } A = \frac{1}{\sin A}\right]$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$ $\left[\because \tan A = \frac{\sin A}{\cos A}\right]$

12. Some Application of Trigonometry

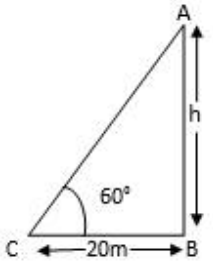
Exercise 12.1

1. Question

A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top the tower is 60°. What is the height of the tower?

Answer

Let the height of the tower = h (m)



In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h}{20}$

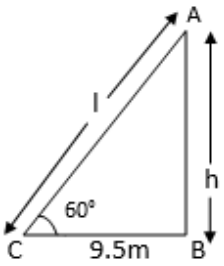
h = 20√3 m

Therefore height of the tower is 20√3 m

2. Question

The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

Answer



Let the length of the ladder=l (m)

In ΔABC,

$\cos 60° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{9.5}{l}$

l =9.5×2 ⇒ 19 m

Therefore length of Ladder is 19 m.

3. Question

A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 60° with the level of the ground. Determine the height of the wall.

Answer

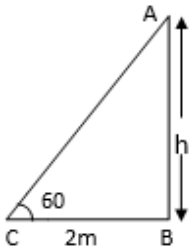
Let the length of the wall = h (m)

In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h}{2}$

h = 2√3 m

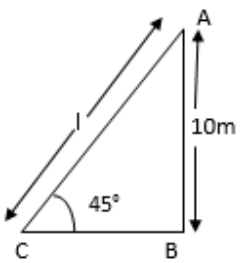


Therefore length of the wall is 2√3 m

4. Question

An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Answer



Let the length of the wire = l (m)

In ΔABC,

Sin 45° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{2}} = \frac{10}{l}$

l ⇒ 10√2 m

⇒ 10x1.41

⇒14.1 m

Therefore Length of wire is 14.1 m

5. Question

A kite is flying at a height of 75 metres from the ground level, attached to a string inclined at 60° to the horizontal. Find the length of the string to the nearest metre.

Answer

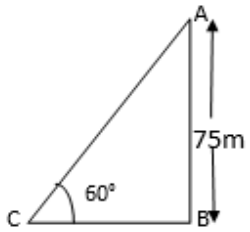
Let the length of the wire = l (m)

In ΔABC,

Sin 60° = $\frac{AB}{AC}$

$\frac{\sqrt{3}}{2} = \frac{75}{l}$

$\sqrt{3}l$ = 2×75



l = $\frac{150}{\sqrt{3}}$

l = $\frac{150\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{150\sqrt{3}}{3}$ ⇒ 50√3 ⇒ 86.6 m.

Therefore length of string is 86.6 m.

6. Question

The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle θ with the ground level such that tanθ =15/8, how high is the kite? Assume that there is no slack in the string.

Answer

Given: The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle θ with the ground level such that $\tan\theta = 15/8$.

To find: how high is the kite.

Solution: Let the height of string = h (m)

$\tan\theta = \frac{15}{8}$ (given)

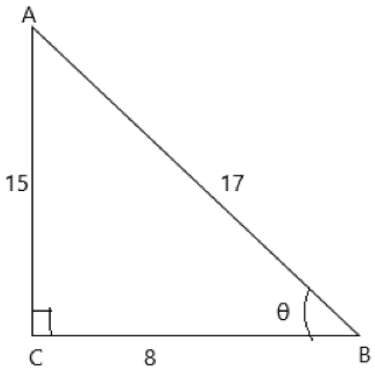
Since $\tan\theta$ = perpendicular/baseSo perpendicular= 15 and base=8So we construct a right triangle ABC right angled at C such that$\angle ABC=\theta$ and AC = Perpendicular = 15BC = base = 8By Pythagoras theorem,$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (15)^2 + (8)^2$

$\Rightarrow AB^2 = 225 + 64$

$\Rightarrow AB^2 = 289$

$\Rightarrow AB = \sqrt{289} \Rightarrow AB = 17$

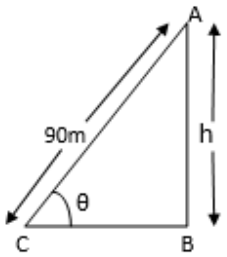


Since Sin θ = perpendicular/hypotenuse

$\Rightarrow \text{Sin}\,\theta = \frac{15}{17}$ ..... (1)

In ΔABC,

$\text{Sin}\,\theta = \frac{AB}{AC}$ $\text{Sin}\,\theta = \frac{h}{90}$ ..... (2)



Equating (1) and (2) we get,

$\frac{15}{17} = \frac{h}{90}$ $17h = 90\times15$

$\Rightarrow h = \frac{90 \times 15}{17} \Rightarrow h = 79.41$ m.

Therefore length of string is 79.41 m.

7. Question

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of elevation of the top and the bottom of the flag-staff are respectively 60° and 45°. Find the height of the flag-staff and that of the tower.

Answer

Let the height of tower = h (m)

Let the height of the flag-staff = t (m)

In ΔDBC,

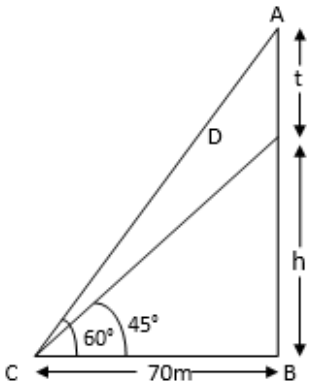
$\tan 45° = \frac{DB}{BC}$

$1 = \frac{h}{70}$

h = 70 m

Therefore height of tower = 70m.

Now in ΔABC,



$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h+t}{70} \Rightarrow \sqrt{3} = \frac{70+t}{70}$ (on substituting value of h =70)

$70+t = 70\sqrt{3}$

$t = 70\sqrt{3}-70$

$t = 70 (\sqrt{3} -1)$

$t = 70 \times (1.732-1)$

$t = 70 \times 0.732 \Rightarrow 51.24$ m.

Therefore height of the flag- staff is 51.24 m.

8. Question

A vertically straight tree, 15m height, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

Answer

Total height of the tree is 15 m.

I.e AB= 15m.

Let height at which tree is broken is h (m)

Therefore BC = h (m)

CD = AB-BC

= 15-h.

In ΔDBC,

$\sin 60° = \frac{BC}{CD}$

$\frac{\sqrt{3}}{2} = \frac{h}{15-h}$

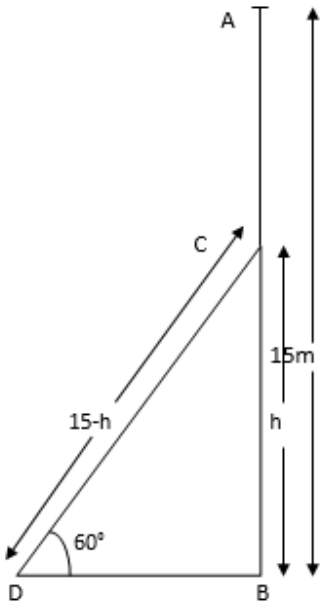
On cross-multiplication

√3(15-h) = 2h

⇒ 15√3-√3h = 2h

⇒ h(2+√3) = 15√3

$h = \frac{15\sqrt{3}}{2+\sqrt{3}}$



on multiplying and dividing by 2-√3

h = $\frac{15\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

h = $\frac{30\sqrt{3}-45}{4-3} \Rightarrow \frac{30\sqrt{3}-45}{1}$ ⇒ 6.96 m.

Therefore the tree is broken at 6.96 m from the ground.

9. Question

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 5 metres. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are respectively 30° and 60°. Find the height of the tower.

Answer

Let the height of the tower = h (m)

Let the point of elevation on the ground is x (m) away from the foot of the tower.

In ΔDBC,

tan 30° = $\frac{BD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

On the cross multiplication

x = h√3 -------(1)

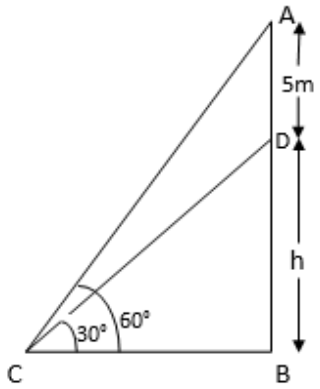
In ΔABC,

tan 60° = $\frac{AB}{BC}$

$\sqrt{3} = \frac{AD+BD}{BC}$

$\sqrt{3} = 5 + \frac{h}{x}$ ---------(2)

On substituting value of x from equn. (1) in eqn. (2)



$\sqrt{3} = \frac{5+h}{h\sqrt{3}}$

$h\sqrt{3} \times \sqrt{3} = 5+h$

$3h = 5+h$

$3h-h = 5$

$2h = 5 \Rightarrow h = \frac{5}{2}$

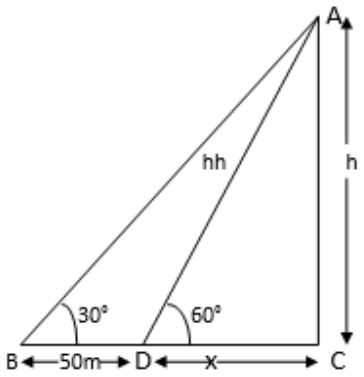
h = 2.5 m.

Therefore height of the tower is 2.5 m.

10. Question

A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

Answer



Let the height of the tower = h (m)

In ΔABD,

tan 60° = $\frac{AB}{BD}$

√3 = $\frac{h}{x}$

h = $x\sqrt{3}$ ---------- (1)

In ΔABC,

tan 30° = $\frac{AB}{BC}$

tan 30° = $\frac{AB}{BD+CD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+50}$ ------(2)

on substituting value of h from equn. (1) In equn. (2)

$$\frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{x+50}$$

On cross multiplication

√3×√3 $x = x + 50$

3 $x = x + 50$

3 $x - x = 50$

2 $x = 50$

$x = \frac{50}{2}$ ⇒ 25 m.

Now substituting value of $x = 25$ in eqn. (1)

h = 25√3 ⇒ 43.3m.

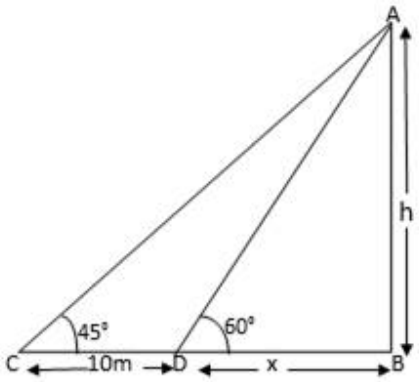
Therefore height of tower is 43.3 m.

11. Question

The shadow of a tower, when the angle of elevation of the sun is 45°, is found to be 10 m longer than when it was 60°. Find the height of the tower.

Answer

Let the height of the tower = h (m)



Let the point of 60° elevation is x (m) away from the foot of the tower.

In ΔABC,

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{CD+BD}$

$1 = \frac{h}{10+x}$

$h = 10+ x$ ----(1)

In ΔABD,

$\tan 60° = \frac{AB}{BD}$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}\, x$

$x = \frac{h}{\sqrt{3}}$ ---------(2)

$substituting value\ of\ x$ From eqn. (2) in eqn. (1)

$h = 10 + \frac{h}{\sqrt{3}}$

h - $\frac{h}{\sqrt{3}}$ = 10

$\frac{\sqrt{3}h-h}{\sqrt{3}}$ = 10

$h\left(\sqrt{3}-1\right)$ = 10√3

$h = \frac{10\sqrt{3}}{\sqrt{3}-1} \Rightarrow \frac{(10\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

h= $\frac{30+10\sqrt{3}}{3-1}$ ⇒ h= $\frac{30+10\sqrt{3}}{2}$

⇒ 15+5√3 ⇒ 23.66 m.

Therefore height of the tower is 23.66 m.

12. Question

A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point.

Answer

Let the height of the parachutist = h (m)

Let the distance of falling point from observation point = x (m)

In ΔABC,

tan 45° = $\frac{AB}{BC}$

1 = $\frac{AB}{CD+BD} \Rightarrow \frac{h}{100+x}$

h = 100+ x ----------(1)

In ΔABD,

tan 60° = $\frac{AB}{BD} \Rightarrow \frac{h}{x}$

√3 = $\frac{h}{x}$

h = √3 x ----------(2)

From eqn. (1) and eqn. (2) we get,

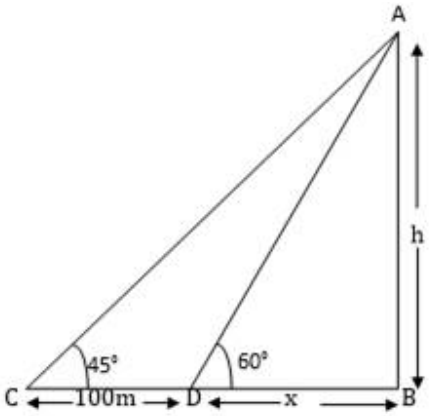
100+ x = √3 x

100 = x (√3-1)

$x = \frac{100}{\sqrt{3}-1} \Rightarrow \frac{100(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{100\times\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$$x = \frac{100\times 2.732}{2}$$

$x =$ 136.6 m.



Now using value of x in eqn. (1)

$h =$ 100+ x

h = 100+136.6

h = 236.6 m.

Therefore height of parachutist is 236.6 m. and distance of point where he falls

is 136.6 m.

13. Question

On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects.

Answer

Let the distance between the objects = x (m.)

In ΔABC,

tan 45° = $\frac{AB}{BC}$

1 = $\frac{AB}{CD+BD} \Rightarrow$

1 = $\frac{150}{x+y}$

$x + y$ = 150 ----------(1)

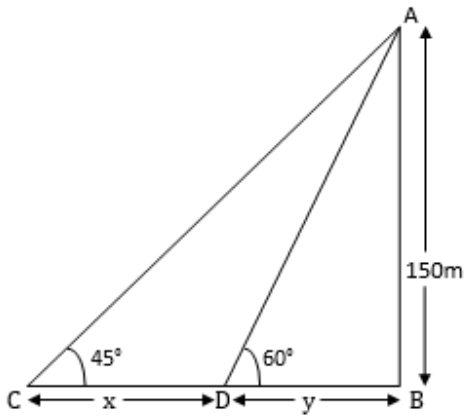
In ΔABD,

tan 60° = $\frac{AB}{BD}$

$\sqrt{3} = \frac{150}{y}$

$y\sqrt{3} = 150$

$y = 150/\sqrt{3}$ -------(2)



substituting value of y in eqn.(1)

$$\frac{150}{\sqrt{3}} + x = 150$$

$x = 150 - \frac{150}{\sqrt{3}}$

$x = \frac{150\sqrt{3}-150}{\sqrt{3}} \Rightarrow \frac{150(\sqrt{3}-1)}{\sqrt{3}} \Rightarrow \frac{(150\sqrt{3})(\sqrt{3}-1)}{\sqrt{3}\times\sqrt{3}}$

$x = \frac{(150\sqrt{3})(\sqrt{3}-1)}{3}$

x = 63.4 m.

Therefore the distance between the points is 63.4 m.

14. Question

The angle of elevation of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 meters towards the foot of the tower, the angle of elevation of the tower becomes 60°. Show that the height of the tower is 129.9 metres (Use $\sqrt{3} = 1.732$).

Answer

Let the height of the tower = h (m)

In ΔABC,

tan 30° = $\frac{AB}{BC}$

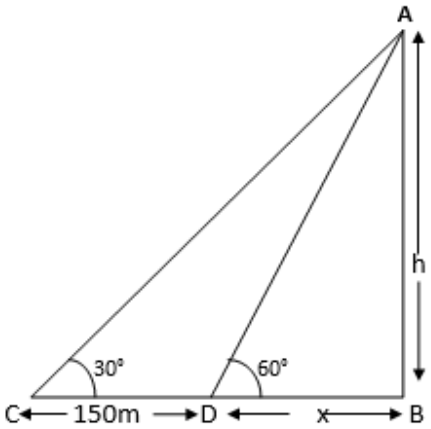
$\frac{1}{\sqrt{3}} = \frac{AB}{CD+BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{150+x}$

√3h = 150+ x

$x = \sqrt{3}\,h - 150$ -------- (1)

In ΔABD,



tan 60° = $\frac{AB}{BD}$

√3 = $\frac{h}{x}$ ⇒ h = √3 x

$x = \frac{h}{\sqrt{3}}$ -------(2)

on substituting value of *x* from eqn.(2)*i* eqn.(1)

$\frac{h}{\sqrt{3}} = \sqrt{3}\,h - 150$

$h = 3h - 150\sqrt{3}$

h-3h = -150$\sqrt{3}$

2h = 150√3

h = $\frac{150\sqrt{3}}{2}$ ⇒ 75$\sqrt{3}$

h= 129.9 m.

Hence height of tower is 129.9 m.

15. Question

The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63°. Find the height of the tower and the distance of the first position from the tower. [Take tan 32° = 0.6248 and tan 63° = 1.9626]

Answer

Let the height of the tower = h (m)

Let the distance of point from the foot of the tower = x (m)

In ΔABC,

$\tan 32° = \frac{AB}{BC}$

$\tan 32° = \frac{AB}{CD+DB}$

$0.6248 = \frac{h}{100+x}$

h = 0.6248(100+ x) ----------(1)

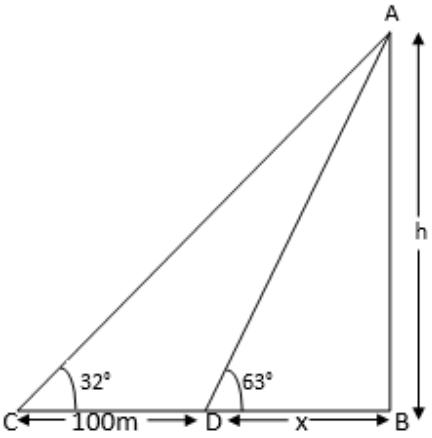
In ΔABD,

$\tan 63° = \frac{AB}{BD}$

$1.9626 = \frac{h}{x}$

h = 1.9626 x ----------(2)

Substituting value of h from eqn. (2) in eqn. (1)



1.9626 x= 0.6248(100+ x)

1.9626 x= 62.48+0.6248 x

1.9626 x-0.6248 x = 62.48

1.3378 x = 62.48

$$x = \frac{62.48}{1.3378}$$

x = 46.70 m.

on substituting value of x in eqn.(2)

h = 1.9626× 46.7

h = 91.66 m.

Distance of the first position from tower=

BC = CD+DB

BC = 100+46.7

$BC = 146.7\ m.$

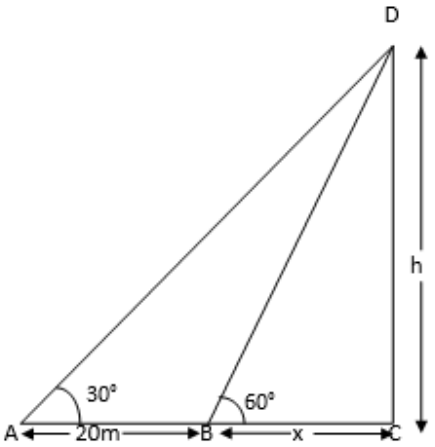
height of tower is 91.66 m.

16. Question

The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B the angle of elevation increases to 60°. Find the height of the tower and the distance of the tower from the point A.

Answer

Let the height of the tower is = h (m)



Distance of point B from foot of the tower is = x (m)

In ΔADC,

tan 30° = $\frac{DC}{AC}$

$\frac{1}{\sqrt{3}} = \frac{h}{20+x}$

√3 h = 20+ x ----------(1)

In ΔDCB,

tan 60° = $\frac{DC}{BC}$

√3 = $\frac{h}{x}$

h = √3 x ----------(2)

On substituting value of h from eqn. (2) in eqn. (1)

√3× √3 x = 20 + x

3 x = 20+ x

3 x- x= 20

x = 10

Therefore distance of point A from tower is

AC = AB+BC

AC = 20+10 ⇒ 30

Ac = 30 m.

Now substituting value of x in eqn. (1)

$\sqrt{3}$ h = 20+10 ⇒ 30

$h = \frac{30}{\sqrt{3}}$ ⇒ 17.32 m.

Therefore height of tower is 17.32 m.

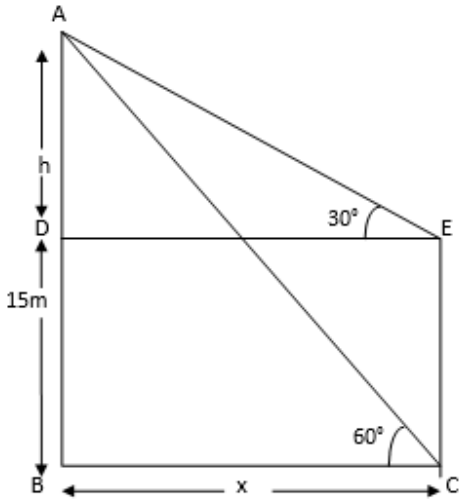
17. Question

From the top of a building 15 m high the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between the tower and building.

Answer

Let the distance between tower and building = x (m)

In ΔABC,



tan 60° = $\frac{AB}{BC}$

√3 = $\frac{AD+BD}{BC}$

$\frac{\sqrt{3}}{1} = \frac{h+15}{x}$

√3 x = h+15

h = √3 x-15 ------(1)

In ΔABE,

tan 30° = $\frac{AD}{DE}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{3}\,h$$ ---------(2)

$$Substiuting\ value\ of\ x\ in\ eqn.(1)$$

$$h = \sqrt{3}\times\sqrt{3}h - 15$$

h = 3h-15 ⇒ 2h = 15

h = $\frac{15}{2}$ ⇒ 7.5 m.

Height of tower = 15+7.5 ⇒ 22.5 m.

$$x = \sqrt{3}\times 7.5$$ ⇒ 12.99 m.

Therefore distance between tower and building is 12.99 m.

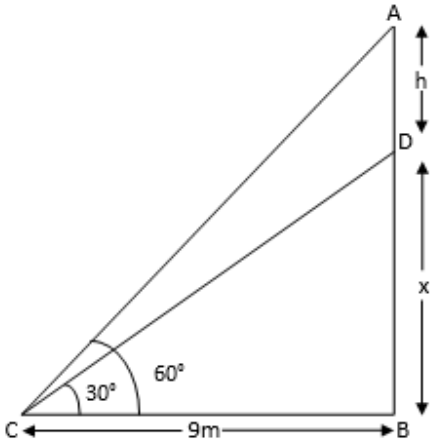
18. Question

On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

Answer

Let the height of the Flag-pole = h(m)

And height of tower = x (m)



In ΔABC,

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{h+x}{9}$

h+ x = 9√3 -------(1)

In ΔDBC,

tan 30° = $\frac{DB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{x}{9}$

√3 x = 9

$x = \frac{9}{\sqrt{3}} \Rightarrow \frac{9\sqrt{3}}{3}$

$x = 3\sqrt{3}$ -------(2)

Now substituting value of x in eqn. (1)

h+3√3 = 9√3

h = 9√3-3√3

h = 6√3 m.

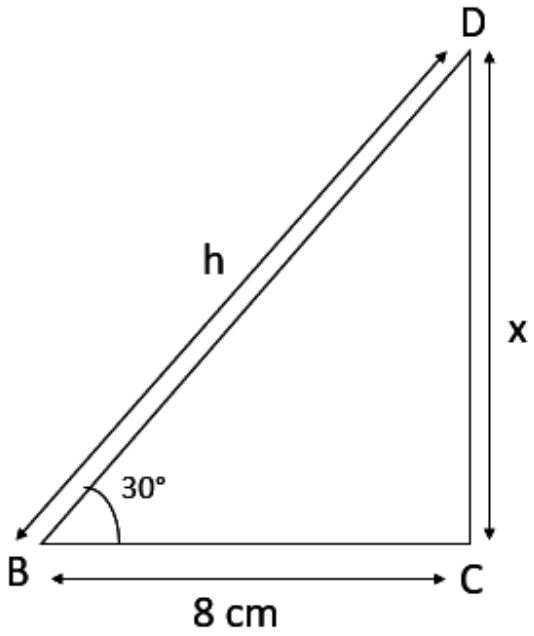
Therefore height of tower is 3√3 m. and height of flag pole is 6√3 m.

19. Question

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer

let the broken part be DB.



Distance from the foot of the tree and point C is 8 cm.

BC = 8 cm

Height of tree = Height of broken part + height of the remaining tree= DC + DB

In ΔDBC,

$$\tan\theta = \frac{perpendicular}{base} \quad \cos\theta = \frac{base}{hypotenuse}$$

tan 30° = $\frac{DB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{x}{8}$

$\sqrt{3}\,x = 8$

$x = \frac{8}{\sqrt{3}}$ ---------(1)

Cos 30° = $\frac{BC}{DC}$

$\frac{\sqrt{3}}{2} = \frac{8}{h}$

$\sqrt{3h} = 16$

h $= \frac{16}{\sqrt{3}}$

Height of tree $= \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}}$

$= \frac{24\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{24\sqrt{3}}{3}$

Height of tree = 8√3 m

20. Question

From a point P on the ground the angle of elevation of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag-staff from P is 45°. Find the length of the flag-staff and the distance of the building from the point P. (Take $\sqrt{3} = 1.732$).

Answer

Let the height of the flag-staff = h (m)

And the distance of point P from foot of building = x (m)

In ΔAPB,

tan 45° $= \frac{AB}{BP}$

tan 45° $= \frac{h+10}{x}$

1 = $= \frac{h+10}{x}$

h+10 = x -------(1)

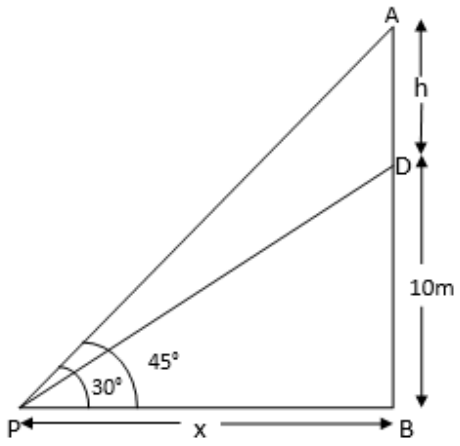
In ΔDPB,

tan 30° $= \frac{DB}{BP}$

$\frac{1}{\sqrt{3}} = \frac{10}{X}$

$x = 10\sqrt{3}$ ⇒17.32 m. ----------(2)

On substituting value of x in eqn. (1)



h = $x - 10$

h = 17.32-10 ⇒ 7.32 m.

Therefore height of flag-staff is 7.32 m. and distance of point P from tower is 17.32 m.

21. Question

A 1.6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles.

Answer

Let the height of the lamp post = h (m)

And height of girl is CD = 1.6 m.

Length of shadow is OD = 4.8 m.

In ΔCDO

$\tan\theta = \frac{CD}{OD} \Rightarrow \frac{1.6}{4.8} \Rightarrow \frac{1}{3}$ ------(1)

In ΔABO

$\tan\theta = \frac{AB}{BO}$

$\frac{1}{3} = \frac{x}{(3.2+4.8)}$

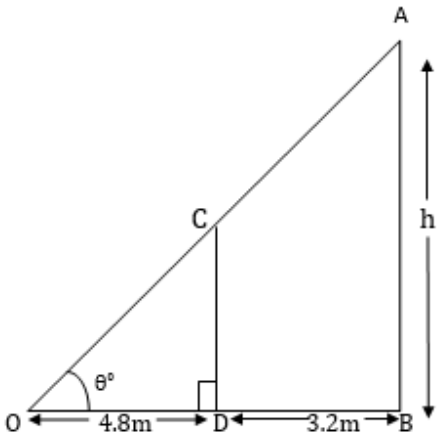
$x = \frac{8}{3}$

$Therefore\ height\ of\ lamp\ post\ is\ \frac{8}{3}\ m.$

By Smilariting

In ΔABO and in ΔCDO

∠ABO = ∠CDO = 90°



∠AOB = ∠COD

ΔABO∼ ΔCDO (AA similarity)

For similar Δs sides are in ratio

Hance,

$$\frac{AB}{CD} = \frac{BO}{DO}$$

$$\frac{x}{1.6} = \frac{8}{4.8}$$

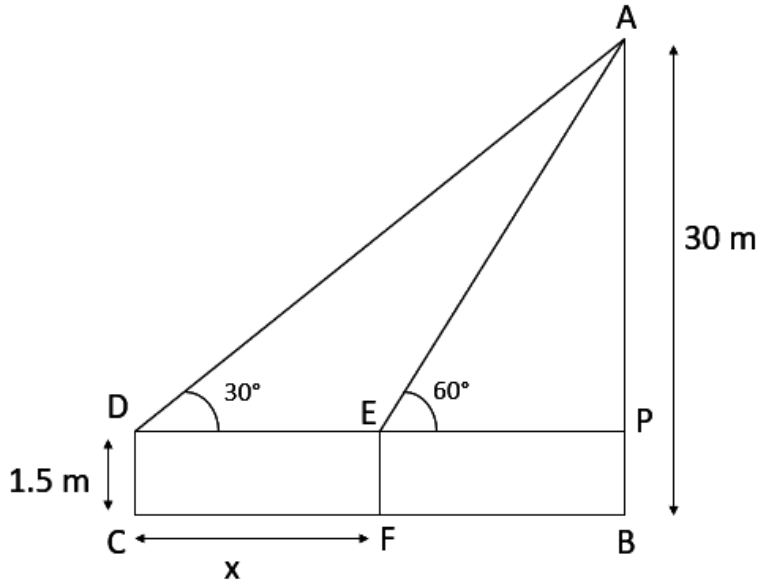
$$x = \frac{8}{3}m.$$

$$Therefore\ height\ of\ lamp\ post\ is \frac{8}{3}m.$$

22. Question

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer



The height of the boy is DC = 1.5 mLet he was at point C initially and then moved to point F.Let CF = x Now DC is parallel to PB.DP is parallel to CB.⇒ DE = CF EP = FBNow,

AP = AB - BP

= 30-1.5 = 28.5 m.

Since the tower is vertical,∠APE = 90°We know, in a right-angled triangle, $\tan\ \theta = \frac{perpendicular}{base}$

In ΔADP,

$$tan\ 30^{\circ} = \frac{AP}{DP} \quad \frac{1}{\sqrt{3}} = \frac{28.5}{DP}$$

DP = 28.5√3 ----------(1)

In ΔAEB,

$$tan\ 60^{\circ} = \frac{AP}{PE} \quad \Rightarrow \sqrt{3} = \frac{AP}{PE}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{PE}$$

$$\Rightarrow PE = \frac{28.5}{\sqrt{3}} \quad PE = \frac{28.5\sqrt{3}}{3}$$

DE = DP - EP $= 28.5\sqrt{3} - \frac{28.5\sqrt{3}}{3}$

$$= 28.5\sqrt{3}\left(1 - \frac{1}{3}\right)$$

$= 28.5\sqrt{3}\left(\frac{2}{3}\right) \quad = \frac{57\sqrt{3}}{3}$

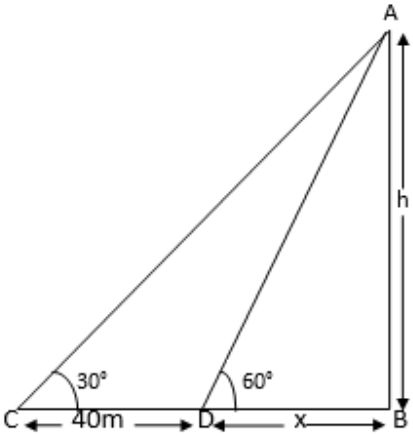
= 19√3 m.

Therefore the walking distance of boy is 19√3 m.

23. Question

The shadow of a tower standing on level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60°. Find the height of the tower.

Answer



Let the height of tower = h (m)Since the tower is vertical to the ground.∠ABC = 90°We know, in a right-angle triangle, $\tan\ \theta = \frac{perpendicular}{base}$

In ΔABC,

$\tan\ 30^{\circ} = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{CD+BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{40+x}$

√3h = 40+ x ----------(1)

In ΔABD,

$tan\ 60^{\circ} = \frac{AB}{BD}$

$\sqrt{3} = \frac{h}{x}$

h =√3 x -------(2)

on substituting the value of h from eqn. (2) in eqn. (1)

h = √3× (√3h-40)

h = 3h-40√3

2h = 40√3

h = 20√3 m.

Therefore the height of the tower is 20√3 m.

24. Question

From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transmission tower.

Answer

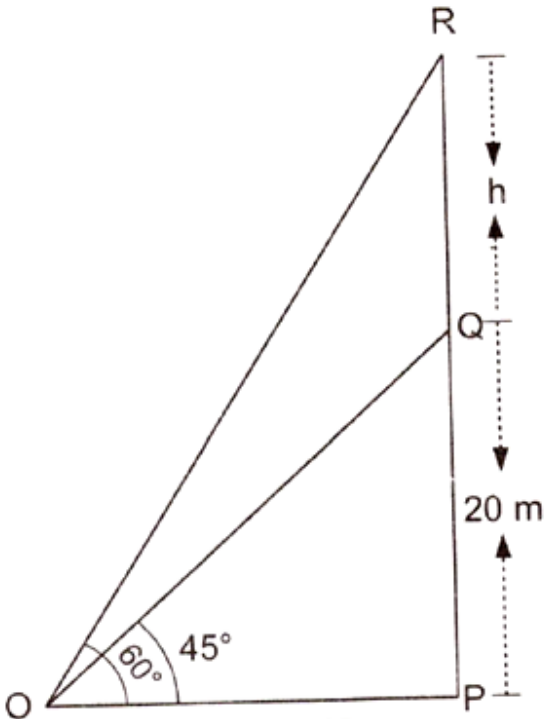


Fig. 12.48

Since the building is vertical.∠QPO = 90°

In a right-angled triangle, we know, $\tan\ \theta = \frac{perpendicular}{base}$

In ΔOPQ,

tan 45° = $\frac{QP}{OP}$

1 = $\frac{20}{OP}$

OP = 20 -------(1)

Now in ΔOPR

tan 60° = $\frac{PR}{OP}$

$\sqrt{3} = \frac{RQ+QP}{OP}$

$\sqrt{3} = \frac{h+20}{20}$

20√3 = h+20

h = 20√3-20

h = 20(√3-1) m.

Therefore the height of transmission tower is 20(√3-1) m.

25. Question

The angles of depression of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of the multistoried building and the distance between the two buildings.

Answer

Let DC is tall building and AB is multistoried building.

AB = AE+EB

AB = h+8 ----------(1)

In ΔABC,

tan 45° = $\frac{AB}{BC}$

1 = $\frac{h+8}{x}$

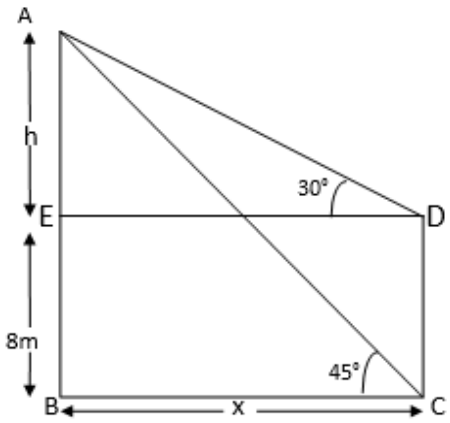
$x = h + 8$ ----------(2)

In ΔAED,

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

√3h = x ---------(3)



Substituting value of x from eqn. (2) in eqn. (1)

$$\sqrt{3}h = h + 8$$

√3h-h = 8

h = $\frac{8}{\sqrt{3}-1}$

h = $\frac{8\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{8(\sqrt{3}+1)}{2}$

h = 4(√3+1)m. --------(4)

Substituting value of h from eqn. (4) in eqn. (3)

√3h = x

x = √3× 4(√3+1)

x = √3 (4√3+4)

$$x = 12 + 4\sqrt{3}$$

$$x = 4(3 + \sqrt{3})m.$$

Therefore height of multistoried building is

= 8+4(√3+1)

= 8+4√3+4

= 12+4√3

= 4(3+√3) m.

Distance between two building is 4(3+√3) m.

26. Question

A statue 1.6 m tall stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the

pedestal is 45°. Find the height of the pedestal.

Answer

Let AD is statue of height 1.6 m. and BD is pedestal of height h (m).

Let the distance between point of elevation and foot of pedestal is x (m).

In ΔDBC,

$\tan 45° = \frac{BD}{BC}$

$1 = \frac{h}{x}$

$h = x$ ----------(1)

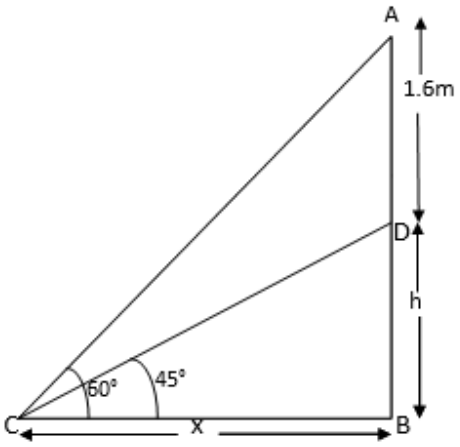
In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AD+BD}{BC}$

$\sqrt{3} = \frac{1.6+h}{x}$

$\sqrt{3}x = 1.6+h$ ----------(2)



On substituting value of x from eqn. (1) in eqn. (2)

$\sqrt{3}h = 1.6+h$

$\sqrt{3}h - h = 1.6$

$h = \frac{1.6}{\sqrt{3}-1}$

on rationalizing we get.

h = $\frac{1.6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}) \Rightarrow \frac{1.6(\sqrt{3}+1)}{3-1} \Rightarrow \frac{1.6(\sqrt{3}+1)}{2}$

$0.8(\sqrt{3}+1) = \frac{8(\sqrt{3}+1)}{10} = \frac{4(\sqrt{3}+1)}{5}$ m.

Therefore height of pedestal is $\frac{4(\sqrt{3}+1)}{5}$ m.

27. Question

A T.V. tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the river.

Answer

Let BC be the height of the T.V tower and AB be the width of the river.

In ΔABC,

tan 60° = $\frac{BC}{AB}$

√3 = $\frac{h}{x}$ ------(1)

In ΔABC,

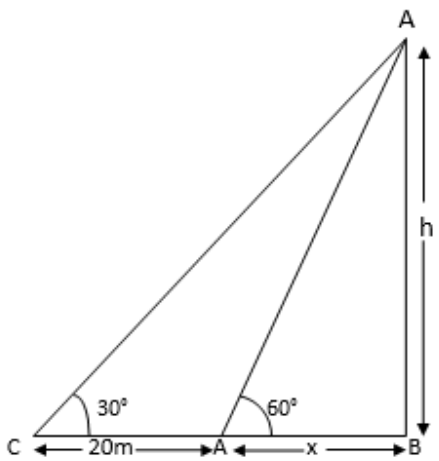
tan 30° = $\frac{BC}{BD}$

tan 30° = $\frac{BC}{DA+AB}$

$\frac{1}{\sqrt{3}} = \frac{h}{20+x}$

√3h = 20+ χ ---------(2)

On substituting value of h in eqn.(2)



√3× √3 x = 20+ x

3 $x = 20 + x$

3 $x - x = 20$

2 $x = 20$

$x = 10\ m.$

On substituting value of x in eqn. (1)

h = √3 x

h = 10√3 m.

Therefore height of T.V tower is 10√3 m. and width of river is 10 m.

28. Question

From the top a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer

Let the distance between the foots of building and cable tower is x (m).

The height of cable tower = AB = AE+EB ⇒ (h+7)m.

In ΔAED,

tan 60° = $\frac{AE}{DE}$

√3 = $\frac{h}{x}$

h = √3 x ---------(1)

The height of cable tower = AB = AE+EB ⇒ (h+7)m.

In ΔDEB,

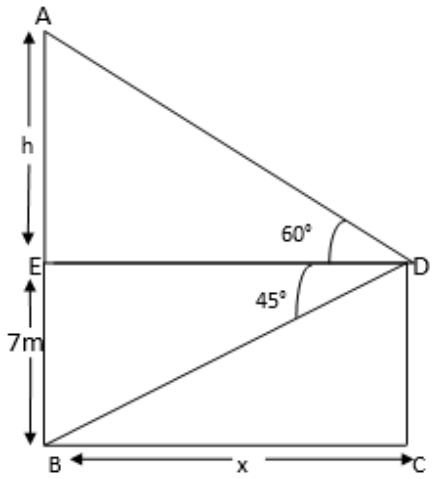
tan 45° = $\frac{BE}{DE}$

1 = $\frac{7}{x}$

$x = 7$ ----------(2)

On substituting value of x in eqn. (1)

h = 7√3



Height of cable tower is (h+7)m.

⇒ 7√3+7

⇒ 7(√3+1)m.

Therefore height of cable tower is 7(√3+1)m.

29. Question

As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer

Let the distance between the two ships be x (m.)

And distance between the ship and foot of light house is y (m.)

In ΔABD

$\tan 45° = \frac{AB}{BD}$

$1 = \frac{AB}{BD}$

AB = BD

$75 = y$

$y = 75\ m.$ --------(1)

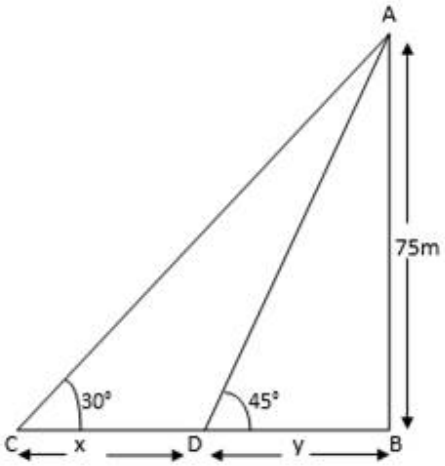
In ΔABC

$\tan 30° = \frac{AB}{BC}$

$\tan 30° = \frac{AB}{BD+DC}$

$$\frac{1}{\sqrt{3}} = \frac{75}{y+x}$$

$y + x$ = 75√3 --------(2)



On substituting value of y in eqn. (2)

$$x = 75\sqrt{3} - 75$$

$$x = 75(\sqrt{3} - 1)m.$$

Therefore distance between the two ships is $75(\sqrt{3} - 1)m.$

30. Question

The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer

Let AB be the building of height 50 m. and tower of height h (m.)

In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{50}{x}$

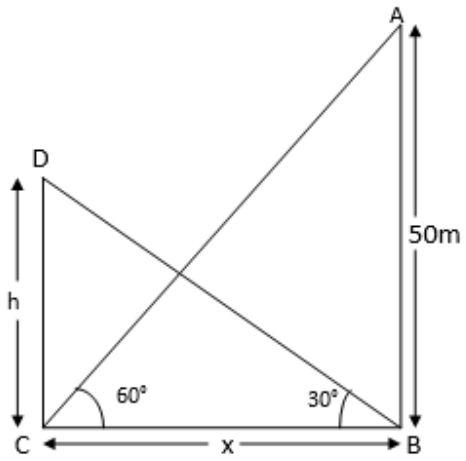
$x = \frac{50}{\sqrt{3}}$ ------(1)

Now in ΔDCB

tan 30° = $\frac{DC}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$x = \sqrt{3}h$ -----(2)



On substituting value of x in eqn. (1)

$$x = \frac{50}{\sqrt{3}}$$

$\sqrt{3}h = \frac{50}{\sqrt{3}}$

$h = \frac{50}{\sqrt{3}\times\sqrt{3}}$

$h = \frac{50}{3}$

Therefore height of tower is $\frac{50}{3}$ m.

31. Question

From a point on a bridge across a river the angles of depression of the banks on opposite side of the river are 30° and 45° respectively. If bridge is at the height of 30m from the banks, find the width of the river.

Answer

Let A and B be the points on the bank n opposite sides f the river and BC be the width of the river.

BC = BD+DC ⇒ $(x + y)$m.

AD be the height of the bridge.

In ΔADC

tan 45° = $\frac{AD}{DC}$

$1 = \frac{30}{y}$

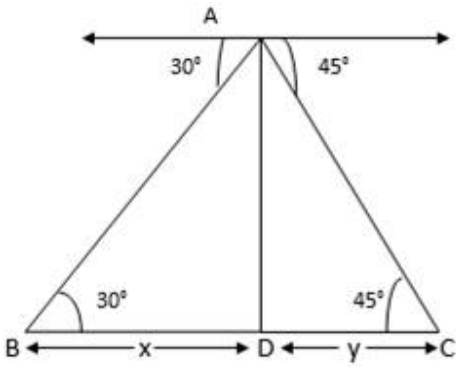
$y = 30\ m.$ ---------(1)

In ΔADB

tan 30° = $\frac{AD}{BD}$

$\frac{1}{\sqrt{3}} = \frac{30}{x}$

$x = 30\sqrt{3}$ ----------(2)



Width of the river = ($x + y$)

⇒ 30√3+30

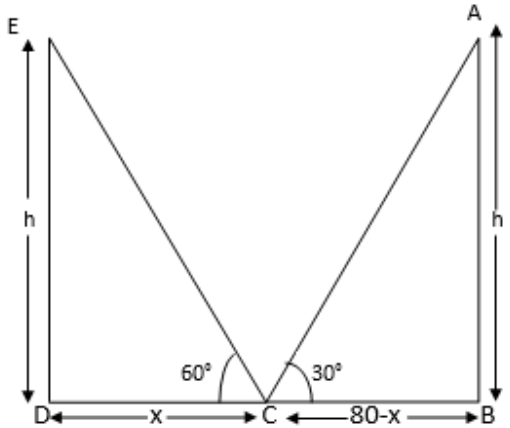
⇒ 30(√3+1)m.

32. Question

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Answer

Let AB and ED are two poles of equal height.



Let C be the point between the poles on the ground.

Since poles are vertical to the ground.∠ADC = ∠ABC = 90°In a right-angled triangle, we know,

$$\tan\,\theta = \frac{perpendicular}{base}$$

In ΔEDC

tan 60° = $\frac{ED}{DC}$

$$\sqrt{3} = \frac{h}{x}$$

h = √3 x --------(1)

In ΔABC

tan 30° = $\frac{AB}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{h}{80 - x}$$

√3h = 80-x----------(2)

On substituting value of h from eqn.(1) in eqn. (2)

√3 × √3x = 80 - x

⇒ 3x = 80 - x

⇒ 4x = 80

⇒ x = 20 m

On substituting value of x in eqn. (1)

h = 20√3Distance of C from pole ED = 20 mDistance of C from pole AB = 80 - 20= 60 m

Therefore the height of the poles is 20√3 m. and distances of the points from one pole is 20 m and from other pole is 60 m.

33. Question

A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with the foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.

Answer

Let width of the river be DC= DB+BC ⇒ $(x + y)$ m.

In ΔABC

tan 60° = $\frac{AB}{BC}$

$$\sqrt{3} = \frac{20}{y}$$

20 = √3 y

$y = \frac{20}{\sqrt{3}}$ -----(1)

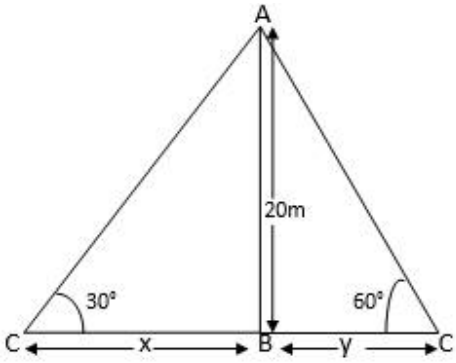
Now in ΔABD

tan 30° = $\frac{AB}{DB}$

$$\frac{1}{\sqrt{3}} = \frac{20}{x}$$

$$x = \frac{20}{x}$$

Therefore width of river = $(x + y)$ m.



$\Rightarrow 20\sqrt{3} + \frac{20}{\sqrt{3}}$

$\Rightarrow \frac{20\sqrt{3}+\sqrt{3}+20}{\sqrt{3}} \Rightarrow \frac{60+20}{\sqrt{3}} \Rightarrow \frac{80}{\sqrt{3}}$ m.

Width of river is $\frac{80}{\sqrt{3}}$ m.

34. Question

A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.

Answer

Let the height of tower is BD = h (m.)

Now in ΔABC

tan 45° = $\frac{AB}{BC} \Rightarrow \frac{AD+BD}{BC}$

1 = $\frac{7+h}{x}$

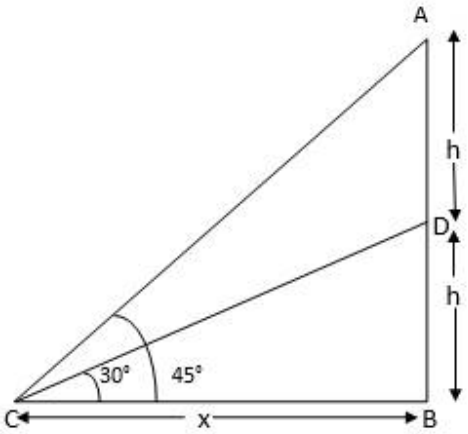
7+h = x ------(1)

Now in ΔDBC

tan 30° = $\frac{BD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = \sqrt{3}h$ ----------(2)



On substituting value of x in eqn.(1)

7+h = √3h

√3h- h = 7

h (√3-1) = 7

h = $\frac{7}{\sqrt{3}-1} \Rightarrow \frac{7}{1.732-1}$

$\frac{7}{0.732}$ = 9.56m.

Therefore height of tower is 9.56m.

35. Question

The length of the shadow of a tower standing on level plane is found to be 2x metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of tower is $x\left(\sqrt{3}+1\right)$ metres.

Answer

Let the height of tower is AB = h (m.)

Now in ΔABD

tan 45° = $\frac{AB}{BD}$

1 = $\frac{h}{y}$

h = y ------(1)

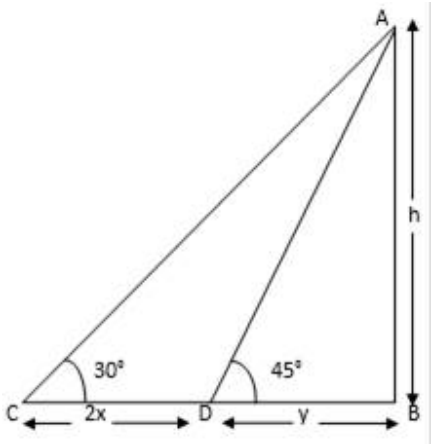
Now in ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BD+DC}$

$\frac{1}{\sqrt{3}} = \frac{h}{y+2x}$

√3h = 2 $x + y$ -------(2)



On substituting value of y from eqn. (1) in eqn. (2)

√3h = 2 $x + y$

√3h = 2 $x + h$

√3h-h = 2 x

h (√3-1) = 2 x

h = $\frac{2x}{\sqrt{3}-1}$

on rationalsing above fraction we get,

h = $\frac{2x(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

h = $\frac{2x(\sqrt{3}+1)}{2}$

$x = (\sqrt{3}+1)$

Therefore height of tower is $(\sqrt{3}+1)$ m.

36. Question

A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 metres. Find the height of the tree.

Answer

In fig.BD is the height of the tree. Let the broken part touches the ground at point C

BD = AB+AD

in ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{10}$

√3h = 10

h = $\frac{10}{\sqrt{3}}$

h = $\frac{10\sqrt{3}}{3}$ m. -------(1)

Again ΔABC.

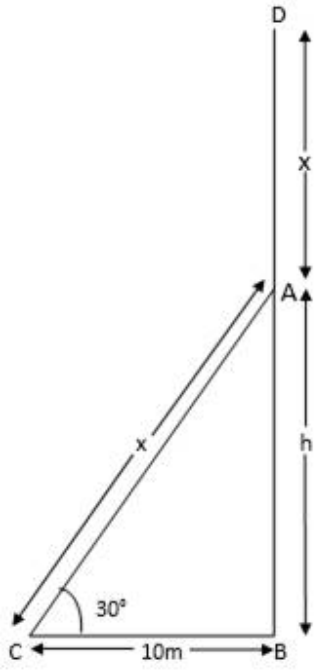
COS 30° = $\frac{BC}{AC}$

$\frac{\sqrt{3}}{2} = \frac{10}{x}$

$\sqrt{3}\,x = 20$

$x = \frac{20}{\sqrt{3}}$

$x = \frac{20\sqrt{3}}{3}$ ------(2)



Therefore height of tree = $(x + h)$ m.

Adding eqn. (1) and (2) we get,

$\frac{10\sqrt{3}}{3} + \frac{20\sqrt{3}}{3}$

$\frac{10\sqrt{3}+20\sqrt{3}}{3} = \frac{30\sqrt{3}}{3} \Rightarrow 10\sqrt{3}$

10× 1.732 ⇒ 17.32 m.

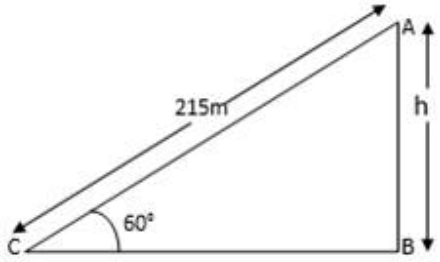
Therefore height of tree is 17.32 m.

37. Question

A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

Answer

In the fig. Let AB be the height of the balloon.



In ΔABC

sin 60° = $\frac{AB}{AC}$

$\frac{\sqrt{3}}{2} = \frac{h}{215}$

2h = 215√3

h = $\frac{215\sqrt{3}}{2}$ ⇒ 186.19 m.

Therefore height of balloon from the ground is 186.19 m.

38. Question

Two men on either side of the cliff 80 m high observes the angles of elevation of the top of the cliff to be 30° and 60° respectively. Find the distance between the two men.

Answer

In fig. AB be the height of the cliff.

Let the distance between the two men is DC.

DC = $(x + y)$ m.

In ΔABC

tan 60° = $\frac{AB}{BC}$

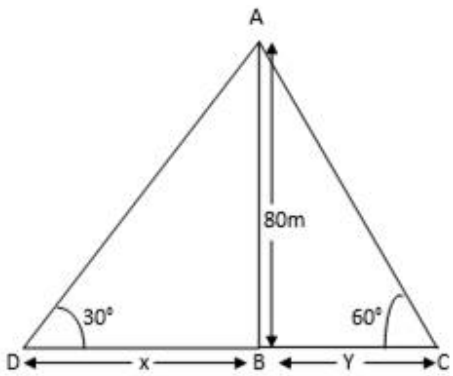
√3 = $\frac{80}{y}$

√3 y = 80

$$y = \frac{80}{\sqrt{3}}$$

$y = \frac{80\sqrt{3}}{3}$ ------(1)

In ΔABD



tan 30° = $\frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{80}{x}$$

$x = 80\sqrt{3}$ --------(2)

Adding eqn. (1) and (2) we get,

$(x + y) = 80\sqrt{3} + \frac{80\sqrt{3}}{3}$

$\frac{3\times80\sqrt{3}+80\sqrt{3}}{3} \Rightarrow \frac{240\sqrt{3}+80\sqrt{3}}{3} \Rightarrow \frac{320\sqrt{3}}{3} \Rightarrow$ 184.75 m.

Therefore distance between two men is 184.75 m.

39. Question

Find the angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to its height.

Answer

Let the position of the sun be at O and AB be the height of the pole.

Now BC is the shadow cast by the pole. From point C, the angle of elevation of the top of the pole (point A) and the sun would be the same.

Since length of shadow is equal to the height of the vertical pole.

Therefore AB = BC

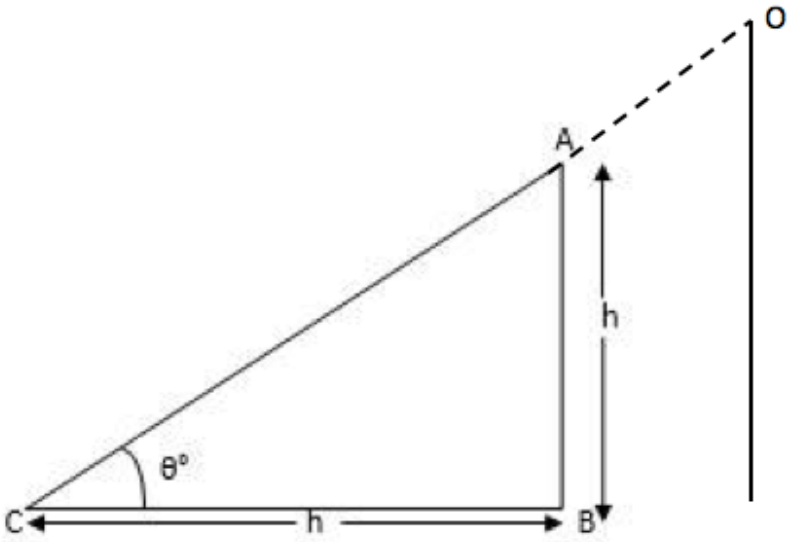
In ΔABD

tan θ = $\frac{AB}{BC}$

tan θ = $\frac{h}{h} \Rightarrow 1$

θ = tan 1 (since tan 45°= 1)

θ = 45°



Therefore angle of elevation is 45°

40. Question

A fire in a building B is reported on teleported on telephone to two fire stations P and Q, 20 km apart from each other on a straight road. P observes that the fire is at an angle of 60° to the road and Q observes that it is at an angle of 45° to the road. Which station should send its team and how much will this team have to travel?

Answer

Let the height of building be 'h'.Let the distance between P and foot of building is 'x' metres.

In ΔPRS

$\tan 60° = \frac{PS}{PR}$

$\sqrt{3} = \frac{h}{x}$

h = √3x ---------(1)

In ΔQRS

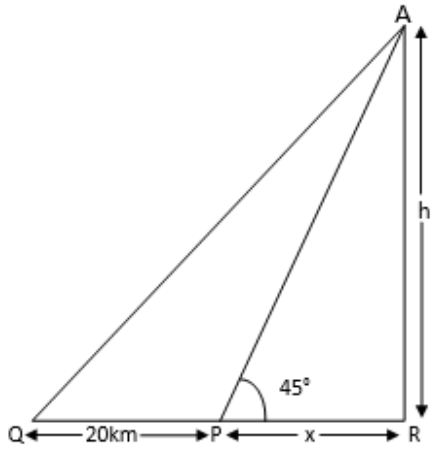
$\tan 45° = \frac{RS}{QR}$

$\tan 45° = \frac{h}{20+x}$

$1 = \frac{h}{20+x}$

h = 20+ χ ---------(2)

on substituting the value of h from eqn. (2) in eqn. (1)



20+ x = √3 x

√3 x – x = 20

$$x = \frac{20}{\sqrt{3}-1}$$

On rationalising above fraction we get,

$x = \frac{20\times\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{20\times\sqrt{3}+1}{2} \Rightarrow 10(\sqrt{3}+1)$

x = 10(1.732+1)

$x = 10\times2.732$

$x = 27.32\ m.$

Therefore Station P has to send the team. And the distance between station P and the building is 27.32 m.

41. Question

A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

Answer

In the fig. AB is the height of the cliff.

AB = AE+EB ⇒ h+10

In ΔAED

tan 45° = $\frac{AE}{DE}$

1 = $\frac{h}{DE}$

DE = h -----(1)

In ΔDEB

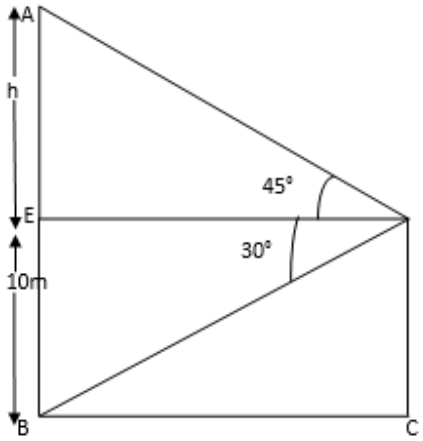
tan 30° = $\frac{EB}{DE}$

$$\frac{1}{\sqrt{3}} = \frac{10}{DE}$$

DE = 10√3 -----(2)

From eqn. (1) and eqn. (2) we get,

h = 10√3



Height of cliff = AE+EB

= 10√3+10

= 27.32 m.

Therefore distance of the cliff from the ship is 10√3 and Height of cliff is 27.32m.

42. Question

A man standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

Answer

In the fig. AB is the height of hill AB = AE+EB ⇒ h+8

In ΔAEB

tan 60° = $\frac{AE}{DE}$

√3 = $\frac{h}{DE}$

h = √3 DE ------- (1)

In ΔDEB

tan 30° = $\frac{EB}{DE}$

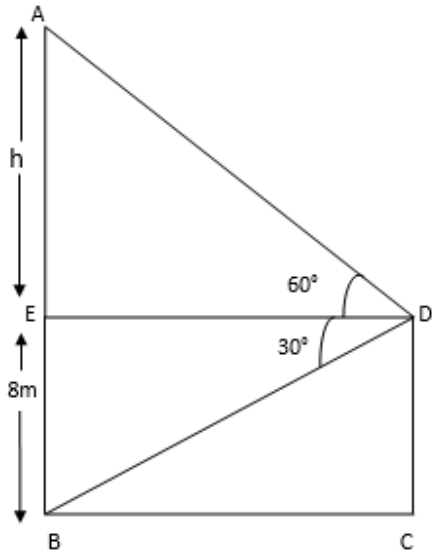
$$\frac{1}{\sqrt{3}} = \frac{8}{DE}$$

DE = 8√3 ------(2)

From eqn. (1) and eqn. (2) we get,

h = √3× 8√3 ⇒ 24 m.

Height of hill = 24+8 ⇒ 32 m.



On substitution value of h in eqn. (1)

24 = √3 DE

DE = $\frac{24}{\sqrt{3}}$

DE = $\frac{24\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{24\sqrt{3}}{3}$

DE = 8√3 m.

Therefore distance between ship and hill is 8√3 m and height of the hill is 32 m.

43. Question

There are two temples, one on each bank of a river, just opposite to each other. One temple is 50 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Answer

In fig let the height of the other temple is h (m.) and distance between two temple is x (m.)

In ΔABC

$\tan 60° = \frac{AB}{BC}$

$\tan 60° = \frac{AB+EB}{BC}$

$\sqrt{3} = \frac{50}{x}$

$x = \frac{50}{\sqrt{3}} \Rightarrow \frac{50\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$x = \frac{50\sqrt{}}{3}$ (1)

In ΔAED

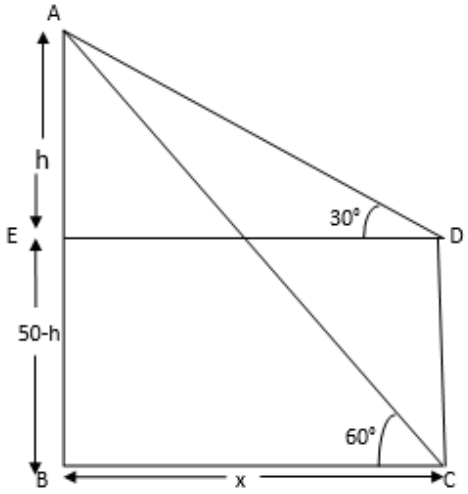
$\tan 30° = \frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$X = h\sqrt{3}$ (2)

From eqn. (1) and eqn. (2) we get,

$\sqrt{3}h = \frac{50\sqrt{3}}{3}$



$h = \frac{50}{3} = 16.67m$

On substituting the value of 'h' in eqn (2)

$X = \frac{50\sqrt{3}}{3} \Rightarrow 28.87m$

Therefore height of the temple is 50-16.67 = 33.33m

And the distance between the two temples is 28.87m

44. Question

The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Answer

In the fig let C be the initial position of the aeroplane. After 15 seconds the position of the aeroplane becomes E.

In ΔABC

$\tan 45° = \frac{BC}{AB}$

$1 = \frac{3000}{y}$

Y = 3000m ------(2)

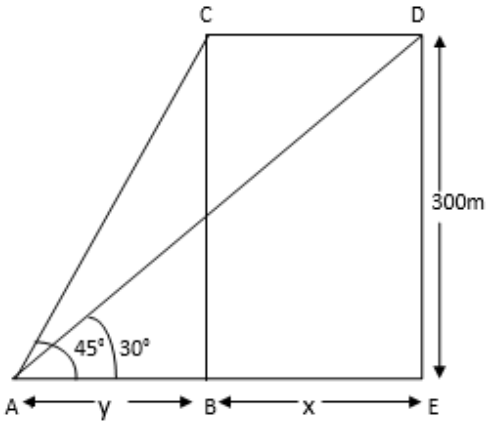
In ΔADE

$\tan 30° = \frac{DE}{AD}$

$\frac{1}{\sqrt{3}} = \frac{3000}{x+y}$

$x + y = 3000\sqrt{3}$

Using equation (1) to replace value of y, we get



x = 3000√3 – 3000

⇒ 3000(√3-1)

⇒ 2196m

Since the distance travelled by aeroplane in 15 seconds is 2196m. Therefore distance travelled by aeroplane in 1 hour =

$\frac{2196\times3600}{15\times1000}$ ⇒ 527.04 km/hr

Therefore speed of aeroplane is 527.04 km/hr

45. Question

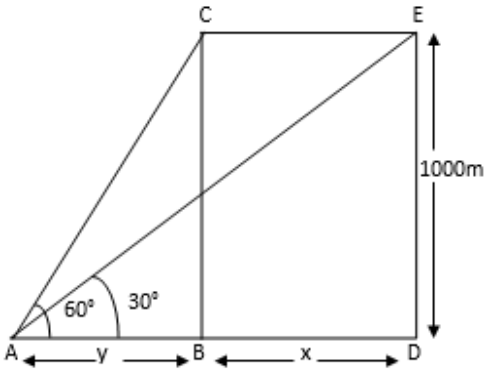
An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km / hr.

Answer

Given: An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°.

To find: the speed of the aeroplane in km / hr.

Solution: Draw the figure according to given information



In the fig let C be the initial position of the aeroplane. After 10 seconds the position of the aeroplane becomes E.

As $\tan\theta = \frac{perpendicular}{base}$

In ΔABC

tan 60° = $\frac{BC}{AB}$

we know tan 60° = √3

⇒ √3 = $\frac{1000}{y}$

It is given it is flying horizontally 1 km above the ground.

⇒ √3y = 1000 m

⇒ y = $\frac{1000}{\sqrt{3}}$ -------(1)

In ΔADE

tan 30° = $\frac{DE}{AD}$

we know tan 30° = $\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1000}{x+y}$

$\Rightarrow x + y = 1000\sqrt{3}$

On substituting value of y from eqn (1)

$x + \frac{1000}{\sqrt{3}} = 1000\sqrt{3}$

$x = 1000\sqrt{3} - \frac{1000}{\sqrt{3}}$

$\Rightarrow x = \frac{1000\sqrt{3} \times \sqrt{3} - 1000}{\sqrt{3}}$

$\Rightarrow x = \frac{3000 - 1000}{\sqrt{3}}$

$\Rightarrow x = \frac{2000}{\sqrt{3}} \Rightarrow x = 1154.7m$

Since the distance travelled by aeroplane in 10 seconds is 1154.7 m. As 1 hr = 3600 sec and 1 km = 1000 mTherefore distance travelled by aeroplane in 1 hour $= \frac{1154.7 \times 3600}{10 \times 1000} \Rightarrow$ 415.69 km/hr

Therefore speed of aeroplane is 415.69 km/hr

46. Question

From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole.

Answer

In the fig CE is the height of the pole and x be the distance between tower and pole.

In ΔADE

$\tan 45° = \frac{AD}{DE}$

$1 = \frac{50-h}{x}$

x = 50-h ----- (1)

In ΔABC

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{50}{x}$

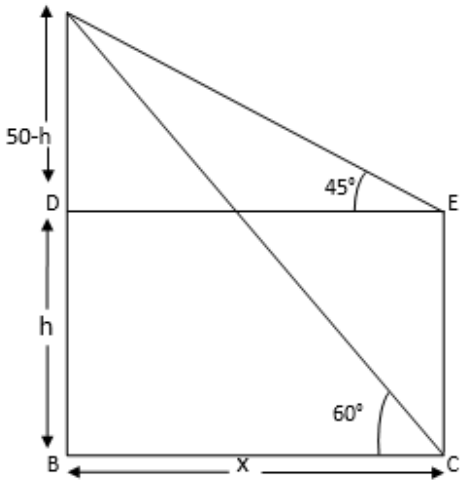
$x = \frac{50}{\sqrt{3}}$ -----(2)

On substituting value of x in eqn (1), we get

$50 - h = \frac{50}{\sqrt{3}}$

h = 50 - $\frac{50}{\sqrt{3}} \Rightarrow \frac{50(\sqrt{3}-1)}{\sqrt{3}}$

⇒ 21.13m



Therefore the height of the pole is 21.13m

47. Question

The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree when seen from the top of the second tree is 45°. If the height of the second tree is 80 m, find the height of the first tree.

Answer

In the fig let CE be the height of the first tree

and AB is the height of the second tree.

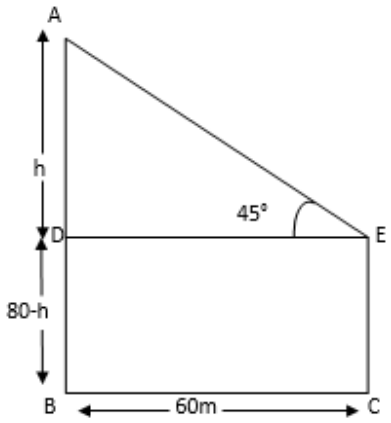
In ΔADE

tan 45° = $\frac{AD}{DE}$

1 = $\frac{h}{60}$

h = 60m

Therefore height of the first tree is 80-60 = 20m

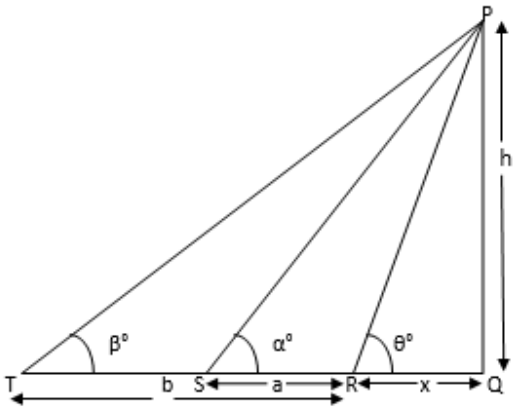


48. Question

A tree standing on a horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, the angles of elevation of the top are respectively α and β. Prove that the height of the top from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

Answer

In the fig let RP be the leaning tree, R & S be the two points at distance 'a' and 'b' from point Q.



In ΔPQR

tan θ° = $\frac{PQ}{QR}$

tan θ° = $\frac{h}{x}$

x = $\frac{h}{tan\theta}$ .....(1)

In ΔPQS

tan α = $\frac{PQ}{QS} \Rightarrow \frac{PQ}{QR+RS}$

tan α = $\frac{h}{x+a}$ .....(2)

In ΔPQT

tan α = $\frac{PQ}{QS} \Rightarrow \frac{PQ}{QR+RS}$

tan β = $\frac{PQ}{PT} \Rightarrow \frac{PQ}{QR+RS+ST}$

tan β = $\frac{h}{x+b}$ …….(3)

On substituting value of x from eqn (1) in eqn (2) we get,

tan α = $\frac{h \tan\theta}{h+a \tan\theta}$

h tan α + a tan θ tan α = h tan θ

h tan α = tan θ(h-a tan α)

tan θ = $\frac{h \tan\alpha}{h- a \tan\alpha}$ …….(4)

Now on substituting value of x in eqn (3)

tan β = $\frac{h \tan\theta}{h+b \tan\theta}$

Now on substituting value of tan θ in eqn (4)

tan β = $\frac{h^2+\tan\theta}{h^2-a\, h \tan\alpha+b\, h \tan\alpha}$

h^2tan β$- a\, h \tan\alpha \tan\beta + b\, h \tan\alpha \tan\beta - h^2 \tan\alpha$ = 0

h(h tan β$- h \tan\alpha + b \tan\alpha \tan\beta - a \tan\alpha \tan\beta)$ = 0

h$(\tan\beta - \tan\alpha)$+tan αtan β(b-a) = 0

h = $\frac{(b-a)\tan\alpha \tan\beta}{\tan\alpha-\tan\beta}$ Proved

49. Question

The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Calculate the height of the tower

Answer

In the fig PQ be the height of the tower.

In ΔQRT

tan 45° = $\frac{QR}{RT}$

1 = $\frac{h}{x}$

h = x …….(1)

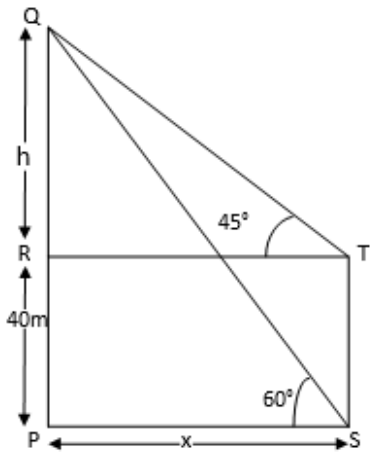
In ΔQRT

tan 60° = $\frac{QR}{PS}$

√3 = $\frac{h+40}{x}$

√3x = h+40 ……..(2)

On substituting the value of x from eqn (1) in eqn (2)



√3h = h+40

h (√3-1) = 40

h = $\frac{40}{\sqrt{3}-1}$ ⇒ 54.64m

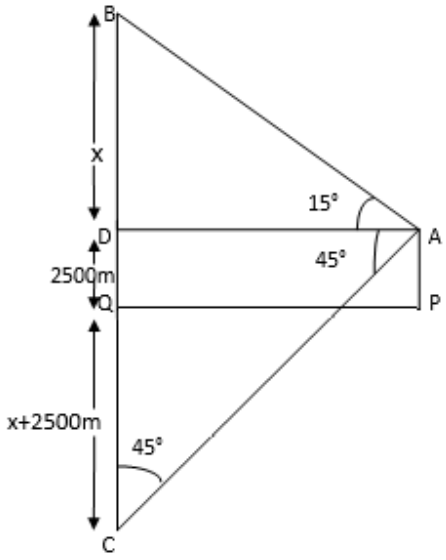
Therefore height of the tower is = h+40 = 54.64+40 ⇒ 94.64m

50. Question

The angle of elevation of a stationery cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level? (Use tan 15° = 0.268)

Answer

In the fig B is the position of the cloud and C is the point of reflection of the cloud in the lake.



In the fig BD = x

DQ = AP= 2500m

QC = BQ = BD + DQ = (2500 + x)m

DC = DQ +QC = 2500 + 2500 + x = (5000 + x)m

In ΔQRT

tan 45° = $\frac{DC}{AD}$

1 = $\frac{5000+x}{AD}$

AD = 5000 + x …….(1)

Tan15° = tan (45° - 30°) = $\frac{tan45°-tan30°}{1+tan45°tan30°}$ [Using formula tan(α-β) = $\frac{tan\alpha-tan\beta}{1+tan\alpha tan\beta}$]

Tan15° = $\frac{1-1/\sqrt{3}}{1+1/\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

In ΔABD

tan 15° = $\frac{BD}{AD}$

$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{x}{5000+x}$

(√3+1)x = (√3+1)(5000+x)

√3x + x = 5000√3 + √3x – 5000 - x

2x = 5000(√3-1)

x = 2500(√3-1)m

Now BQ = BD + DQ = x + 2500

(On substituting value of x)

BQ = 2500(√3-1) + 2500

⇒ 2500√3 – 2500 + 2500

2500√3m

Therefore height of cloud is 2500√3m

51. Question

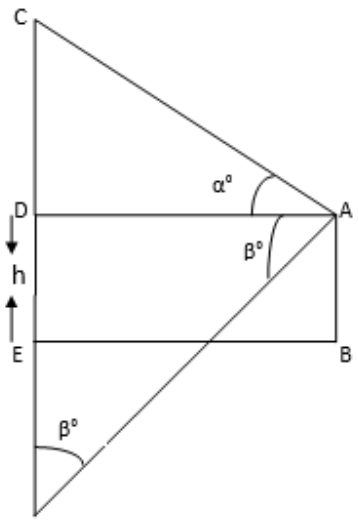
If the angle of elevation of a cloud from a point h metres above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is $\frac{2\ h\ \sec\alpha}{\tan\beta - \tan\alpha}$

Answer

In the fig A is the point of observation and C is the position of the cloud.

Let the distance between the cloud and point of observation is x.

In ΔACD



Sin α = $\frac{CD}{AC}$

CD = AC Sin α ⇒ x Sin α

Cos α = $\frac{AD}{AC}$

AD = x Cos α ……(1)

CE = CD + DE = (h + x Sin α)

EF = CE = (h + x Sin α)

DF = DE + EF = (h + h + x Sin α) = (2h + x Sin α) ……..(2)

In ΔADF

tan β = $\frac{DF}{AD}$

On substituting value of DF & AD

from above eqns (1) and (2)

tan β = $\frac{2h+x\ Sin\ \alpha}{x\ Cos\ \alpha}$

$\frac{Sin\ \beta}{Cos\ \beta} = \frac{2h+x\ Sin\ \alpha}{x\ Cos\ \alpha}$

2h Cos β + x Sin α Cos β = x Sin β Cos α

x (Cos α Sin β - Sin α Cos β) = 2h Cos β

x = $\frac{2h\ Cos\ \beta}{\text{Cos } \alpha \text{ Sin } \beta - \text{Sin } \alpha \text{ Cos } \beta}$

On dividing numerator and denominator by Cos α Cos β, we get

x = $\frac{2h\ Sec\ \alpha}{\tan\beta - \tan\alpha}$

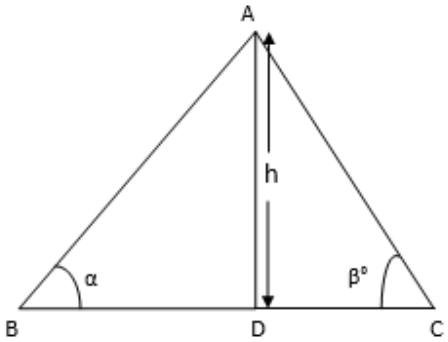
Therefore the distance between cloud and point of observation is $\frac{2h\ Sec\ \alpha}{\tan\beta - \tan\alpha}$ m

52. Question

From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by $\frac{\tan\alpha \tan\beta}{\tan\alpha + \tan\beta}$

Answer

In the fig let B & C be two mile stones. And height of the aeroplane is AD



In ΔABD

tan α = $\frac{AD}{BD}$

tan α = $\frac{h}{BD}$

h = BD tan α ……(1)

In ΔACD

tan β = $\frac{AC}{CD}$

tan β = $\frac{h}{CD}$

CD = $\frac{h}{\tan \beta}$ ……(2)

On adding eqn (1) and (2)

BD + CD = $\frac{h\,(\tan \alpha + \tan \beta)}{\tan \alpha \tan \beta}$

Now as B and C are milestones, the distance between them = 1

Therefore, BD + CD = 1

1 = $\frac{h\,(\tan \alpha + \tan \beta)}{\tan \alpha \tan \beta}$

h = $\frac{(\tan \alpha \tan \beta)}{\tan \alpha + \tan \beta}$ Proved

53. Question

PQ is a post of given height a, and AB is a tower at some distance. If α and β are the angles of elevation of B, the top of the tower, at P and Q respectively. Find the height of the tower and its distance from the post.

Answer

In the fig PQ is the post off height 'a' and AB is the tower of height 'h'

In ΔBAP

tan α = $\frac{AB}{AP}$

tan α = $\frac{h}{AP}$

AP = h tan α ……(1)

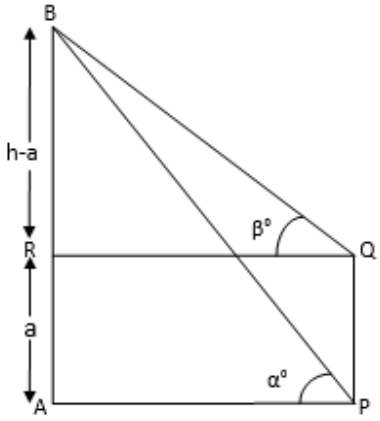
In ΔBRQ

tan β = $\frac{BR}{QR}$

tan β = $\frac{h-a}{AP}$

AP = $\frac{h-a}{\tan \beta}$ ……(2)

From eqn (1) and (2) we get,



$\frac{h-a}{\tan\beta} = \frac{h}{\tan\alpha}$

h tan β = (h-a) tan α

h tan β = h tan α – a tan α

h (tan α - tan β) = a tan α

h = $\frac{a\tan\alpha}{\tan\alpha-\tan\beta}$

Therefore height of the tower is $\frac{a\tan\alpha}{\tan\alpha-\tan\beta}$

Now AP = $\frac{h}{\tan\alpha} = \frac{\frac{a\tan\alpha}{\tan\alpha-\tan\beta}}{\tan\alpha}$

⇒ $\frac{a\tan\alpha}{\tan\alpha(\tan\alpha-\tan\beta)}$

⇒ $\frac{a\tan\alpha}{\tan\alpha(\tan\alpha-\tan\beta)}$

Therefore distance is ⇒ $\frac{a}{\tan\alpha-\tan\beta}$

54. Question

A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Answer

In the fig let the length of ladder is h (m)

In ΔAEB

Sin α = $\frac{AE}{AB}$

Sin α = $\frac{AE}{h}$

AE = h Sin α

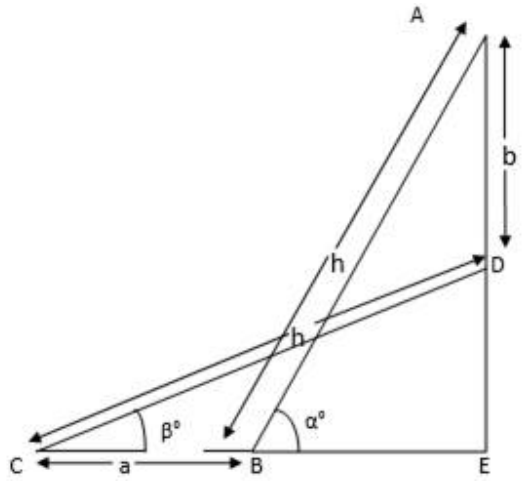
Cos α = $\frac{BE}{AB}$

Cos α = $\frac{BE}{h}$

BE = h Cos α

In ΔDEC

Sin β = $\frac{DE}{CD}$



Sin β = $\frac{DE}{h}$

DE = h Sin β

Cos β = $\frac{CE}{CD}$

Cos β = $\frac{CE}{h}$

CE = h Cos β

Now

$\frac{BC}{AD} = \frac{a}{b}$

$\frac{CE-BE}{AE-DE} = \frac{a}{b}$

$\frac{a}{b} = \frac{h\,Cos\beta - h\,Cos\,\alpha}{h\,Sin\,\alpha - h\,Sin\,\beta}$

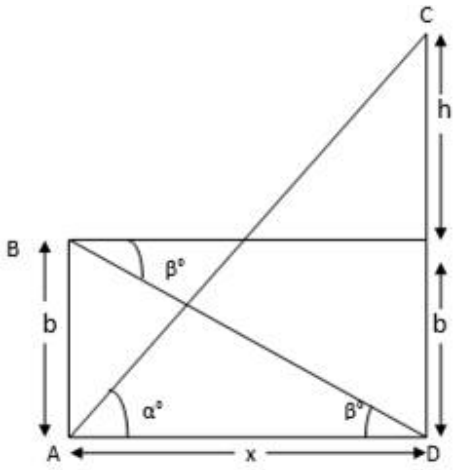
$\frac{a}{b} = \frac{Cos\beta - Cos\,\alpha}{Sin\,\alpha - Sin\,\beta}$ Proved

55. Question

A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b metres just above A is β. Prove that the height of the tower is b tan α cot β.

Answer

In the fig let CD is the height of tower



CD = a + b

In ΔABD

Cot β = $\frac{AD}{AB}$

Cot β = $\frac{x}{b}$

x = b Cot β ……(1)

In ΔADC

tan α = $\frac{CD}{AD}$

tan α = $\frac{CD}{b\,Cot\,\beta}$

⇒ CD = b Cot β tan α

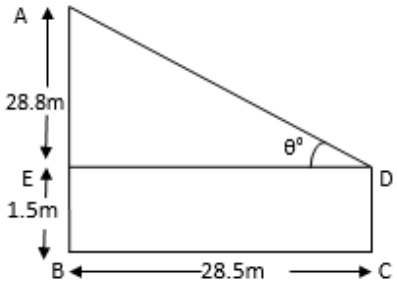
Therefore height of the tower is b Cot β tan α

56. Question

An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the tower from his eye.

Answer

In the fig let DC is the observer of the height 1.5m.



In ΔAED

$\tan \theta = \frac{AE}{DE}$

$\tan \theta = \frac{28.5}{28.5}$

$\tan \theta = 1$

$\theta = \tan^{-1} 1$

$\theta = 45°$

Hence the angle of the observation of

the tower from observer's eye is 45°

57. Question

A carpenter makes stools for electricians with a square top of side 0.5 m and at a height of 1.5 m above the ground. Also, each leg is inclined at an angle of 60° to the ground. Find the length of each leg and also the lengths of two steps to be put at equal distances.

Answer

In the fig let the height of the stool from the ground is 1.5m i.e. AL = BM = 1.5m

In ΔACL

$\tan 60° = \frac{AL}{CL}$

$\sqrt{3} = \frac{1.5}{CL}$

$CL = \frac{1.5}{\sqrt{3}} \Rightarrow \frac{1.5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

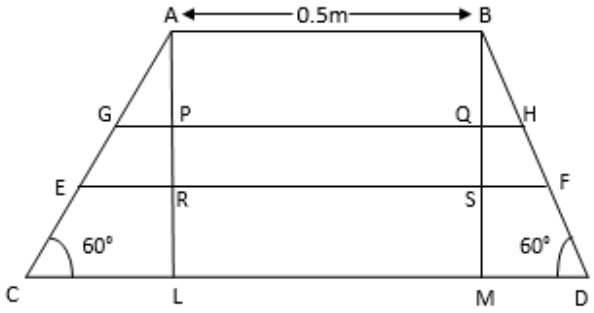
$\Rightarrow \frac{1.5\sqrt{3}}{3} = 0.866m$

Again in ΔACL

$\text{Sin } 60° = \frac{AL}{AC}$

$\frac{\sqrt{3}}{2} = \frac{1.5}{AC}$

AC = $\frac{3}{\sqrt{3}}$ = √3m ⇒ 1.732m



Lengths of steps are GH and EF and steps are put at equal distance such that AP =PR = RL

Now consider ΔAGP and ΔACL

$\frac{AP}{AL} = \frac{GP}{CL}$

$\frac{AP}{3AP} = \frac{GP}{0.866}$

GP = 0.2887m

Length of step = GH = GP + PQ + QH = GP + AB + GP (Since QH = GP)

GH = 2GP + AB = 2 × 0.2887 + 0.5 = 1.0774m

Similarly we can say that,

ER = $\frac{2CL}{3}$ ⇒ $\frac{2\times0.866}{3}$ = 0.577m

Length of the step is EF = ER + RS + SF

⇒ ER + AB + ER = 2ER + AB

EF = 2× 0.5773 + 0.5 ⇒ 1.6546m

58. Question

A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Answer

In the fig 'C' is the position of the kites. Let the length of second kite string is h.

In ΔABC

Sin 30° = $\frac{BC}{AC}$

$\frac{1}{2} = \frac{10+x}{100}$

20 + 2x = 100

x = 40m ……..(1)

In ΔCFD

Sin 45° = $\frac{CF}{CD}$

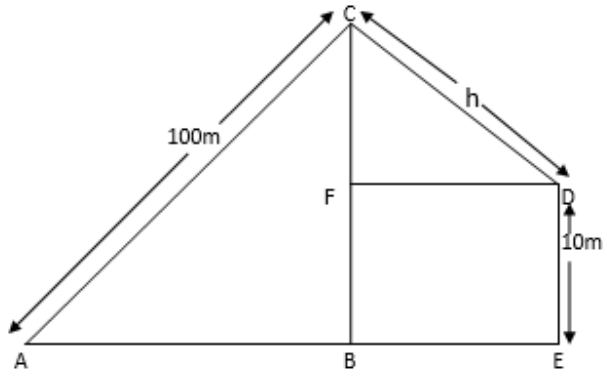
$\frac{1}{\sqrt{2}} = \frac{x}{h}$

h = √2 x ……..(2)

On substituting value

of x from eqn (1) in eqn (2)

h = √2 × 40 ⇒ 40√2 m



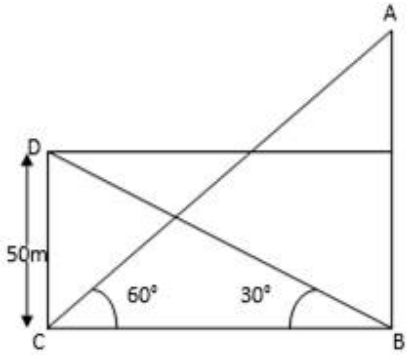
Therefore length of string of second kite is 40√2 m

59. Question

The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill?

Answer

In the fig DC is the tower and AB is the hill.



In ΔDCB

tan 30° = $\frac{DC}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50}{BC}$

BC = 50√3 m

In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{AB}{50\sqrt{3}}$

AB = 50√3× √3 m ⇒ 150m

Therefore the distance between tower and hill is 50√3m and height of hill is 150m

60. Question

Two boats approach a light house in mid-sea from opposite directions. The angles of elevation of the top of the light house from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the light house.

Answer

In the fig AB is the light house of height h (m)

In ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{100-x}$

x = 100 - √3 h ……(1)

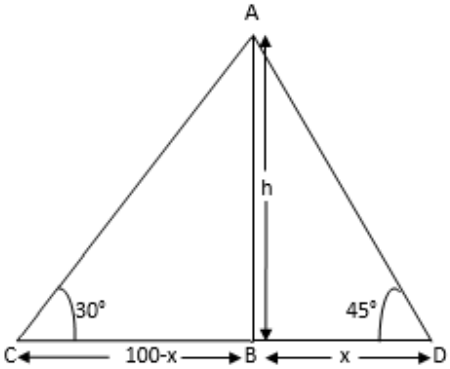
In ΔABD

tan 45° = $\frac{AB}{BD}$

1= $\frac{h}{x}$

x = h ……(2)

On substituting value of x from eqn (2) in eqn (1)



h = 100 - √3 h

h + √3 h = 100

h (1 + √3) = 100

h = $\frac{100}{1+\sqrt{3}} = \frac{100(\sqrt{3}-1)}{2}$ ⇒ 50 (√3 - 1)

Therefore height of the light house is 50 (√3 - 1)m

61. Question

From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find

(i) the horizontal distance between AB and CD.

(ii) the height of the lamp post.

(iii) the difference between the heights of the building and the lamp post.

Answer

In the fig AB is the building of height 60m and CD is the lamp post of height h (m)

(i) In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{60}{BC}$

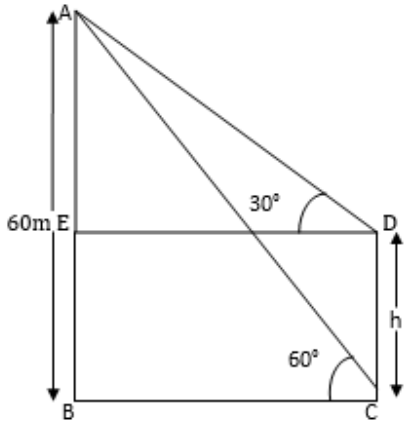
BC = $\frac{60}{\sqrt{3}}$

On multiplying and dividing by √3, we get

BC = $\frac{60\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$ ⇒ 20√3

Therefore distance between building

and lamp post is 20√3 m



(ii) In ΔAED

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$

$\sqrt{3}AE$ = 20√3

AE = $\frac{20\sqrt{3}}{\sqrt{3}}$ = 20

Therefore height of lamp post is CD = AB-AE ⇒ 60-20 = 40 m

Therefore height of lamp post is 40 m

(iii) The difference between the height of the building and the lamp post is 60-40 = 20m

62. Question

From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light house be h metres and the line joining the ships passes through the foot of the light house, show that the distance between the ship is $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \tan\beta} metres$

Answer

In the fig AB is the light house of height h (m)

In ΔADC

tan β = $\frac{AD}{DC}$

tan β = $\frac{h}{y}$

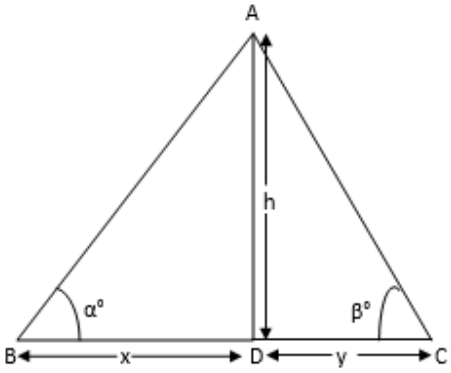
h = y tan β or y = $\frac{h}{\tan\beta}$.....(1)

In ΔADB

tan α = $\frac{AD}{BD}$

tan α = $\frac{h}{x}$

h = x tan α or x = $\frac{h}{\tan \alpha}$ ……(2)



The distance between the two ships is BC = x + y

On adding eqn (1) & (2) we get,

x + y = $\frac{h}{\tan \alpha} + \frac{h}{\tan \beta}$

⇒ $\frac{h \tan \beta + h \tan \alpha}{\tan \alpha \tan \beta}$

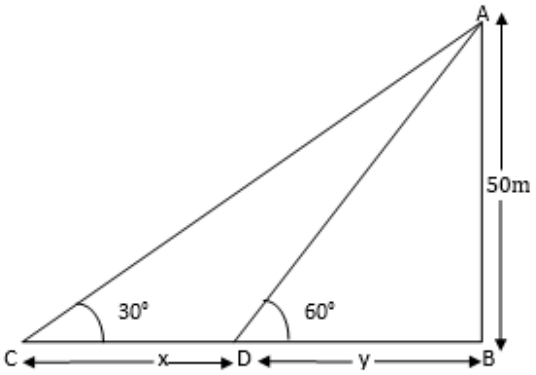
$\frac{h(\tan \beta + \tan \alpha)}{\tan \alpha \tan \beta}$ meters *PROVED*

63. Question

A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60° respectively. What is the distance between the two cars and how far is each car from the tower?

Answer

In the fig AB is the tower on the highway.



In ΔABC

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50}{x+y}$

$x + y = 50\sqrt{3}$ ……..(1)

In ΔABD

$\tan 60° = \frac{AB}{BD}$

$\sqrt{3} = \frac{50}{y}$

$y = \frac{50}{\sqrt{3}}$

On multiplying and dividing by √3, we get

$y = \frac{50\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{50\sqrt{3}}{3}$ ……(2)

Therefore the distance between

the first car and tower is 28.87m

On substituting value of y

from eqn (2) in eqn (1)

$x + \frac{50\sqrt{3}}{3} = 50\sqrt{3}$

$x = 50\sqrt{3} - \frac{50\sqrt{3}}{3}$

$\Rightarrow \frac{3\times50\sqrt{3} - 50\sqrt{3}}{3}$

$\Rightarrow \frac{150\sqrt{3} - 50\sqrt{3}}{3} = \frac{100\sqrt{3}}{3} = 57.73m$

The distance between two cars is 57.73m

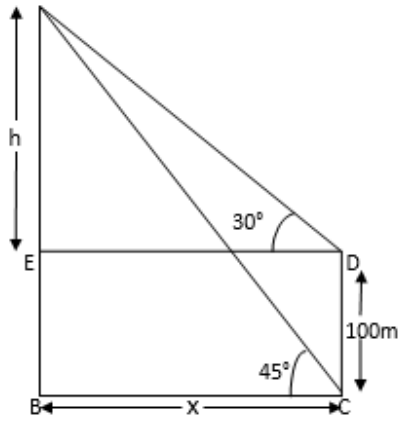
The distance between second car and tower is (x + y) = 57.73 + 28.87 = 86.60m

64. Question

The angles of elevation of the top of a rock from the top and foot of a 100 m high tower are respectively 30° and 45°. Find the height of the rock.

Answer

In the fig AB is the Rock and CD is the tower.



AB = AE + EB ⇒ h + 100

In ΔABC

tan 45° = $\frac{AB}{BC}$

1 = $\frac{100+h}{x}$

x = 100 + h .....(1)

In ΔAEB

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{100+h}$

√3h= 100 + h

h(√3-1) = 100

h = $\frac{100}{\sqrt{3}-1}$

On multiplying and dividing by √3 + 1, we get

h = $\frac{100(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{100(\sqrt{3}+1)}{2}$

⇒ 50 (√3 + 1) = 136.6m

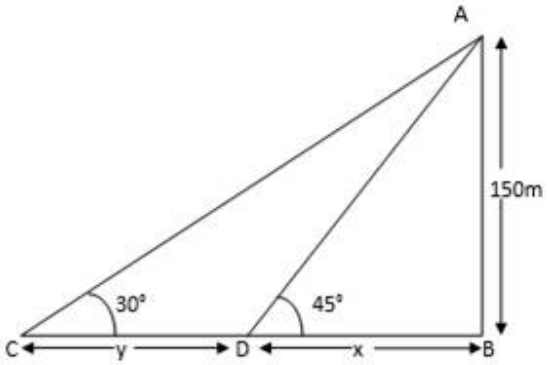
Therefore height of the rock = h +100 ⇒ 136.6 + 100 = 236.6m

65. Question

As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships.

Answer

In ΔABD



tan 45° = $\frac{AB}{BD}$

1 = $\frac{150}{x}$

x = 150m ……..(1)

In ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{150}{x+y}$

x + y = 150√3 ……..(2)

On substituting value of x from eqn (1) in eqn (2), we get

150 + y = 150√3

y = 150 (√3 - 1) ⇒ 109.8m

Therefore the distance between two ships is 109.8m

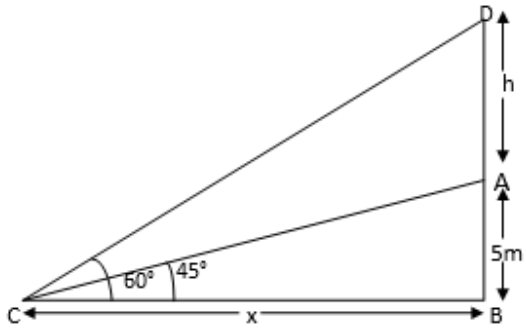
66. Question

A flag-staff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is 60° and from the same point, the angle of elevation of the top of the tower is 45°. Find the height of the flag-staff.

Answer

In the fig let AD is the Flag-Staff of height h (m)

In ΔABC



$\tan 45° = \frac{AB}{BC}$

$1 = \frac{5}{x}$

x = 5m

In ΔDBC

$\tan 60° = \frac{DB}{BC}$

$\sqrt{3} = \frac{h+5}{5}$

$h + 5 = 5\sqrt{3}$

$h = 5\sqrt{3}-5$

$h = 5(\sqrt{3}-1)$

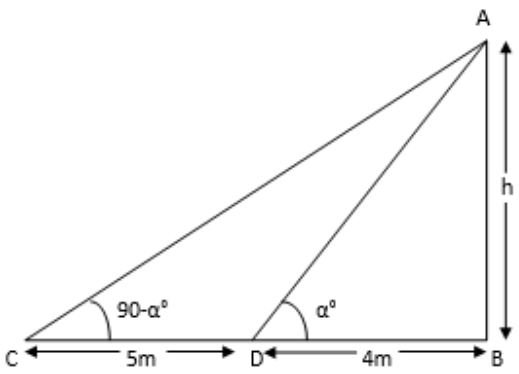
h = 3.66 m

Therefore height of flag-staff is 3.66m

67. Question

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer



In the fig let AB is the Tower of height h (m).Since the tower is vertical to the ground.

10. Circles

Exercise 10.1

1. Question

Fill in the blanks:

(i) The common point of a tangent and the circle is called…… .

(ii) A circle may have …… parallel tangents.

(iii) A tangent to a circle intersects it in …… points(s).

(iv) A line intersecting a circle in two points is called a …… .

(v) The angle between tangent at a point on a circle and the radius through the point is …… .

Answer

(i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(ii) A circle may have two parallel tangents

(iii) A tangent to a circle intersects it in one point.

(iv) Secant is a line intersecting a circle in two points

(v) The angle between tangent at a point on a circle and the radius through the point is 90°

2. Question

How many tangents can a circle have?

Answer

A circle can have infinite tangents.

3. Question

O is the centre of a circle of radius 8 cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB.

Answer

$$OB^2 = 8^2 + 15^2$$
$$OB = \sqrt{64 + 225}$$
$$OB = 17cm$$

4. Question

If the tangent at a point P to a circle with centre O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm. Find the radius of the circle.

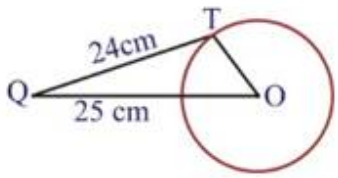
Answer

since QT is a tangent to the circle at T and OT is radius,

Therefore OT perpendicular QT

It is given that OQ=25 cm and QT= 24 cm

By Pythagoras theorem we have



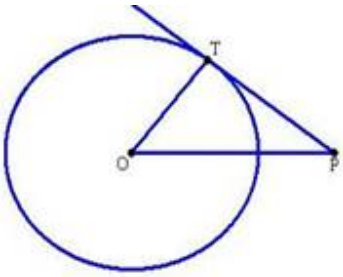
$$OB^2 = OA^2 + AB^2$$
$$OQ^2 = QT^2 + OT^2$$
$$OT^2 = 25^2 - 24^2$$
$$= 49 \times 1 = 49$$
$$OT = 7$$

Exercise 10.2

1. Question

If PT is a tangent at T to a circle whose centre is O and OP = 17 cm, OT = 8 cm, find the length of the tangent segment PT.

Answer



Given that O is the center of the circle and OP =17 cm and the radius of the circle OT=8cm.

We need to find the length of the segment PT.

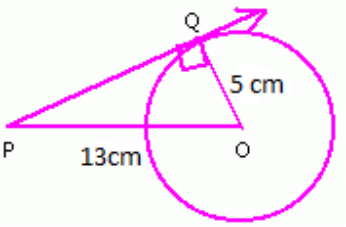
The line PT is the tangent line to the circle at the point T, the line through the centre is perpendicular to PT.

$$OP^2 = OT^2 + PT^2$$
$$17^2 = 8^2 + PT^2$$
$$PT^2 = 289 - 64$$
$$= 225$$
$$PT^2 = \sqrt{225} = 15$$

2. Question

Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle.

Answer



Given: PQ is a tangent to the circle intersect at OP=13cm and OQ=5 cm

Proof: In right triangle OQP

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{169 - 25} = 12cm$$

Therefore the length of the tangent from the point is $12cm$

3. Question

A point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Answer

Let PQ be a tangent to the circle from point P and OQ be the radius at the point of contact.

$\therefore \quad \angle OQP = 90°$

$\Rightarrow \quad OP^2 = OQ^2 + PQ^2$

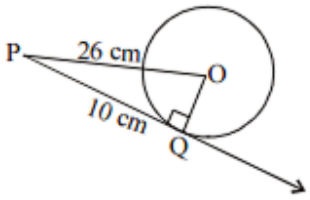
$(\because$ using Pythagoras theoren

$\Rightarrow \quad OQ^2 = OP^2 - PQ^2 = 26^2 - 10^2$

$= (26 + 10)(26 - 10) = 36 \times 16$

$\Rightarrow \quad OQ = 6 \times 4 = 24$

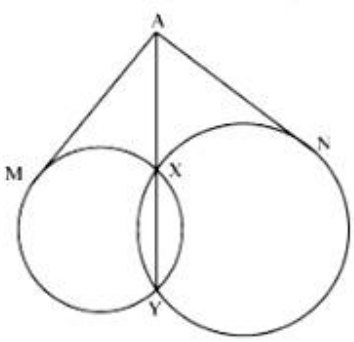
$\therefore$ Radius of the circle = **24 cm.**



4. Question

If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.

Answer



Let the two circle intersect at a point X and Y , XY is the common chord.

Suppose A is a point on their common chord and AM and AN be the tangent drawn from A to the circle

AM is the tangent and AXY is a secant.

$AM^2 = AX \times AY$ .....(i)

AN is the tangent and AXY is the secant.

$AN^2 = AX \times AY$ .....(i)

Therefore, from equations (i) and (ii), we get,

AM = AN.

5. Question

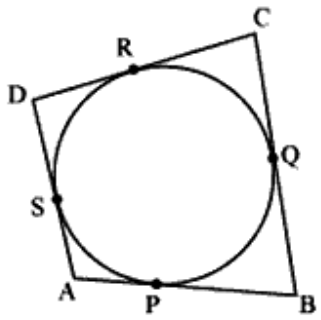
If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair.

Answer

Given: the sides of a quadrilateral touch a circle

To prove: the sum of a pair of opposite sides is equal to the sum of the other pair.

Proof:



From the theoram which states that the lengths of the two tangents drawn from an external point to a circle are equalFrom points A the tangents drawn are AP and AS,AP = AS .... (1)From points B the tangents drawn are BP and BQ,BP = BQ ..... (2)From points D the tangents drawn are DR and DS,DR = DS ....(3)From points C the tangents drawn are CR and CQ,CR = CQ ..... (4)Add 1,2,3 and 4 to getAP+BP+DR+CR = AS+BQ+DS+CQ(AP+BP)+(DR+CR)= (AS+DS)+ (BQ+CQ)AB+ DC = AD + BC

Hence proved

6. Question

If AB, AC, PQ are tangents in Fig. 10.51 and AB = 5 cm, find the perimeter of $\triangle APQ$.

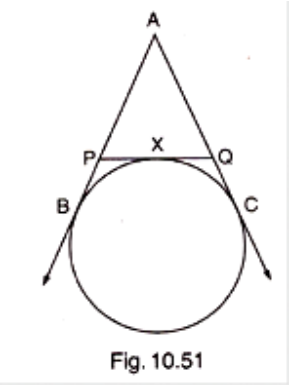


Fig. 10.51

Answer

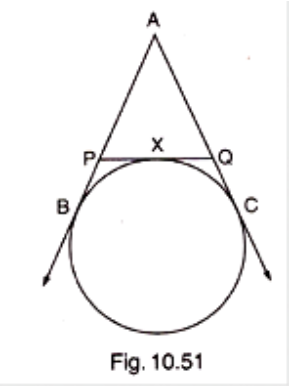


Fig. 10.51

Given: AB and Ac are tangent to the circle with centre O

PQ is tangent to the circle at X which intersect AB and Ac in P and Q

To find : Perimeter of triangle APQ

Proof:

$\mathrm{AB} = \mathrm{AC}$

$QC = QX$

$PB = PX$

$AB = AC = 5cm$

$Perimeter\ of\ \Delta APQ = AQ + QP + AP$

$= AC + AB = 5 + 5 = 10cm$

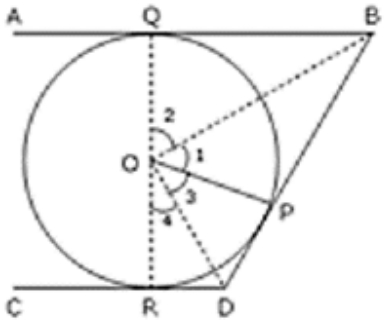
7. Question

Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Answer

Three tangent AB,CD and BD of a circle such as AB and CD are two parallel tangent BD intercept an angle BOD at the centre.

To Prove: $\angle BOD = 90^\circ$



$Construction: Join\ OQ\ and\ OR$

$\text{Pr}oof: OP \perp BD$

$In\ rtght\ angle\ \triangle OQB\ and\ OPB.$

$\triangle s\ OQB \cong OPB.$

$\text{Sin}ce\ OQ = OP, \angle Q = \angle P\ and\ OB\ is\ common$

$So\ \angle 1 = \angle 2$

$Simillarly\ in\ \triangle s\ ORD\ and\ OPD\ we\ have\ \angle 3 = \angle 4$

$\angle BOD = \angle 1 + \angle 3$

$$= \frac{1}{2}(\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2}(180^\circ) = 90^\circ$$

8. Question

In Fig. 10.52, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30^0$, find $m\angle PRS$.

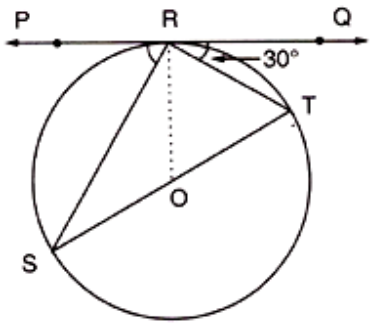


Fig. 10.52

Answer

$Given\ that, \angle TRQ = 30^\circ$

$\text{Sin}ce, ST\ is\ a\ diameter\ and\ angle\ in\ a\ semi-circle\ is\ a\ rt.\ angle$

$Therefore, \angle SRT = 90^\circ$

$Now,$

$\angle TRQ + \angle SRT + \angle PRS = 180^\circ$

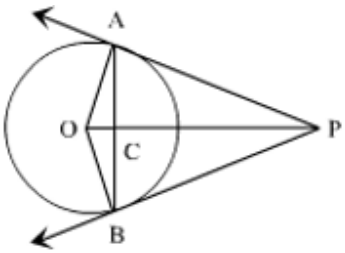
$30^\circ + 90^\circ + \angle PRS = 180^\circ$

$\angle PRS = 60^\circ$

9. Question

If PA and PB are tangents from an outside point P. such that PA = 10 cm and $\angle APB = 60^0$. Find the length of chord AB.

Answer



Given: PA and PB are tangent of a circle PA= 10 cm and angle APB= 60^o

Let O be the center of the given circle and C be the point of intersection of OP and AB

In triangle PAC and triangle PBC

PA = PB (tangent from an external point are equal)

APC =BPC (tangent from an external point are equally inclined to the segment joining center to the point)

PC =PC (common)

$\triangle PAC \cong \triangle PBC\,(by\ SAS)$

$AC = BC$

$\angle APB = \angle APC + \angle BPC$

$\angle APC = \frac{1}{2}\angle APB = \frac{1}{2}\times 60^0 = 30^0$

$\angle APC + \angle BCP = 180^\circ = 90^\circ$

$In\ rt\ triangle\ ACP$

$\sin 30^\circ = \frac{AC}{AP}$

$\frac{1}{2} = \frac{AC}{10} = 5cm$

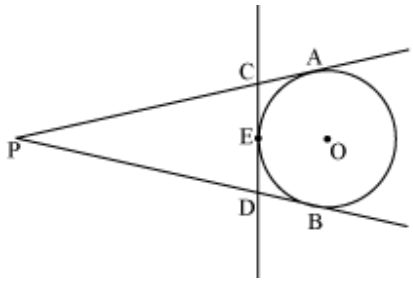
$AC = BC$

$AB = 5 + 5 = 10\,cm$

10. Question

From an external point P, tangents PA are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of $\triangle PCD$.

Answer



Given: PA and PB are tangent to the circle with centre O

CD is tangent to the circle at E which intersect PA and PB in C and D

To find : Perimeter of triangle PCD

Proof:

$\mathrm{PA} = \mathrm{PB}$

$CA = DE$

$DB = DE$

$PA = PB = 14cm$

$Perimeter\ of\ \Delta PCD = PC + CD + PD$

$= PA + PB = 14 + 14 = 28cm$

11. Question

In Fig. 10.53, ABC is a right triangle right-angled at B such that BC = 6 cm and AB = 8 cm. Find the radius of its incircle.

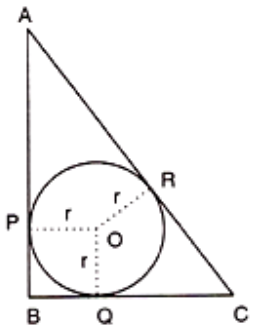
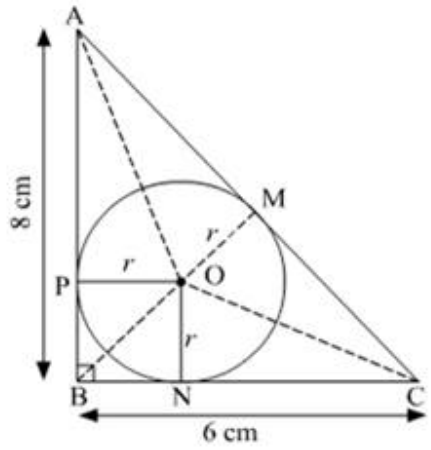


Fig. 10.53

Answer

Let ABC be the right angled triangle such that angle B=90^{o}, BC=6cm, AB= 8cm. Let O be the centre and r be the radius of the in circle.



AB, BC and CA are tangent to the circle at P,N and M

OP=ON=OM=r (radius of the circle)

$$Area\ of\ \triangle ABC = \frac{1}{2} \times 6 \times 8 = 24\ cm^2$$

By Pythagoras theorem,

$$CA^2 = AB^2 + BC^2$$

$$CA^2 = 8^2 + 6^2$$

$$CA = 10\ cm$$

$$Area\ of\ \triangle ABC = Area\ of\ \triangle OAB + Area\ of\ \triangle OBC + Area\ of\ \triangle OCA$$

$$24 = \frac{1}{2} r \times AB + \frac{1}{2} r \times BC + \frac{1}{2} r \times CA$$

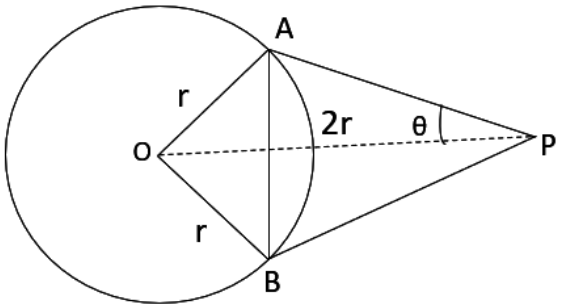
$$r = \frac{2 \times 24}{AB + BC + CA}$$

$$r = \frac{48}{8 + 6 + 10} = \frac{48}{24} = 2\ cm$$

12. Question

From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that $\triangle APB$ is equilateral.

Answer



AP is the tangent to the circle,

According to the theorem which states that tangent to a circle is perpendicular to the

radius through the point of contact.

$\Rightarrow OA \perp AP$

$\angle OAP = 90^\circ$

Also $OB \perp BP$

$\Rightarrow \angle OBP = 90^\circ$

In Δ OAP sin θ =perpendicular/hypotenuse

$$\sin\angle OPA = \frac{r}{2r} = \frac{1}{2}$$

As

$$sin\ 30^\circ = \frac{1}{2}$$

$$\Rightarrow \angle OPA = 30^\circ$$

Similarly $\angle OPB = 30^\circ$Now $\angle APB = \angle OPA + \angle OPB$

$= 30^\circ + 30^\circ$

$= 60^\circ$ .... (1)

In Δ PAB,As PA and PB are drawn from external point P,By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal

.⇒ PA=PBAlso $\angle PAB = \angle PBA$ .... (2)

As $\angle PAB + \angle PBA + \angle APB = 180^\circ$ (sum of angles of triangle)

$\angle PAB + \angle PBA = 180^\circ - \angle APB$

$\angle PAB + \angle PBA = 180^\circ - 60^\circ$

$\Rightarrow 2\angle PAB = 120^\circ$

$\Rightarrow \angle PAB = 60^\circ$ .... (3)

From 1 and 2 and 3,$\angle PAB = \angle PBA = \angle APB = 60^\circ$Hence ΔPAB is an equilateral triangle.

13. Question

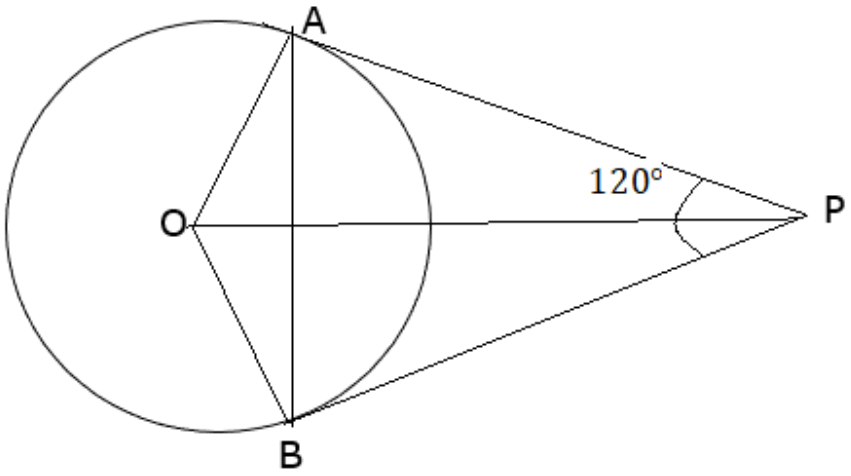
Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$. Prove that OP = 2 AP.

Answer

Given: Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$.

To prove: OP = 2 AP

Proof: Construct the figure according to the conditions given.



HereIn triangle OAP and OBP,PA = PB (Length of Tangents from external point are equal)

OA = OB (Radii of same circle)

OP = OP (common)

Δ OAP ~ Δ OBP (By SSS criterion)

∠OPA = ∠OPB = 60^o.

In Triangle OAP , ∠OAP = 90^o (By theoram which states that tangent to a circle is perpendicular to the radius through the point of contact)We know in a right angle triangle

$$\sin\theta = \frac{perpendicular}{hypotenuse}$$

⇒ sin 60^o= AP / OP , i.e 1/2 = AP / OP

So,OP = 2 AP

Hence proved.

14. Question

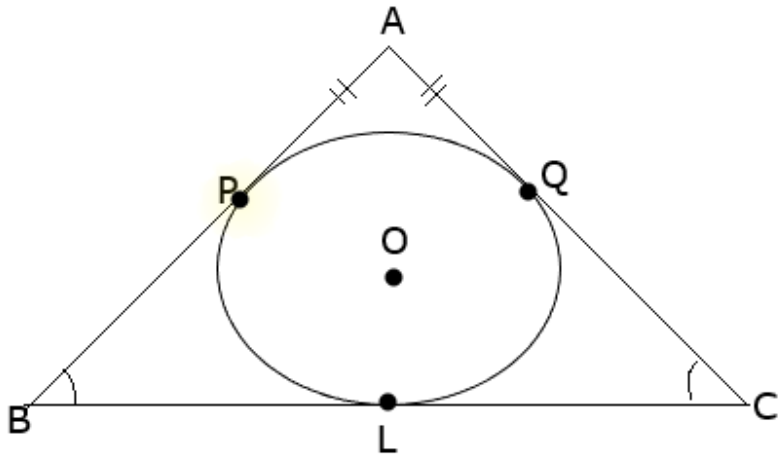
If $\triangle ABC$ is isosceles with AB = AC and C (O, r) is the incircle of the $\triangle ABC$ touching BC at L, prove that L bisect BC.

Answer

Given: If $\triangle ABC$ is isosceles with AB = AC and C (O, r) is the incircle of the $\triangle ABC$ touching BC at L.

To prove: L bisect BC.

Proof: Construct the figure according to given condition.



AB = AC (given)From the theorem which states that the lengths of two tangents drawn from external point to a circle are equal. ..... (1) As tangents AP and AQ are drawn from the external point A.AP = AQ Also,AB = AC⇒ AP + PB = AQ + QC⇒ AP + PB = AP + QC⇒ PB = QCFrom (1) as tangents BP and BL are drawn from external point B,And tangents CQ and CL are drawn from external point C.⇒ BP = BL ..... (3) CQ = CL ..... (4)As we have proved PB = QCFrom 3 and 4BL = CL⇒ L bisects BC.Hence proved.

15. Question

In Fig. 10.54, a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.

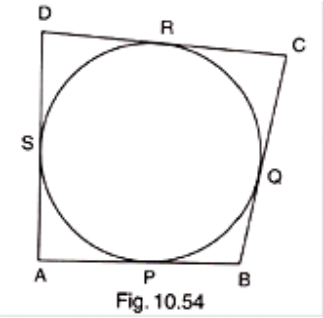


Fig. 10.54

Answer

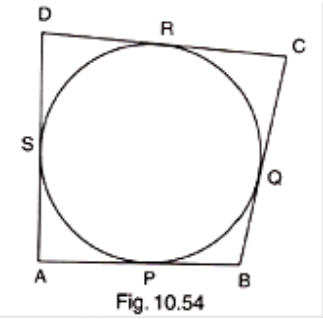


Fig. 10.54

$Here,$

$AP = AS$

$Let,\ AP = AS = X$

$Simillarly, BP = BQ$

$CQ = CR$

$RD = DS$

$\text{Sin}\, ce, AP = X$

$\Rightarrow BP = AB - AP = 6 - X$

$Now,\ BP = BQ = 6 - X$

$CQ = BC - BQ = 7 - (6 - X)$

$= 1 + X$

$now, CQ = CR = 1 + X$

$RD = CD - CR = 4 - (1 + X)$

$= 3 - X$

$RD = DS = 3 - X$

$AD = AS + SD$

$X + 3 - X = 3$

$AD = 3cm$

$Here,$

$AP = AS$

$Let,\ AP = AS = X$

$Simillarly, BP = BQ$

$CQ = CR$

$RD = DS$

$\text{Sin}\, ce, AP = X$

$\Rightarrow BP = AB - AP = 6 - X$

$Now,\ BP = BQ = 6 - X$

$CQ = BC - BQ = 7 - (6 - X)$

$= 1 + X$

$now, CQ = CR = 1 + X$

$RD = CD - CR = 4 - (1 + X)$

$= 3 - X$

$RD = DS = 3 - X$

$AD = AS + SD$

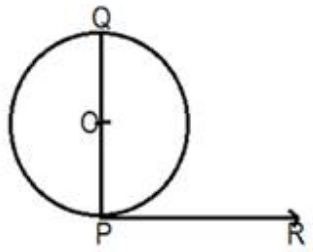
$X + 3 - X = 3$

$AD = 3cm$

16. Question

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

Answer



Draw a circle with centre O, draw a tangent PR touching circle at P.Draw QP perpendicular to RP at a point P, QP lies in the circle.Now,∠OPR = 90ºAlso, ∠QPR = 90º Therefore,∠OPR = ∠QPRThis is possible only when O lies on QP.Hence, it is proved that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

17. Question

In fig. 10.55, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^0$.

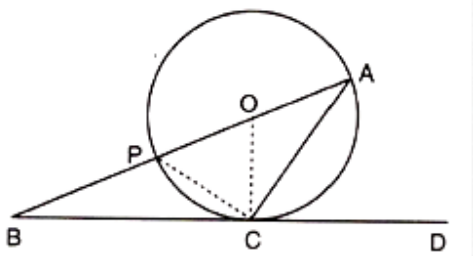


Fig. 10.55

Answer

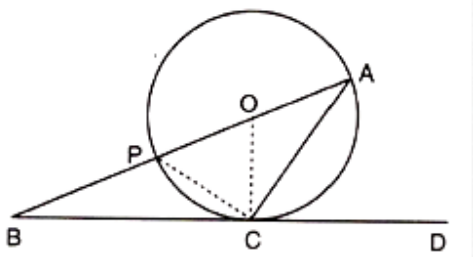


Fig. 10.55

Given: In the above figure, O is the centre of the circle and BCD is tangent to it at C.To prove: ∠BAC + ∠ACD = 90°Proof:

In ΔOAC

OA = OC [radii of same circle]

⇒ ∠OCA = ∠OAC [angles opposite to equal sides are equal]

⇒ ∠OCA = ∠BAC [1]

Also,

OC ⊥ BD [Tangent at any point on a circle is perpendicular to the radius through point of contact]

⇒ ∠OCD = 90°

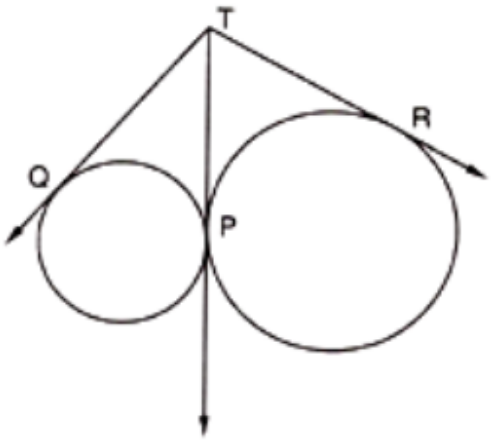
⇒ ∠OCA + ∠ACD = 90°

⇒ ∠BAC + ∠ACD = 90° [From 1]

Hence Proved

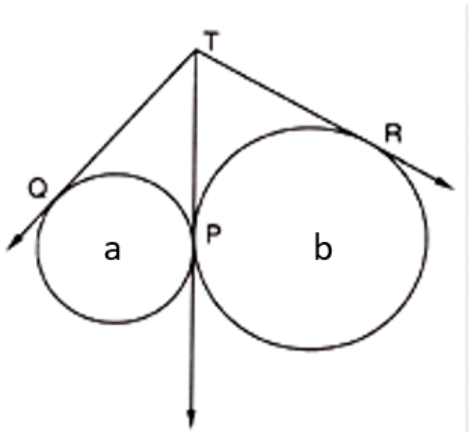
18. Question

Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that TQ = TR



Answer

Let us label two circles as 'a' and 'b'



As TQ and TP are tangents to circle a,And TP and TR are tangents to circle b.By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal.

TQ=TP ...(1)TP=TR ...(2)From 1 and 2,TQ=TRHence proved

19. Question

In Fig 10.57, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^{o}$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius r of the circle.

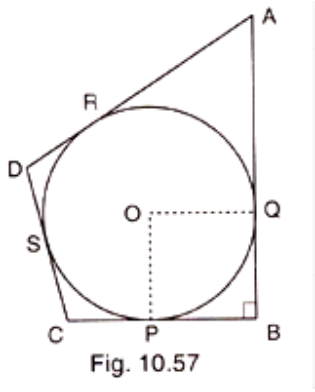
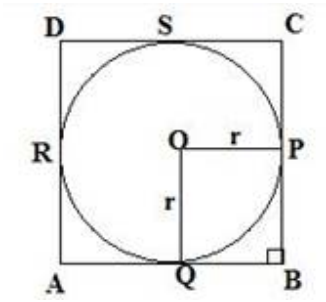


Fig. 10.57

Answer



Given: ABCD is a quadrilateral in which

$\angle B = 90^{\circ}$

$AD = 23cm, DS = 5cm \text{ and } AB = 29cm$

Let the radius of the incircle be r cm.

AD = DS = 5cm (tangent from an external point)

Since AD=23 cm

So,

AR + RD + AD

AR + 5 = 23 cm

AR = 18 cm ------(i)

and AQ = AR

since AR = 18 cm

So, AQ + QB = AB

Now OP and OQ are radius of the circle. So from tangent P and Q

$\angle OPB = \angle OQB = 90^{\circ}$

$OPBQ \text{ is a square}$

$OP = QB$

$Radius \text{ of the circle} = 11cm$

20. Question

In Fig. 10.58, there are two concentric circles with centre O of radii 5 cm and 3 cm. from an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

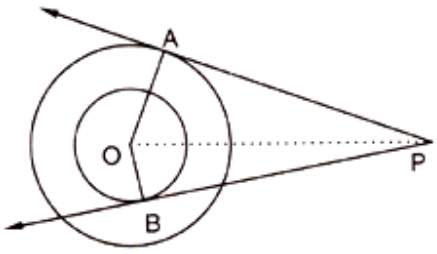
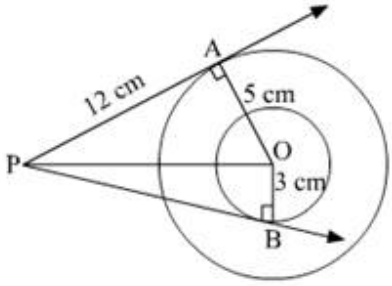


Fig. 10.58

Answer



PA and PB are the tangent drawn from the external point P to outer and inner circle respectively

$\angle OAP = 2\ \angle OBP = 90^\circ$

$Given\ OA = 5\,cm, OB = 3cm\ and\ AP = 12cm$

$In\ \triangle OAP$

$OP^2 = (12cm)^2 + (5cm)^2 = 169cm^2$

$OP = 13cm$

$IN\ \triangle OBP,$

$PB^2 = OP^2 - OB^2$

$PB^2 = (13cm)^2 - (3cm)^2 = 160cm^2$

$PB = 4\sqrt{10}cm$

$Thus\ the\ length\ of\ PB = 4\sqrt{10}cm$

21. Question

In Fig. 10.59, AB is a cord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

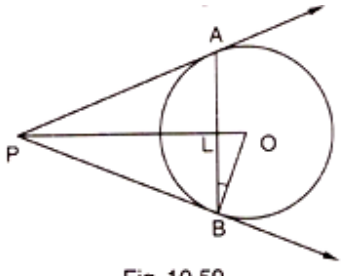
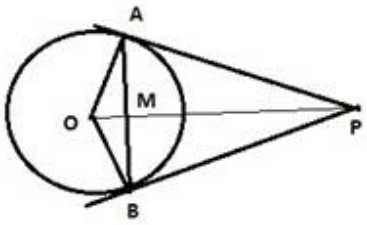


Fig. 10.59

Answer



OA=10 cm

As we know Perpendicular from centre to the chord bisects the chord.

So AM=MB=8cm

Using Pythagoras theorem in triangle AOM

$$OM = \sqrt{10^2 - 8^2} = 6\,cm$$

$$\tan \angle AOM = \frac{8}{6} = \frac{4}{3}$$

$$Now in \triangle OAP$$

$$\tan \angle AOM = \frac{PA}{OA}$$

$$PA = \frac{40}{3}$$

22. Question

In Fig. 10.60, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at A. Prove that PL + LM = PN + MN.

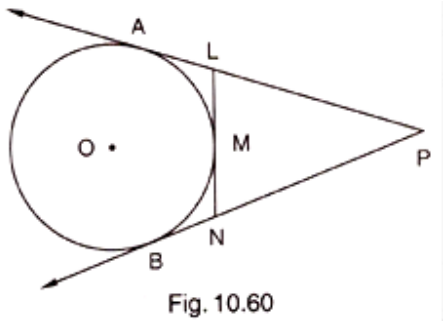


Fig. 10.60

Answer

$$PA = PB - - - - - (i)$$

As tangent drawn from external points to a circle are equal in length

$$PL + AL = PN + BN - - - - - - (ii)$$

PLA and PNB are the two tengent which are equal

$$AL = ML \; and \; BN = MN - - - - - - - (iii)$$

$$From (ii) \; and \; (iii)$$

$$PL = ML$$

23. Question

In Fig. 10.61, BDC is a tangent to the given circle at point D such that BD = 30 cm and CD = 7 cm. The other tangents BE and CF are drawn respectively from B and C to the circle and meet when produced at A making BAC a right angle triangle. Calculate (i) AF (ii) radius of the circle.

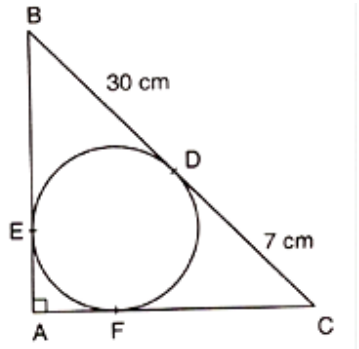


Fig. 10.61

Answer

Given : AB, BC and AC are tangents to the circle at E, D and F.

BD = 30 cm and DC = 7 cm and ∠BAC = 90°

Recall that tangents drawn from an exterior point to a circle are equal in length

Hence BE = BD = 30 cm

Also FC = DC = 7 cm

Let AE = AF = x → (1)

Then AB = BE + AE = (30 + *x*)

AC = AF + FC = (7 + *x*)

BC = BD + DC = 30 + 7 = 37 cm

Consider right Δ ABC, by Pythagoras theorem we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow (37)^2 = (30 + x)^2 + (7 + x)^2$

$\Rightarrow 1369 = 900 + 60x + x^2 + 49 + 14x + x^2$

$\Rightarrow 2x^2 + 74x + 949 - 1369 = 0$

$\Rightarrow 2x^2 + 74x - 420 = 0$

$\Rightarrow x^2 + 37x - 210 = 0$

$\Rightarrow x^2 + 42x - 5x - 210 = 0$

$\Rightarrow x (x + 42) - 5 (x + 42) = 0$

$\Rightarrow (x - 5) (x + 42) = 0$

$\Rightarrow (x - 5) = 0$ or $(x + 42) = 0$

⇒ x = 5 or x = − 42

⇒ x = 5 [Since x cannot be negative]

∴ AF = 5 cm [From (1)]

Therefore AB =30 +x = 30 + 5 = 35 cm

AC = 7 + x = 7 + 5 = 12 cm

Let 'O' be the centre of the circle and 'r' the radius of the circle.

Join point O, F; points O, D and points O, E.

From the figure,

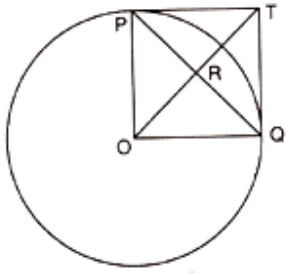
Area of (ΔABC) = Area (ΔAOB) + Area (ΔBOC) + Area (ΔAOC)

∴ r = 5

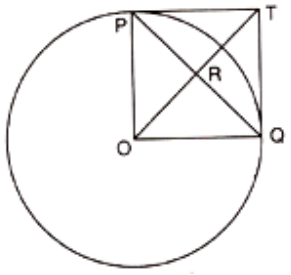
Thus the radius of the circle is 5 cm

24. Question

In Fig. 10.62, $PO \perp OQ$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.



Answer



To prove: PQ and OT are the right bisectors.

Proof: To prove PQ and OT are the right bisectors,

We need to prove ∠PRT= ∠TRQ=∠QRO=∠ORP = 90^o

As it is given that $PO \perp OQ$,

⇒ ∠POQ = 90^o

In Δ POT and Δ OQT

OP=OQ (Radius)∠OPT = ∠OQT = 90^{o} (Tangent to a circle at a point is perpendicular to the radius through the point of contact)OT=OT (common)

∴ Δ POT ≅ Δ OQTThus PT=OQ (BY C.P.C.T) ..... (1) Now in Δ PRT and Δ ORQ ∠TPR = ∠OQR (alternate angles)∠PTO = ∠TOQ (alternate angles)PT=OQ (from (1))∴ Δ PRT ≅ Δ ORQThus TQ = OP (By C.P.C.T 0Hence PT=TQ=OQ=OPThus it is a square,⇒ The diagnols bisect at 90^{o}.

Hence proved

25. Question

In Fig. 10.63, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^{o}$. Prove that OA = 2AB.

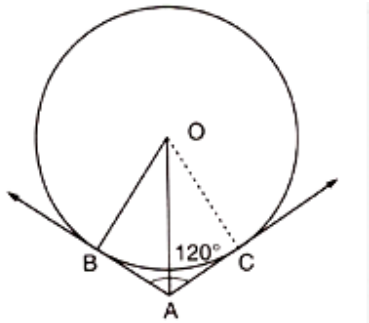
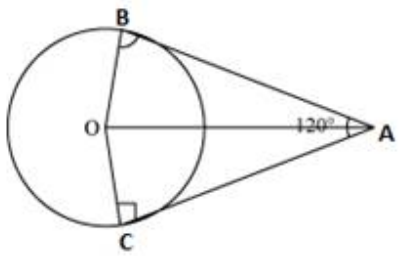


Fig. 10.63

Answer



It can be clearly show that OA bisects angle CAB,

$\angle OBC = \angle OBD = 60^{\circ}$

$In \triangle OAB$

$\angle OAB = 60^{\circ}, \angle OBA = 90^{\circ}$

$\angle BOA + \angle OAB + \angle OBA = 180^{\circ}$

$\angle BOA = 180^{\circ} - 150^{\circ} = 30^{\circ}$

$\sin(\angle BOA) = \frac{AB}{AO}$

$\sin 30^{\circ} = \frac{AB}{AO}$

$\frac{1}{2} = \frac{AB}{AO}$

$AO = 2AB$

26. Question

In Fig. 10.64, BC is a tangent to the circle O. OE bisects AP. Prove that $\triangle AEO \sim \triangle ABC$.

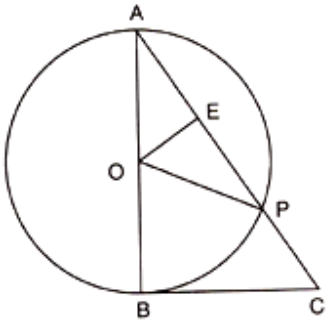
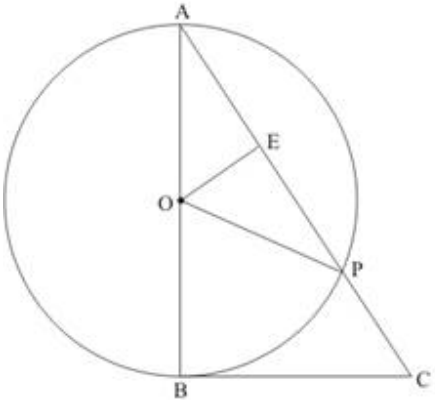


Fig. 10.64

Answer



Triangle AOP is an isosceles triangle because OA=OP as they are the radius of the circle. We know that radius of the circle is always perpendicular to the tangent at the point of contact.

Here OB is the radius and BC is the tangent and B is the point of contact, Therefore

$\angle ABC = 90^{\circ}$

Also from the property of isoceles triangle we have found that

$\angle OEA = 90^{\circ}$

Therefore,

$\angle ABC = \angle OEA$

$\angle A$ *is common angle to both triangle*

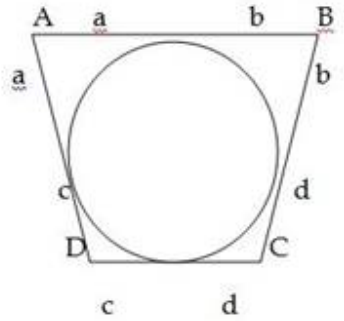
THerefore, from AA postulates of similar triangle

$\triangle AOE \sim \triangle ABC$

27. Question

The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm and 7 cm respectively. Determine the length of the fourth side.

Answer



Given

$AB = 7, BC = 5, CD = 4$

Length of the tangent drawn from an external point to the circle are equal

Therefore

$AB + CD = BC + AD$

$7 + 4 = 5 + AD$

$AD = 11 - 5 = 6$

28. Question

In Fig. 10.65, common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS.

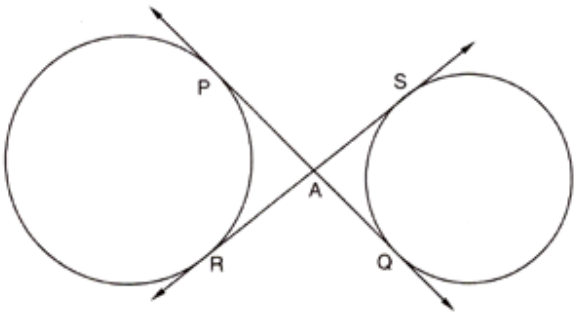
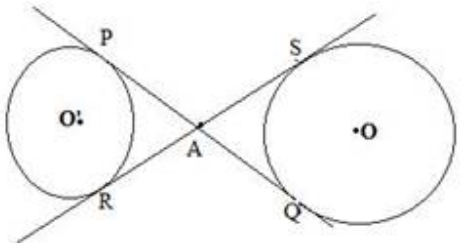


Fig. 10.65

Answer



Given: PQ and RS are the two common tangent to the two circle

To Proof: A is the point of intersection of PQ and RS

We know that , length of two tangent drawn from an exterior point to acirclr are equal.

Therefore

PA = RA---------- (i)

QA = SA ---------- (ii)

Adding two equations we get

PA + QA =RA + SA

PQ =RS (proved)

29. Question

Equal circles with centres O and O’ touch each other at X. OO’ produced to meet a circle with centre O’, at A. AC is tangent to the circle whose centre is O. O’ D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.

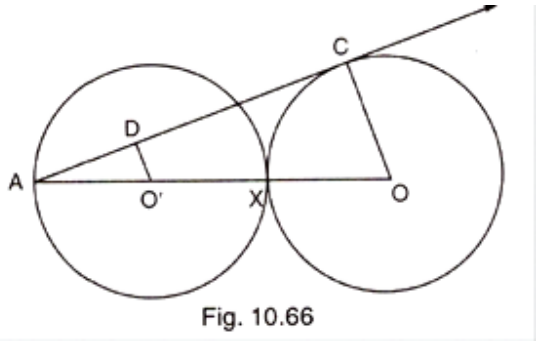


Fig. 10.66

Answer

We know that ∠ADO’ = 90° (since O’D is perpendicular to AC)

As we know radius is perpendicular to the tangent.

So, OC ⊥ AC

⇒ ∠ACO = 90°

In ΔADO’ and ΔACO,

∠ADO’ = ∠ACO (each 90°)

∠DAO = ∠CAO (common)

By AA criteria,

ΔADO’ ~ ΔACO

As we know corresponding sides of a triangle are in ratio. $\frac{AO'}{AO}=\frac{DO'}{CO}$

AO = AO’ + O’X + OX

As radii of two circles are equal.

⇒ AO = AO’ + AO’ + AO’

= 3 AO’

$$\frac{AO'}{AO}=\frac{AO'}{3AO}=\frac{1}{3} \quad \frac{DO'}{CO}=\frac{AO'}{3AO}=\frac{1}{3} \quad \Rightarrow \frac{DO'}{CO}=\frac{1}{3}$$

30. Question

In Fig. 10.67, OQ:PQ=3:4 and perimeter of $\triangle POQ$ = 60 cm. Determine PQ, OR and OP.

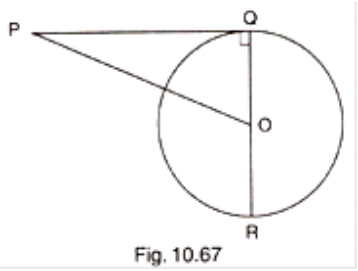
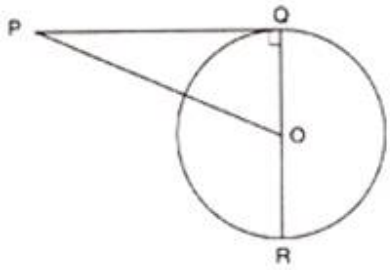


Fig. 10.67

Answer



Given that OQ: PQ=3:4

Let ratio coefficient =x, so

OQ=3x and PQ=4x

We know that a tangent to a circle is perpendicular to the radius at the point of tangency

So

$\angle OQP = 90^\circ$

Then applying Pythagoras theorem in triangle POQ

$OP^2 = OQ^2 + PQ^2$

$OP^2 = (3x)^2 + (4x)^2$

$OP^2 = 9x^2 + 16x^2$

$OP^2 = 25x^2$

$OP = 5x$

$Perimeter\ of\ a\ triangle\ POQ\ is = 60\,cm,\ So$

$3x + 4x + 5x = 60$

$12x = 60$

$x = 5$

$So,$

$OQ = 3x = 15cm$

$PQ = 4x = 20cm$

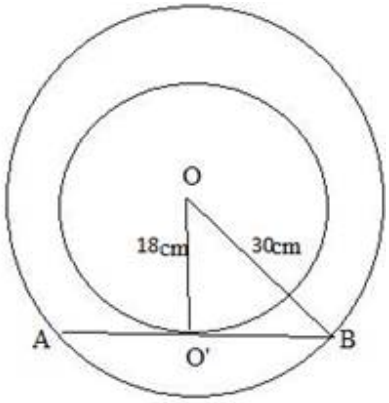
$OP = 5x = 25cm$

$QR = 2(OQ) = 2 \times 15 = 30\,cm$

31. Question

Two concentric circles are of diameters 30 cm and 18 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer



In the diagram AB is the chord touching the smaller circle. We have the right angled triangle OO'B

By Pythagoras theorem

$$O'B = \sqrt{(30)^2 + (18)^2} = 24\,cm$$

Now since the chord of the larger circle which touches the smaller circle is bisected at the point of contact

We have

AB= 2 × 24= 48 cm

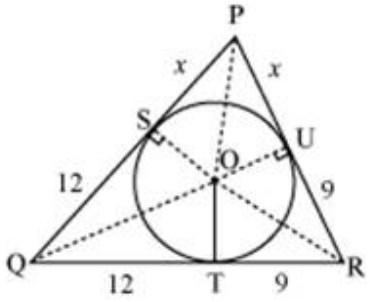
So ans is 18 cm.

32. Question

A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If area of $\triangle PQR$ is 336 cm^2, find the sides PQ and PR.

Answer

Let PQ and PR touch the circle at points S and U respectively. Join O with P, Q, R, S and U



We have OS = OT = OU = 6cm

QT = 12 cm and TR = 9cm

QR= QT +TR = 12cm + 9cm = 21 cm

Now QT = QS=12 cm (tangent from the same point)

TR =RU=9cm

Let PS =PU =x cm

Then PQ=PS+SQ- (12 + x)cm and PR = PU+RU= (9+x)cm

It is clear that

$$ar(\triangle OQR)+ar(\triangle OPR)+ar(\triangle OPQ)=ar(\triangle PQR)$$

$$\frac{1}{2}\times QR\times OT+\frac{1}{2}\times PR\times OU+\frac{1}{2}\times PQ\times OS=189cm^2$$

$$\frac{1}{2}\times(12+x)\times 6+\frac{1}{2}\times(9+x)\times 6+\frac{1}{2}\times 21\times 6=189$$

$$\frac{1}{2}\times 6(12+x+9+x+21)=189$$

$$3(42+2x)=189$$

$$42+2x=63$$

$$x=\frac{21}{2}=10.5$$

$$\textbf{Thus PQ}=(12+10.5)\ \textbf{cm}=22.5\ \textbf{cm and PR}=(9+10.5)\ \textbf{cm}=19.5\ \textbf{cm}$$

33. Question

In Fig. 10.68, a $\triangle ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the lengths of sides AB and AC, when area of $\triangle ABC$ is 84 cm^2.

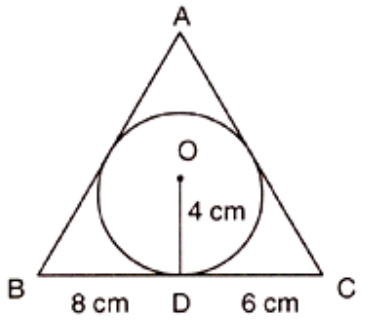
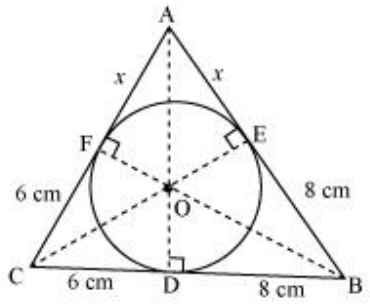


Fig. 10.68

Answer



Firstly consider that the given circle will touch the given circle will touch the sides AB and AC of the triangle at a point E and F respectively.

Let AF=x

Now in triangle ABC

CF = CD=6cm

(Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point C)

BE = BD =8cm (Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point B)

AE = AF =X

Now AB= AE + EB =x + 8

Also BC = BD+ DC = 8+6 =14 and CA= CF+FA = 6+ x

Now we get all side of the triangle and its area can be find by using hero’s formula

$Semi-perimeter = s = \frac{28+2x}{2} = 14+x$

$Area\ of\ \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(14+x)\{(14+x)-14\}\{(14+x)(6+x)\}\{(14+x)-(8+x)\}}$

$= 4\sqrt{3(14x+x^2)}$

$Ar\ of\ \triangle OBC = \frac{1}{2}\times 4\times 14 = 28$

$Ar\ of\ \triangle OCA = \frac{1}{2}\times 4\times (6+x) = 12+2x$

$Ar\ of\ \triangle OAB = \frac{1}{2}\times 4\times (8+x) = 16+2x$

$Area\ of\ \triangle ABC = Ar\ of\ \triangle OBC + Ar\ of\ \triangle OCA + Ar\ of\ \triangle OAB$

$4\sqrt{3(14x+x^2)} = 28+12+2x+16+2x$

$\sqrt{3(14x+x^2)} = 14+x$

Squarring both side and solving we get

$x(x+14)-7(x+14) = 0$

$or\ x = -14\ and\ 7$

$x = -14\ is\ not\ possible$

$so\ x = 7$

$hence\ AB = 7+8 = 15\ cm$

$CA = 6+7 = 13cm$

34. Question

In Fig. 10.69, AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^0$, find $\angle ATQ$.

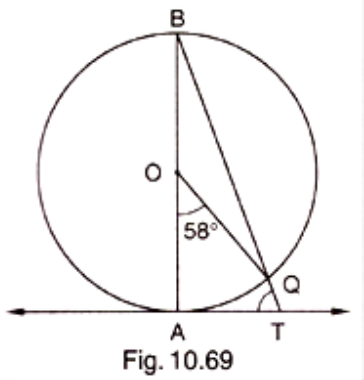


Fig. 10.69

Answer

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58 = 29^\circ$$

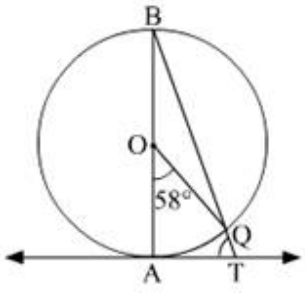
$$\angle A = 90^\circ$$

So

$$\angle BAT + \angle ABT + \angle ATQ = 180$$

$$\angle ATQ = 180 - 90 + 29 = 61^\circ$$

$$\angle ATQ = 61^\circ$$



35. Question

In Fig. 10.70, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.

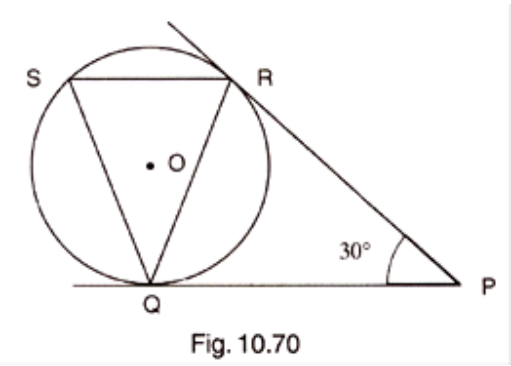


Fig. 10.70

Answer

As we know that the tangents drawn from an external point to a circle are equal.

Therefore, PQ = PR

Also, from the figure, PQR is an isosceles triangle, because PQ = PR

Therefore, ∠RQP =∠QRP (Because the corresponding angles of the equal sides of the isosceles triangle are equal)And, from the angle sum property of a triangle,

$\angle RQP + \angle QRP + \angle RPQ = 180^o$

$\angle RQP + \angle RQP + \angle RPQ = 180^o$

$2\angle RQP + \angle RPQ = 180^o$

$2\angle RQP + 30^o = 180^o$

$2\angle RQP = 180^o - 30^o$

$2\angle RQP = 150^o$

$\angle RQP = 150^o/2$

Therefore, $\angle RQP = 75^o$

SR || QP and QR is a transversal

$\because \angle SRQ = \angle PQR$ …[Alternate interior angle]

$\therefore \angle SRQ = 75°$

$\Rightarrow \angle ORP = 90°$…[Tangent is Perpendicular to the radius through the point of contact]

$\angle ORP = \angle ORQ + \angle QRP$

$\Rightarrow 90° = \angle ORQ + 75°$

$\Rightarrow \angle ORQ = 15°$

Similarly, $\angle RQO = 15°$

In Δ QOR,

$\angle QOR + \angle QRO + \angle OQR = 180°$

$\Rightarrow \angle QOR + 15° + 15° = 180°$

$\Rightarrow \angle QOR = 150°$

$\Rightarrow \angle QSR = \angle QOR/2$

$\Rightarrow \angle QSR = 150°/2 = 75°$

In ΔRSQ,

$\angle RSQ + \angle QRS + \angle RQS = 180°$

$\Rightarrow 75° + 75° + \angle RQS = 180°$

$\angle RQS = 30^o$

CCE - Formative Assessment

1. Question

In Fig. 10.72, PA and PB are tangents to the circle drawn from an external point P. CD are a third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length PC?

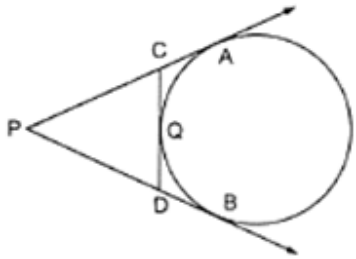


Fig. 10.72

Answer

Given:

PB = 10 cm

CQ = 2 cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

PA = PB = 10 cm (tangent from P)

And,

CA = CQ= 10 cm (tangent from C)

Now,

PC = PA – CA

= 10 cm – 2 cm

= 8 cm

Hence, PC = 8 cm

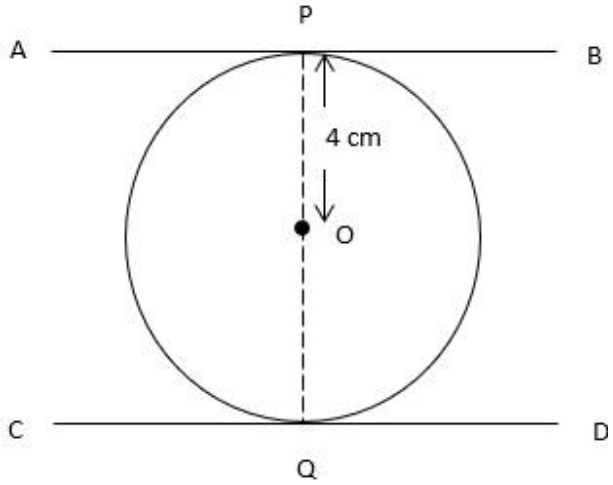
2. Question

What is the distance between two parallel tangents of a circle of radius 4 cm?

Answer

Given:

Radius of circle (say PO) = 4 cm



Let AB ∥ CD be two tangents which meets the circle at P and Q respectively. And, O be the center of circle.

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

PQ = 2 × PO

= 2 × 4 cm

= 8 cm

Hence, Distance between tangents = 8 cm

3. Question

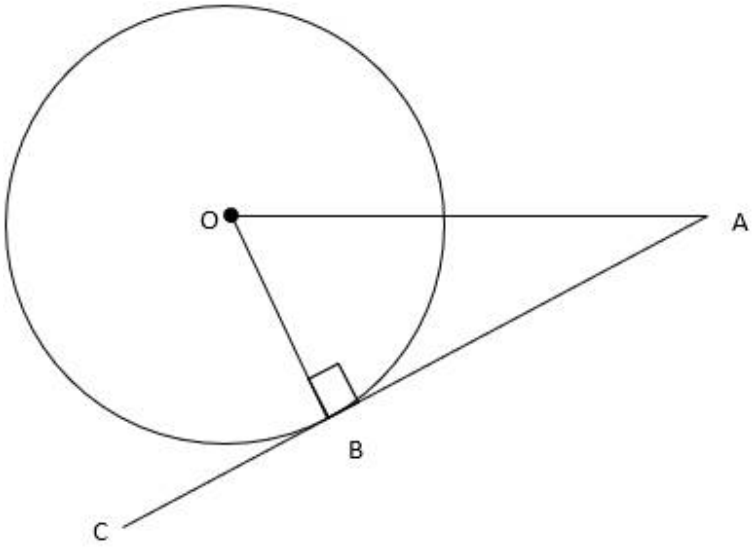
The length of tangent from a point A at a distance of 5 cm from the center of the circle is 4 cm. What is the radius of the circle?

Answer

Given:

OA (say) = 5 cm

AB = 4 cm



Let AC be the tangent which meets the circle at the point B and O be the center of circle.

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠OBA.

Therefore, by Pythagoras Theorem,

$AB^2 + OB^2 = AO^2$

$\Rightarrow OB^2 = AO^2 - AB^2$

$\Rightarrow OB = \sqrt{(AO^2 - AB^2)}$

$\Rightarrow OB = \sqrt{(5^2 - 4^2)}$

$\Rightarrow OB = \sqrt{(25 - 16)}$

$\Rightarrow OB = \sqrt{9}$

$\Rightarrow OB = 3$ cm

Hence, Radius = 3 cm

4. Question

Two tangents TP and TQ are drawn from an external point T to a circle with center O as shown in Fig. 10.73. If they are inclined to each other at an angle of 100°, then what is the value of ∠ POQ?

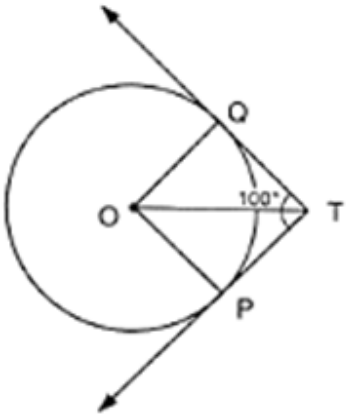


Fig. 10.73

Answer

Given:

∠QTP = 100°

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360°.

By property 1,

∠OPT = 90° and ∠OQT = 90°

And,

By property 2,

∠QTP + ∠OPT + ∠OQT + ∠POQ = 360°

⇒ ∠POQ = 360° – ∠QTP + ∠OPT + ∠OQT

⇒ ∠POQ = 360° – 100° + 90° + 90°

⇒ ∠POQ = 360° – 280°

⇒ ∠POQ = 80°

Hence, ∠POQ = 80°

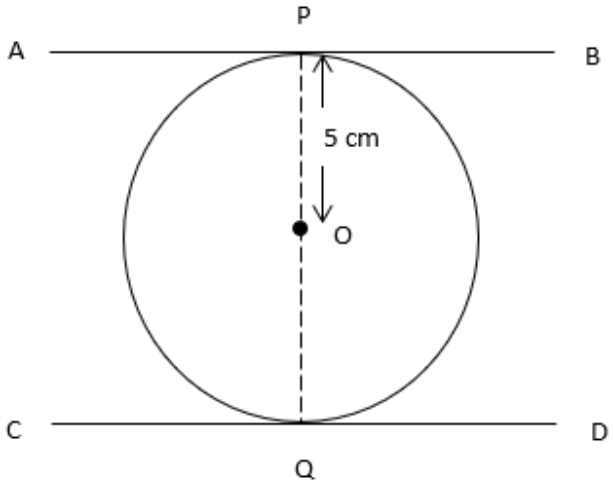
5. Question

What the distance between two parallel tangents to a circle of radius 5 cm?

Answer

Given:

Radius of circle (say PO) = 5 cm



Let AB ∥ CD be two tangents which meets the circle at P and Q respectively. And, O be the center of circle.

Property: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

PQ = 2 × PO

= 2 × 5 cm

= 10 cm

Hence, Distance between tangents = 10 cm

6. Question

In Q. No. 1, if PB = 10 cm, what is the perimeter of Δ PCD?

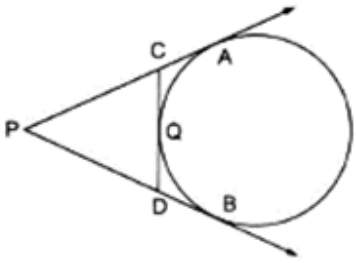


Fig. 10.72

Answer

Given:

PB = 10 cm

CQ = 2 cm

Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Using the above property,

PA = PB = 10 cm (tangent from P)

DB = DQ= 10 cm (tangent from D)

And,

CA = CQ= 10 cm (tangent from C)

Now,

Perimeter of ΔPCD = PC + CD + DP

= PC + CQ + QD + DP

= PC + CA + DB + PD [∵CA = CQ and DB = DQ]

= PA + PB [∵PA = PC + CA and PB = PD + BD]

= 10 cm + 10 cm

= 20 cm

Hence, Perimeter of ΔPCD = 20 cm

7. Question

In Fig. 10.74, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR.

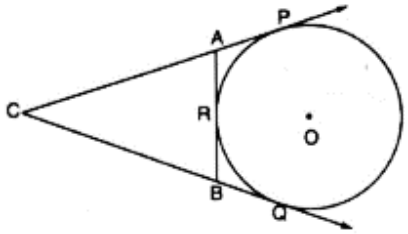


Fig. 10.74

Answer

Given:

CP = 11 cm

BC = 7 cm

Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Using the above property,

CP = CQ = 11 cm (tangent from C)

BQ = BR (tangent from B)

And,

AP = AR (tangent from A)

Now,

BR = BQ = CQ – CB

= 11 cm – 7 cm

= 4 cm

Hence, BR = 4 cm

8. Question

In Fig. 10.75, Δ ABC is circumscribing a circle. Find the length of BC.

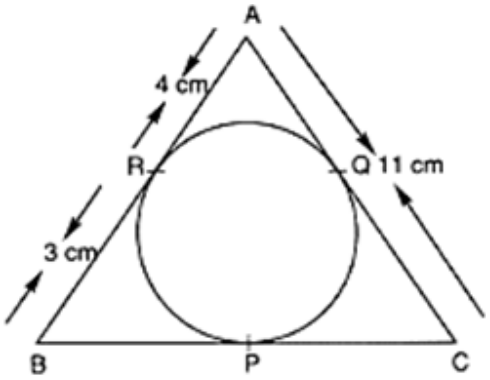


Fig. 10.75

Answer

Given:

AR = 4 cm

BR = 3 cm

AC = 11 cm

Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Using the above property,

AR = AQ = 4 cm (tangent from A)

BR= BP (tangent from B)

And,

CP = CQ (tangent from C)

Also,

CQ = CA – AQ = 11 cm – 4 cm = 7 cm

Now,

BC = BP + PC

= BR + CQ [∵ BR = BP and CP = CQ = 7 cm]

= 3 cm + 7 cm

= 10 cm

Hence, BC = 10 cm

9. Question

In Fig. 10.76, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.

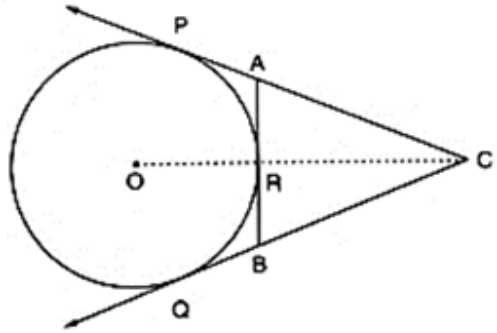


Fig. 10.76

[Hint: We have, CP = 11 cm

$\therefore$ CP = CQ = CQ = 11 cm

Now, BR = BQ [Tangents drawn from B)

$\Rightarrow$ BQ = 4 cm

$\therefore$ BC = CQ - BQ = (11 - 4)cm = 7 cm

Answer

Given:

BR = 4 cm

CP = 11 cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

BR = BQ = 4 cm (tangent from B)

And,

CP = CQ = 11 cm (tangent from C)

Now,

BC = CQ – BQ

= 11 cm – 4 cm

=7 cm

Hence, BC = 7 cm

10. Question

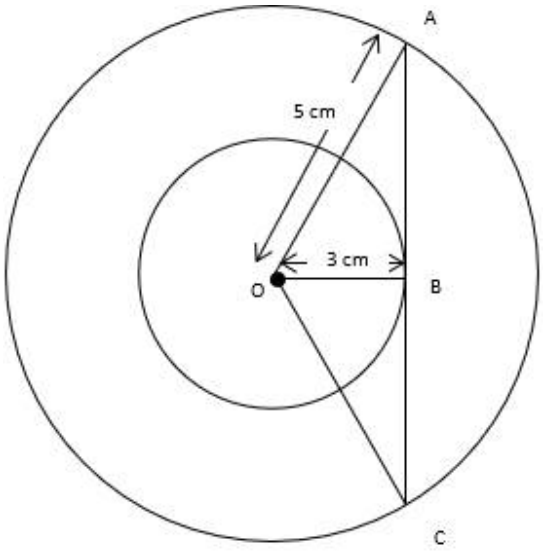
Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer

Given:

AO (say) = CO (say) = 5 cm

BO (say) = 3 cm



Let AC be the tangent which meets the circle at the point B and O be the center of circle.

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠OBA and ΔCOB is right-angled at ∠OBC.

Therefore,

By Pythagoras Theorem in ΔAOB,

$AB^2 + OB^2 = AO^2$

$\Rightarrow AB^2 = AO^2 - OB^2$

$\Rightarrow AB = \sqrt{(AO^2 - OB^2)}$

$\Rightarrow AB = \sqrt{(5^2 - 3^2)}$

$\Rightarrow AB = \sqrt{(25 - 9)}$

$\Rightarrow AB = \sqrt{16}$

⇒ AB= 4 cm

Similarly,

By Pythagoras Theorem in ΔCOB,

$AB^2 + OB^2 = CO^2$

$\Rightarrow CB^2 = CO^2 - OB^2$

$\Rightarrow CB = \sqrt{(CO^2 - OB^2)}$

$\Rightarrow CB = \sqrt{(5^2 - 3^2)}$

$\Rightarrow CB = \sqrt{(25 - 9)}$

$\Rightarrow CB = \sqrt{16}$

⇒ CB= 4 cm

Now,

AC = AB + BC

= 4 cm + 4 cm

= 8 cm

Hence, Length of chord = 8 cm

11. Question

In Fig. 10.77, PA and PB are tangents to the circle with centre O such that ∠APB = 50°. Write the measure of ∠OAB

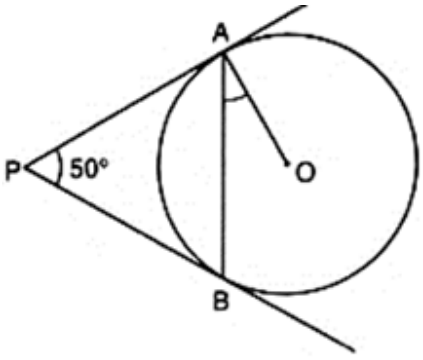


Fig. 10.77

Answer

Given:

∠APB = 50°

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a triangle = 180°.

By property 1,

AP = BP (tangent from P)

Therefore, ∠PAB = ∠PBA

Now,

By property 3 in ΔPAB,

∠PAB + ∠PBA + ∠APB = 180°

⇒ ∠PAB + ∠PBA = 180° – ∠APB

⇒ ∠PAB + ∠PBA = 180° – 50°

⇒ ∠PAB + ∠PBA = 130°

$$\Rightarrow \angle PAB = \angle PBA = \frac{130^\circ}{2} = 65^\circ$$

By property 2,

∠PAO = 90°

Now,

∠PAO = ∠PAB + ∠OAB

⇒ ∠OAB = ∠PAO – ∠PAB

⇒ ∠OAB = 90° – 65° = 25°

Hence, ∠OAB = 25°

1. Question

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12 cm. Length PQ is cm

A. 12 cm

B. 13 cm

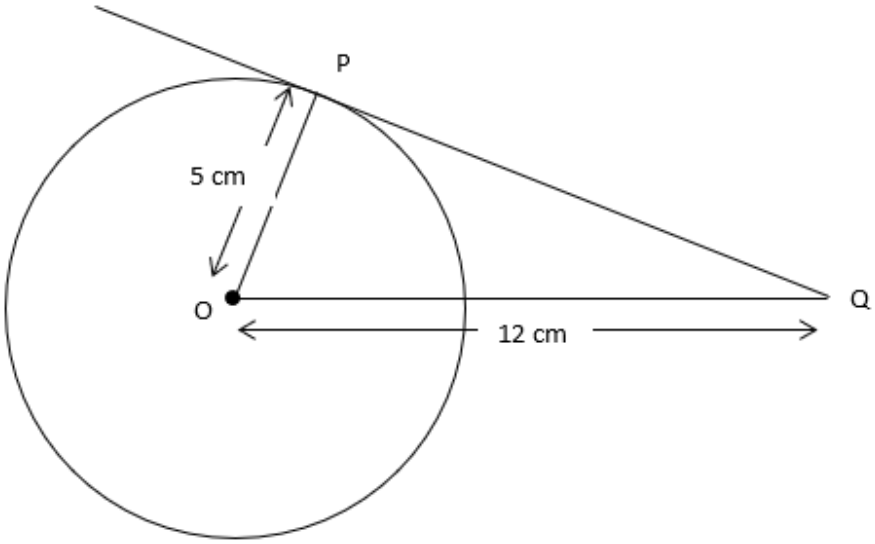
C. 8.5 cm

D. $\sqrt{119}$ cm

Answer

Given:

OQ = 12 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at ∠OPQ.

Therefore,

By Pythagoras Theorem in ΔPOQ,

$OP^2 + PQ^2 = OQ^2$

$\Rightarrow PQ^2 = OQ^2 - OP^2$

$\Rightarrow PQ = \sqrt{(OQ^2 - OP^2)}$

$\Rightarrow PQ = \sqrt{(12^2 - 5^2)}$

$\Rightarrow PQ = \sqrt{(144 - 25)}$

$\Rightarrow PQ = \sqrt{119}$ cm

Hence, $PQ = \sqrt{119}$ cm

2. Question

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

A. 7 cm

B. 12 cm

C. 15 cm

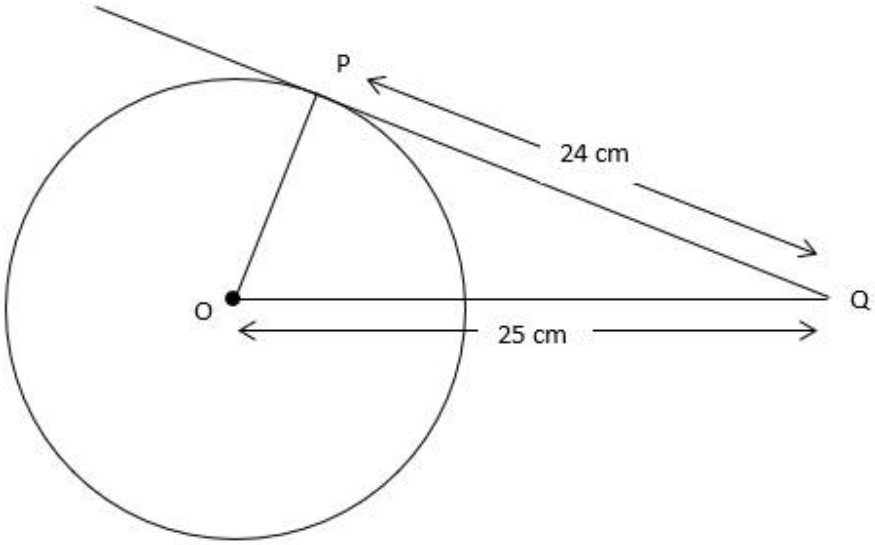
D. 24.5 cm

Answer

Given:

OQ = 25 cm

PQ = 24 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at ∠OPQ.

Therefore,

By Pythagoras Theorem in ΔPOQ,

$OP^2 + PQ^2 = OQ^2$

$\Rightarrow OP^2 = OQ^2 - PQ^2$

$\Rightarrow OP = \sqrt{(OQ^2 - PQ^2)}$

$\Rightarrow OP = \sqrt{(25^2 - 24^2)}$

$\Rightarrow OP = \sqrt{(625 - 576)}$

$\Rightarrow OP = \sqrt{49}$ cm

$\Rightarrow OP = 7$ cm

Hence, OP = 7 cm

3. Question

The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

A. √7 cm

B. 7 cm

C. 5 cm

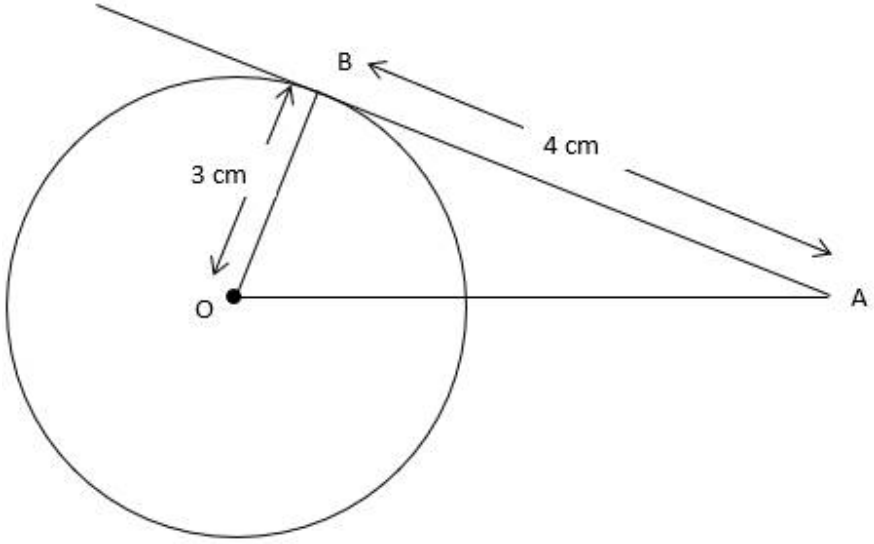
D. 25 cm

Answer

Given:

AB (say) = 4 cm

Radius (OB) = 3 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠ABO.

Therefore,

By Pythagoras Theorem in ΔPOQ,

$OA^2 = OB^2 + BA^2$

$\Rightarrow OA = \sqrt{(OB^2 + BA^2)}$

$\Rightarrow OA = \sqrt{(3^2 + 4^2)}$

$\Rightarrow OA = \sqrt{(9 + 16)}$

$\Rightarrow OA = \sqrt{25}$ cm

$\Rightarrow OA = 5$ cm

Hence, distance of A from center = 5 cm

4. Question

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° then ∠POA is equal to

A. 50°

B. 60°

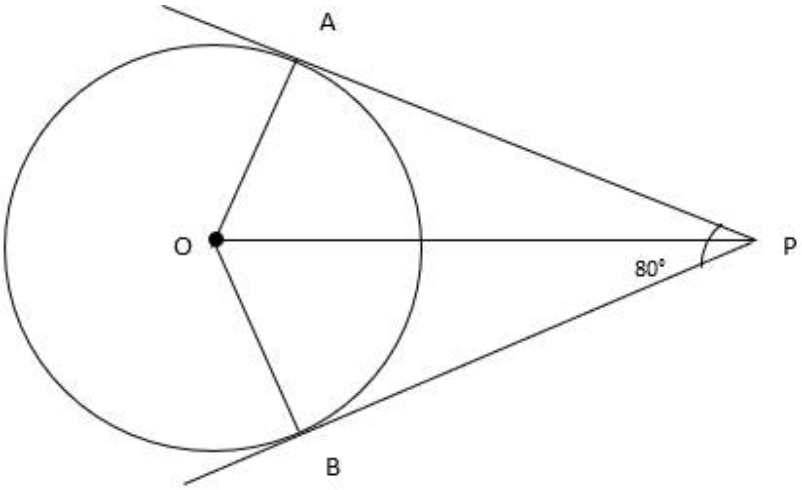
C. 70°

D. 80°

Answer

Given:

∠APB = 80°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

<u>Property 3:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1,

∠PAO = 90°

∠PBO = 90°

By property 2,

∠APB + ∠PAO + ∠PBO + ∠AOB = 360°

⇒ ∠AOB = 360° - ∠APB + ∠PAO + ∠PBO

⇒ ∠AOB = 360° - (80° + 90° + 90°)

⇒ ∠AOB = 360° - 260°

⇒ ∠AOB = 100°

Now, in ΔPOA and ΔPOB

OA = OB [∵ radius of circle]

PA = PB [By property 3 (tangent from P)]

OP = OP [∵ common]

∴ By SSS congruency,

ΔPOA ≅ ΔPOB

Hence, by CPCTC

∠POA = ∠POB

Now,

∠AOB = 100°

⇒ ∠POA + ∠POB = 100° [∵∠AOB = ∠POA + ∠POB]

⇒ ∠POA + ∠POA = 100° [∵∠POA = ∠POB]

⇒ 2∠POA = 100°

$$\Rightarrow \angle POA = \frac{100^\circ}{2}$$

⇒ ∠POA = 50°

Hence, ∠POA = 50°

5. Question

If TP and TQ are two tangents to a circle with centre O so that ∠POQ = 110°, then, ∠PTQ is equal to

A. 60°

B. 70°

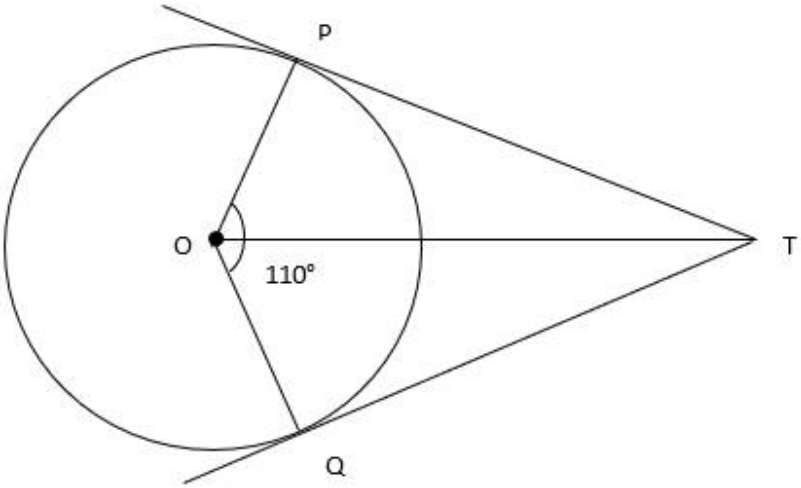
C. 80°

D. 90°

Answer

Given:

∠POQ = 110°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

∠TPO = 90°

∠TQO = 90°

By property 2,

∠POQ + ∠ TPO + ∠ TQO + ∠PTQ = 360°

⇒ ∠PTQ = 360° - ∠POQ + ∠ TPO + ∠ TQO

⇒ ∠PTQ = 360° - (110° + 90° + 90°)

⇒ ∠PTQ = 360° - 290°

⇒ ∠PTQ = 70°

Hence, ∠PTQ = 70°

6. Question

PQ is a tangent to a circle with centre 0 at the point P. If A Δ OPQ is an isosceles triangle, then ∠OQP is equal to

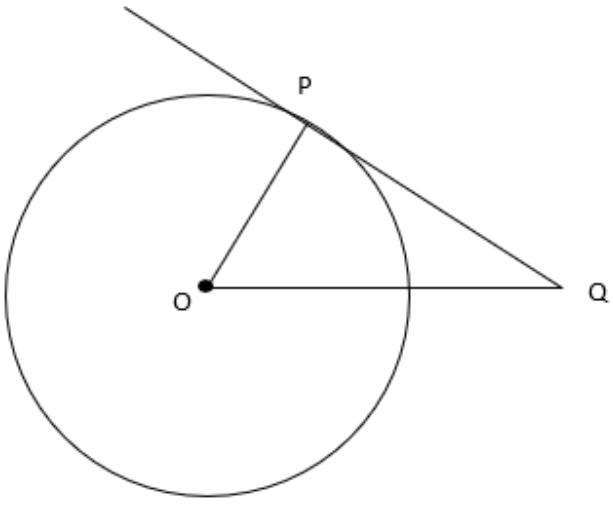
A. 30°

B. 45°

C. 60°

D. 90°

Answer



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a triangle = 180°.

By property 1, ΔPOQ is right-angled at ∠OPQ (i.e., ∠OPQ = 90°).

∵ ΔPOQ is an isosceles triangle

∴ ∠POQ = ∠OQP

By property 2,

∠POQ + ∠OQP + ∠QPO = 180°

⇒ ∠POQ + ∠OQP = 180° - ∠QPO

⇒ ∠POQ + ∠OQP = 180° - 90°

⇒ ∠POQ + ∠OQP = 180° - 90°

⇒ ∠POQ + ∠OQP = 90°

⇒ ∠OQP + ∠OQP = 90° [∵∠POQ = ∠OQP]

⇒ 2∠OQP = 90°

$\Rightarrow \angle OQP = \frac{90°}{2}$⇒ ∠OQP = 45°

Hence, ∠OQP = 45°

7. Question

Two equal circles touch each other externally at C and AB is a common tangent to the circles. Then, ∠ACB =

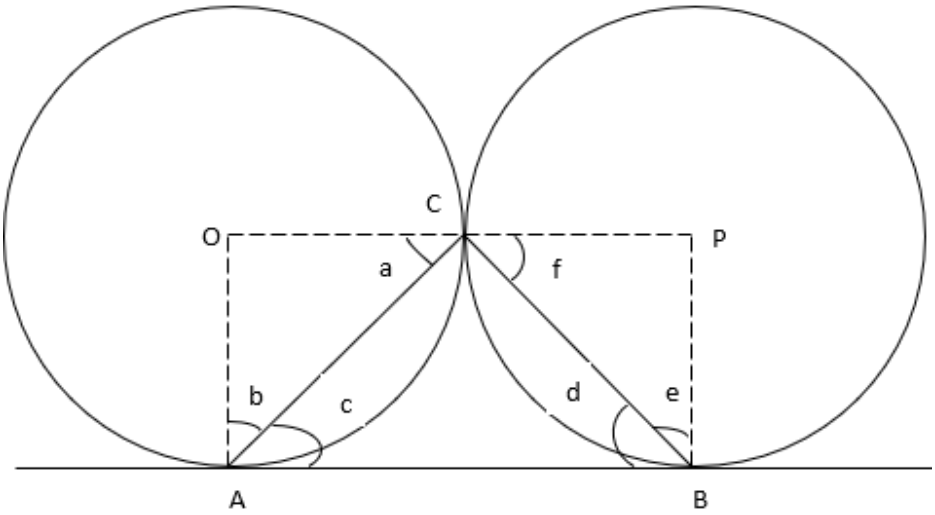
A. 60°

B. 45°

C. 30°

D. 90°

Answer



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a straight line = 180°.

Property 3: Sum of all angles of a triangle = 180°.

By property 1, ΔOAB is right-angled at ∠OAB (i.e., ∠OAB = 90°) and ΔPBA is right-angled at ∠ PBA (i.e., ∠ PBA = 90°)

Clearly,

∠b + ∠c = ∠OAB

⇒ ∠b + ∠c = 90°

⇒ ∠b = 90° - ∠c

Similarly,

∠d + ∠e = ∠PBA

⇒ ∠d + ∠e = 90°

⇒ ∠e = 90° - ∠d

Now,

∠a = ∠b = 90° - ∠c [∵ OA = OC (Radius)]

And,

∠e = ∠f = 90° - ∠d [∵ PB = PC (Radius)]

By property 2,

∠a + ∠f + ∠ACB = 180°

⇒ ∠ACB = 180° – ∠a – ∠f

⇒ ∠ACB = 180° – (90° - ∠c) – (90° - ∠d)

⇒ ∠ACB = 180° – 90° + ∠c – 90° + ∠d

⇒ ∠ACB = ∠c + ∠d

Now, in ΔACB

By property 3,

∠ACB + ∠c + ∠d = 180°

⇒ ∠ACB + ∠ACB = 180° [∵∠ACB = ∠c + ∠d]

⇒ 2∠ACB = 180°

$$\Rightarrow \angle ACB = \frac{180^\circ}{2}$$

⇒ ∠ACB = 90°

Hence, ∠ACB = 90°

8. Question

ABC is a right angled triangle, right angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O is inscribed in ΔABC. The radius of the circle is

A. 1 cm

B. 2 cm

C. 3 cm

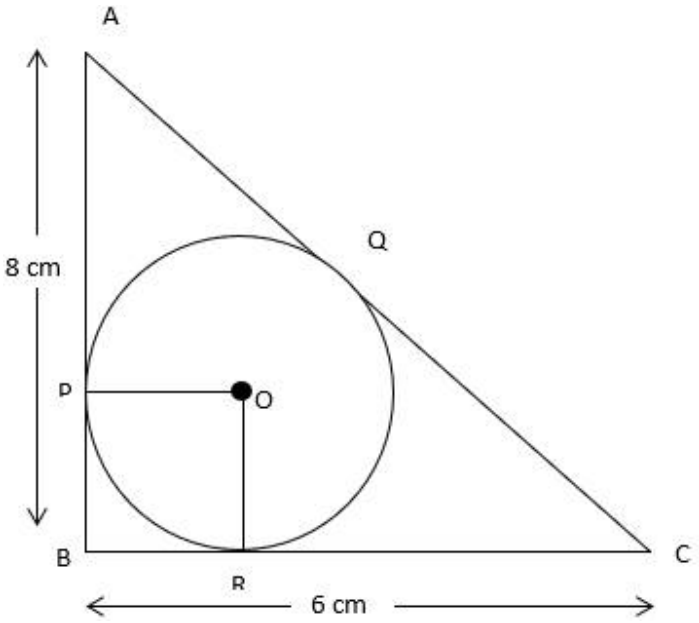
D. 4 cm

Answer

Given:

BC = 6 cm

AB = 8 cm



<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

AP = AQ (Tangent from A)

BP = BR (Tangent from B)

CR = CQ (Tangent from C)

∵ ABC is a right-angled triangle, ∴ by Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 8^2 + 6^2$

$\Rightarrow AC^2 = 64 + 36$

$\Rightarrow AC^2 = 100$

$\Rightarrow AC = \sqrt{100}$

$\Rightarrow AC = 10$ cm

Clearly,

AQ + QC = AC = 10 cm

$\Rightarrow$ AP + RC = 10 cm [$\because$ AQ = AP and QC = RC]

Also,

AB + BC = 8 cm + 6 cm = 14 cm

$\Rightarrow$ AP + PB + BR + RC = 14 cm [$\because$ AB = AP + PB and BC = BR + RC]

$\Rightarrow$ AP + RC + PB + BR = 14 cm

$\Rightarrow$ 10 cm + BR + BR = 14 cm [$\because$ AP + RC = 10 cm and PB = BR]

$\Rightarrow$ 10 cm + 2BR = 14 cm

$\Rightarrow$ 2BR = 14 cm – 10 cm = 4 cm

$$\Rightarrow \mathrm{BR} = \frac{4}{2}\mathrm{cm}$$

$\Rightarrow$ BR = 2 cm

Now,

$\angle$BPO = 90° [By property 3]

$\angle$BRO = 90° [By property 3]

$\angle$PBM = 90° [Given]

Now by property 2,

$\angle$BPO + $\angle$BRO + $\angle$PBM + $\angle$ROP = 360°

$\Rightarrow \angle$ROP = 360° - $\angle$BPO + $\angle$BRO + $\angle$PBM

$\Rightarrow \angle$ROP = 360° - (90° + 90° + 90°)

$\Rightarrow \angle$ROP = 360° - 270°

$\Rightarrow \angle$ROP = 90°

Now, $\because \angle$ROP = 90° and BP = BR which are adjacent sides

$\therefore$ Quadrilateral PBRO is a square

⇒ PO = BR = 2 cm

Hence, Radius = 2 cm

9. Question

PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120° , then ∠OPQ is

A. 60°

B. 45°

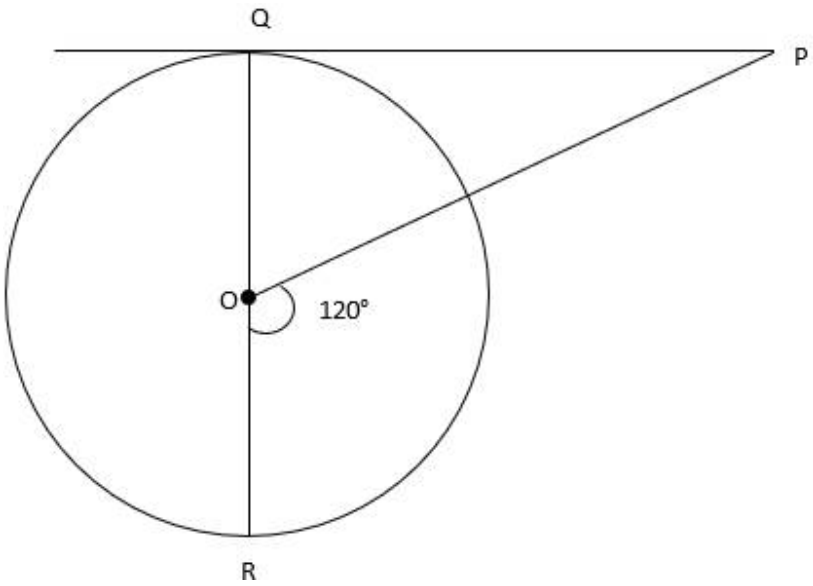
C. 30°

D. 90°

Answer

Given:

∠POR = 120°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a straight line = 180°.

<u>Property 3:</u> Sum of all angles of a triangle = 180°.

By property 1,

∠PQO = 90°

By property 2,

∠POQ + ∠POR = 180°

⇒ ∠POQ + 120° = 180°

⇒ ∠POQ = 180° - 120°

⇒ ∠POQ = 60°

Now by property 3 in ΔOPQ,

∠POQ + ∠PQO + ∠OPQ = 180°

⇒ ∠OPQ = 180° - ∠POQ + ∠PQO

⇒ ∠OPQ = 180° - (60° + 90°)

⇒ ∠OPQ = 180° - 150°

⇒ ∠OPQ = 30°

Hence, ∠OPQ = 30°

10. Question

If four sides of a quadrilateral ABCD are tangential to a circle, then

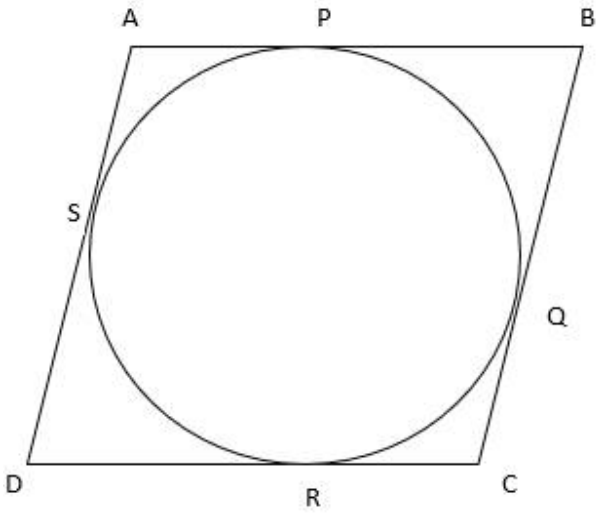
A. AC + AD = BD + CD

B. AB + CD = BC + AD

C. AB + CD = AC + BC

D. AC + AD = BC + DB

Answer



Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AS (tangent from A)

BP = BQ (tangent from B)

CR = CQ (tangent from C)

DR = DS (tangent from D)

Now we add above 4 equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ AB + CD = AD + BC

[∵ AP + BP = AB

CR + DR = CD

AS + DS = AD

BQ + CQ = BC]

Hence, the right option is AB + CD = AD + BC

11. Question

The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

A. √7 cm

B. 2√7 cm

C. 10 cm

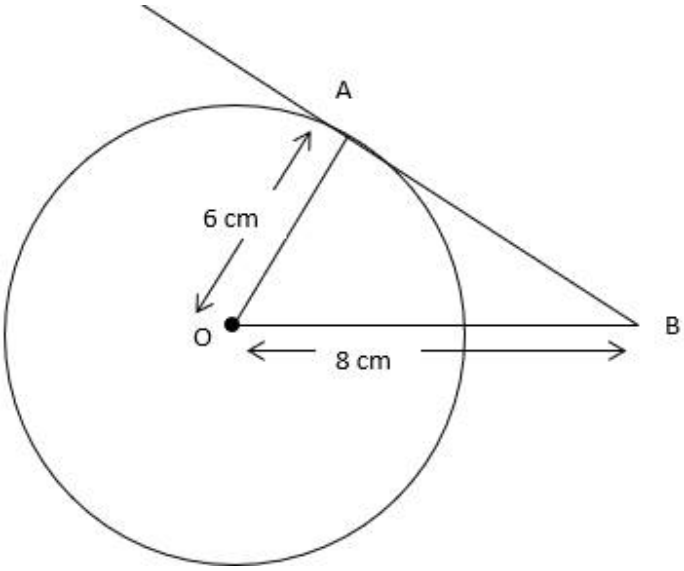
D. 5 cm

Answer

Given:

OA = 6 cm

OB = 8 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠OAB (i.e., ∠OAB = 90°).

Therefore by Pythagoras theorem,

$OA^2 + AB^2 = OB^2$

$\Rightarrow AB^2 = OB^2 - OA^2$

$\Rightarrow AB^2 = 8^2 - 6^2$

$\Rightarrow AB^2 = 64 - 36$

$\Rightarrow AB^2 = 28$

$\Rightarrow AB = \sqrt{28}$

$\Rightarrow AB = 2\sqrt{7}$

Hence, length of tangent is $2\sqrt{7}$ cm.

12. Question

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm, then AB is equal to

A. 4 cm

B. 6 cm

C. 8 cm

D. 12 cm

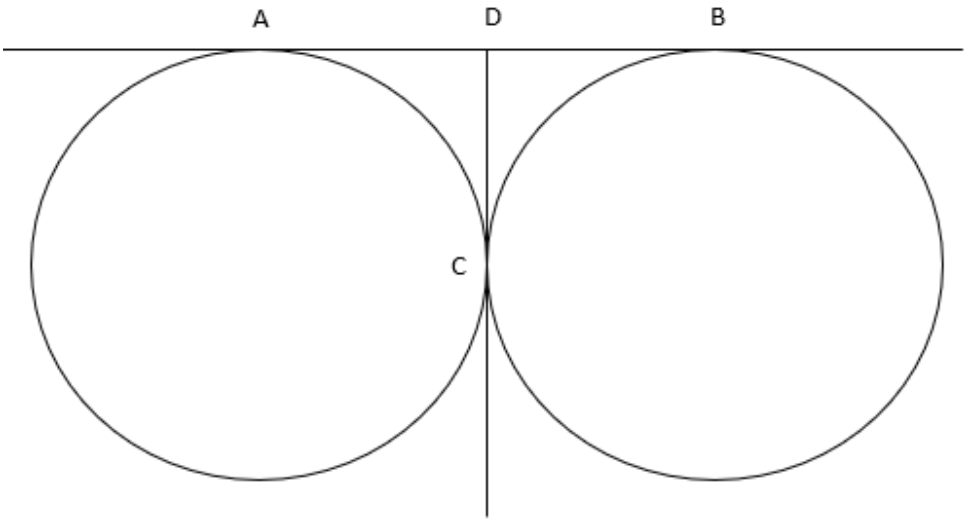
Answer

Given:

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm

To find: length of AB

Solution:



Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Since c is the external point to the circles and two common tangents touch each other at c.

By the above property,

AD = BD = CD = 4 cm (tangent from D)

Now clearly,

AB = AD + BD

⇒ AB = AD + BD

⇒ AB = 4 cm + 4 cm

⇒ AB = 8 cm

Hence, AB = 8 cm

13. Question

In Fig. 10.78, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,

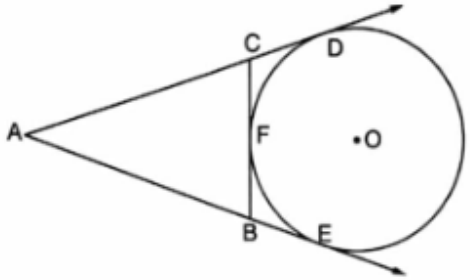


Fig. 10.78

A. AD = AB + BC + CA

B. 2AD = AB + BC + CA

C. 3AD = AB + BC + CA

D. 4AD = AB + BC + CA

Answer

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AE = AD (tangent from A)

AB = AC (tangent from A)

CD = CF (tangent from C)

BF = BE (tangent from B)

Now adding the above equations,

AB + BC + CA = AB + BF + FC + CA

⇒ AB + BC + CA = AB + BE + CD + CA

⇒ AB + BC + CA = AE + AD [∵ AE = AB + BE and AD = AC + CD]

⇒ AB + BC + CA = AD + AD [∵ AD = AE]

⇒ AB + BC + CA = 2AD

Hence, 2AD = AB + BC + CA

14. Question

In Fig. 10.79, RQ is a tangent to the circle with centre O. If SQ = 6 cm and QR = 4 cm, then OR =

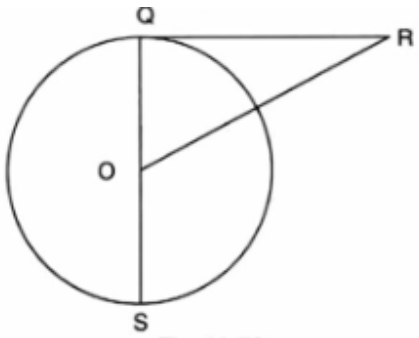


Fig. 10.79

A. 8 cm

B. 3 cm

C. 2.5 cm

D. 5 cm

Answer

Given:

SQ = 6 cm

QR = 4 cm

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔROQ is right-angled at ∠OQR (i.e., ∠OQR = 90°).

Diameter QS = 6 cm

$$\text{Radius} = \frac{\text{Diameter}}{2}$$

$$\Rightarrow \text{Radius} = \frac{6\text{ cm}}{2}$$

⇒Radius (OQ) = 3 cm

Now by Pythagoras theorem,

$OR^2 = OQ^2 + QR^2$

$\Rightarrow OR^2 = 3^2 + 4^2$

$\Rightarrow OR^2 = 9 + 16$

$\Rightarrow OR^2 = 25$

$\Rightarrow OR = \sqrt{25}$

$\Rightarrow OR = 5$ cm

Hence, OR= 5 cm.

15. Question

In Fig. 10.80, the perimeter of ΔABC is

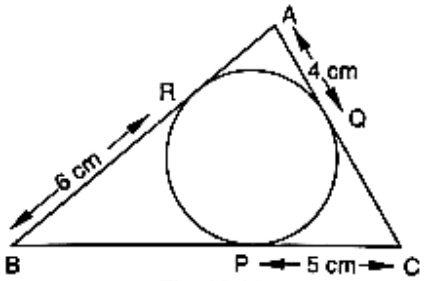


Fig. 10.80

A. 30 cm

B. 60 cm

C. 45 cm

D. 15 cm

Answer

Given:

AQ = 4 cm

BR = 6 cm

PC = 5 cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AR = AQ = 4 cm (tangent from A)

BR = BP = 6 cm (tangent from B)

CP = CQ = 5 cm (tangent from C)

Now,

Perimeter of ΔABC = AB + BC + CA

⇒ Perimeter of ΔABC = AR + RB + BP + PC + CQ + QA

[∵ AB = AR + RB

BC = BP + PC

CA = CQ + QA]

⇒ Perimeter of ΔABC = 4 cm + 6 cm + 6 cm + 5 cm + 5 cm + 4 cm

⇒ Perimeter of ΔABC = 30 cm

Hence, Perimeter of ΔABC = 30 cm

16. Question

In Fig. 10.81, AP is a tangent to the circle with centre O such that OP = 4 cm and ∠OPA = 30°. Then, AP =

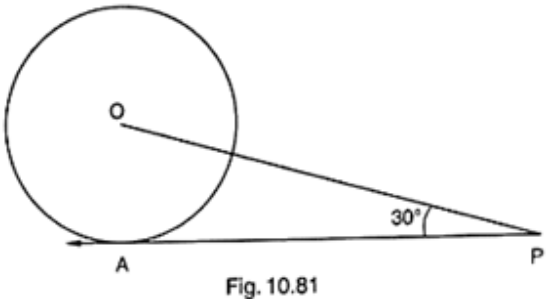


Fig. 10.81

A. 2√2 cm

B. 2 cm

C. 2√3cm

D. 3√2 cm

Answer

Given:

OP = 4 cm

∠OPA = 30°

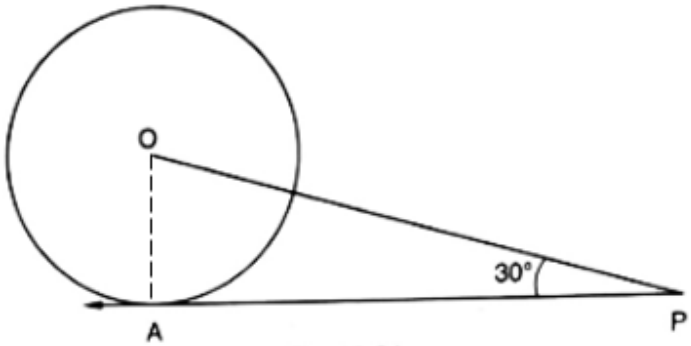


Fig. 10.81

Property: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

By above property, ΔPOA is right-angled at ∠OAP (i.e., ∠OAP = 90°).

Now we know that,

$$\text{Cos}\,\theta = \frac{\text{Base}}{\text{Hypotnuse}}$$

Therefore,

$$\text{Cos}\,\angle P = \frac{AP}{OP}$$

$$\Rightarrow \text{Cos}\,30° = \frac{AP}{4}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{4}$$

$$\Rightarrow AP = \frac{4 \times \sqrt{3}}{2}$$

⇒ AP = 2√3 cm

Hence, AP = 2√3 cm

17. Question

AP and PQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If OA = 15 cm, then AP + AQ =

A. 12 cm

B. 18 cm

C. 24cm

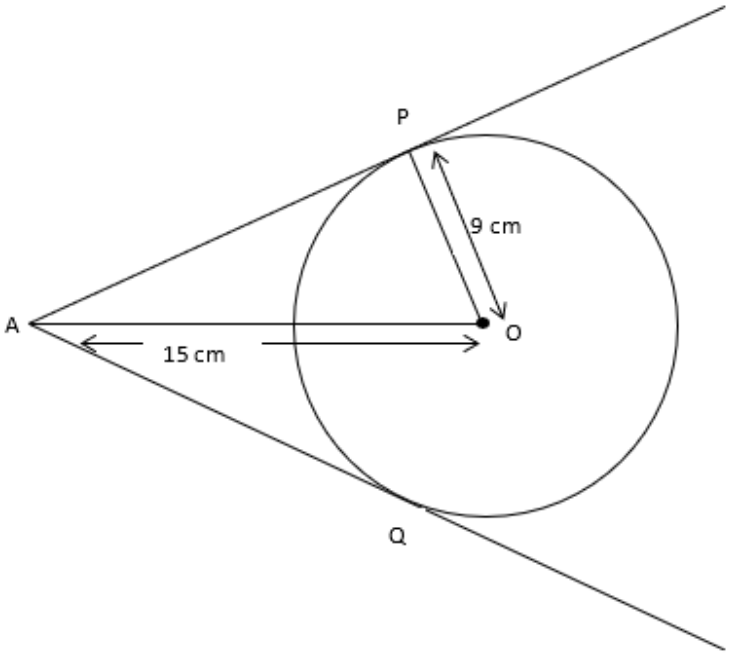
D. 36 cm

Answer

Given:

Radius = 9 cm

OA = 15 cm



<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AQ (tangent from A)

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOA is right-angled at ∠OAP (i.e., ∠OPA = 90°).

Therefore by Pythagoras theorem,

$AP^2 + PO^2 = AO^2$

$\Rightarrow AP^2 = AO^2 - PO^2$

$\Rightarrow AP^2 = 15^2 - 9^2$

$\Rightarrow AP^2 = 225 - 81$

$\Rightarrow AP^2 = 144$

$\Rightarrow AP = \sqrt{144}$

$\Rightarrow AP = 12$

AP + AQ = 12 cm + 12 cm = 24 cm

Hence, AP + AQ = 24 cm

18. Question

At one end of a diameter PQ of a circle of radius 5 cm, tangent XPY is drawn to the circle. The length of chord AB parallel to XY and at a distance of 8 cm from P is

A. 5 cm

B. 6 cm

C. 7 cm

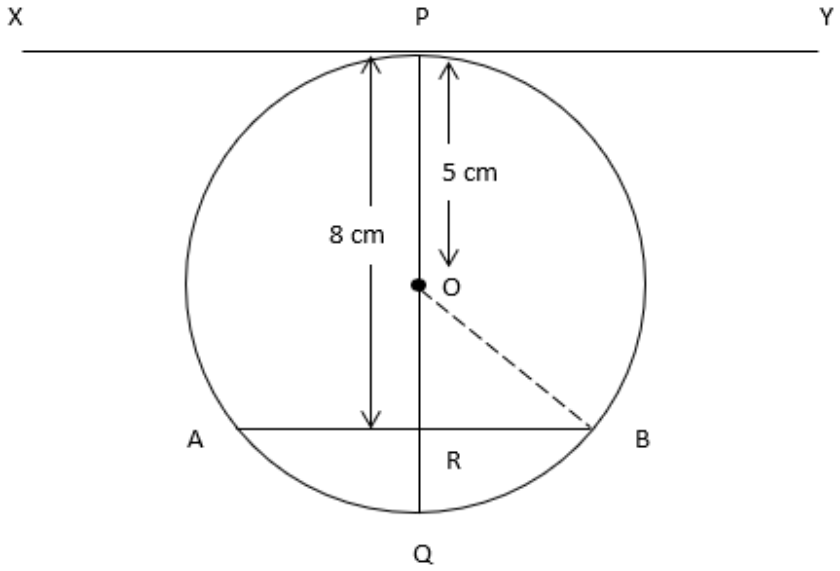
D. 8 cm

Answer

Given:

Radius = OP = 5 cm

Distance of AB and XY = 8 cm



∵ Distance of AB and XY = 8 cm

And AB is parallel to XY

∴ PR = 8 cm

Join OB

Now,

OB = OP = 5 cm [radius]

Also,

OR = PR – PO

⇒ OR = 8 cm – 5 cm

⇒ OR = 3 cm

∴ By Pythagoras theorem in ΔORB,

$OB^2 = OR^2 + RB^2$

$\Rightarrow 5^2 = 3^2 + RB^2$

$\Rightarrow RB^2 = 5^2 - 3^2$

$\Rightarrow RB^2 = 25 - 9$

$\Rightarrow RB^2 = 16$

$\Rightarrow RB = 4$

Now,

AB = AR + RB

$\Rightarrow AB = 2RB$

$\Rightarrow AB = 2 \times 4$

$\Rightarrow AB = 8$ cm

Hence, Length of chord = 8 cm

19. Question

If PT is tangent drawn from a point P to a circle touching it at T and O is the centre of the circle, then ∠OPT + ∠POT =

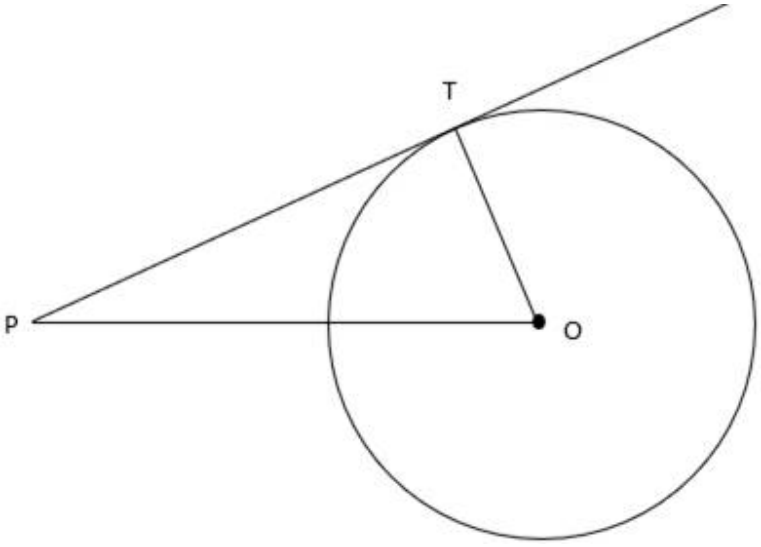
A. 30°

B. 60°

C. 90°

D. 180°

Answer



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a triangle = 180°

By property 1, ΔPTO is right-angled at ∠OTP (i.e., ∠OTP = 90°).

By property 2,

$\angle OTP + \angle POT + \angle TPO = 180°$

$\Rightarrow 90° + \angle POT + \angle TPO = 180°$

$\Rightarrow \angle POT + \angle TPO = 180° - 90°$

$\Rightarrow \angle POT + \angle TPO = 90°$

Hence, $\angle POT + \angle TPO = 90°$

20. Question

In the adjacent figure, if AB = 12 cm, BC = 8 cm and AC = 10 cm, then AD =

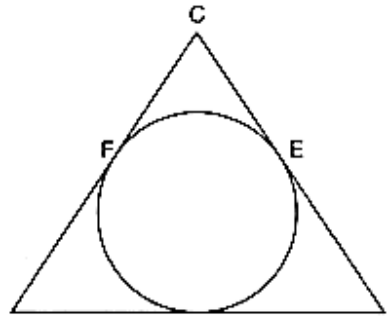


Fig. 10.82

A. 5 cm

B. 4 cm

C. 6 cm

D. 7 cm

Answer

Given:

AB = 12 cm

BC = 8 cm

AC = 10 cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AD = AF (tangent from A)

BD = BE (tangent from B)

CF = CE (tangent from C)

Clearly,

AB = AD + DB = 12 cm

BC = BE + EC = 8 cm

AC = AF + FC = 10 cm

Now,

AB – BC = 12 cm – 8 cm

⇒ (AD + DB) – (BE + EC) = 12 cm – 8 cm

⇒ AD + DB – BE – EC = 12 cm – 8 cm

⇒ AD + BE – BE – CF = 12 cm – 8 cm [∵ DB = BE and CF = CE]

⇒ AD – CF = 12 cm – 8 cm

⇒ AD – (10 cm – AF) = 12 cm – 8 cm [∵AF + FC = 10 cm ⇒ FC = 10 cm – AF]

⇒ AD – (10 cm – AF) = 4 cm

⇒ AD – 10 cm + AF = 4 cm

⇒ AD + AD = 4 cm + 10 cm [∵ AD = AF]

⇒ 2AD = 14 cm

$$\Rightarrow AD = \frac{14\ cm}{2}$$

⇒ AD = 7 cm

Hence, AD = 7 cm

21. Question

In Fig. 10.83, if AP = PB, then

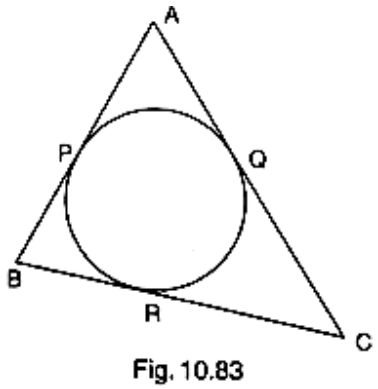


Fig. 10.83

A. AC = AB

B. AC = BC

C. AQ = QC

D. AB = BC

Answer

Given:

AP = PB

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AQ (tangent from A)

BR = BP (tangent from B)

CQ = CR (tangent from C)

Clearly,

AP = BP = BR

AQ = AP = BR

Now,

AQ + QC = BR + RC

⇒ AC = BC [∵AC = AQ + QC and BC = BR + RC]

Hence, AC = BC

22. Question

In Fig. 10.84, if AP = 10 cm, then BP =

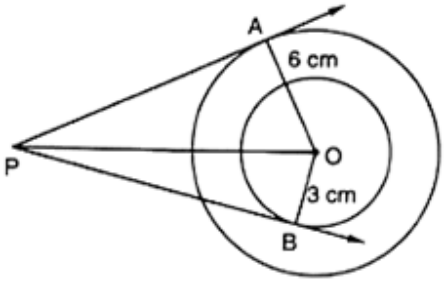


Fig. 10.84

A. $\sqrt{109}\,\text{cm}$

B. $\sqrt{127}\,\text{cm}$

C. $\sqrt{119}\,\text{cm}$

D. $\sqrt{109}\,\text{cm}$

Answer

Given:

AP = 10 cm

OA = 6 cm

OB = 3 cm

<u>Property :</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPAO is right-angled at ∠PAO (i.e., ∠PAO = 90°) and ΔPBO is right-angled at ∠PBO (i.e., ∠PBO = 90°).

Therefore by Pythagoras theorem in ΔPAO,

$OP^2 = OA^2 + AP^2$

$\Rightarrow OP^2 = 6^2 + 10^2$

$\Rightarrow OP^2 = 36 + 100$

$\Rightarrow OP = \sqrt{136}$

Now by Pythagoras theorem in ΔPBO,

$OP^2 = OB^2 + BP^2$

$BP^2 = OP^2 - OB^2$

$\Rightarrow BP^2 = (\sqrt{136})^2 - 3^2$

$\Rightarrow BP^2 = 136 - 9$

$\Rightarrow BP = \sqrt{127}$

Hence, BP= √127 cm

23. Question

In Fig. 10.85, if PR is tangent to the circle at P and Q is the centre of the circle, then ∠POQ =

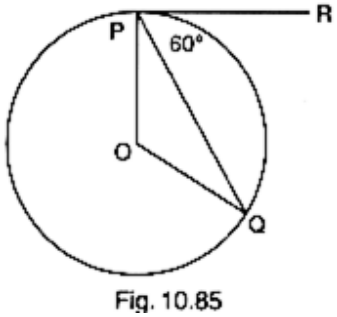


Fig. 10.85

A. 110°

B. 100°

C. 120°

D. 90°

Answer

Given:

∠RPQ = 60°

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a triangle = 180°.

By property 1, ΔOPR is right-angled at ∠OPR (i.e., ∠OPR = 90°).

OP = OQ [∵ radius of circle]

∴ ∠OPQ = ∠OQP = 30°

Now by property 2,

∠OPQ + ∠OQP + ∠POQ = 180°

⇒ 30° + 30° + ∠POQ = 180°

⇒ 60° + ∠POQ = 180°

⇒ ∠POQ = 180° - 60°

⇒ ∠POQ = 120°

Hence, ⇒ ∠POQ = 120°

24. Question

In Fig. 10.86, if quadrilateral PQRS circumscribes a circle, then PD + QB =

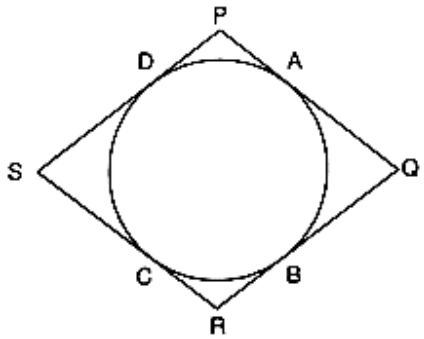


Fig. 10.86

A. PQ

B. QR

C. PR

D. PS

Answer

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

PD = PA (tangent from P)

QB = QA (tangent from Q)

RC = RB (tangent from R)

SC = SD (tangent from S)

Now,

PD + QB = PA + QA

⇒ PD + QB = PQ [∵PQ = PA + QA]

Hence, PD + QB = PQ

25. Question

In Fig. 10.87, two equal circles touch each other at T, if QP = 4.5 cm, then QR =

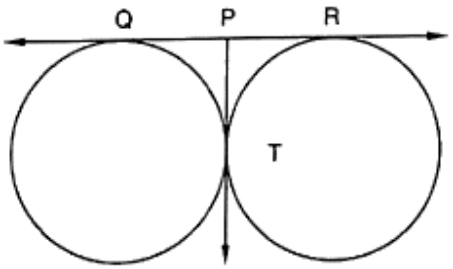


Fig. 10.87

A. 9 cm

B. 18 cm

C. 15 cm

D. 13.5 cm

Answer

Given:

QP = 4.5 cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

PQ = PT = PR = 4.5 cm (tangent from P)

Now,

QR = PQ + PR

QR = PQ + PQ [∵PQ = PR]

QR = 2PQ

QR = 2 × 4.5 cm

QR = 9 cm

Hence, QR = 9 cm

26. Question

In Fig. 10.88, APB is a tangent to a circle with centre O at point P. If ∠QPB = 500, then the measure of ∠POQ is

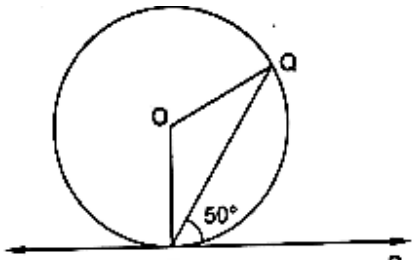


Fig. 10.88

A. 100°

B. 120°

C. 140°

D. 150°

Answer

Given:

∠QPB =50°

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a triangle = 180°.

By property 1, ΔOPB is right-angled at ∠OPB (i.e., ∠OPB = 90°).

∠OPQ = ∠OPB − ∠QPB

⇒ ∠OPQ = 90° − 50° = 40°

And,

∠OPQ = ∠OQP [∵ OP = OQ (radius of circle)]

Now by property 2,

∠OPQ + ∠OQP + ∠POQ = 180°

⇒ 40° + 40° + ∠POQ = 180°

⇒ 80° + ∠POQ = 180°

⇒ ∠POQ = 180° - 80°

⇒ ∠POQ = 100°

11. Constructions

Exercise 11.1

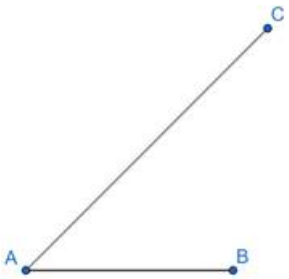
1. Question

Determine a point which divides a line segment of length 12 cm internally in the ratio 2 : 3. Also, justify your construction.

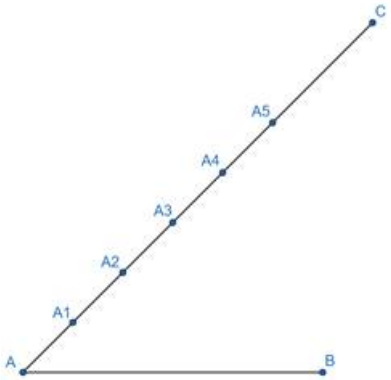
Answer



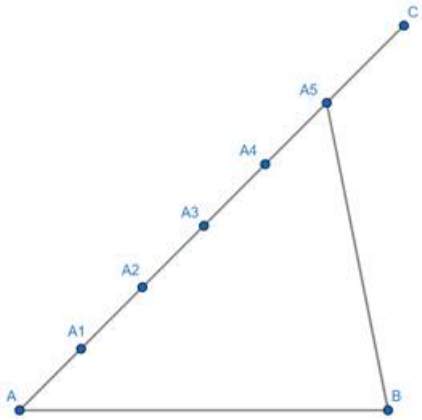
We need to divide this line segment AB of length 12 cm internally in the ratio 2 : 3.



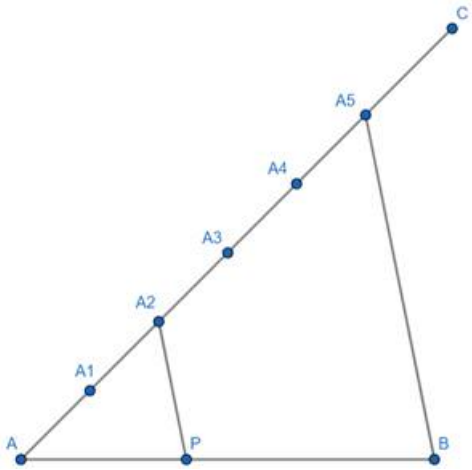
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



Step 2: We plot (2 + 3 =) 5 points A_1, A_2, A_3, A_4, and A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.



Step 3: We join points A_5 and B.



Step 4: We draw line segment A_2P such that $A_2P \parallel A_5B$ and P is the point of intersection of this line segment with AB.

Point P divides AB in the ratio 2 : 3.

Justification –

In ΔAA_2P and ΔAA_5B,

i. $\angle A$ is common.

ii. $\angle AA_2P = \angle AA_5B$ (corresponding angles $\because A_2P \parallel A_5B$)

Hence, $\Delta AA_2P \sim \Delta AA_5B$

So, ratio of lengths of corresponding sides must be equal.

$$\Rightarrow \frac{AA_2}{AP} = \frac{AA_5}{AB}$$

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = x$

So, the previous relation can be re – written as –

$$\frac{2x}{AP} = \frac{5x}{AP + PB}$$

$\Rightarrow 2(AP + PB) = 5AP$

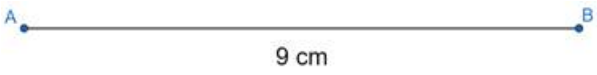
$\Rightarrow 2PB = 3AP$

$\Rightarrow AP/PB = 2/3$, or, $AP : PB = 2 : 3$

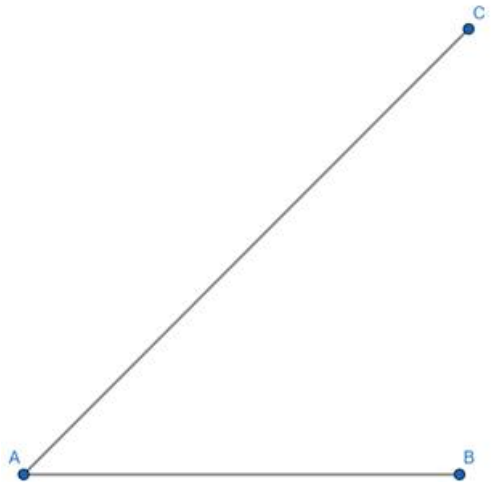
2. Question

Divide a line segment of length 9 cm internally in the ratio 4 : 3. Also, give justification of the construction.

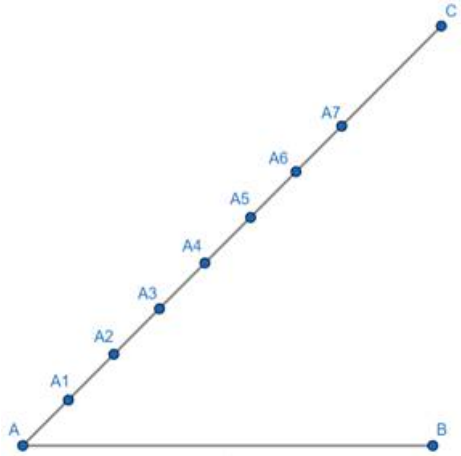
Answer



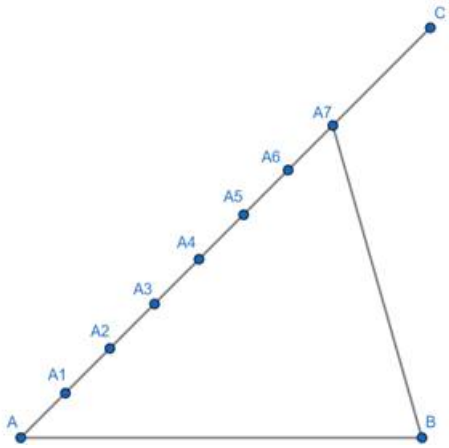
We need to divide this line segment AB of length 9 cm internally in the ratio 4 : 3.



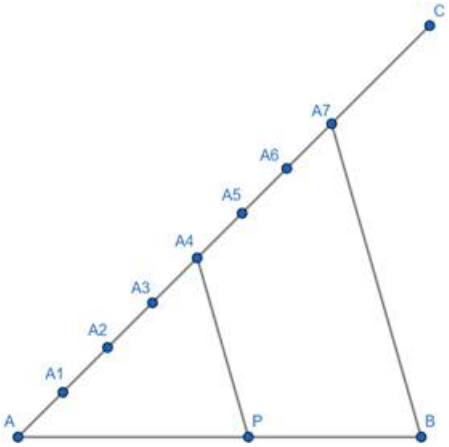
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



Step 2: We plot (4 + 3 =) 7 points A_1, A_2, A_3, A_4, A_5, A_6, and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$



Step 3: We join points A_7 and B.



Step 4: We draw line segment A_4P such that $A_4P \parallel A_7B$ and P is the point of intersection of this line segment with AB.

Point P divides AB in the ratio 4 : 3.

Justification –

In ΔAA_4P and ΔAA_7B,

iii. $\angle A$ is common.

iv. $\angle AA_4P = \angle AA_7B$ (corresponding angles $\because A_4P \parallel A_7B$)

Hence, $\Delta AA_4P \sim \Delta AA_7B$

So, ratio of lengths of corresponding sides must be equal.

$$\Rightarrow \frac{AA_4}{AP} = \frac{AA_7}{AB}$$

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = x$

So, the previous relation can be re – written as –

$$\frac{4x}{AP} = \frac{7x}{AP + PB}$$

$\Rightarrow 4(AP + PB) = 7AP$

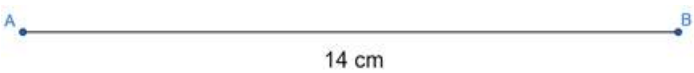
$\Rightarrow 4PB = 3AP$

$\Rightarrow AP/PB = 4/3$, or, $AP : PB = 4 : 3$

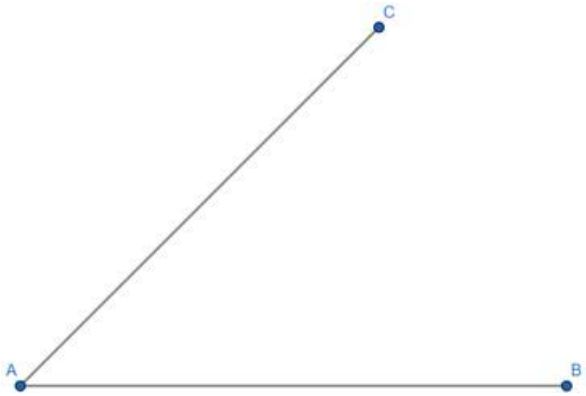
3. Question

Divide a line segment of length 14 cm internally in the ratio 2 : 5. Also, justify your construction.

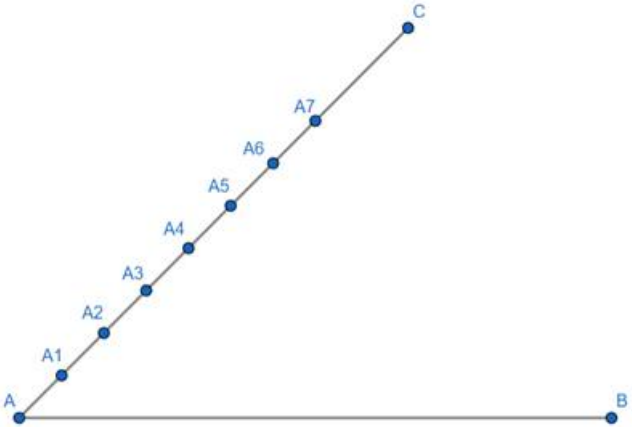
Answer



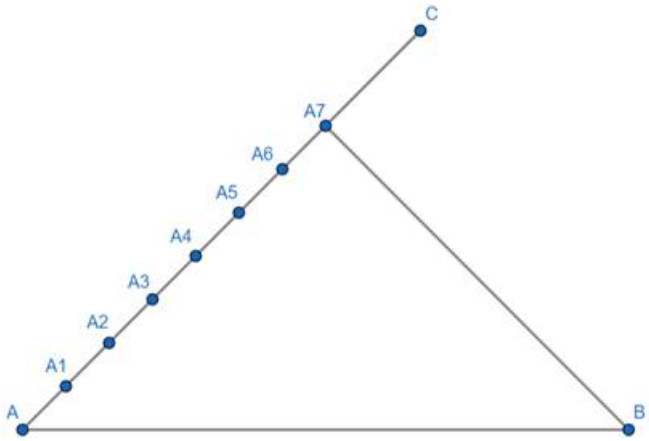
We need to divide this line segment AB of length 14 cm internally in the ratio 2 : 5.



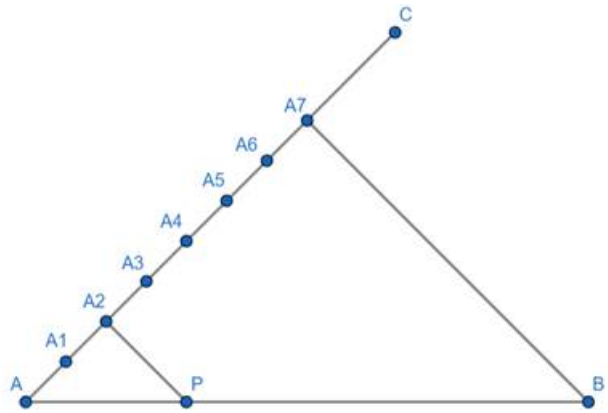
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



Step 2: We plot (2 + 5 =) 7 points A_1, A_2, A_3, A_4, A_5, A_6, and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$



Step 3: We join points A_7 and B.



Step 4: We draw line segment A_2P such that $A_2P \parallel A_7B$ and P is the point of intersection of this line segment with AB.

Point P divides AB in the ratio 2 : 5.

Justification –

In ΔAA_2P and ΔAA_7B,

v. $\angle A$ is common.

vi. $\angle AA_2P = \angle AA_7B$ (corresponding angles $\because A_2P \parallel A_7B$)

Hence, $\Delta AA_2P \sim \Delta AA_7B$

So, ratio of lengths of corresponding sides must be equal.

$\Rightarrow \frac{AA_2}{AP} = \frac{AA_7}{AB}$

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = x$

So, the previous relation can be re – written as –

$$\frac{2x}{AP} = \frac{7x}{AP + PB}$$

$\Rightarrow 2(AP + PB) = 7AP$

$\Rightarrow 2PB = 5AP$

$\Rightarrow AP/PB = 2/5$, or, $AP : PB = 2 : 5$

Exercise 11.2

1. Question

Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are (2/3) of the corresponding sides of it.

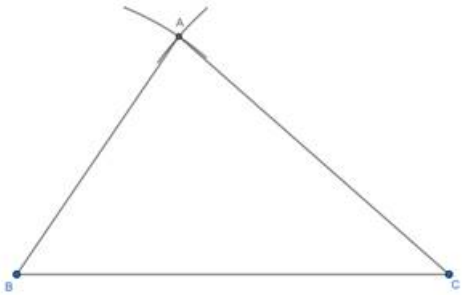
Answer

The steps involved in the required construction are:

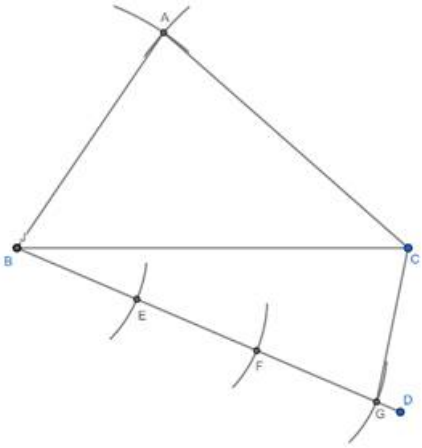
1) Draw a line segment BC=6 cm.



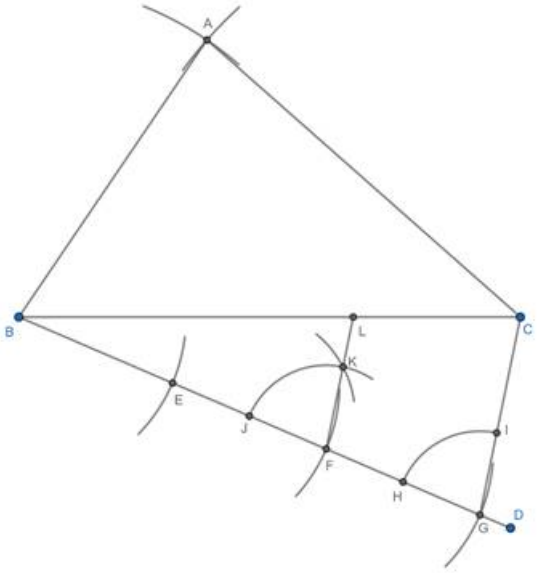
2) Taking B as the center and radius 4 cm, draw an arc. Now taking C as the center and radius 5 cm draw another arc, intersecting the previous arc at A. Join AB and AC.



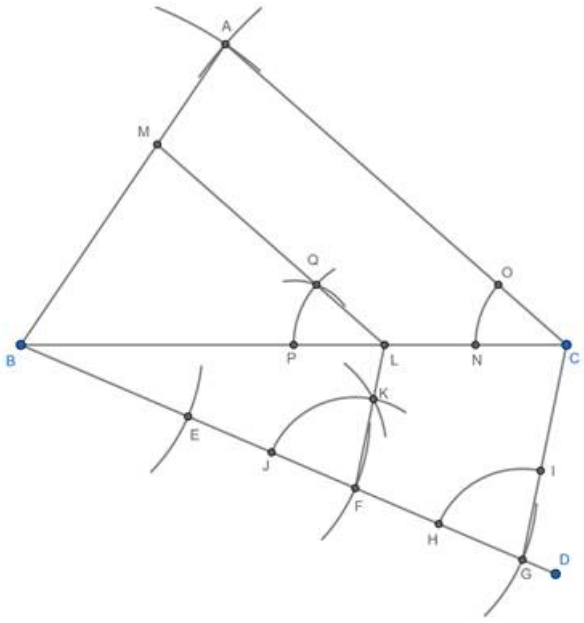
3) Draw any line segment BD, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BD at E. Taking E as the center and radius BE, draw an arc, intersecting BD at F. Taking F as the center and radius BE, draw an arc, intersecting BD at G. Join CG.



4) Taking G as the center and any radius, draw an arc., intersecting BD and CG at H and I respectively. Taking F as the center and radius GH, draw an arc., intersecting BD at J. Taking J as the center and radius HI, draw an arc, intersecting previous arc at K. Join and extend FK, intersecting BC at L.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at N and O respectively. Taking L as the center and radius CN, draw an arc., intersecting BC at P. Taking P as the center and radius NO, draw an arc, intersecting previous arc at Q. Join and extend LQ, intersecting AB at M.



6) ΔBLM is the required triangle.

2. Question

Construct a triangle similar to a given ΔABC such that each of its sides is $(5/7)^{th}$ of the corresponding sides of ΔABC. It is given that AB = 5 cm, BC = 7 cm and $\angle ABC = 50^{0}$.

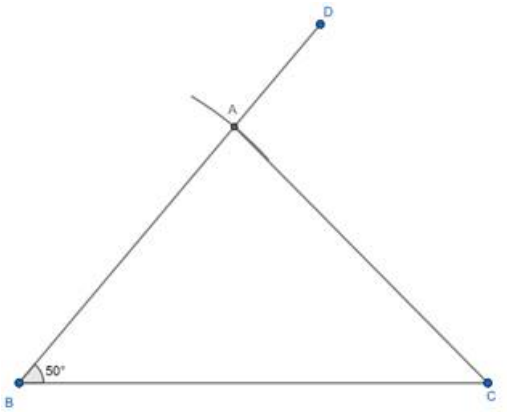
Answer

The steps involved in the required construction are:

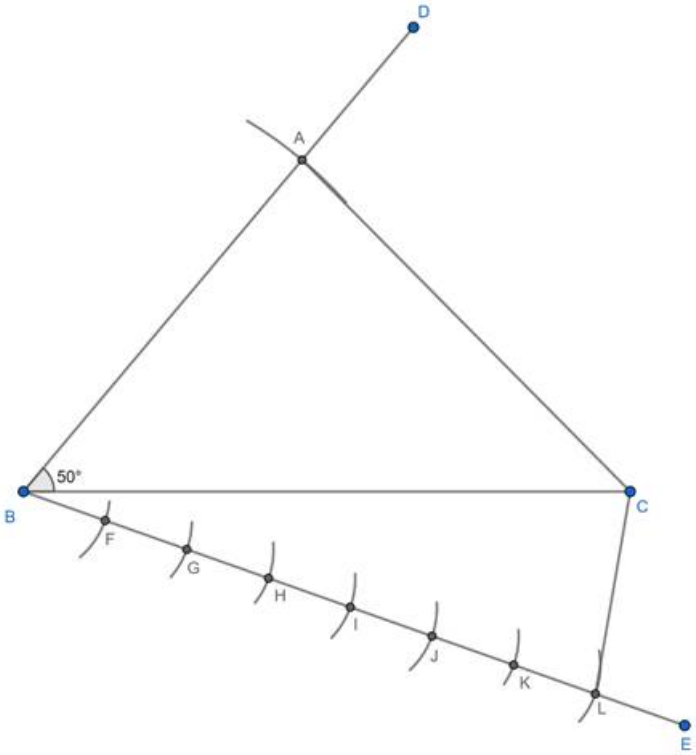
1) Draw a line segment BC=6 cm.



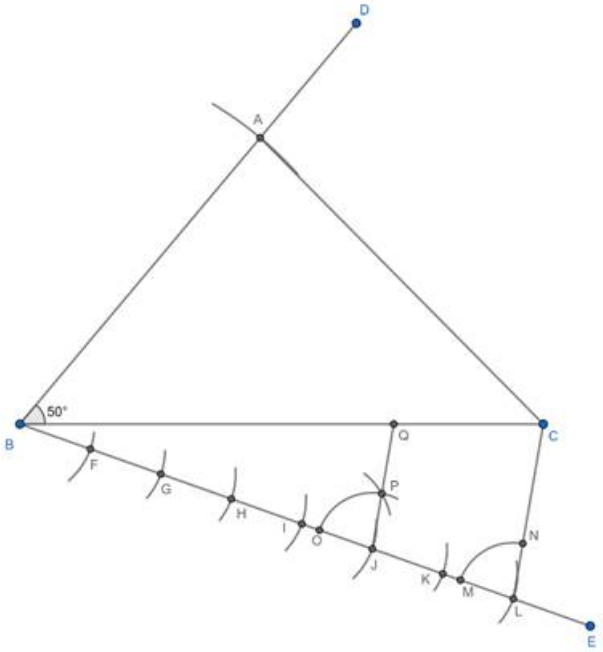
2) Using a protractor, draw ∠CBD=50°. Taking B as the center and radius 5 cm draw an arc, intersecting BD at A. Join AC.



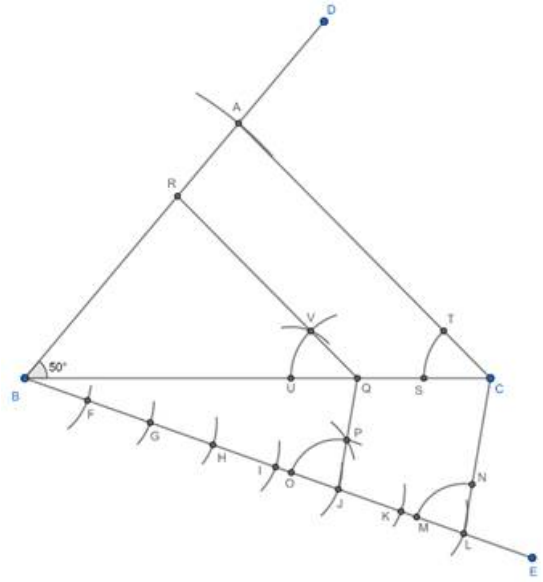
3) Draw any line segment BE, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BE at F. Taking F as the center and radius BF, draw an arc, intersecting BE at G. Similarly, repeat the process 5 more times to get points H, I, J, K and L. Join CL.



4) Taking L as the center and any radius, draw an arc., intersecting BE and CL at M and N respectively. Taking J as the center and radius LM, draw an arc., intersecting BE at O. Taking O as the center and radius MN, draw an arc, intersecting previous arc at P. Join and extend JP, intersecting BC at Q.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at S and T respectively. Taking Q as the center and radius CS, draw an arc., intersecting BC at U. Taking U as the center and radius ST, draw an arc, intersecting previous arc at V. Join and extend QV, intersecting AB at R.



6) ΔBQR is the required triangle.

3. Question

Construct a triangle similar to a given $\triangle ABC$ such that each of its sides is $(2/3)^{rd}$ of the corresponding sides of $\triangle ABC$. It is given that BC = 6 cm, $\angle B = 50^{o}$ and $\angle C = 60^{o}$.

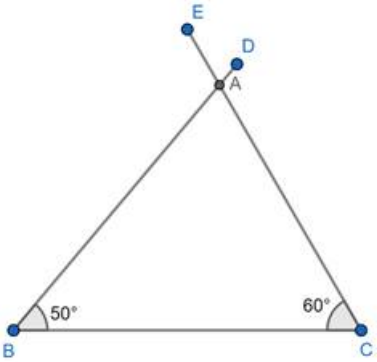
Answer

The steps involved in the required construction are:

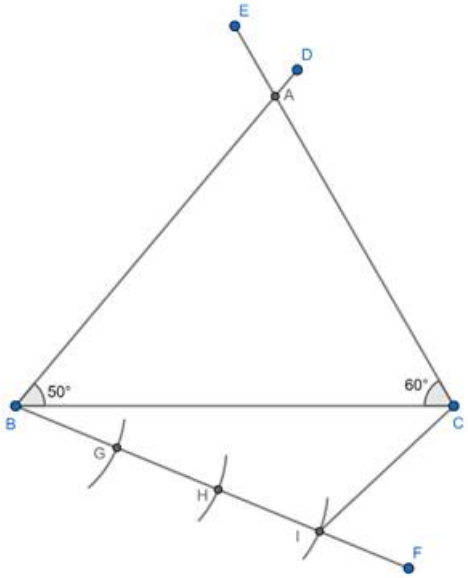
1) Draw a line segment BC=6 cm.



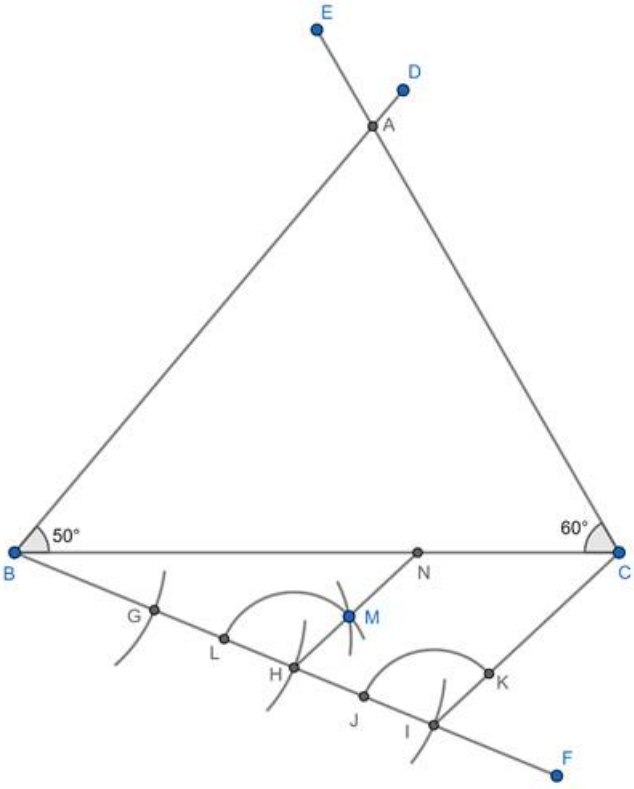
2) Using a protractor, draw ∠CBD=50° and ∠BCE=60°. BD and CE intersect at point A.



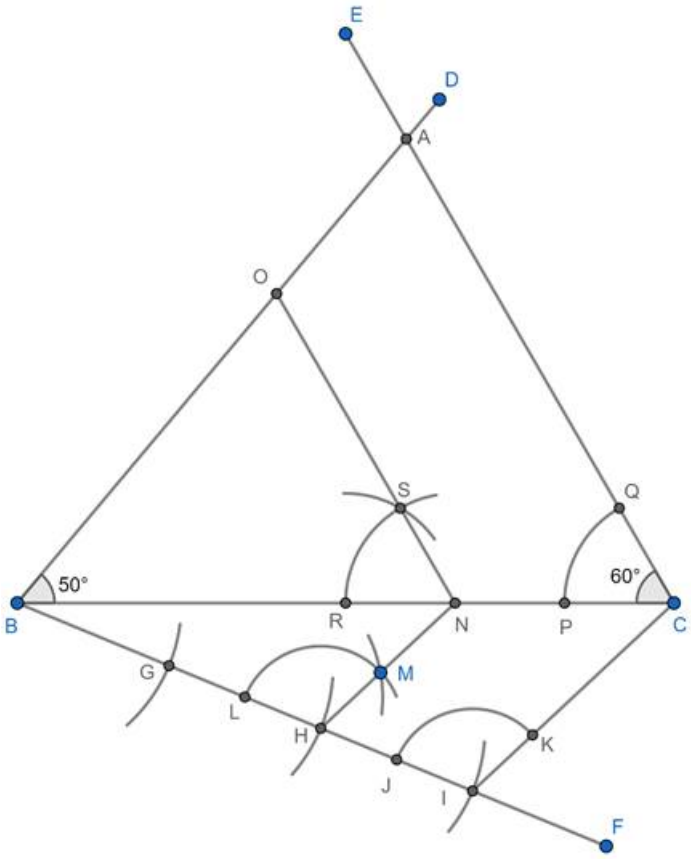
3) Draw any line segment BF, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BF at G. Taking G as the center and radius BG, draw an arc, intersecting BF at H. Taking H as the center and radius BG, draw an arc, intersecting BF at I. Join CI.



4) Taking I as the center and any radius, draw an arc., intersecting BF and CI at J and K respectively. Taking H as the center and radius IJ, draw an arc., intersecting BF at L. Taking L as the center and radius JK, draw an arc, intersecting previous arc at M. Join and extend HM, intersecting BC at N.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at P and Q respectively. Taking N as the center and radius CP, draw an arc., intersecting BC at R. Taking R as the center and radius PQ, draw an arc, intersecting previous arc at S. Join and extend NS, intersecting AB at O.



6) ΔBNO is the required triangle.

4. Question

Draw a ΔABC in which BC = 6 cm, AB = 4 cm and AC = 5 cm. Draw a triangle similar to ΔABC with its sides equal to $(3/4)^{th}$ of the corresponding sides of ΔABC.

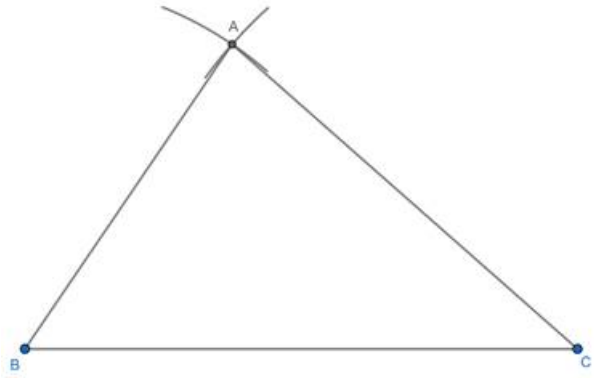
Answer

The steps involved in the required construction are:

1) Draw a line segment BC=6 cm.

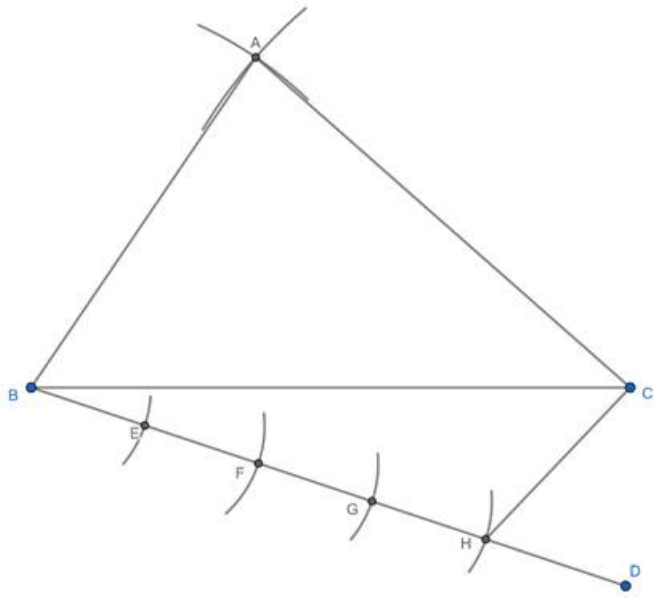


2) Taking B as the center and radius 4 cm, draw an arc. Now taking C as the center and radius 5 cm draw another arc, intersecting the previous arc at A. Join AB and AC.

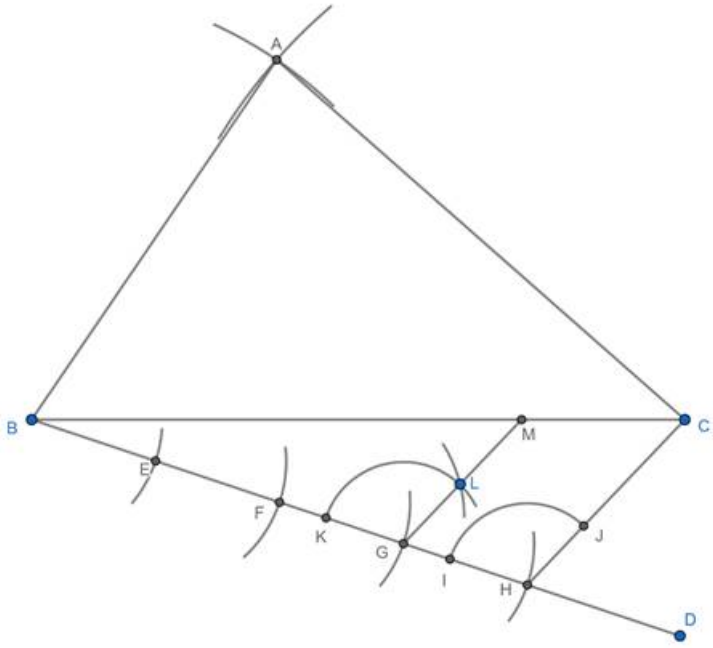


3) Draw any line segment BD, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BD at E. Taking E as the center and radius BE, draw an arc,

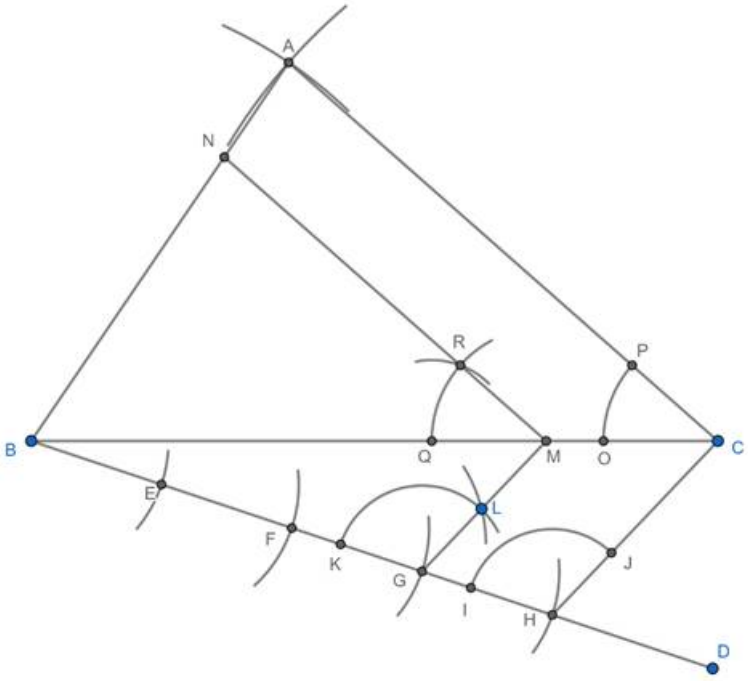
intersecting BD at F. Similarly, repeat the process 2 more times to get points G and H. Join CH.



4) Taking H as the center and any radius, draw an arc., intersecting BD and CH at I and J respectively. Taking G as the center and radius HI, draw an arc., intersecting BD at K. Taking K as the center and radius IJ, draw an arc, intersecting previous arc at L. Join and extend HL, intersecting BC at M.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at O and P respectively. Taking M as the center and radius CO, draw an arc., intersecting BC at Q. Taking Q as the center and radius OP, draw an arc, intersecting previous arc at R. Join and extend MR, intersecting AB at N.



6) ΔBMN is the required triangle.

5. Question

Construct a triangle with sides 5cm, 6 cm and 7 cm and then another triangle whose sides are 7/5 of the corresponding sides of the first triangle.

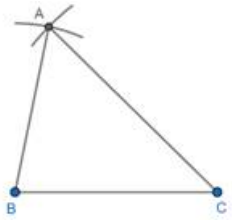
Answer

The steps involved in the required construction are:

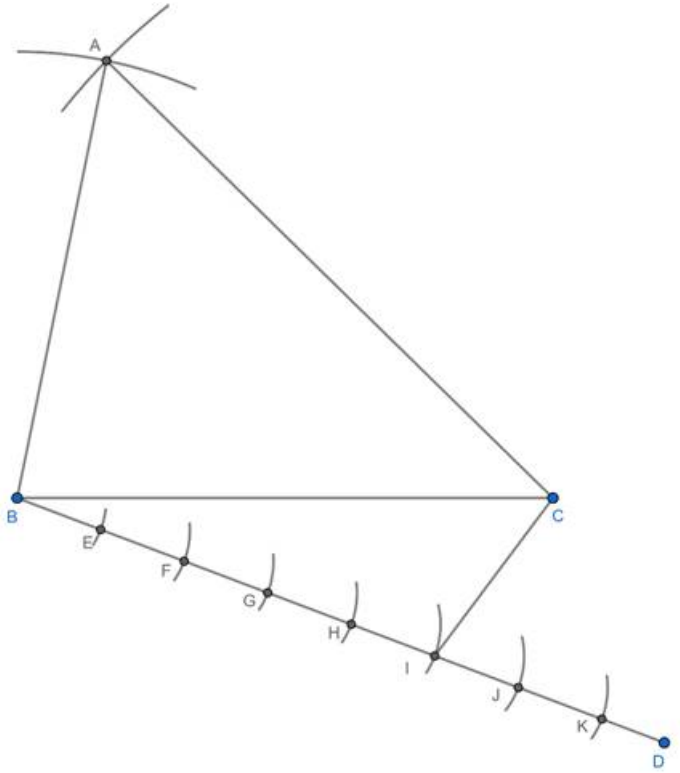
1) Draw a line segment BC=6 cm.



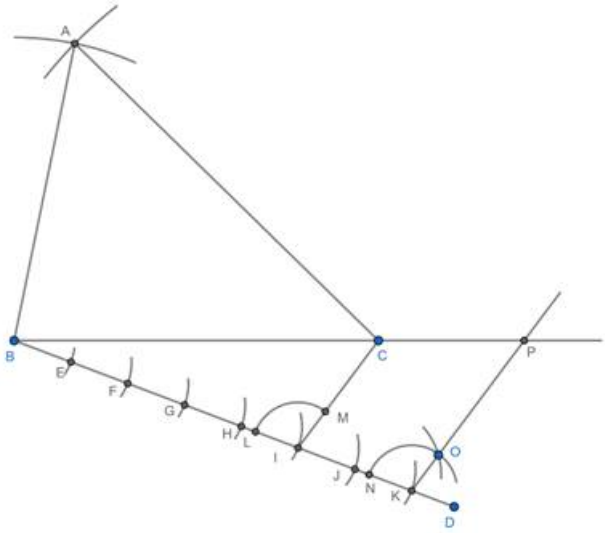
2) Taking B as the center and radius 5 cm, draw an arc. Now taking C as the center and radius 7 cm draw another arc, intersecting the previous arc at A. Join AB and AC.



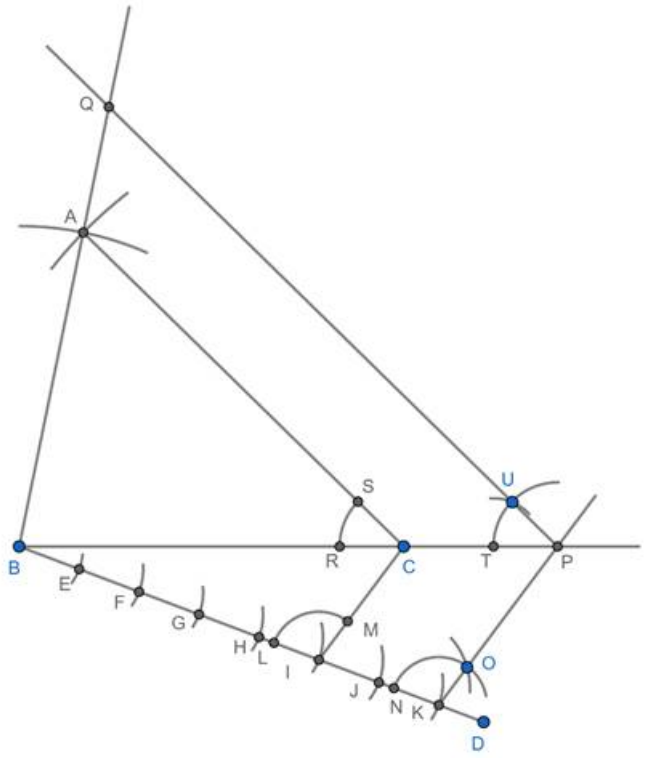
3) Draw any line segment BD, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BD at E. Taking E as the center and radius BE, draw an arc, intersecting BD at F. Similarly, repeat the process 5 more times to get points G, H, I, J and K. Join CI.



4) Taking I as the center and any radius, draw an arc., intersecting BD and CI at L and M respectively. Taking K as the center and radius IL, draw an arc., intersecting BD at N. Taking N as the center and radius LM, draw an arc, intersecting previous arc at O. Join and extend KO, intersecting extended BC at P.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at R and S respectively. Taking P as the center and radius CR, draw an arc., intersecting BP at T. Taking T as the center and radius RS, draw an arc, intersecting previous arc at U. Join and extend PU, intersecting extended AB at Q.



6) ΔBPQ is the required triangle.

6. Question

Draw a right triangle ABC in which AC = AB = 4.5 cm and $\angle A = 90^0$. Draw a triangle similar to ΔABC with its sides equal to $(5/4)$th of the corresponding sides of ΔABC.

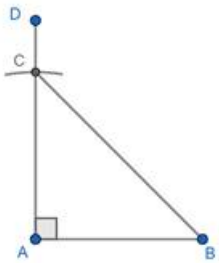
Answer

The steps involved in the required construction are:

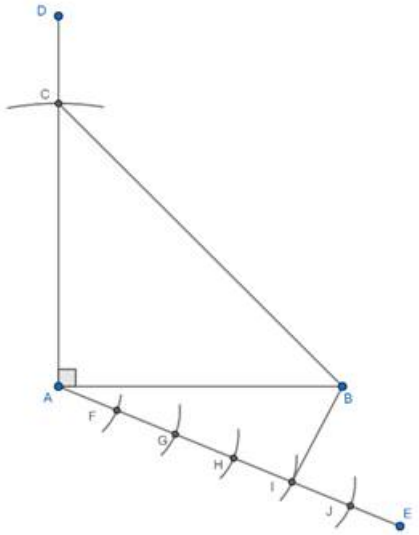
1) Draw a line segment AB=4.5 cm.



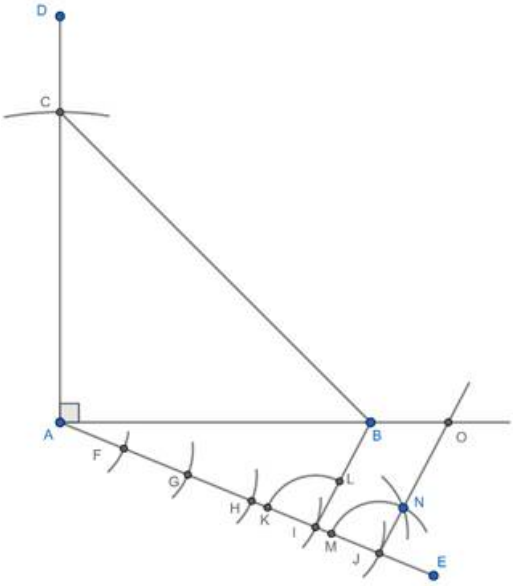
2) Using a protractor, draw ∠BAD=90°. Taking A as the center and radius 4.5 cm, draw an arc, intersecting AD at C. Join BC.



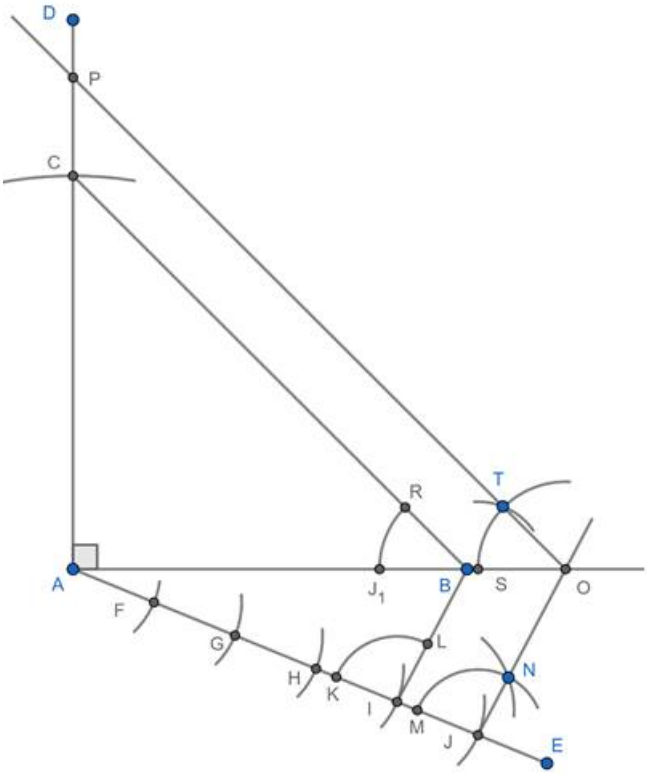
3) Draw any line segment AE, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AE at F. Taking F as the center and radius AF, draw an arc, intersecting AE at G. Similarly, repeat the process 3 more times to get points H, I and J. Join BI.



4) Taking I as the center and any radius, draw an arc., intersecting AE and BI at K and L respectively. Taking J as the center and radius IK, draw an arc., intersecting AE at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend JN, intersecting extended AB at O.



5) Taking B as the center and any radius, draw an arc., intersecting BA and BC at Q and R respectively. Taking O as the center and radius BQ, draw an arc., intersecting AO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting AD at P.



6) ΔAOP is the required triangle.

7. Question

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 5 cm and 4 cm. Then construct another triangle whose sides are 5/3 times the corresponding sides of the given triangle.

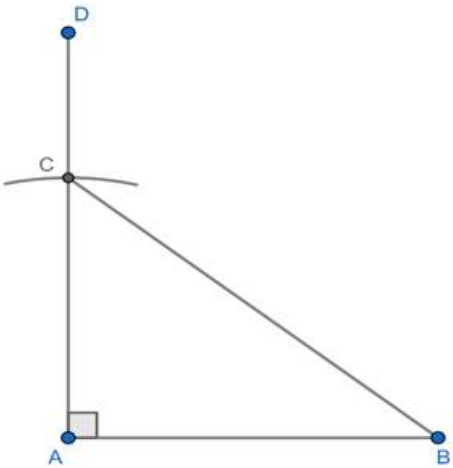
Answer

The steps involved in the required construction are:

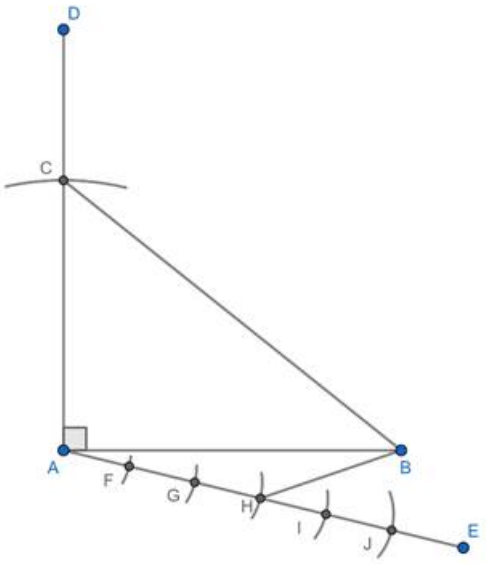
1) Draw a line segment AB=5 cm.



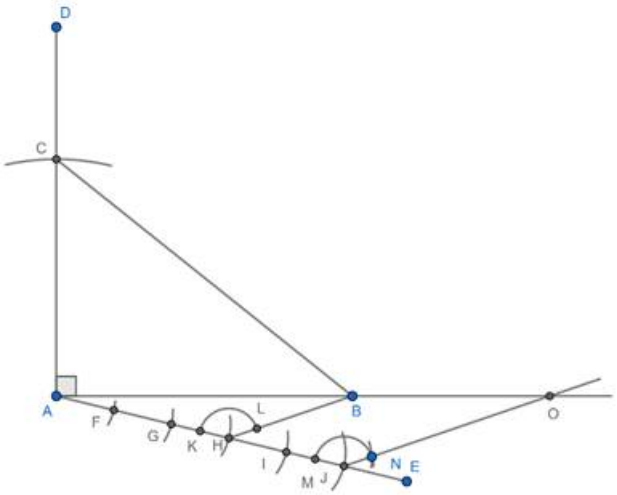
2) Using a protractor, draw ∠BAD=90°. Taking A as the center and radius 4 cm, draw an arc, intersecting AD at C. Join BC.



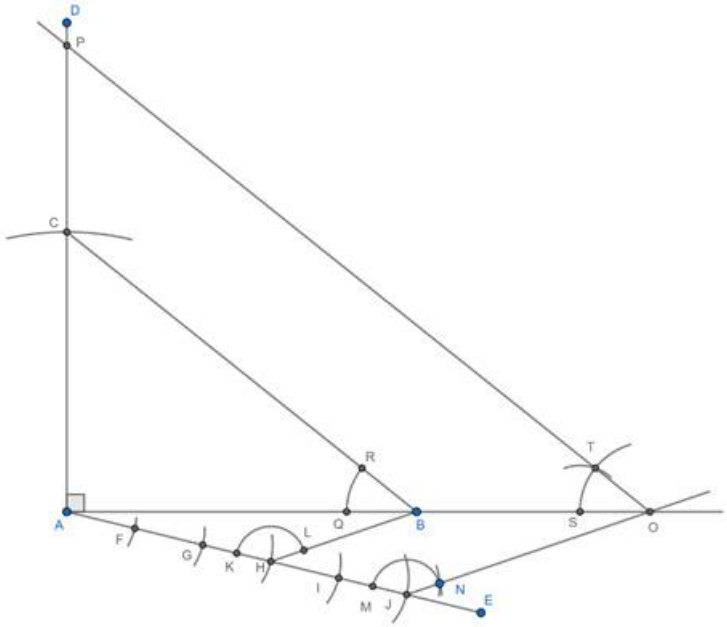
3) Draw any line segment AE, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AE at F. Taking F as the center and radius AF, draw an arc, intersecting AE at G. Similarly, repeat the process 3 more times to get points H, I and J. Join BH.



4) Taking H as the center and any radius, draw an arc., intersecting AE and BH at K and L respectively. Taking J as the center and radius HK, draw an arc, intersecting AE at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend JN, intersecting extended AB at O.



5) Taking B as the center and any radius, draw an arc., intersecting BA and BC at Q and R respectively. Taking O as the center and radius BQ, draw an arc., intersecting AO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting AD at P.



6) ΔAOP is the required triangle.

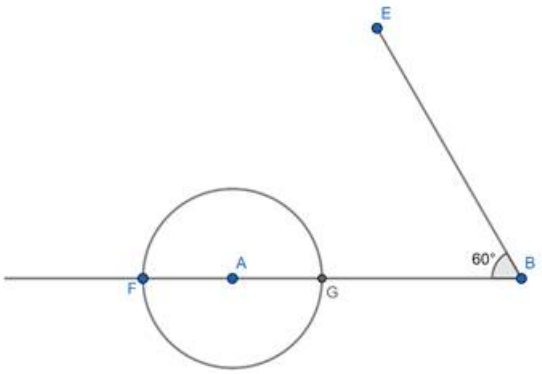
8. Question

Construct a $\triangle ABC$ in which AB = 5 cm. $\angle B = 60^0$ altitude CD = 3 cm. Construct a $\triangle AQR$ similar to $\triangle ABC$ such that side of $\triangle AQR$ is 1.5 times that of the corresponding sides of $\triangle ACB$.

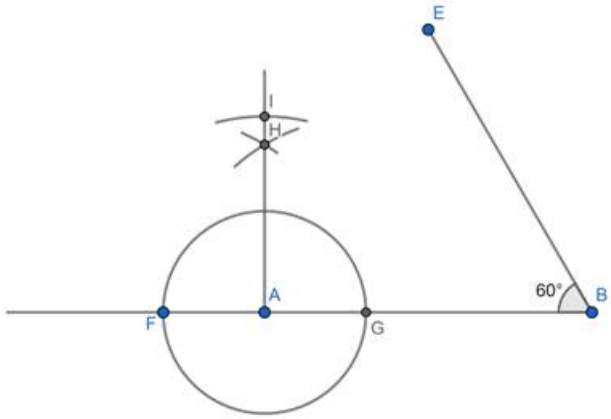
Answer

The steps involved in the required construction are:

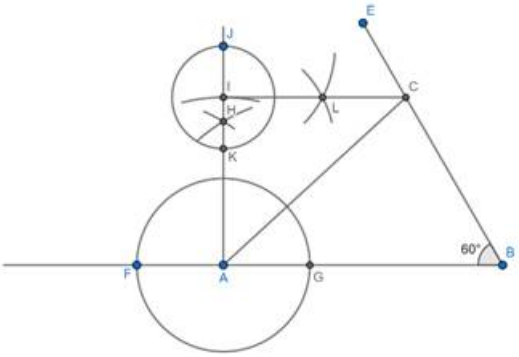
1) Draw a line segment AB= 5 cm. Using a protractor, draw ∠ABE=60°. Keeping A as the center, draw a circle of any radius, intersecting extended AB at F and G.



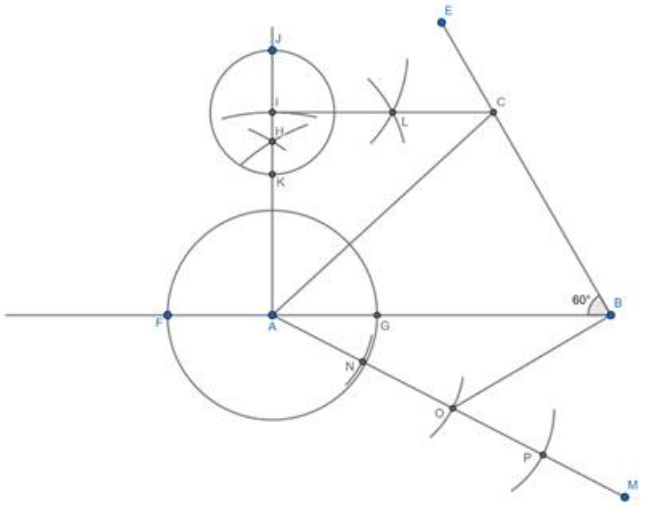
2) Keeping F as the center and any radius draw a circular arc. Now, Keeping G as the center and same radius as before, draw another arc, intersecting previous arc at H. Join and extend AH. Keeping A as the center and radius 3 cm, draw an arc, intersecting extended AH at I.



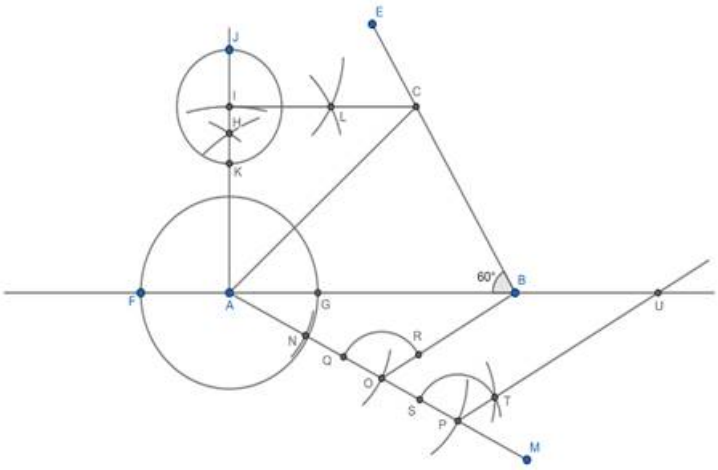
3) Keeping I as the center, draw a circle of any radius, intersecting extended AH at J and K. Keeping J as the center and any radius draw a circular arc. Now, Keeping K as the center and same radius as before, draw another arc, intersecting previous arc at L. Join and extend IL, intersecting BE at C. Join AC.



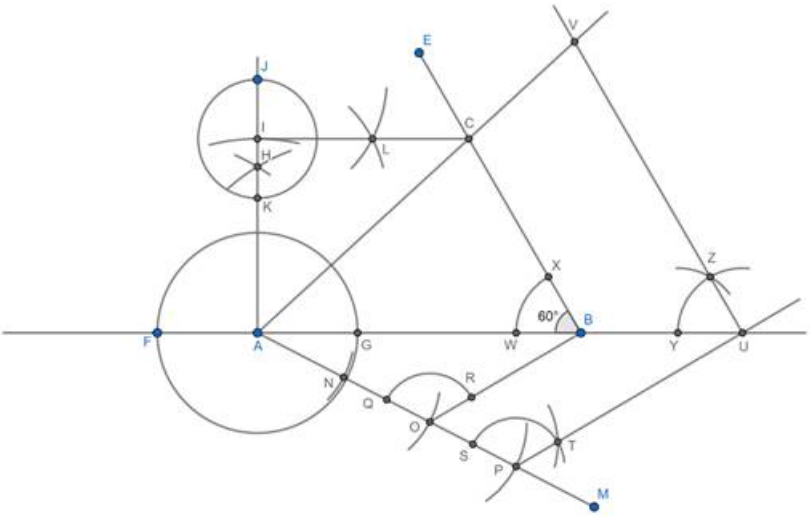
4) Draw any line segment AM, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AM at N. Taking N as the center and radius AN, draw an arc, intersecting AM at O. Taking O as the center and radius AN, draw an arc, intersecting AM at P. Join BO.



5) Taking O as the center and any radius, draw an arc., intersecting AM and BO at Q and R respectively. Taking P as the center and radius OQ, draw an arc, intersecting AM at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend PT, intersecting extended AB at U.



6) Taking B as the center and any radius, draw an arc., intersecting BA and BC at W and X respectively. Taking U as the center and radius BW, draw an arc., intersecting AU at Y. Taking Y as the center and radius WX, draw an arc, intersecting previous arc at Z. Join and extend UZ, intersecting extended AC at V.



7) ΔAUV is the required triangle.

9. Question

Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are 3/2 times the corresponding sides of the isosceles triangle.

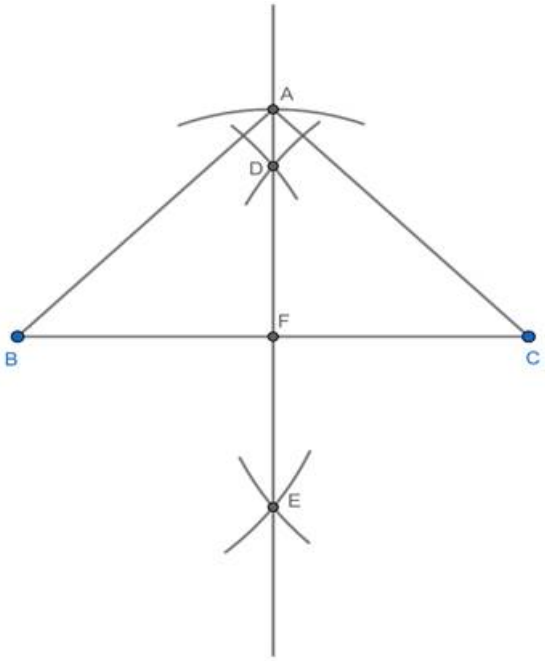
Answer

The steps involved in the required construction are:

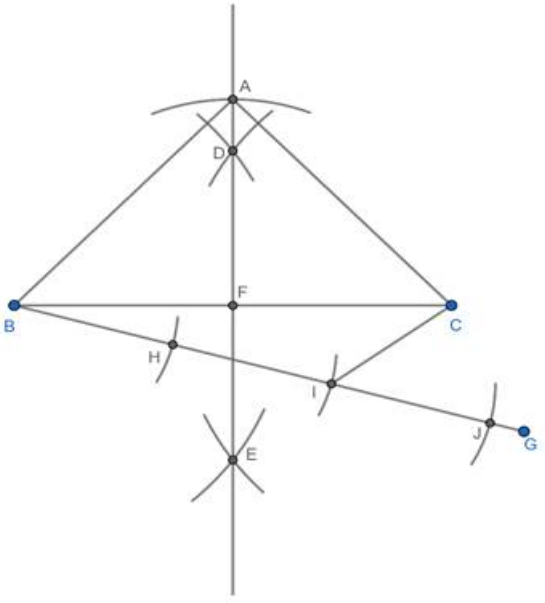
1) Draw a line segment BC= 8 cm.



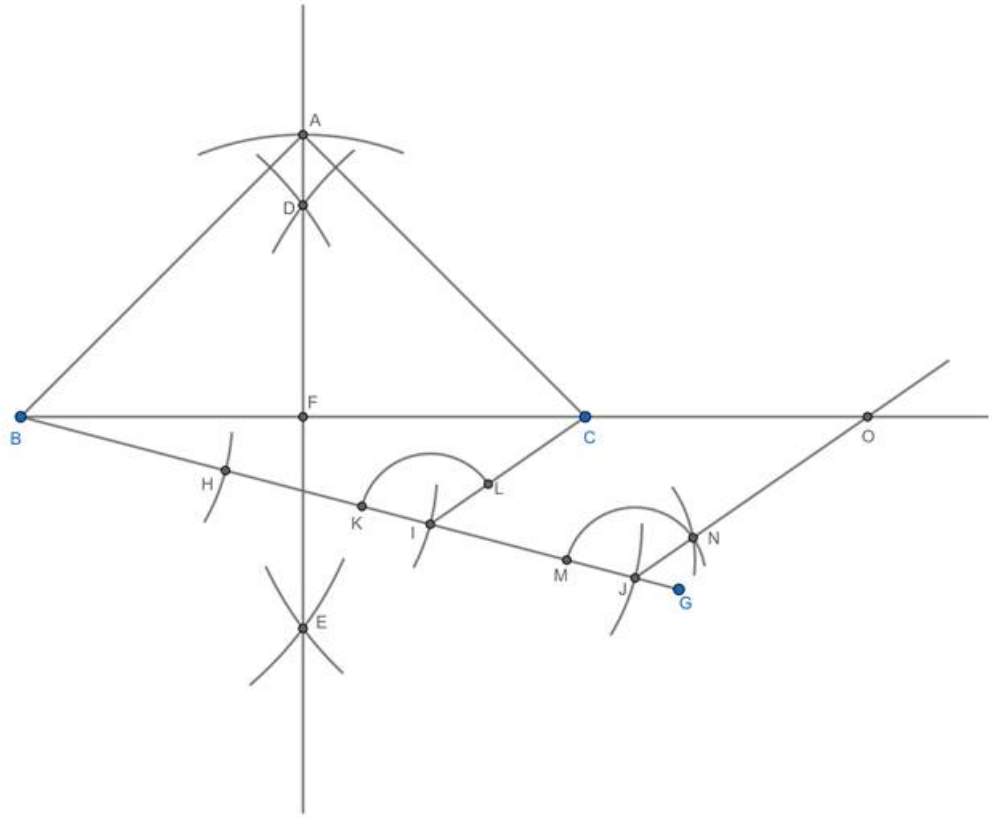
2) Taking B as the center and any radius greater than $\frac{BC}{2}$, draw two arcs on each side of BC. Taking C as the center and same radius, draw 2 more arcs, intersecting previous arcs at D and E. DE, intersecting BC at F. Taking F as the center and radius 4 cm, draw an arc, intersecting FD at A. Join AB and AC.



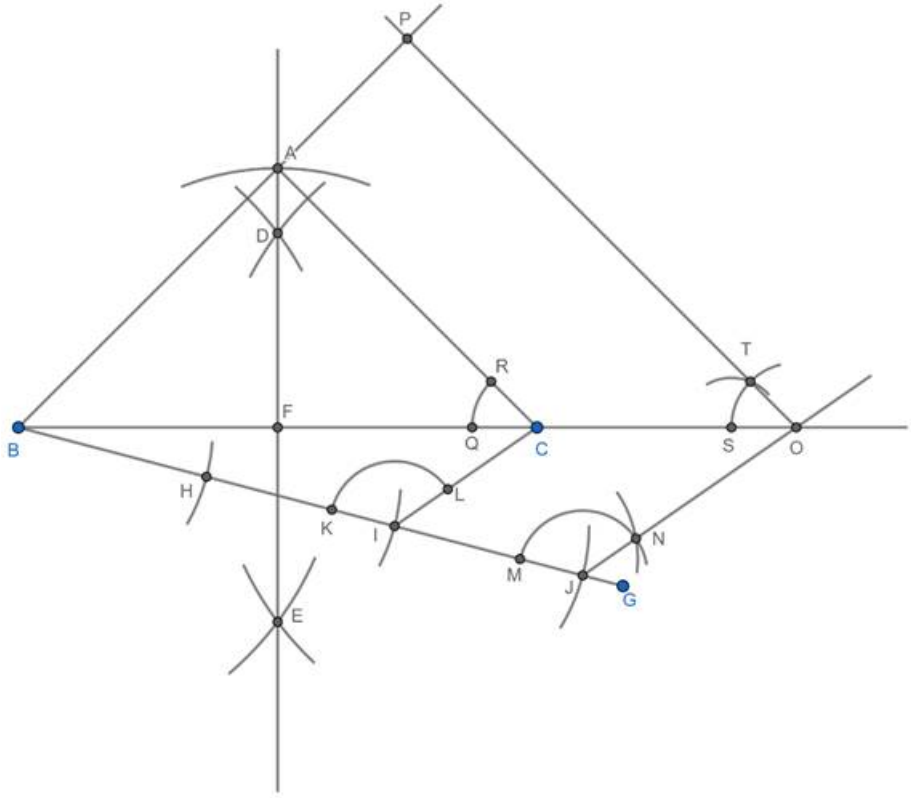
3) Draw any line segment BG, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BG at H. Taking H as the center and radius BH, draw an arc, intersecting BG at I. Taking I as the center and radius BH, draw an arc, intersecting BG at J. Join CI.



4) Taking I as the center and any radius, draw an arc., intersecting BG and CI at K and L respectively. Taking J as the center and radius IK, draw an arc, intersecting BG at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend JN, intersecting extended BC at O.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at Q and R respectively. Taking O as the center and radius CQ, draw an arc., intersecting BO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting extended AB at P.



6) ΔBOP is the required triangle.

10. Question

Draw a $\triangle ABC$ with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^{o}$. Then, construct a triangle whose sides are $(3 / 4)^{th}$ of the corresponding sides of the $\triangle ABC$.

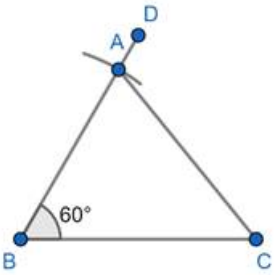
Answer

The steps involved in the required construction are:

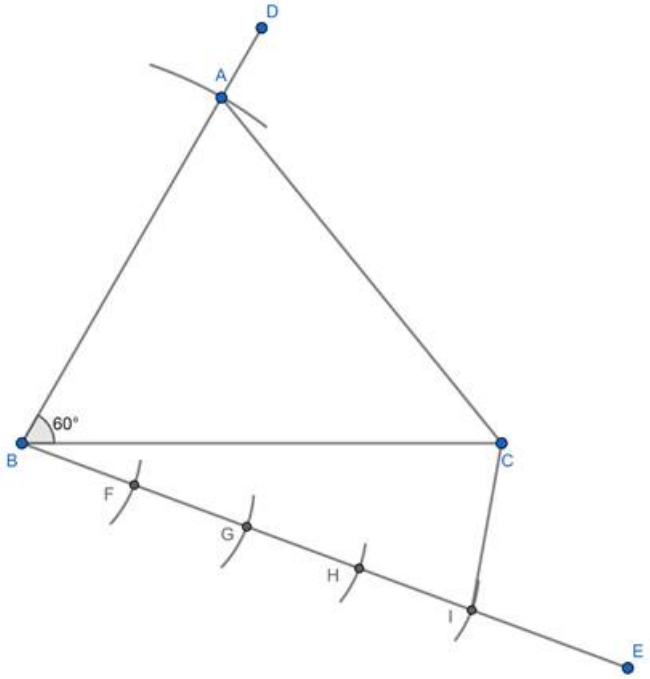
1) Draw a line segment BC= 6 cm.



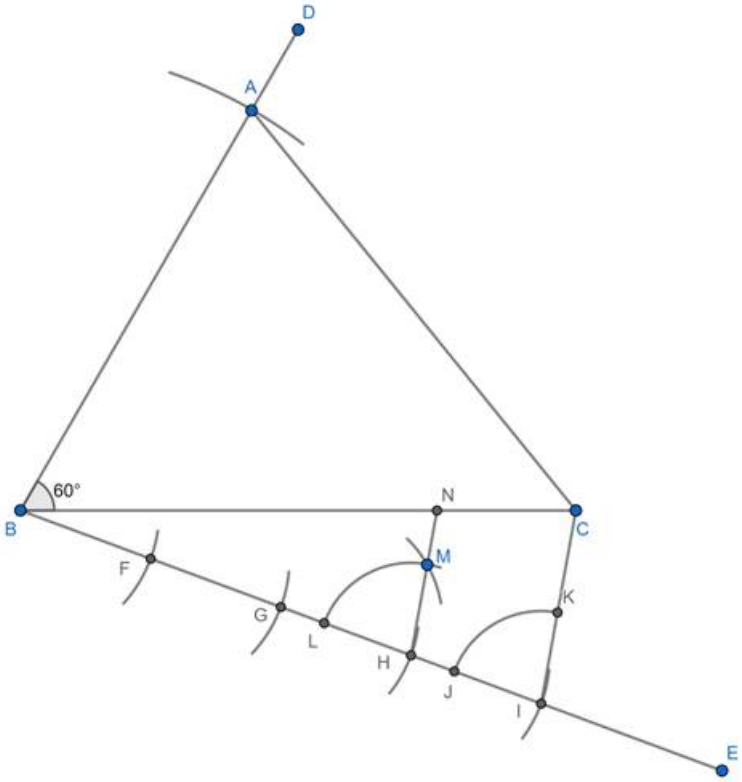
2) Using a protractor, draw ∠CBD=60°. Taking B as the center and radius 5 cm draw an arc, intersecting BD at A. Join AC.



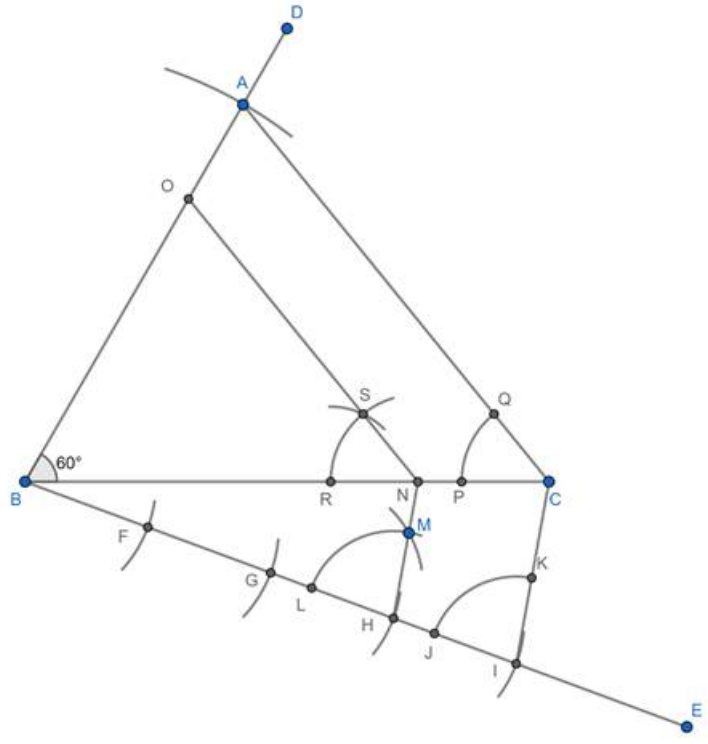
3) Draw any line segment BE, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BE at F. Taking F as the center and radius BF, draw an arc, intersecting BE at G. Similarly, repeat the process 2 more times to get points H and I. Join CI.



4)Taking I as the center and any radius, draw an arc., intersecting BD and CI at J and K respectively. Taking H as the center and radius IJ, draw an arc., intersecting BD at L. Taking L as the center and radius JK, draw an arc, intersecting previous arc at M. Join and extend HM, intersecting BC at N.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at P and Q respectively. Taking N as the center and radius CP, draw an arc., intersecting BC at R. Taking R as the center and radius PQ, draw an arc, intersecting previous arc at S. Join and extend NS, intersecting AB at O.



6) ΔBNO is the required triangle.

11. Question

Construct a triangle similar to $\triangle ABC$ in which AB = 4.6 cm, BC = 5.1 cm, $\angle A = 60^0$ with scale factor 4 : 5.

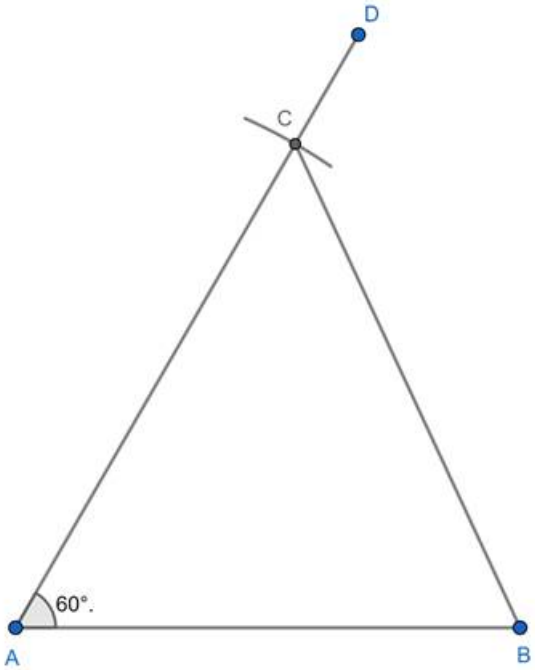
Answer

The steps involved in the required construction are:

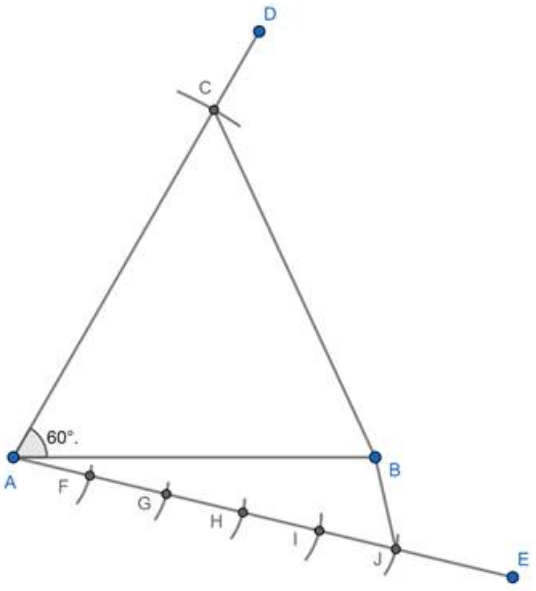
1) Draw a line segment AB=4.6 cm.



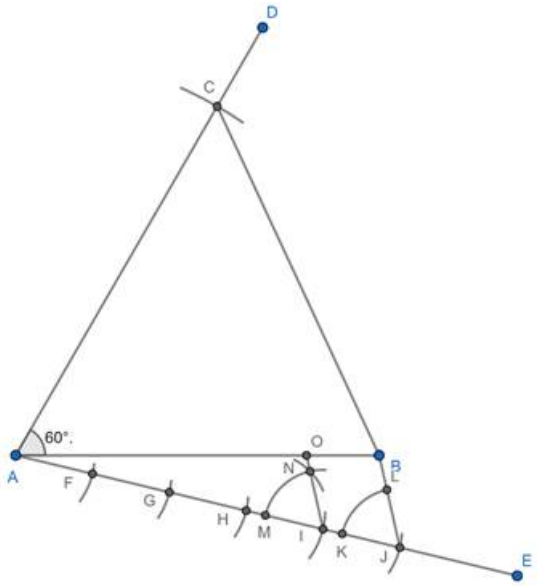
2) Using a protractor, draw ∠BAD=60°. Taking B as the center and radius 5.1 cm, draw an arc, intersecting AD at C. Join BC.



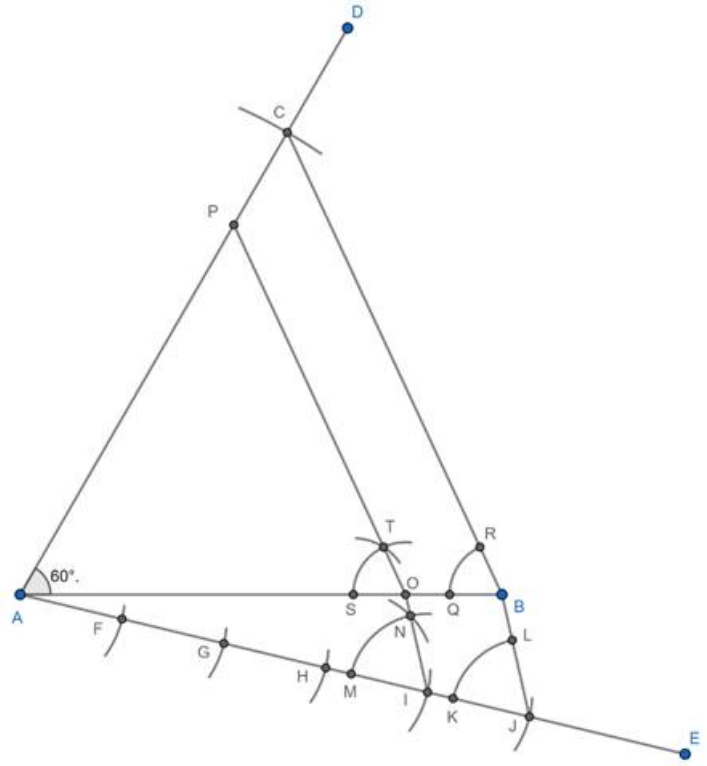
3) Draw any line segment AE, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AE at F. Taking F as the center and radius AF, draw an arc, intersecting AE at G. Similarly, repeat the process 3 more times to get points H, I and J. Join BJ.



4) Taking J as the center and any radius, draw an arc., intersecting AE and BJ at K and L respectively. Taking I as the center and radius JK, draw an arc., intersecting AE at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend IN, intersecting AB at O.



5) Taking B as the center and any radius, draw an arc., intersecting BA and BC at Q and R respectively. Taking O as the center and radius BQ, draw an arc., intersecting AO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting AC at P.



6) ΔAOP is the required triangle.

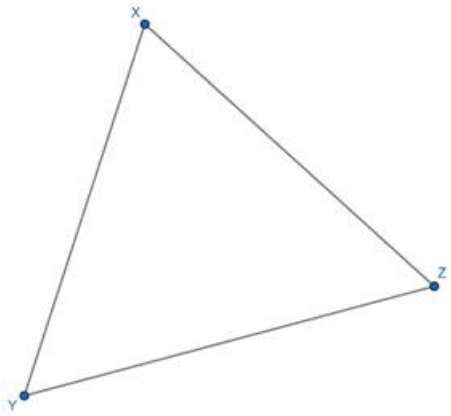
12. Question

Construct a triangle similar to a given ΔXYZ with its sides equal to $(3 / 4)^{th}$ of the corresponding sides of ΔXYZ. Write the steps of construction.

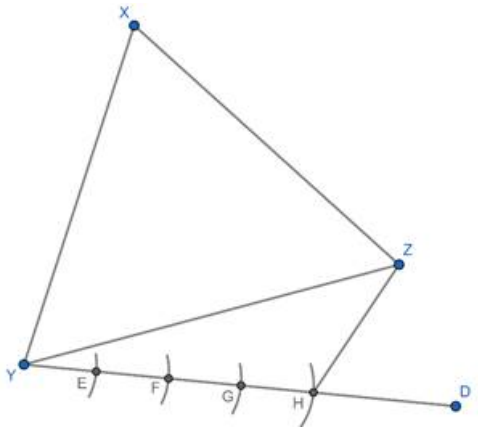
Answer

The steps involved in the required construction are:

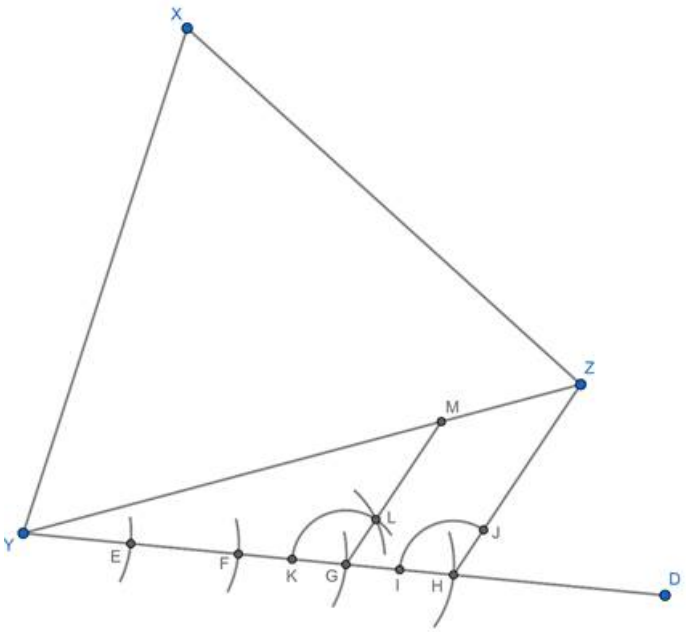
1) Draw any random triangle ΔXYZ.



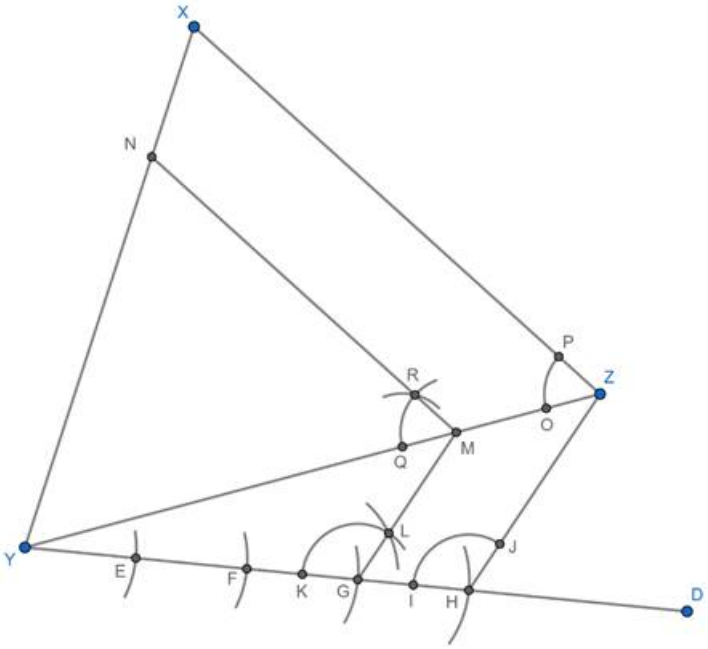
2) Draw any line segment YD, making an acute angle with YZ and opposite to the vertex X. Taking Y as the center and any radius, draw an arc, intersecting YD at E. Taking E as the center and radius YE, draw an arc, intersecting YD at F. Similarly, repeat the process 2 more times to get points G and H. Join ZH.



3) Taking H as the center and any radius, draw an arc., intersecting YD and ZH at I and J respectively. Taking G as the center and radius HI, draw an arc., intersecting YD at K. Taking K as the center and radius IJ, draw an arc, intersecting previous arc at L. Join and extend HL, intersecting YZ at M.



4) Taking Z as the center and any radius, draw an arc., intersecting YZ and ZA at O and P respectively. Taking M as the center and radius ZO, draw an arc., intersecting YZ at Q. Taking Q as the center and radius OP, draw an arc, intersecting previous arc at R. Join and extend MR, intersecting XY at N.



5) ΔYMN is the required triangle.

13. Question

Draw a right triangle in which sides (other than the hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are 3/4 times the corresponding sides of the first triangle.

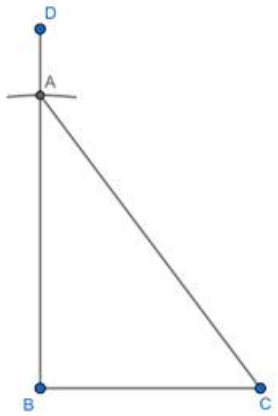
Answer

The steps involved in the required construction are:

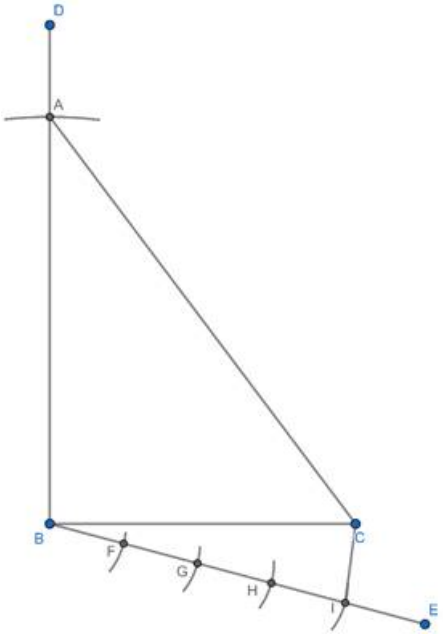
1) Draw a line segment BC= 6 cm.



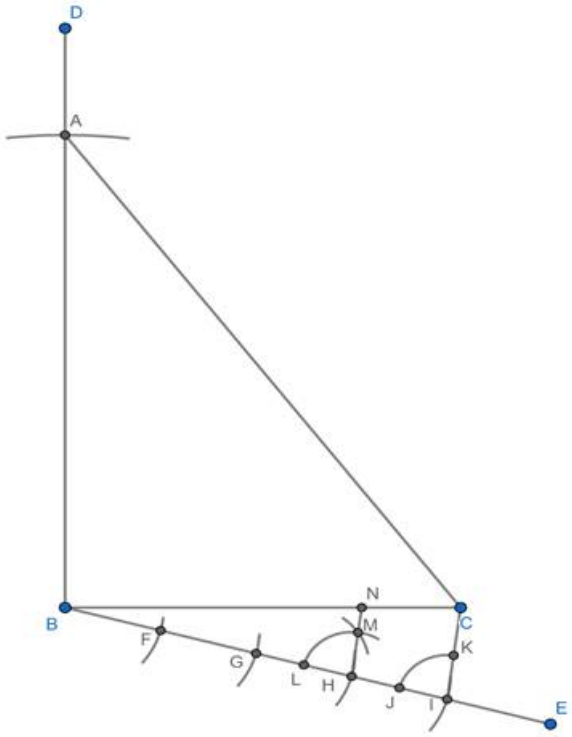
2) Using a protractor, draw ∠CBD=90°. Taking B as the center and radius 8 cm draw an arc, intersecting BD at A. Join AC.



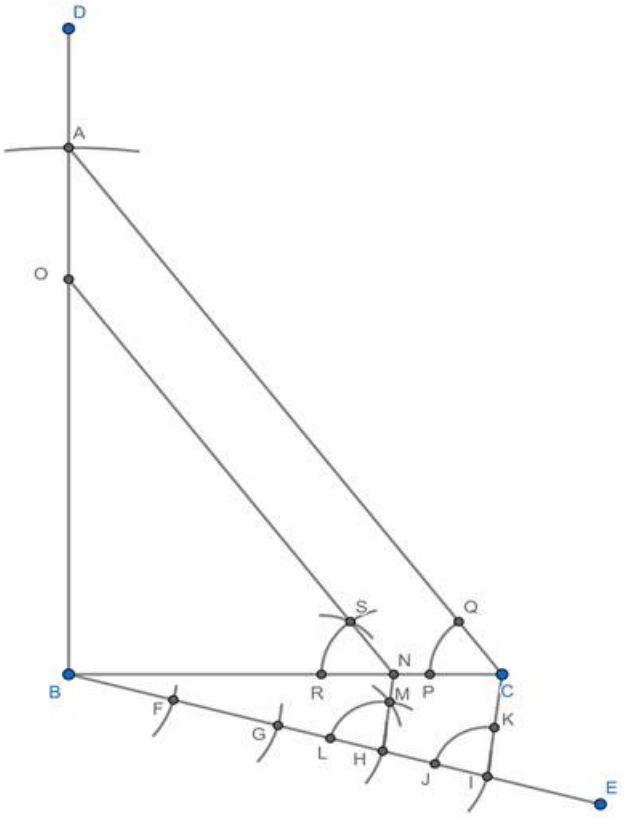
3) Draw any line segment BE, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BE at F. Taking F as the center and radius BF, draw an arc, intersecting BE at G. Similarly, repeat the process 2 more times to get points H and I. Join CI.



4)Taking I as the center and any radius, draw an arc., intersecting BD and CI at J and K respectively. Taking H as the center and radius IJ, draw an arc., intersecting BD at L. Taking L as the center and radius JK, draw an arc, intersecting previous arc at M. Join and extend HM, intersecting BC at N.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at P and Q respectively. Taking N as the center and radius CP, draw an arc., intersecting BC at R. Taking R as the center and radius PQ, draw an arc, intersecting previous arc at S. Join and extend NS, intersecting AB at O.



6) ΔBNO is the required triangle.

14. Question

Construct a triangle with sides 5 cm, 5.5 cm and 6.5 cm. Now construct another triangle, whose sides are 3/5 times the corresponding sides of the given triangle.

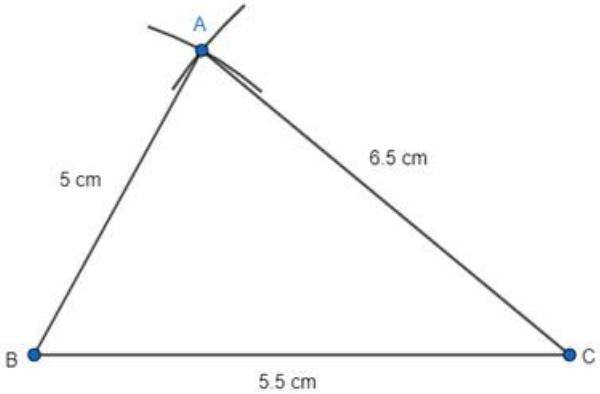
Answer

Step 1. At first drawn a base line BC of length 5.5 cm with the help of scale.

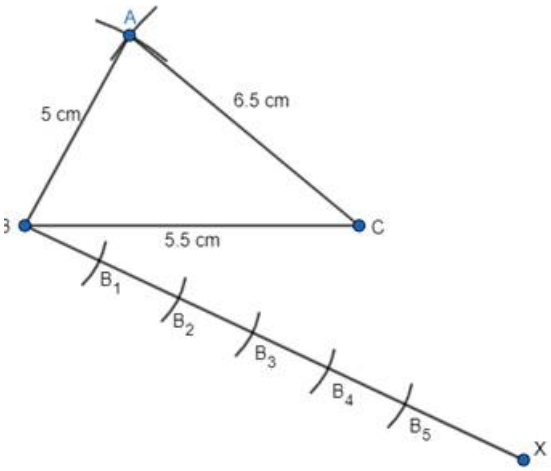


Step 2. Taking B as center draw an arc of radius 5 cm with the help of compass. Similarly taking C as center draw a arc of radius 6.5 cm with the help of compass.

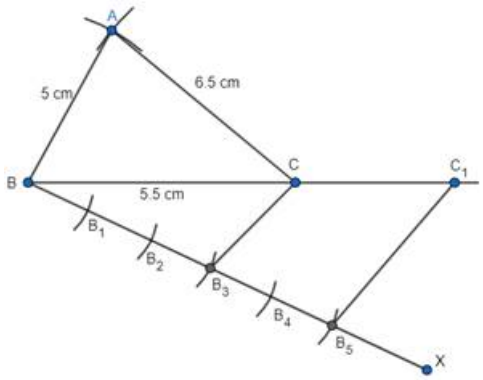
Join AB and AC thus completing the triangle ABC.



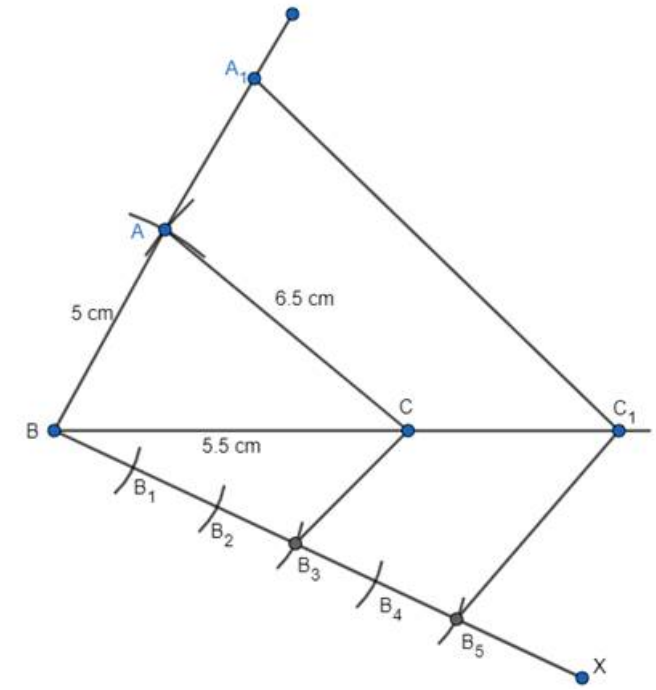
Step 3. A ray BX is drawn making an acute angle with BC opposite to vertex A. Five points B_1 , B_2 , B_3, B_4, and B_5 at equal distance is marked on BX.



Step 4. B_3 is joined with C to form B_3 C as 3 point is smaller. B_5 C_1 is drawn parallel to B_3 C as 5 point is greater.



Step 5:$C_1A_1$is drawn parallel to CA.



Thus, A_1BC_1 is the required triangle.

Justification:

Since the scale factor is $\frac{3}{5}$,

We need to prove,

$$\frac{A_1B}{AB} = \frac{A_1C_1}{AC} = \frac{BC_1}{BC} = \frac{3}{5}$$

By construction,

$\frac{BC_1}{BC} = \frac{BB_5}{BB_3} = \frac{3}{5}$ ... (1)

Also, A_1C_1 is parallel to AC.

So, this will make same angle with BC.

$\therefore \angle A_1C_1B = \angle ACB$ .... (2)

Now,

In ΔA_1BC_1 and ΔABC

$\angle B = \angle B$ (common)

$\angle A_1C_1B = \angle ACB$ (from 2)

$\Delta A_1BC_1 \sim \Delta ABC$

Since corresponding sides of similar triangles are in same ratio.

$$\frac{A_1B}{AB} = \frac{A_1C_1}{AC} = \frac{BC_1}{BC}$$

From (1)

$$\frac{A_1B}{AB} = \frac{A_1C_1}{AC} = \frac{BC_1}{BC} = \frac{3}{5}$$

Hence construction is justified.

15. Question

Construct a triangle PQR with sides QR = 7 cm, PQ = 6 cm and $\angle PQR = 60^0$. Then construct another triangle whose sides are 3/5 of the corresponding sides of ΔPQR.

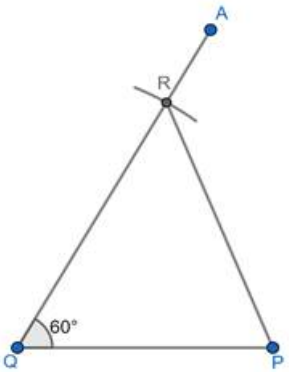
Answer

The steps involved in the required construction are:

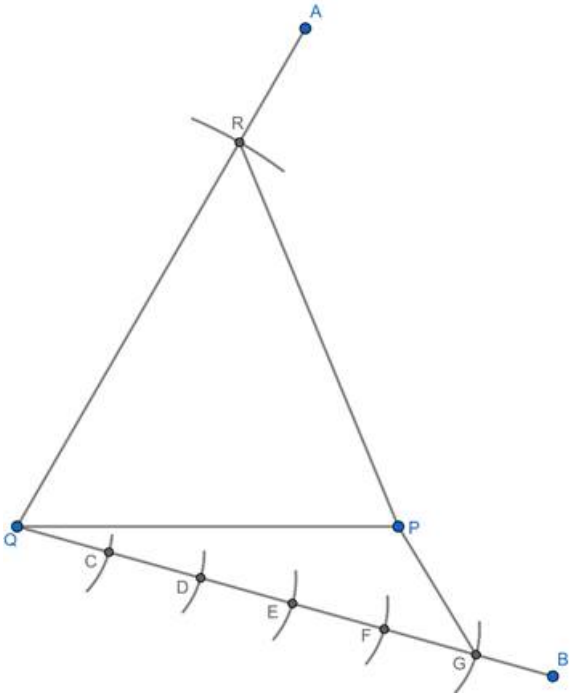
1) Draw a line segment PQ=6 cm.



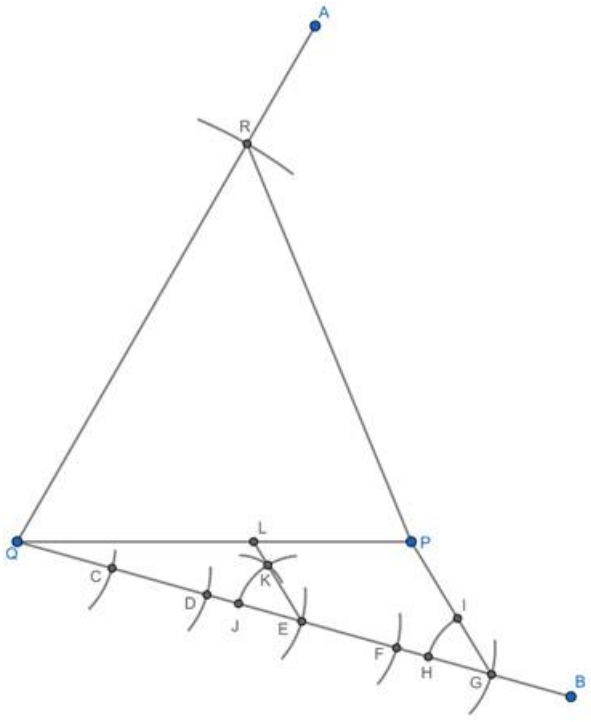
2) Using a protractor, draw ∠PQA=60°. Taking Q as the center and radius 7 cm, draw an arc, intersecting QA at R. Join PR.



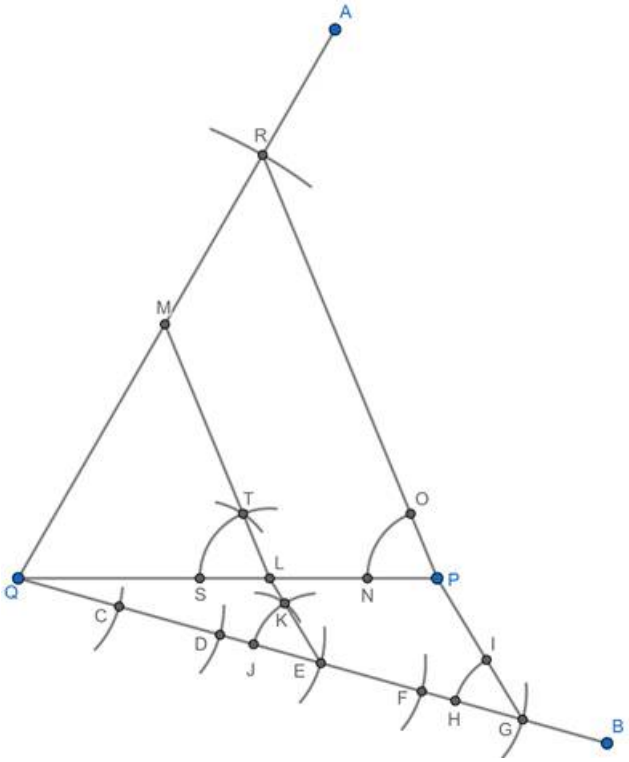
3) Draw any line segment QB, making an acute angle with PQ and opposite to the vertex R. Taking Q as the center and any radius, draw an arc, intersecting QB at C. Taking C as the center and radius QC, draw an arc, intersecting QB at D. Similarly, repeat the process 3 more times to get points E, F and G. Join PG.



4) Taking G as the center and any radius, draw an arc., intersecting QB and PG at H and I respectively. Taking E as the center and radius GH, draw an arc, intersecting QB at J. Taking J as the center and radius HI, draw an arc, intersecting previous arc at K. Join and extend EK, intersecting extended PQ at L.



5) Taking P as the center and any radius, draw an arc., intersecting PQ and PR at N and O respectively. Taking L as the center and radius PN, draw an arc., intersecting PQ at S. Taking S as the center and radius NO, draw an arc, intersecting previous arc at T. Join and extend LT, intersecting QR at M.



6) ΔQLM is the required triangle.

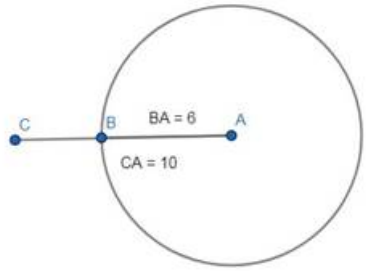
Exercise 11.3

1. Question

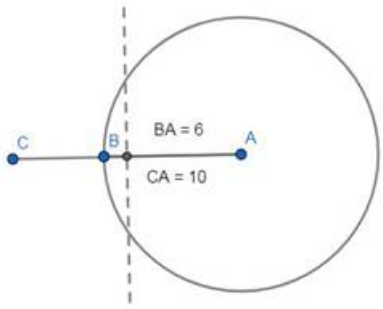
Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Answer

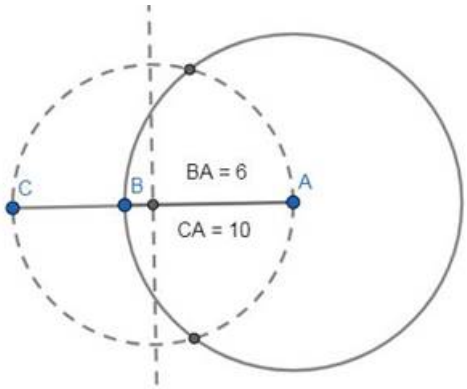
Step1: Draw circle of radius 6cm with center A, mark point C at 10 cm from center



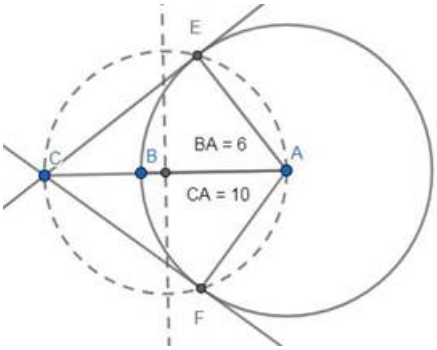
Step 2: find perpendicular bisector of AC



Step3: Take this point as center and draw a circle through A and C



Step4:Mark the point where this circle intersects our circle and draw tangents through C



Length of tangents = 8cm

AE is perpendicular to CE (tangent and radius relation)

In ΔACE

AC becomes hypotenuse

$AC^2 = CE^2 + AE^2$

$10^2 = CE^2 + 6^2$

$CE^2 = 100-36$

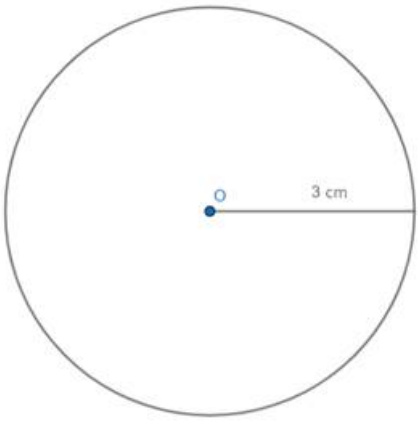
$CE^2 = 64$

CE = 8cm

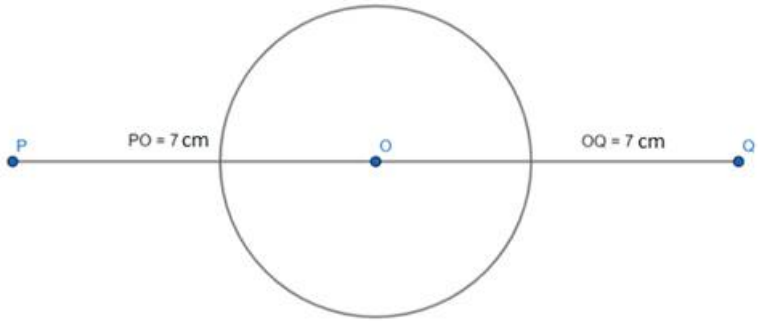
2. Question

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

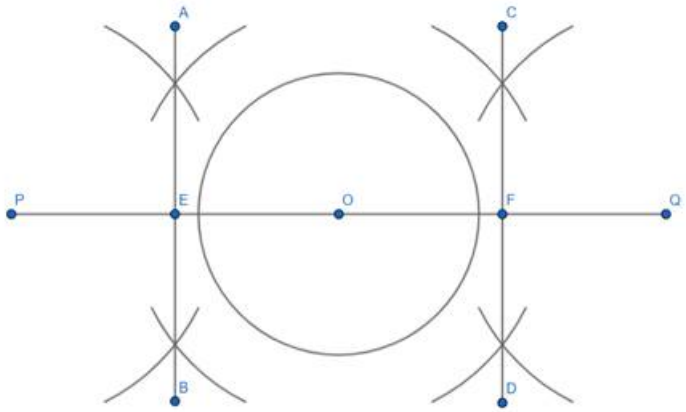
Answer



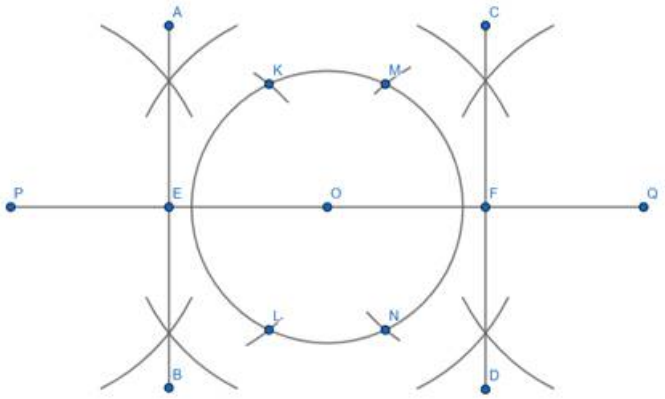
Step 1: We construct a circle with centre O and radius 3 cm.



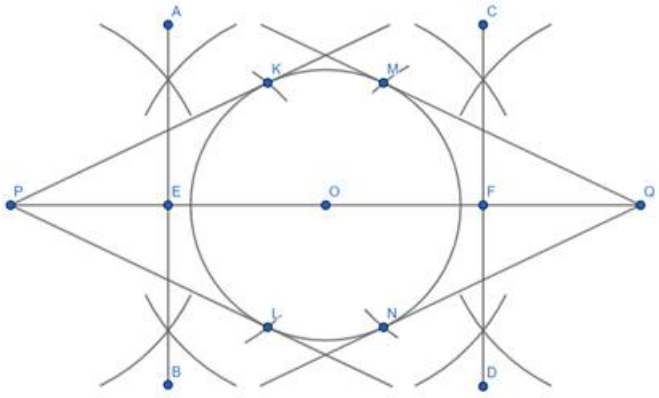
Step 2: We draw a diameter through O and extend it from both ends to points P and Q such that OP = OQ = 7 cm.



Step 3: We construct perpendicular bisectors AB and CD of segments OP and OQ respectively. E and F are the corresponding intersection points.



Step 4: We take OE as radius and construct arcs taking E as centre to cut the circle at points K and L. Similarly, we take OF as radius and construct arcs taking F as centre to cut the circle at points M and N.



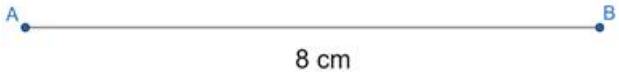
Step 5: We join points P and K, P and L, Q and M, and Q and N to get the tangents PK, PL, QM, QN from points

P and Q to the circle.

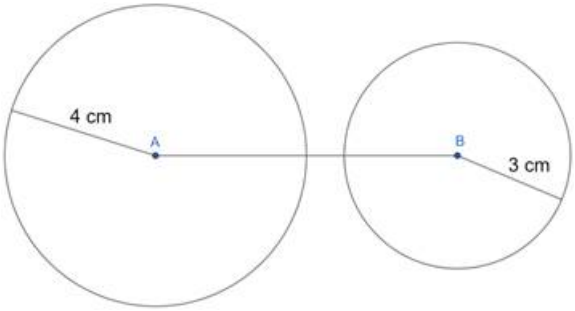
3. Question

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

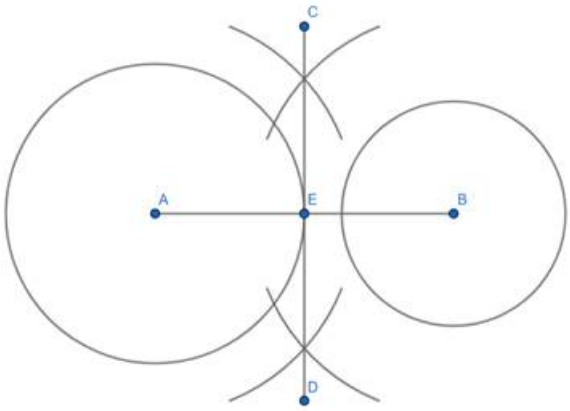
Answer



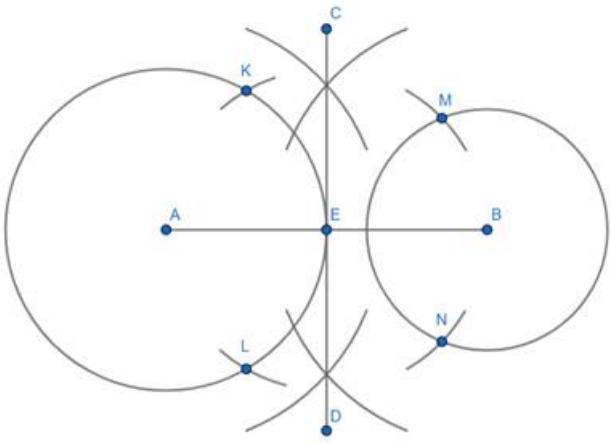
Step 1: We construct line segment AB of length 8 cm.



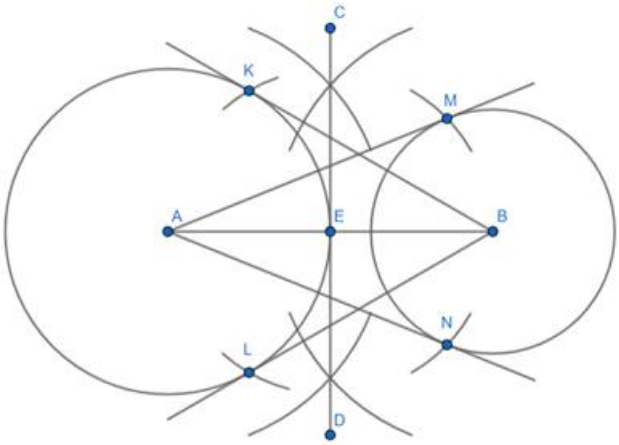
Step 2: Taking A as centre, we construct a circle of radius 4 cm and taking B as centre, we construct a circle of radius 3 cm.



Step 3: We construct perpendicular bisector CD of line segment AB. They intersect at point E.



Step 4: Taking AE as radius and E as centre, we construct arcs to cut circle A at points K and L. Similarly, Taking BE as radius and E as centre, we construct arcs to cut circle B at points M and N.

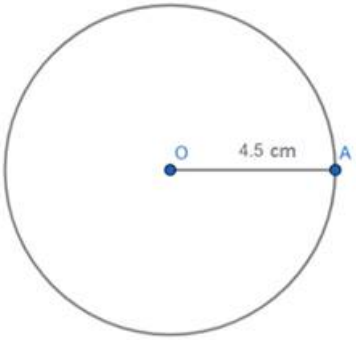


Step 5: We join points A and M, A and N, B and K, B and L to get tangents AM and AN from point A to circle B, and tangents BK and BL from point B to circle A.

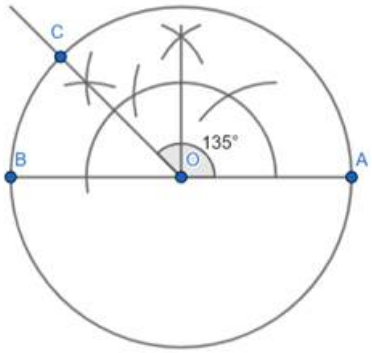
4. Question

Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

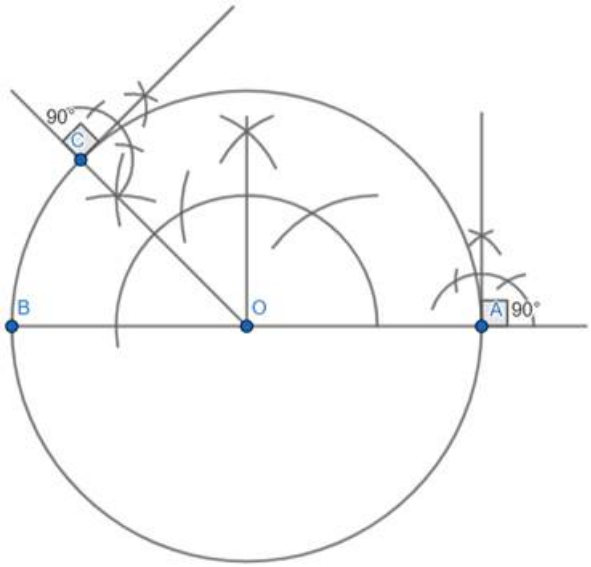
Answer



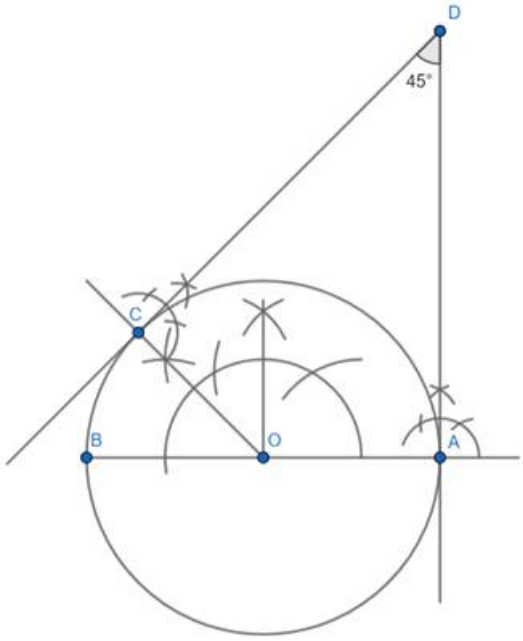
Step 1: We construct a circle with radius 4.5 cm, centred at O.



Step 2: We construct an angle of 135° at centre O, such that ∠AOC = 135°, where C is another point on the circle.



Step 3: We construct perpendiculars to OC and OA at points C and A respectively.

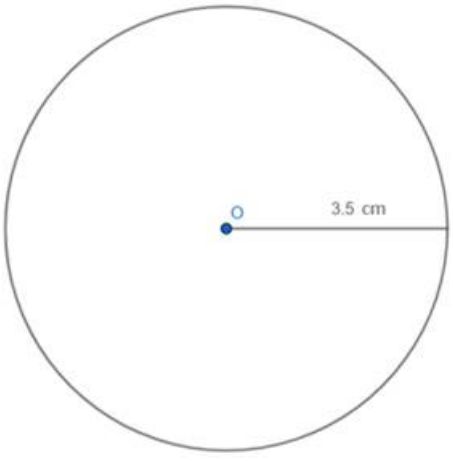


Step 4: We extend the perpendiculars to meet at point D, and we get tangents AD and CD to the circle, enclosing 45° between them.

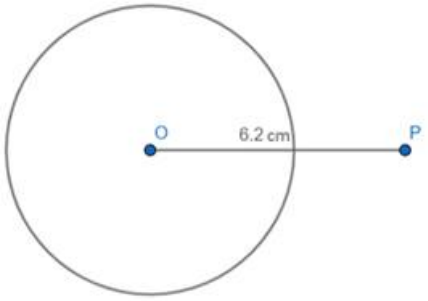
5. Question

Draw two tangents to a circle of radius 3.5 cm from a point P at a distance of 6.2 cm from its centre.

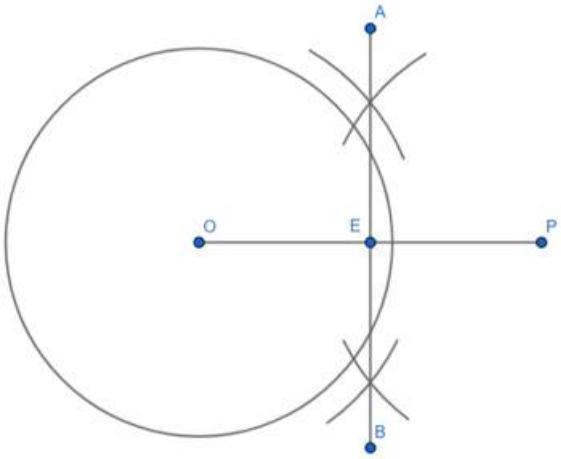
Answer



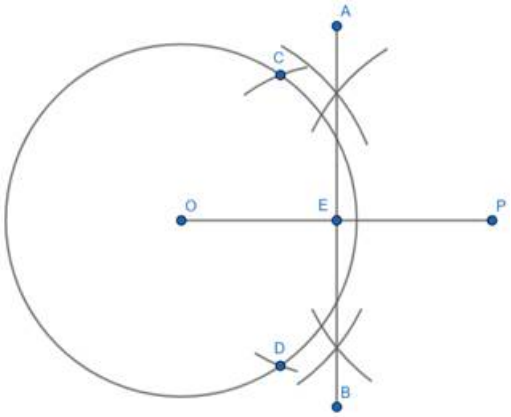
Step 1: Construct a circle of radius 3.5 cm, centred at a point O.



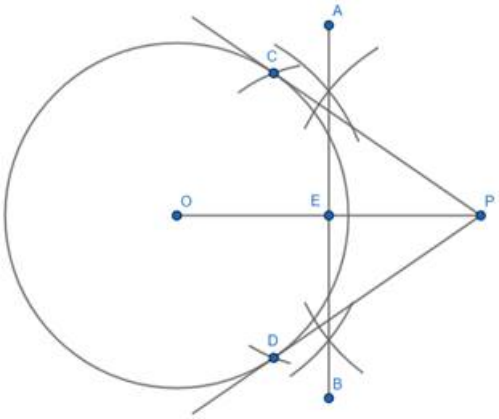
Step 2: Construct line segment OP, such that point P is at a distance of 6.2 cm from O.



Step 3: Construct perpendicular bisector AB of line segment OP. Point E is where they intersect.



Step 4: Taking OE as the radius and centre at E, we draw arcs to cut the circle at points C and D.



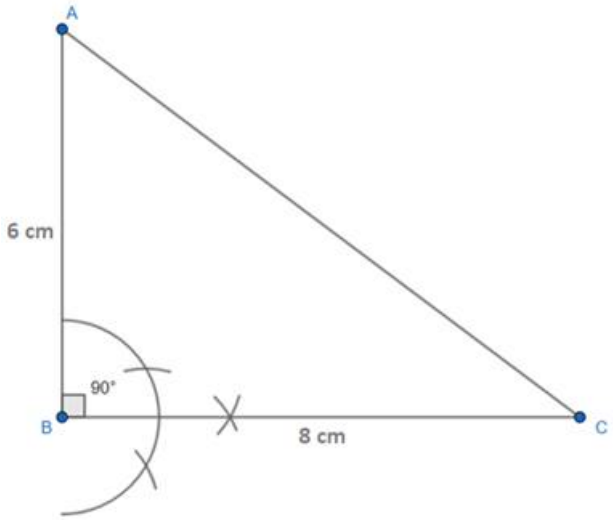
Step 5: We join points P and C, and P and D to get tangents PC and PD to the circle.

6. Question

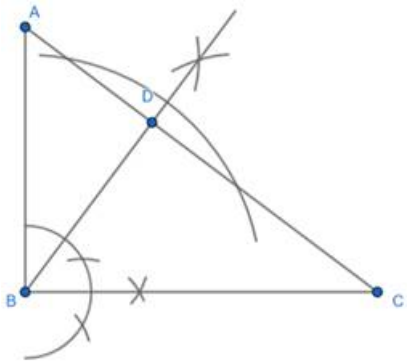
Draw a right triangle ABC in which AB = 6 cm, BC = 8 cm and $\angle B = 90^{o}$. Draw BD perpendicular from B on AC

and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

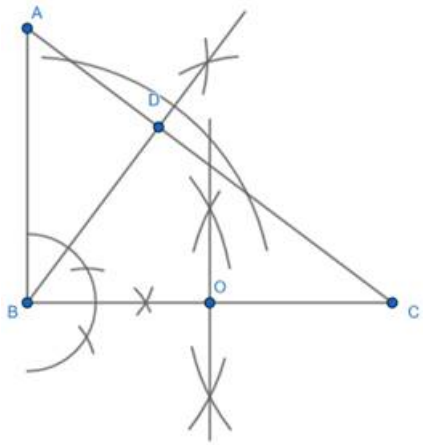
Answer



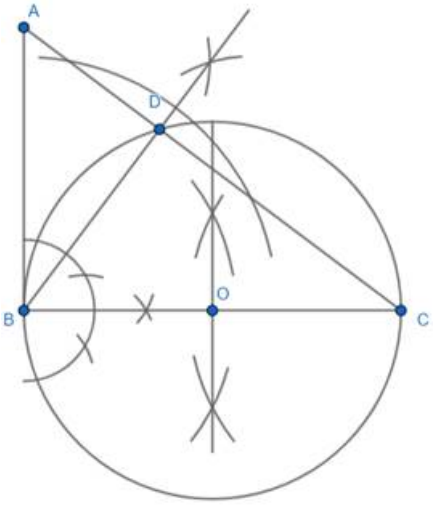
Step 1: Construct line segment AB of length 6 cm. Then construct perpendicular BC of length 8 cm. Finally join points A and C to complete the triangle.



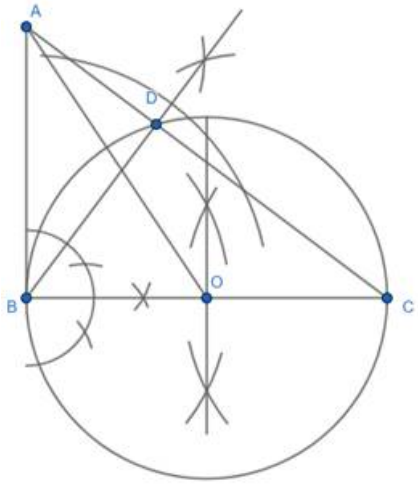
Step 2: Construct a perpendicular from point B onto AC. Point D is where this perpendicular meets AC.



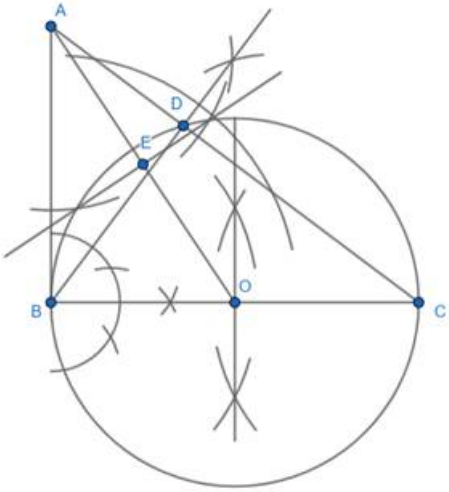
Step 3: Construct perpendicular bisector of line segment BC, which intersects it at pt. O.



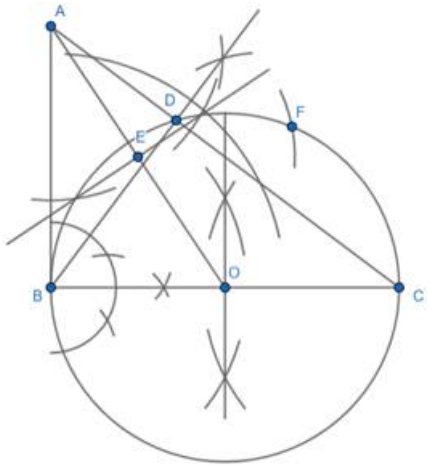
Step 4: Taking centre at O and radius as OB, we construct a circle. It passes through points C and D too.



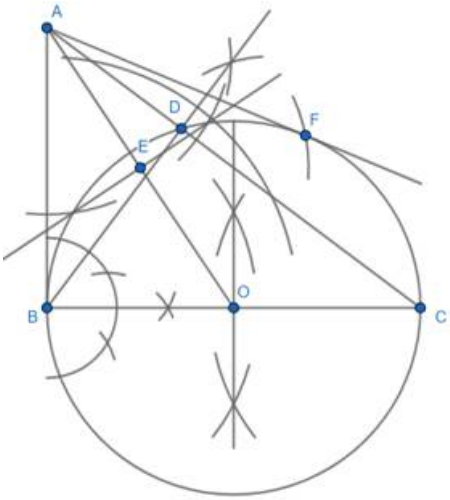
Step 5: We join points A and O.



Step 6: We construct the perpendicular bisector of AO, which intersects it at point E.



Step 7: Taking centre at E and radius as OE, we draw an arc cutting the circle at F. (We cut it at one point only because AB is already a tangent to the circle)



Step 8: We join points A and F, thus obtaining tangents AB and AF to the circle.

15. Areas Related to Circles

Exercise 15.1

1. Question

Find the circumference and area of a circle of radius 4.2cm.

Answer

Given,

Radius of circle = 4.2cm

Circumference of circle = 2πr

$= 2\times\frac{22}{7}\times4.2$

= 26.4cm

Area of circle = $2\pi r^2$

$=\frac{22}{7}\times4.2\times4.2$

$=55.44cm^2$

2. Question

Find the circumference of a circle whose area is $301.84cm^2$.

Answer

Given,

Area of circle = $301.84cm^2$

= πr^2 = 301.84

$= r^2 = \frac{301.84\times7}{22}$ = 96.24

= $r^2 = \sqrt{96.24}$ = 9.81cm

Circumference of the circle = 2πr

$= 2\times\frac{22}{7}\times9.81$

= 61.6 cm

3. Question

Find the area of a circle whose circumference is 44cm.

Answer

Circumference of the circle = 44cm

2πr = 44cm

$$r = \frac{44\times7}{2\times22} = 7cm$$

Area of circle = $\pi r^2 = \frac{22}{7}\times7\times7$

Area of circle = 154 cm^2

4. Question

The circumference of a circle exceeds the diameter by 16.8cm. Find the circumference of the circle.

Answer

Given : The circumference of a circle exceeds the diameter by 16.8 cm.**To find :** The circumference of the circle.**Solution :**

Let diameter of circle = X cm

So, acc. to given condition

Circumference = x+16.8 cm

Circumference of circle is 2πr.

⇒2πr = x + 16.8 Diameter = 2r

$$\Rightarrow \frac{22}{7}\times x = x + 16.8$$

$$\Rightarrow \frac{22}{7}x - x = 16.8$$

$$\Rightarrow \frac{22x - 7x}{7} = 16.8$$

$$\Rightarrow \frac{15x}{7} = 16.8$$

⇒ 15 x=16.8 × 7⇒15 x=117.6

$$\Rightarrow x = \frac{117.6}{15}$$

$$\Rightarrow x = 7.84$$

Circumference= x + 16.8 (x = 2r)

Circumference=7.84 + 16.8 = 24.64 cm

5. Question

A horse is tied to a pole with 28m long string. Find the area where the horse can graze. $(Take\ \pi = 22/7)$.

Answer

Length of string = radius of area which horse can graze

r = 28m

so,

Area where the horse can graze = πr^2

$= \frac{22}{7} \times 28 \times 28$ = 2464 m^2

6. Question

A steel wire when bent in the form of square encloses an area of 121cm^2. If the same wire is bent in the form of a circle, find the area of the circle.

Answer

Area of square = 121 cm^2

a^2 = 121

a = $\sqrt{121}$ = 11cm

Perimeter of square = length of wire

4a = 4 ×11 = 44cm

Perimeter of circle = 2πr

2πr = 44

$r = \frac{44 \times 7}{2 \times 22}$ = 7cm

Area of circle = πr^2

Area of circle = $\frac{22}{7} \times 7 \times 7$ = 154cm^2

7. Question

A horse is placed for grazing inside a rectangular field 40m by 36m and is tethered to one corner by a rope 14m long. Over how much area can it graze? $(Take\ \pi = 22/7)$.

Answer

Given,

Length of field = 40m

Breadth of field = 36m

Length of rope (radius) =14m

So,

Area horse can graze = $\frac{\pi r^2}{4}$

Area horse can graze = $\frac{22\times14\times14}{7\times4}$ = 154 m^2

8. Question

A sheet of paper is in the form of a rectangle ABCD in which AB=40cm and AD=28cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper.

Answer

Area of rectangle = length × breadth

Area of rectangle = 40 × 28

Area of rectangle = 1120 cm^2

Diameter of semi circular portion = 28cm

Radius of semi circular portion = $\frac{28}{2}$ = 14cm

So,

Area of semi circular portion = $\frac{\pi r^2}{2}$

= $\frac{22\times14\times14}{7\times2}$ = 308 cm^2

Area of remaining portion = 1120 – 308 = 812cm^2

9. Question

The circumference of two circles are in the ratio 2:3. Find the ratio of their areas.

Answer

Ratio of circumferences of two circles with radius r_1 and r_2 respectively

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3}$$

$$= \frac{r_1}{r_2} = \frac{2}{3}$$

Ratio of area = $\frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{2^2}{3^2} = \frac{4}{9}$ = 4:9

10. Question

The side of a square is 10cm. Find the area of circumscribed and inscribed circles.

Answer

Side of square = 10cm

Radius of inscribed circle = $\frac{side}{2}$

Radius of inscribed circle = $\frac{10}{2}$ = 5cm

Area of inscribed circle = $\pi r^2 = \frac{22\times5\times5}{7}$

$= \frac{550}{7}$ = 78.5 cm^2

Radius of circumscribed circle = $\frac{diagonal\ of\ square}{2}$

$$= \frac{\sqrt{2}\,a}{2} = \frac{\sqrt{2}\times10}{2} = 5\sqrt{2}$$

Area of circumscribed circle = πr^2

$$= \frac{22}{7}\times5\sqrt{2}\times5\sqrt{2} = \frac{22\times50}{7}$$

$$= \frac{1100}{7} = 157cm^2$$

11. Question

The sum of the radii of two circles is 140cm and the difference of their circumferences is 88cm. Find the diameters of the circles.

Answer

Let radius of first circle = $r_1$cm

Let radius of second circle = $r_2$cm

So,

$r_1 + r_2$ = 140cm …. (i)

$2\pi r_1 - 2\pi r_2$ = 88cm

$$r_1 - r_2 = \frac{88\times7}{2\times22} = 14cm$$

$r_1 - r_2$ = 14cm …. (ii)

By adding equation 1 & 2

$r_1+r_2 = 140$ cm
$r_1-r_2 = 14$ cm

$2r_1 = 154$

---

r_1 = 77cm

From equation 1

77 + r_1 = 140 cm

r_2 = 140 – 77 = 63cm

r_2 = 63cm

So,

Diameter of first circle = 2×r_1 = 2×77 = 154cm

Diameter of second circle = 2×r_2 = 2×63 = 126cm

12. Question

The area of a circle inscribed in an equilateral triangle is 154cm^2. Find the perimeter of the triangle. $\left(Use\ \pi = 22/7\ and\ \sqrt{3} = 1.73\right)$

Answer

Area of inscribed circle = 154cm^2

= πr^2 = 154cm^2

$$r^2 = \frac{154\times7}{22}$$

$$r = \sqrt{49} = 7cm$$

Radius of inscribed circle = 7cm

$$r = \frac{side\ of\ equilateral\ triangle}{2\sqrt{3}}$$

$7 = \frac{a}{2\sqrt{3}}$ (a = side of triangle)

a = $14\sqrt{3}cm$

Perimeter of equilateral triangle = 3a

$$3a = 3\times14\sqrt{3}$$

$= 42\times1.73$ (given)

= 72.66 = 72.7cm^2

13. Question

A field is in the form of a circle. A fence is to be erected around the field. The cost of fencing would be Rs.2640 at the rate of Rs.12 per metre. Then, the field is to be thoroughly ploughed at the cost of Re.0.50 per m^2. What is the amount required to plough the field?$(Take\ \pi = 22/7)$

Answer

Total cost of fencing = Rs 2640

Per meter rate of fencing = Rs 12

So,

Circumference of field = $\frac{2640}{12} = 220\ m$

$$r = \frac{220\times7}{2\times22} = 35$$

Radius of field = 35m

Area of field = $\frac{22}{7}\times35\times35$ = 3850m^2

Cost of plugging 1 m^2 field = 0.50 Rs

Total cost of plugging the field = 3850×0.50 = Rs 1925.00

14. Question

If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.

Answer

When a square inscribed in a circle then,

Diameter of circle = diagonal of square

Let side of the square be = a cm

Diagonal of square be =$\sqrt{2a}\ cm$

Area of square = a^2 cm^2

Diameter of circle = $\sqrt{2}a\ cm$

∴ radius of circle = $\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

Area of circle = $\pi\times\frac{a^2}{2}\ cm$

Ratio of area of circle and square = $\frac{\pi a^2}{2} : a^2$

= π : 2

15. Question

A park is in the form of a rectangle 120m×100m. At the centre of the park there is a circular lawn. The area of park excluding lawn is 8700m^2. Find the radius of the circular lawn. $(Use\ \pi = 22/7)$

Answer

Total area of rectangular park = 120×100 = 12000 m^2

Area of park excluding circular lawn = 8700m^2

So,

Area of circular lawn = 1200 – 8700 = 3300m^2

= πr^2 = 3300

$$r^2 = \frac{3300\times7}{22} = 1050m$$

r = 32.40 m

16. Question

The radii of two circles are 8cm and 6cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles.

Answer

Radius of first circle = 8cm

Area of first circle = πr^2

$$= \frac{22}{7}\times8\times8cm^2$$

Radius of second circle = 6cm

Area of second circle = $\frac{22}{7}\times6\times6cm^2$

Total area = $\frac{22}{7}\times8^2 + \frac{22}{7}\times6^2$

$$= \frac{22}{7}(64+36) = \frac{22}{7}\times100\ cm^2$$

$$\pi r^2 = \frac{22}{7}\times100$$

r^2 = 100

r = 10cm

17. Question

The radii of two circles are 19cm and 9cm respectively. Find the radius and area of the circle which has its circumference equal to the sum of the circumferences of the two circles..

Answer

Radius of the first circle = 19cm

Circumference of first circle = 2πr

= 2π × 19cm

Radius of second circle = 2πr

=2π × 9cm

Total circumference = 2π × 19 + 2π × 9

= 2π (19+9)

$$= 2\times\frac{22}{7}\times 28 = 176\ cm$$

2πr = 176

$$r = \frac{176\times 7}{2\times 22} = 28cm$$

Area of circle = $\frac{22}{7}\times 28\times 28 = 2464cm^2$

18. Question

A car travels 1 kilo meter distance in which each wheel makes 450 complete revolutions. Find the radius of its wheels.

Answer

Total distance covered = 1km = 100000cm

Distance covered by circular wheel in 1 revolution = circumference of circle

Circumference of circle = 2πr

Total no. of revolution = 450

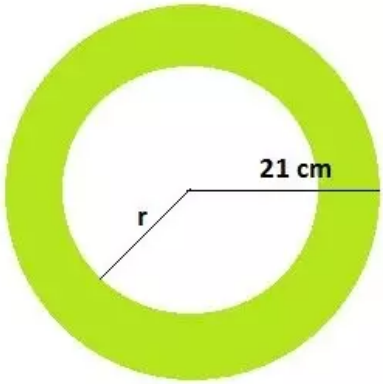
= 2πr × 450 = 100000

$$r = \frac{100000\times 7}{450\times 2\times 22} = 35.35\ cm$$

19. Question

The area enclosed between the concentric circles is 770cm^2. If the radius of the outer circle is 21cm, find the radius of the inner circle.

Answer



Area enclosed between two concentric circle = 770 cm^2

Radius of outer circle = 21cm

Let radius of inner circle = r cm

Area enclosed = area of outer circle – area of inner circle

Area enclosed = 770 п 21^2 – п r^2 = 770

п (441 – r^2) = 770

$$441 - r^2 = \frac{770 \times 7}{22}$$

$$441 - r^2 = 245$$

$$r^2 = 441 - 245$$

$$r^2 = 196$$

$$r = \sqrt{196}$$

$$r = 14$$

r = 14 cm

Exercise 15.2

1. Question

Find, in terms of π, the length of the arc that subtends an angle of 30° at the centre of a circle of radius 4cm.

Answer

Given,

Angle = 30°

Radius of circle = 4cm

180° = п radius

$$1° = \frac{\pi}{180°}$$

$$30° = \frac{30°\pi}{180°} = \frac{\pi}{6} \text{ radius}$$

Arc length = radius × angle subtended by arc at center

$$= 4 \times \frac{\pi}{6} = \frac{2\pi}{3}$$

2. Question

Find the angle subtended at the centre of a circle of radius 5cm by an arc of length $(5\pi / 3)$ cm.

Answer

Arc length = $\frac{5\pi}{3}$ cm

Radius of circle = 5cm

Formula:

Arc length = r×q

r = radius of circle

q = angle subtended by arc at the center

$$= \frac{5\pi}{3} = 5 \times q$$

$$q = \frac{5\pi}{3 \times 5} = \frac{\pi}{3} = \frac{180}{3} = 60°$$

3. Question

An arc of length 20π cm subtends an angle of 144° at the centre of a circle. Find the radius of the circle.

Answer

Arc length = 20π cm

Angle subtend at center = 144°

$$= \frac{\pi \times 144°}{180°} = \frac{4\pi}{5}$$

Arc length = radius × angle

$$\text{radius} = \frac{\text{arc length}}{\text{angle}} = \frac{20\pi \times 5}{4\pi} = 25\text{cm}$$

4. Question

An arc of length 15cm subtends an angle of 45° at the centre of a circle. Find in terms of π, the radius of the circle.

Answer

Arc length = 15cm

Angle subtend = 45°

$$= \frac{45\times\pi}{180^\circ} = \frac{\pi}{4} \text{ radius}$$

$$\text{radius of circle} = \frac{\text{arc length}}{\text{angle subtend at centre}}$$

$$= \frac{15\times4}{\pi} = \frac{60^\circ}{\pi}\text{cm}$$

5. Question

Find the angle subtended at the centre of a circle of a circle of radius 'a' by an arc of length $(a\pi/4)$ cm.

Answer

Radius of circle = a

Length of arc = $\frac{\frac{a\pi}{4}}{a} = \frac{\pi}{4}$

$$= \frac{180^\circ}{4} = 45^\circ$$

So,

Angle subtended at the center = 45°

6. Question

A sector of a circle of radius 4cm contains an angle of 30°. Find the area of the sector.

Answer

Given,

Radius of sector = 4cm

Angle of sector = 30°

Area of sector = $\frac{\theta}{360^\circ}\times\pi r^2$

$$= \frac{30}{360}\times\pi\times16$$

$$= \frac{1}{12}\times\pi\times16 = \frac{4\pi}{3}\text{cm}^2$$

7. Question

A sector of a circle of radius 8 cm contains an angle of 135°. Find the area of the sector.

Answer

Radius of sector = 8cm

Angle = 135°

Area of sector = $\frac{\theta\pi r^2}{360^\circ}$

Area of sector = $\frac{135^\circ}{360^\circ}\times\pi\times8\times8 = 24\pi cm^2$

8. Question

The area of a sector of a circle of radius 2 cm is π cm^2. Find the angle contained by the sector.

Answer

Given,

Area of sector = пcm^2

Radius = 2cm

Area of sector = $\frac{\theta}{360^\circ}\times\pi r^2$

$$\pi = \frac{\theta}{360^\circ}\times\pi\times4$$

$$Q = \frac{360^\circ\times\pi}{\pi\times4} = 90^\circ$$

9. Question

The area of a sector of a circle of radius 5cm is 5π cm^2. Find the angle contained by the sector.

Answer

Area of sector = 5п cm^2

Radius = 5cm

$$5п = \frac{\theta}{360^\circ}\times\pi\times25$$

$$Q = \frac{5\pi\times360^\circ}{25\pi} = 72^\circ$$

10. Question

AB is a chord of a circle with centre O and radius 4cm. AB is of length 4 cm. Find the areas of the sector of the circle formed by chord AB.

Answer

Length of the chord = 4cm

Radius of circle = 4cm

(This chord and radius makes an equilateral triangle)

So,

Q = 60° (in equilateral triangle)

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \pi \times 4 \times 4$

$= \frac{1}{6} \times \pi \times 16 = \frac{8\pi}{3} cm^2$

11. Question

In a circle of radius 35cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and area of the sector.

Answer

Given,

Radius of circle = 35cm

Angle subtend by arc = 72°

Length of arc = r×q

Since,

180° = π radius

$1^\circ = \frac{\pi}{180^\circ}$

$72^\circ = \frac{\pi \times 72^\circ}{180^\circ} = \frac{2\pi}{5}$ radius

Length of the arc = $35 \times 2 \times \frac{22}{7} \times \frac{1}{5} = 44cm$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{72^\circ}{360^\circ} \times \frac{22}{7} \times 35 \times 35$

= 770 cm^2

12. Question

The perimeter of a sector of a circle of radius 5.7m is 27.2m. Find the area of the sector.

Answer

Given,

Perimeter of sector of circle = 272m

Radius of sector = 5.7m

Perimeter of sector = $\frac{\theta}{360} \times 2\pi r + 2r = 27.2$

$= \frac{\theta}{360} \times 2\pi r = 27.2 - 11.4$

$= \frac{\theta}{360} = \frac{15.8}{2\pi r}$ (Equation first)

Area of sector = $\frac{\theta}{360} \times 2\pi r^2$ (Second equation)

Put value of $\frac{\theta}{360}$ from equation first to second,

$= \frac{15.8}{2\pi r} \times 2\pi r^2 = \frac{15.8 \times 5.7}{2} = 45.03 cm^2$

13. Question

The perimeter of a certain sector of a circle of radius 5.6m is 27.2m. Find the area of the sector.

Answer

Given,

Perimeter of sector = 27.2m

Radius of sector = 5.6m

$= \frac{\theta}{360} \times 2\pi r + 2r = 27.2$

$= \frac{\theta}{360} \times 2\pi r = 27.2 - 11.2$

$= \frac{\theta}{360} = \frac{16}{2\pi r}$ (Equation first)

Area of sector = $\frac{\theta}{360} \times \pi r^2$ (Equation second)

Put value of $\frac{\theta}{360}$ from equation first to equation second

$= \frac{16}{2\pi r} \times \pi r^2 = \frac{16 \times 5.6}{2} = 44.8\ m^2$

14. Question

A sector is cut-off from a circle of radius 21cm. The angle of the sector is 120°. Find the length of its arc and the area.

Answer

Given,

Radius of sector = 21cm

Angle of sector = 120°

Length of arc = $\frac{120°\pi}{180°}\times 21$

$= \frac{2}{3}\times\frac{22}{7}\times 21 = 44\text{cm}$

Area of sector = $\frac{\theta}{360}\times\pi r^2$

$= \frac{120}{360}\times\frac{22}{7}\times 21\times 21$

$= \frac{1}{3}\times 22\times 3\times 21 = 462\text{cm}^2$

15. Question

The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 7.00AM and 7.05AM.

Answer

Length of minute hand = $\sqrt{21}\text{cm}$

Angle subtend by minute hand in 1 minute = $\frac{360°}{60°}$ = 6°

Angle subtend by minute hand in 5 minute (7-7.05) = 5×6 = 30°

So,

Area described by minute hand in 5 minute = $\frac{\theta}{360°}\times\pi r^2$

$= \frac{30}{360}\times\frac{22}{7}\times\sqrt{21}\times\sqrt{21}$

$= \frac{1}{12}\times\frac{22}{7}\times 21$

= 5.5cm^2

16. Question

The minute hand of a clock is 10cm long. Find the area of the face of the clock described by the minute hand between 8AM and 8.25AM.

Answer

Given,

Length of minute hand = 10cm

Angle subtend by minute hand in 25 minute (8-8.25) = 25×6 = 150°

So,

Area described by minute hand between (8-8.25) = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{150}{360} \times \frac{22}{7} \times 10 \times 10 = 130.95 cm^2$$

17. Question

A sector of 56° cut out from a circle contains area 4.4cm^2. Find the radius of the circle.

Answer

Given,

Angle of sector = 56°

Area of sector = 4.4cm^2

From formula,

$$= \frac{56}{360} \times \pi r^2 = 4.4$$

$$r^2 = \frac{4.4 \times 7 \times 360}{22 \times 56} = 9$$

$$r = \sqrt{9} = 3c$$

18. Question

In a circle of radius 6cm, a chord of length 10cm makes an angle of 110° at the centre of the circle. Find:

(i)the circumference of the circle,

(ii)the area of the circle,

(iii)the length of the arc AB,

(iv)the area of the sector OAB.

Answer

Given,

Radius of circle = 6cm

Length of chord = 10cm

Angle subtend by chord = 110°

I. Circumference of circle = 2πr

= 2×3.14×6 = 37.68cm

II. Are of circle = πr^2

= 3.14×6× = 113.1cm^2

III. Length of arc = radius × angle subtend

$$= 6\times\frac{120\pi}{180}$$

$$= 6\times\frac{2}{3}\times\frac{22}{7} = 11.51\text{cm}$$

IV. Area of sector = $\frac{\theta}{360}\times\pi r^2$

$$= \frac{110}{360}\times\frac{22}{7}\times6\times6 = \frac{242}{7} = 34.5\text{cm}^2$$

19. Question

Fig.15.17, shows a sector of a circle, centre O, containing an angle θ°. Prove that:

(i) Perimeter of the shaded region is $r\left(\tan\theta+\sec\theta+\frac{\pi\theta}{180}-1\right)$

(ii) Area of the shaded region is $\frac{r^2}{2}\left(\tan\theta-\frac{\pi\theta}{180}\right)$

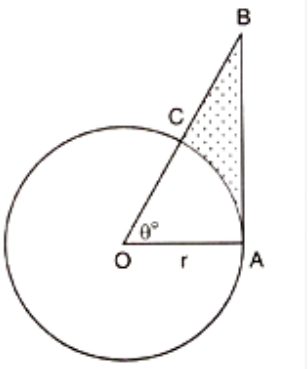


Fig. 15.17

Answer

Angle subtend at centre of circle = θ

Angle OAB = 90°

(At point of contract, tangent is perpendicular to radius)

OAB is right angle triangle

$$\cos\theta = \frac{r}{OB} = OB = r\sec\theta$$

$$\tan\theta = \frac{AB}{r} = AB = r\tan\theta$$

Perimeter of shaded region = AB+ BC+(CA arc)

$$= r\ \tan\theta + (OB - OC) + \frac{\theta}{360} \times 2\pi r$$

$$= r\ \tan\theta + r\ \sec\theta - r + \frac{\pi\theta r}{180}$$

$$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$$

Area of shaded region = (area of triangle AOB) – (area of sector)

$$= \left(\frac{1}{2} \times OA \times AB\right) - \frac{\theta}{360} \times \pi r^2$$

$$= \frac{1}{2} \times 2 \times r\tan\theta - \frac{r^2}{2}\left(\frac{\theta}{180} \times \pi\right)$$

$$= \frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$$

20. Question

Figure 15.18 shows a sector of a circle of radius r cm containing an angle θ°. The area of the sector is A cm^2 and perimeter of the sector is 50cm.

(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$ (ii) $A = 25r - r^2$

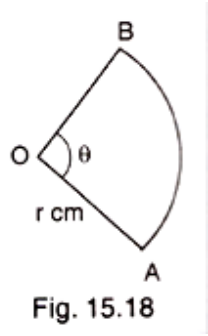


Fig. 15.18

Answer

Given,

Radius of the sector = r cm

Angle subtend = θ

Area of sector = A cm^2

Perimeter of sector = 50cm

Area of sector = $\frac{\theta}{360}\pi r^2$

Perimeter of sector = $\frac{\theta}{360}2\pi r + 2r$

$$= \frac{\theta}{360} 2\pi r + 2r = 50$$

$$= 2r\left(\frac{\pi\theta}{360} + 1\right) = 50$$

$$= r\times\left(\frac{\pi\theta}{360} + 1\right) = \frac{50}{2} = 25$$

$$r = \frac{25}{\left(\frac{1+\pi\theta}{360}\right)} \text{ or } \frac{1+\pi\theta}{360} = \frac{25}{r}$$

(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$

$$= \frac{\pi\theta}{360} = \frac{25}{r} - 1 \rightarrow \left(\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)\right)$$

$= \frac{\theta}{360} = \frac{25-r}{\pi r} \rightarrow$ First equation

$\text{area} = \frac{\theta}{360}(\pi r^2) \rightarrow$ Second equation

Put value of $\frac{\theta}{360}$ from equation first to equation second

$$\text{area} = \frac{25 - r}{\pi r}(\pi r^2) = (25 - r)r$$

Area = $25r-r^2$

21. Question

The length of the minute hand of a clock is 14cm. Find the area swept by the minute hand in 5minutes.

Answer

The length of minute hand = 14cm

Time = 5 minute

Angle subtend by minute hand at center in 60 minute = 360°

In one minute = $\frac{360}{60} = 60°$

In five minute =5×6 = 30°

Area swept in 5 minute = $\frac{\theta}{360}\pi r^2$

$$= \frac{30}{360}\times\frac{22}{7}\times 14\times 14 = \frac{154}{3}$$

= $51.30cm^2$

22. Question

In a circle of radius 21cm, an arc subtends an angle of 60° at the centre. Find (i)the length of the arc (ii)area of the sector formed by the arc$(Use\ \pi = 22/7)$

Answer

Given,

Radius of circle = 21cm

Angle subtend by arc = 60°

$$= \frac{60\pi}{180} = \frac{\pi}{3}\ \text{radius}$$

Length of the arc = $\frac{\pi}{3} \times 21 = 22\text{cm}$

Area of sector formed by arc = $\frac{\theta}{360}\pi r^2$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 = 1 = 231\text{cm}^2$$

Exercise 15.3

1. Question

AB is a chord of a circle with centre O and radius 4cm. AB is of length 4cm and divides the circle into two segments. Find the area of the minor segment.

Answer

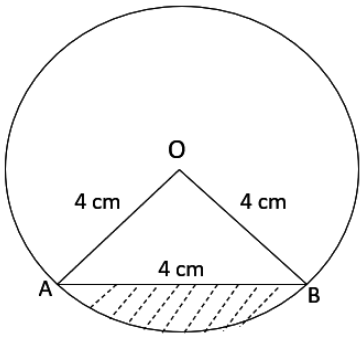
Given: AB is a chord of a circle with centre O and radius 4cm. AB is of length 4cm and divides the circle into two segments.

To find: the area of the minor segment.

Solution:

Radius of circle = 4cm

Length of chord = 4cm



(Hence it makes an equilateral triangle at centre, in which all angle must be = 60°)

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{60}{360}\times\pi\times4\times4$

$= \frac{1}{6}\times\pi\times4\times4 = \frac{8\pi}{3}\text{cm}^2$

Area of equilateral ΔOAB = $\frac{\sqrt{3}}{4}a^2$ $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times16 = 4\sqrt{3}\text{ cm}^2$

Area of minor segment = area of sector – area of ΔOAB

$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)cm^2$

1. Question

AB is a chord of a circle with centre O and radius 4cm. AB is of length 4cm and divides the circle into two segments. Find the area of the minor segment.

Answer

Given,

Radius of circle = 4cm

Length of chord = 4cm

(Hence it makes an equilateral triangle at centre, in which all angle must be = 60°)

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{60}{360}\times\pi\times4\times4$

$\frac{1}{6}\times\pi\times4\times4 = \frac{8\pi}{3}\text{cm}^2$

Area of ΔOAB = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times16 = 4\sqrt{3}\text{ cm}^2$

Area of minor segment = area of sector – area of ΔOAB

$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)cm^2$

2. Question

A chord PQ of length 12 cm subtends an angle of 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ.

Answer

Length of chord PQ = 12cm

Angle subtend at the center = 120°

Let radius of circle = r cm

Area of sector = $\frac{120}{360}\pi r^2 = \frac{\pi r^2}{3} cm^2$

Length of triangle POQ = r cos 60

$= r \times \frac{1}{2} = \frac{r}{2} cm$

Length of base PQ = 2×RQ

$= 2 \times r \sin 60 = 2 \times r \times \frac{\sqrt{3}}{2} = \sqrt{3} r$

Put value of r in respective place,

Area of minor segment = area of sector – area of ΔPOQ

$= \frac{\pi r^2}{3} - \frac{1}{2} \times 12 \times \frac{r}{2}$

$= \frac{\pi \times 48}{3} - 3 \times 4\sqrt{3}$

$16\pi - 12\sqrt{3}$

$= 4(4\pi - 3\sqrt{3}) cm^2$

3. Question

A chord of a circle of radius 14cm makes a right angle at the centre. Find the areas of the minor and major segments of the circle.

Answer

Radius of the circle = 14cm

Angle subtend at center = 90°

By Pythagoras theorem = $AB^2 = OA^2 + OB^2$

$= 14^2 + 14^2$

$AB = 14\sqrt{2}$

Area of sector OAB = $\frac{90}{360} \times \pi r^2$

$= \frac{1}{4}\pi r^2$

$=\frac{1}{4}\times\frac{22}{7}\times14\times14 = 154cm^2$

Area of triangle AOB $=\frac{1}{2}\times14\times14 = 98\ cm^2$

So area of minor segment – OACB =area of sector – area of triangle

= 154 – 98 = 56cm^2

Area of major segment = area of circle - area of minor segment

$= \frac{22}{7}\times14\times14 - 56$

= 44×14 – 56 = 560cm^2

4. Question

A chord 10cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find area of both the segments. $(Take\ \pi = 3.14)$

Answer

Length of chord = 10cm

Radius of circle = $5\sqrt{2}cm$

(This triangle POQ satisfy Pythagoras theorem)

= PQ^2 = PO^2 + OQ^2

$= 10^2 = (5\sqrt{2^2}) + (5\sqrt{2}^2)$

= 100 = 50 + 50

So,

Angle AOQ = 90°

Area of sector = $\frac{90}{360}\pi\times50 = \frac{25}{2}\pi\ cm^2$

Area of triangle POQ = $\frac{1}{2}\times5\sqrt{2}\times5\sqrt{2} = 25cm^2$

Area of minor segment = $\frac{25}{2}\pi - 25 = 14.25cm^2$

5. Question

A chord AB of a circle, of radius 14 cm makes an angle of 60° at the centre of the circle. Find the area of the minor segment of the circle. $(Use\ \pi = 22/7)$

Answer

Radius of circle = 14cm

Angle = 60°

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{60}{360}\times\pi\times14\times14 = \frac{98}{3}\pi$ = 102.57cm^2

Area of triangle OAB = $\frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}\times14\times14\times\sin\theta$

$= \frac{1}{2}\times14\times14\times\frac{\sqrt{3}}{2} = 49\sqrt{3} = 84.77cm^2$

So,

Area of minor segment = 102.57 – 84.77 = 17.80cm^2

Exercise 15.4

1. Question

A plot is in the form of a rectangle ABCD having semi-circle on BC as shown in Fig.15.64. If AB=60m and BC=28m, find the area of the plot.

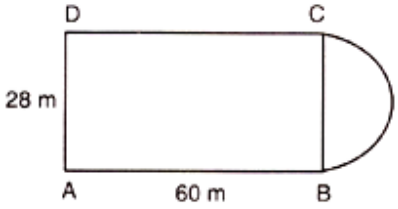


Fig. 15.64

Answer

Given,

AB = 60m

BC = 28m

Area of rectangular portion = 28m×60m = 1680m^2

Diameter of semicircle = length of side BC

Radius = $\frac{28}{2} = 14m$

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22\times14\times14}{7\times2} = 308m^2$

Total area of plot = 1680+308 = 1988m^2

2. Question

A play ground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36m and 24.5m, find the area of the play ground. $(Take\ \pi = 22/7)$.

Answer

Given:

AB = 36m

BC = 24.5m

Area of rectangular portion = 36×24.5 = 882m^2

Radius of semicircular portion = $\frac{24.5}{2} = 12.25m$

Area of both semicircular portion = $2\times\frac{\pi r^2}{2}$

$= \frac{22}{7}\times 12.25\times 12.25$ = 471.625

Area of play ground = 882+471.625 = 1353.62

3. Question

The outer circumference of a circular race-track is 525m. The track is everywhere 14m wide. Calculate the cost of leveling the track at the rate of 50paise per square meter $(Use\ \pi = 22/7)$

Answer

Given,

Circumference of outer circle = 525m

Let radius of outer circle = R_2m

Let radius of inner circle = R_1m

So,

R_2-R_1 = 14 (equation 1)

= 2пR_2 = 525

$$R_2 = \frac{525}{2}\times\frac{7}{22} = 83.52m$$

Put value of R_1 in equation first

= 83.52 – R_1 = 14

= - R_1 = 14 – 83.52

= R_1 = 69.52m

Area of path = $\pi R_2^2 - \pi R_1^2$

$$= \pi(R_2^2 - R_1^2) = \pi(R_2 + R_1)(R_2 - R_1)$$

$$= \frac{22}{7} \times (83.52 + 69.52)(83.52 - 69.52)$$

$$= \frac{22}{7} \times 153.04 \times 14 = 6733.76m^2$$

Cost of leveling the path = 6733.76×.50 = Rs 3388

4. Question

A rectangular piece is 20m long and 15m wide. From its four corners, quadrants of radii 3.5m have been cut. Find the area of the remaining part.

Answer

Length of rectangle = $20m^2$

Breadth of rectangle = $15m^2$

Area of rectangle = $20 \times 15 = 300m^2$

Radius of quadrant = $3.5m^2$

Area of quadrant = $\frac{1}{4} \times \pi r^2$

Area of quadrant = $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{19.25}{2} m^2$

Area of 4 quadrant = $4 \times \frac{19.25}{2} m^2$ = $2 \times 19.25 = 38.50m^2$

Area of remaining part = (area of rectangle-area of 4 quadrant)

Area of remaining part = $300 - 38.50 = 261.5m^2$

5. Question

Four equal circles, each of radius 5cm, touch each other as showing fig.15.65. Find the area included between them. $(Take\ \pi = 3.14)$

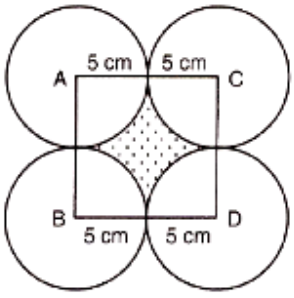


Fig. 15.65

Answer

Given,

Radius of each circle = 5cm

So,

Side of square = 10cm

Area of square = $(10)^2$ = 100cm^2

Area of each quadrant of circle with radius 5cm = $\frac{90}{360}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 25cm^2$

Area of 4 quadrants = $4 \times \frac{1}{4} \times \frac{22}{7} \times 25 = 25\pi cm^2$

Area of remaining portion = 100-25п = 21.5cm^2

6. Question

Four cows are tethered at four corners of a square plot of side 50m, so that they just cannot reach one another. What area will be left un-grazed?

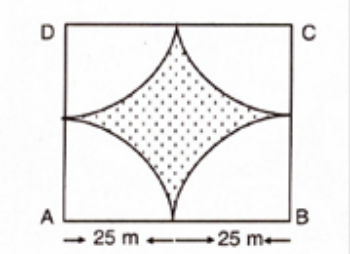


Fig. 15.66

Answer

Side of square = 50m

Area of square = $(5)^2$ = 2500m^2

Radius of quadrant circle = 25m

Area of one quadrant = $\frac{\pi r^2}{4} = \frac{625\pi}{4} m^2$

Area of 4 quadrants = $\frac{625\pi}{4} \times 4 = 625\pi = 1964.28m^2$

So,

Area which left un-grazed = 2500-1964.28 = 535.72m^2

7. Question

A road which is 7m wide surrounds a circular park whose circumference is 352m. Find the area of the road.

Answer

Given,

Circumference of park = 352m

Width of road = 7m

Let radius of park = r

2πr = 352

$$r = \frac{352 \times 7}{2 \times 22} = 56m$$

Area of circle = $\frac{22}{7} \times 56 \times 56 = 9856m^2$

Radius of circle included path of width, 7m = 56+7 = 63m

Area of circle included path = $\frac{22}{7} \times 63 \times 63 = \ 12474m^2$

So,

Area of path = 12474 – 9856 = 2618m^2

8. Question

Four equal circles, each of radius a, touch each other. Show that the area between them is $\frac{6}{7}a^2$ $(Take\ \pi = 3.14)$.

Answer

Radius of each circle = a meter

If we join the centre of each circle it makes a square of side = 2a

Area of square = $(2a)^2 = 4a^2m^2$

Area of each quadrant of circle = $\frac{\pi r^2}{4} = \frac{\pi a^2}{4}m^2$

Area of 4 quadrants = $4 \times \frac{\pi a^2}{4} = \pi a^2 m^2$

So,

Area between circles = $4a^2 - \pi a^2$

$$= 4a^2 - \frac{22}{7}a^2 = \frac{28a^2 - 22a^2}{7} = \frac{6a^2}{7}m^2$$

9. Question

A square water tank has its side equal to 40m. There are four semi-circular grassy plots all round it. Find the cost of surfing the plot at Rs.1.25 per square meter $(Take\ \pi = 3.14)$.

Answer

Side of water tank = 40m

Side of semi circular grassy plots = $\frac{40}{2} = 20m$

Area of one grassy plot = $\frac{\pi r^2}{2}$

$= \frac{22}{7} \times \frac{20 \times 20}{2} = \frac{400\pi}{2} = 200\pi$

Area of grassy plots = 4×200π = 800π

Area of grassy plots = 800 × 3.14 = 2512 cm^2

Cost of surfing 1m^2 plot = 1.25 Rs

Cost of surfing 2512m^2 = 2512 × 1.25 = 3140 Rs

10. Question

A rectangular park is 100m by 50m. It is surrounded by semi-circular flower bed sall round. Find the cost of leveling the semi-circular flower bed sall 60paise per square meter. $(Use\ \pi = 3.14)$

Answer

Length of rectangular park = 100m

Breadth of rectangular park = 50m

Radius of flower bed along length of park = $\frac{100}{2} = 50m$

Area of flower bed along length of park = $\frac{2 \times \pi r^2}{2}$

$= \frac{22}{7} \times 50 \times 50 = 7850m^2$

Radius of flower bed along width = $\frac{50}{2} = 25m$

Area of flower bed along width = $2 \times \frac{22}{7} \times \frac{25 \times 25}{2} = 1962.5m^2$

Total area of flower beds = 7850+1962.50 = 4812.50m^2

So,

Cost of leveling semicircular flower beds = 9812.50×.60 = Rs 5887.50

11. Question

Prove that the area of a circular path of uniform width h surrounding a circular region of radius is $\pi h(2r + h)$.

Answer

Area of inner circle with radius r = πr^2

Radius of outer circle = r+h

Area of outer circle = $\pi(r+h)^2$

Area of circular path with width = h

= $\pi(r+h)^2 - \pi r^2$

By using $(a+b)^2 = a^2 + b^2 + 2ab$

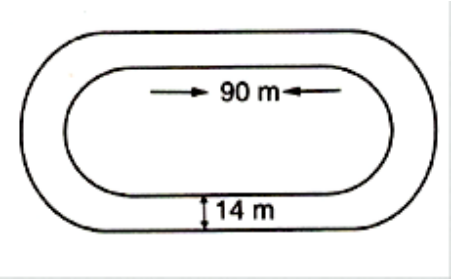
= $\pi(r^2 + h^2 + 2rh) - \pi r^2$

=$\pi r^2 + \pi h^2 + 2\pi rh - \pi r^2$

= $\pi h(2r+h)$… Proved

12. Question

The inside perimeter of a running track (showninFig.15.67) is 400m. The length of each of the straight portion is 90m and the ends are semi-circles. If the track is everywhere 14m wide, find the area of the track. Also find the length of the outer running track.



Answer

Given,

Inside perimeter of track = 400m

Length of straight portion = 90m

Width of path = 14m

Total length of straight path = 90+90 = 180m

Remaining length = 400 – 180 = 220m

This length includes two semi circles or a complete circle.

So,

$2\pi r = 220m$

$$= r = \frac{220 \times 7}{2 \times 22} = 35m$$

Then,

Area of path = (area of rectangles ABCD + rectangle EFGH + two semicircles)

= 14×90+14×90+π [$(25+14)^2 - 35^2$]

[$(a^2 - b^2) = (a+b)(a-b)$]

$$= 2520 + \frac{22}{7} \times 84 \times 14^2$$

Area of path = 6216m^2

Length of outer track = 90+90+2πr

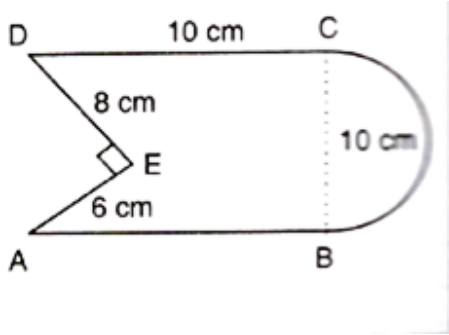
r = 35+14 = 49

$$= 180 + 2\frac{22}{7} \times 49^2$$

= 180 +308 = 488m^2

13. Question

Find the area of Fig15.68, in square cm, correct to one place of decimal. $(Take\ \pi = 22/7)$



Answer

Area of semicircle with diameter = 10cm

$$r = \frac{10}{2} = 5cm$$

$$= \frac{\pi r^2}{2} = \frac{22 \times 5 \times 5}{7 \times 2} = 39.28\ cm^2$$

Area of triangle AED = $\frac{1}{2} \times 8 \times 6 = 24cm^2$

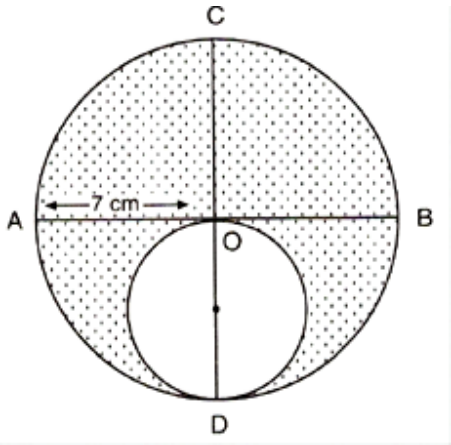
Area of square ABCD = 10×10 = 100cm^2

Area of figure excluded triangle = 100-24 = 76 cm^2

Total area of figure = 39.28+76 = 115.3 cm^2

14. Question

In Fig.15.69, AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If OA=7cm, find the area of the shaded region.



Answer

Area of semicircle ACB = $\frac{\pi r^2}{2}$

$$= \frac{22}{7} \times \frac{7 \times 7}{2} = 77cm^2$$

= area of circle with diameter OD = $\pi r^2 \left(r = \frac{7}{2} = 3.5\right)$

$$= \frac{22}{7} \times 3.5 \times 3.5 = 38.5cm^2$$

Remaining shaded portion in lower semi circle = 77 – 38.5 = 38.5cm^2

Total shaded portion area = 77 + 38.5 = 115.5 cm^2

15. Question

In Fig.15.70, OACB is a quadrant of a circle with centre O and radius 3.5cm. If OD=2cm, find the area of the (i) quadrant OACB (ii) shaded region.

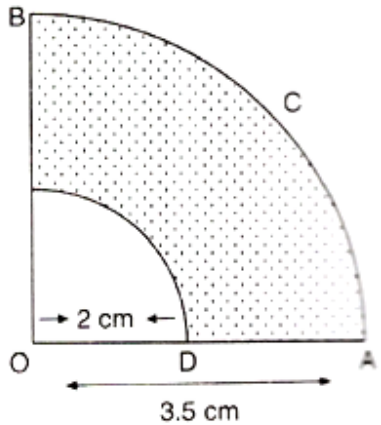


Fig. 15.70

Answer

Given,

Area of quadrant OACB = $\frac{\theta}{360}\pi r^2$

$$= \frac{90}{360} \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= \frac{1}{4} \times 11 \times 3.5 = 9.625 cm^2$$

Area of shaded region = area of quadrant OACB – area of quadrant ODEF

$$= 9.625 - \frac{90}{360} \times \frac{22}{360} \times 2 \times 2$$

$$= 9.625 - \frac{1}{4} \times 3.14 \times 4 = 6.482 cm^2$$

16. Question

From each of the two opposite corners of a square of side 8cm, a quadrant of a circle of radius 1.4cm is cut. Another circle of radius 4.2cm is also cut from the centre as shown in Fig.15.71. Find the area of the remaining (shaded) portion of the square. $(Use\ \pi = 22/7)$.

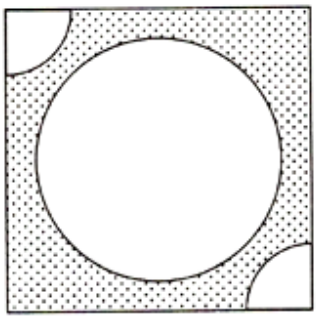


Fig. 15.71

Answer

Given,

Side of square = 8cm

Radius of quadrant circle = 1.4 cm

Radius of inner-circle = 4.2

Area of square = $(side)^2 = 8^2 = 64cm$

Area of one quadrant of circle = $\frac{\theta}{360} \times \pi r^2$

Area of one quadrant of circle = $\frac{90}{360} \times \frac{22}{7} \times 1.4 \times 1.4 = 1.54 cm^2$

So,

Area of 2 quadrant = $2\times1.54 = 3.08cm^2$

Area of inner circle = $\pi r^2 = 3.14\times4.2\times4.2 = 55.44\ cm^2$

Area of shaded portion = area of square – (area of quadrants + area of inner circle)

= 64 – (3.08 + 55.44)

= 64 – 58.52 = $5.48\ cm^2$

17. Question

Find the area of the shaded region in Fig.15.72, if AC=24cm, BC=10cm and O is the centre of the circle. $(Use\ \pi = 3.14)$

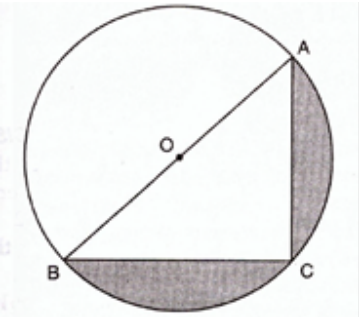


Fig. 15.72

Answer

Given,

AC = 24 cm

BC = 10 cm

By Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$= 24^2+10^2 = 576 + 100 = 676$

$AB = \sqrt{676} = 26cm$

Radius of semi-circle with diameter AB = $\frac{26}{2}$ = 13cm

Area of semi-circle = $\frac{\pi r^2}{2} = \frac{3.14\times13\times13}{2} = 265.33cm^2$

Area of triangle ABC = $\frac{1}{2}\times AC\times BC = \frac{1}{2}\times24\times10 = 120cm^2$

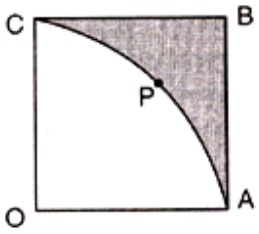
So,

Area of shaded region = area of semi-circle – area of triangle

= $265.33 - 120 = 145.33\ cm^2$

18. Question

In Fig.15.72(a), OABC is a square of side 7cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. $(Use\ \pi = 22/7)$



Answer

Given,

Side of square = 7cm

Area of square = $(side)^2 = 7^2 = 49cm^2$

Area of quadrant OAPC = $\frac{\theta}{360}\pi r^2$

$$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{4} \times 154 = 38.5 cm^2$$

Area of shaded region = (area of square – area of quadrant)

$= 49 - 38.5 = 10.5\ cm^2$

19. Question

A circular pond is of diameter 17.5m. It is surrounded by a 2m wide path. Find the cost of constructing the path at the rate of Rs. 25 per square meter $(Use\ \pi = 3.14)$

Answer

Given,

Diameter of circular pond = 17.5m

Radius of circular pond = $\frac{17.5}{2}$ = 8.75m

Radius of outer circle = (radius of inner circle + width of circular path)

= 8.75+2 = 10.25m

Area of circular path = (area of outer circle – area of inner circle)

$= \pi(R^2 - r^2)$

$= \pi(R+r)(R-r)$

$= \frac{22}{7}(10.75+8.75)(10.75-8.75)$

$= \frac{22}{7} \times 19.50 \times 2 = 3061.50\ m^2$

20. Question

A regular hexagon is inscribed in a circle. If the area of hexagon is $24\sqrt{3}\ cm^2$,find the area of the circle.

$(Use\ \pi = 3.14)$

Answer

Given,

Area of regular hexagon = $24\sqrt{3}$ cm^2

From formula

$$\frac{3\sqrt{3}}{2}\times a^2 = 24\sqrt{3}$$

$$a^2 = \frac{24\sqrt{3}\times 2}{3\sqrt{3}} = 16$$

$$a = \sqrt{16} = 4cm$$

So,

Area of circum circle of regular hexagon = π(side)^2

= 3.14×4×4cm^2 = 50.24cm^2

21. Question

A path of width 3.5m runs around a semi-circular grassy plot whose perimeter is 72m. find the area of the path. $(Use\ \pi = 22/7)$

Answer

Given,

Perimeter of semi-circle = 72m

Width of path around it = 3.5m

Perimeter of semi-circle = πr+2r

$= \frac{22}{7}r + 2r = 72$

$= 22r+14r = 72\times 7$

$r = \frac{72\times 7}{36}$ = 14cm

Radius including the width of path(R) = r+3.5 = 14+3.5 = 17.5m

So, area of path = $\frac{\pi R^2}{2} - \frac{\pi r^2}{2}$

$= \frac{\pi}{2}\left((17.5^2) - (14^2)\right)$

$= \frac{\pi}{2}\left((17.5 + 14)(17.5 - 14)\right)$

$= \frac{3.14}{2} \times 31.5 \times 3.5$ = 173.25m^2

22. Question

Find the area of a shaded region in the Fig.15.73, where a circular arc of radius 7cm has been drawn with vertex A of an equilateral triangle ABC of side 14cm as centre. $\left(Use\ \pi = 22/7\ and\ \sqrt{3} = 1.73\right)$

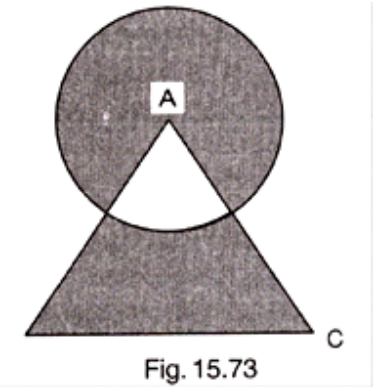


Fig. 15.73

Answer

Given,

Radius = 7cm

Side of equilateral triangle = 14cm

Area of circle = πr^2

Area of circle = $\frac{22}{7} \times 7 \times 7 = 154cm^2$

Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{\sqrt{3}}{4} \times 196 = 84.77cm^2$

We know that an equilateral triangle always subtend an angle of 60 at centre area of sector = $\frac{q}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{1}{6} \times 154$ = 25.666cm^2

This area is common in both the figure so,

Area of shaded region = (area of circle + area of equilateral triangle - 2×area of sector)

= (154+84.77-2×25.67)

= (238.77-51.33) = 187.44cm^2

23. Question

A child makes a poster on a chart paper drawing a square ABCD of side 14cm. She draws four circles with centre A,B,C and D in which she suggests different ways to save energy. The circles are drawn in such away that each circle touches externally two of the three remaining circles (Fig.15.74). In the shaded region she writes a message 'Save Energy'. Find the perimeter and area of the shaded region. $(Use\ \pi = 22/7)$

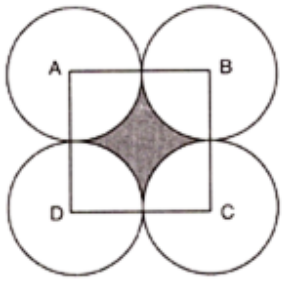


Fig. 15.74

Answer

Given,

Side of square = 14cm

Radius of each circle = $\frac{14}{2}$ = 7cm

Area of square = (side)^2 = 14^2 = 196cm^2

Area of 4 quadrants of circle = $\frac{90}{360}\times\frac{22}{7}\times7\times7$

$$= 4\times\frac{1}{4}\times154 = 154cm^2$$

Area of shaded region = area of square – area of 4 quadrants

= 196 – 154 = 42cm^2

Perimeter of shaded region = $\frac{90}{360}$ ×2πr

$$=\frac{1}{4}\times2\times\frac{22}{7}\times7 = 11cm$$

So, total perimeter of 4 circles = 4×11 = 44cm

16. Surface Areas and Volumes

Exercise 16.1

1. Question

How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm?

Answer

Sum of volumes of balls that can be made from a large sphere will have same volume, as that of the sphere.

As we know that, volume of a sphere = $\frac{4}{3}\pi r^3$. Given that, radius of large sphere, r_1 is 8 cm and that of the small spheres, r_2 is 1 cm.

Therefore, number of small balls that can be made from the big sphere

$$= \frac{volume\ of\ big\ sphere}{volume\ of\ one\ small\ sphere}$$

$$\Rightarrow \frac{\frac{4}{3}\pi {r_1}^3}{\frac{4}{3}\pi {r_2}^3} = \frac{{r_1}^3}{{r_1}^3} = \frac{8^3}{1^3} = 512 \text{ spheres}$$

2. Question

How many spherical bullets each of 5 cm in diameter can be cast from a rectangular block of metal 11 dm × 1 m × 5 dm?

Answer

Sum of volumes of spherical bullets that can be cast from a rectangular block of metal will have same volume, as that of the block.

As we know that, volume of the rectangular block of metal=lbh. Also, volume of spherical bullets = $\frac{4}{3}\pi r^3$

Given that, dimension of rectangular block of metal is 11 dm × 1 m × 5 dm = 110 cm × 100 cm × 50 cm and diameter of spherical bullets = 5cm (radius = $\frac{d}{2} = \frac{5}{2}$cm)

Therefore, number of small bullets that can be made from the block

$$= \frac{volume\ of\ rectangular\ block}{volume\ of\ one\ small\ spherical\ bullet}$$

$$= \frac{110 \times 100 \times 50}{\frac{4}{3}\pi\left(\frac{5}{2}\right)^3}$$

$$= \frac{110 \times 25 \times 50 \times 3 \times 8 \times 7}{22 \times 125}$$

$$= 50 \times 3 \times 8 \times 7 = 8400$$

3. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of the two of the balls are 1.5 cm and 2 cm respectively. Determine the diameter of the third ball.

Answer

Given that, radius of spherical ball = 3 cm

We know that, Volume of the sphere = $\frac{4}{3}\pi r^3$

So, its volume, v = $\frac{4}{3}\pi(3)^3$

The spherical ball of radius 3 cm is melted and recast into three spherical balls.

Let the radius of the third ball be r.

Then, volume of third spherical ball = $\frac{4}{3}\pi(r)^3$

Volume of first ball = $\frac{4}{3}\pi(1.5)^3$

Volume of second ball = $\frac{4}{3}\pi(2)^3$

The volume of the original spherical ball is equal to that of the total volumes of the three balls.

$$\Rightarrow \frac{4}{3}\pi(1.5)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi(r)^3 = \frac{4}{3}\pi(3)^3$$

$$\Rightarrow (1.5)^3 + (2)^3 + (r)^3 = (3)^3$$

$$\Rightarrow (r)^3 = (3)^3 - (1.5)^3 - (2)^3$$

$$\Rightarrow r = \sqrt[3]{27 - 3.375 - 8}$$

$$\Rightarrow r = \sqrt[3]{15.625}$$

⇒ r = 2.5 cm

Thus, diameter = 5 cm

4. Question

2.2 cubic dm of brass is to be drawn into a cylindrical wire 0.25 cm in diameter. Find the length of the wire.

Answer

Given: 2.2 dm^3 of brass is to be drawn into a cylindrical wire 0.25 cm in diameter.

Diameter of cylindrical wire = 0.25cm

Radius of wire, r = $\frac{d}{2} = \frac{0.25}{2} = 0.125cm$

Let length of wire be h

Volume of the cylinder = $\pi r^2 h$

Volume of brass of 2.2dm^3 is equal to volume of cylindrical wire

$\frac{22}{7}(0.125 \times 10^{-3}$

$\Rightarrow h = \frac{2.2\times10^{-3}\times7}{22(0.125\times10^{-2})^2}$

⇒ h = 448m

5. Question

What length of a solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of length 16 cm, external diameter 20 cm and thickness 2.5 mm?

Answer

The solid cylinder should be to hollow cylinder

Given that, diameter of solid cylinder = 2 cm

Length of hollow cylinder = 16cm

External diameter = 20cm

Thickness = 2.5mm=0.25cm

Volume of solid cylinder = $\pi r^2 h$ ------(i)

Radius of the cylinder = 1cm

So, volume of the solid cylinder = $\pi(1)^2 h = \pi h$

Let length of the solid cylinder be h

Volume of hollow cylinder = $\pi h(R^2 - r^2)$

Thickness = R − r

⇒ r = 10 − 0.25

⇒ Internal radius = 9.75cm

So, volume of hollow cylinder = π × 16(100 – 95.0625)

= π × 16 × 4.9375

≈ π × 16 × 4.94 -------(2)

As we know that, volume of both solid and hollow cylinder must be the same, therefore

π h = π × 16 × 4.94

⇒ h ≈ 79cm

∴ Length of the solid cylinder = 79cm

6. Question

A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm which are filled completely. Find the diameter of the cylindrical vessel.

Answer

Given: diameter of the cylindrical vessel = height of the vessel

∴ h = 2r

Given: water filled in the cylindrical vessel is poured in two cylindrical vessels

Volume of a cylindrical vessel = π r^2 h

So, volume = πr^2(2r) = 2 π r^3

Volume of vessel with diameter 42 cm = $\pi \times \left(\frac{42}{2}\right)^2 \times 21$

= π $(21)^3$ cm^3

Since cylinders are filled by water completely, so volume of large cylinder = sum of these small cylinders

∴ 2 π r^3 = 2 × π $(21)^3$

⇒ r = 21cm

∴ diameter of the cylindrical vessel = 42cm

7. Question

50 circular plates each of diameter 14 cm and thickness 0.5 cm are placed one above the other to form a right circular cylinder. Find its total surface area.

Answer

Diameter of each circular plate = 14 cm

∴ their radius, r = 7 cm

Thickness of each plate = 0.5 cm

50 plates are placed one above the other to form a right circular cylinder

∴ height of the cylinder formed, h = 50 × 0.5 cm

Or h = 25cm

Total surface area of the cylinder = 2πr(r + h)

$$= 2 \times \frac{22}{7} \times 7(7 + 25)$$

= 1408 cm^2

8. Question

25 circular plates, each of radius 10.5 cm and thickness 1.6 cm, are placed one above the other to form a solid circular cylinder. Find the curved surface area and the volume of the cylinder so formed.

Answer

Radius of each circular plate = 10.5 cm

Thickness of each plate = 1.6 cm

25 plates are placed one above the other to form a solid circular cylinder

∴ height of the cylinder formed, h = 25 × 1.6 cm = 40cm

Total curved surface area of the cylinder = 2πrh

$$= 2 \times \frac{22}{7} \times 10.5 \times 40$$

= 2640 cm^2

Volume of cylinder = π r^2 h

= π $(10.5)^2$ × 40

= 13860 cm^3

9. Question

A path 2 m wide surrounds a circular pond of diameter 40 m. How many cubic metres of gravel are required to grave the path to a depth of 20 cm?

Answer

Diameter of circular pond is given as 40 cm

∴ Radius of the pond, r = 20cm

Also, thickness, t = R − r

⇒ 2 = R − r

⇒ 2 = R − 20

⇒ R = 22cm

Volume of a hollow cylinder = π (R^2 – r^2)h

= 3.14 × (22^2 – 20^2) × 0.2

≈ 52 .8

∴ 52.8 cubic metres of gravel are required to grave the path to a depth of 20 cm

10. Question

A 16 m deep well with diameter 3.5 m is dug up and the earth from it is spread evenly to form a platform 27.5 m by 7 m. Find the height of the platform.

Answer

Let us assume shape of well is like a solid right circular cylinder

Radius of cylinder, r = 3.5/2 = 1.75m

Depth of well, h = 16m

Volume of right circular cylinder, V’ = π r^2 h

$= \frac{22}{7} \times 1.75^2 \times 16$ ---------(I)

Given that, length of platform, l = 27.5 cm

Breadth of the platform, b = 7cm

Let height of the platform be x m

Volume of the rectangle, V’’ = lbh

= 27.5 × 7 × x

= 192.5x ----------(II)

Since well is spread evenly to form the platform, so volume V’ = V’’

$\Rightarrow \frac{22}{7}(1.75)^2 \times 16 = 192.5x$

⇒ x = 0.8m

∴ Height of the platform = 80cm

11. Question

A well of diameter 2 m is dug 14 m deep. The earth taken out of it is spread evenly all around it to form and embankment of height 40 cm. Find the width of the embankment.

Answer

Let us assume shape of well is like a solid circular cylinder

Radius of cylinder, r = 2/2 = 1m

Depth(height) of well, h = 14m

Volume of right circular cylinder, V’ = π r^2 h

$= \pi \times 1^2 \times 14$ ---------(I)

Given that, length of embankment, l = 40 cm

Let width of the embankment be x m

Volume of the embankment, V’’ = π r^2 h

= π $((1+x)^2 - 1)^2$ x 0.4 ----------(II)

Since well is spread evenly to form the embankment, so their volumes, V’ = V’’

⇒ π × 14 = π $((1+x)^2 - 1)^2$ x 0.4

⇒ x = 5m

∴ Height of the embankment, x = 5cm

12. Question

Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm.

Answer

Given that, side of the cube = 9cmLet radius of cone be x.

Since largest cone is curved from cube.

Diameter of base of cone = side of cube

⇒ 2x = 9

⇒ x = $\frac{9}{2}$ cm

since sides of cube are equal and cone is cut from the cube.Maximum height of the cone can be 9 cm.So,

Height of cone is: h = 9 cm

Volume of largest cone = $\frac{1}{3}\pi r^2 h$

V= $\frac{1}{3}\pi\left(\frac{9}{2}\right)^2 \times 9$

V= $\frac{\pi}{12} \times 9^3$

$$\Rightarrow V = \frac{22 \times 9 \times 9 \times 9}{7 \times 12}$$

$$\Rightarrow V = \frac{22^{11} \times 9 \times 9 \times 9}{7 \times 12_{6}}$$

$$\Rightarrow V = \frac{11 \times 9 \times 9 \times 9}{7 \times 6}$$

$$\Rightarrow V = \frac{11 \times \cancel{9}^{3} \times 9 \times 9}{7 \times \cancel{6}_{2}}$$

$$\Rightarrow V = \frac{11 \times 3 \times 9 \times 9}{7 \times 2}$$

$$\Rightarrow V = \frac{2673}{14}$$

V = 190.92 cm^3

∴ volume of largest cone, v = 190.92 cm^3

13. Question

A cylindrical bucket, 32 cm high and 18 cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer

Volume of cylindrical bucket = π r^2h

Here, r = 18cm and h = 32cm

So, volume of cylindrical bucket = π $(18)^2(32)$ cm^2

When emptied on ground, volume of conical heap of sand = volume of cylindrical bucket

$$\therefore \frac{1}{3}\pi r^2 \times 24 = \pi \times 18^2 \times 32$$

$$\Rightarrow r = \sqrt{\frac{3 \times 18^2 \times 32}{24}} = 36cm$$

Now, $l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2} \approx 43.27$

14. Question

Rain water, which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if a rainfall of 1 cm has fallen? [*Use* $\pi = 22/7$]

Answer

Volume of rainfall = lbh

= 6 × 4 × 0.01

= 0.24 m^3

∵ Volume of water in cylindrical vessel = volume of rainfall

∴ $\pi r^2 h = 0.24$

$$\Rightarrow h = \frac{0.24 \times 7}{22 \times 0.2^2}$$

⇒ h ≈ 1.91m = 191cm

15. Question

A conical flask is full of water. The flask has base-radius r and height h. The water is poured into a cylindrical flask of base-radius rm. Find the height of water in the cylindrical flask.

Answer

Volume of conical flask = $\frac{1}{3}\pi r^2 h$

∵ Volume of water in cylindrical flask = volume of conical flask

∴ $\pi(rm)^2 h' = \frac{1}{3}\pi r^2 h$

$$\Rightarrow h' = \frac{1}{3}\left(\frac{r}{rm}\right)^2 h$$

$$\Rightarrow h' = \frac{h}{3m^2}$$

16. Question

A rectangular tank 15 m long and 11 m broad is required to receive entire liquid contents from a full cylindrical tank of internal diameter 21 m and length 5 m. find the least height of the tank that will serve the purpose.

Answer

Volume of liquid in the new rectangular tank = volume of liquid filled in full cylindrical tank

$lbh = \pi r^2 h'$

$$\Rightarrow h = \frac{\pi r^2 h'}{lb}$$

Here, r = $\frac{21}{2}$ m, h' = 5m, l = 15m and b = 11m, then

$$h = \frac{\frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 5}{15 \times 11} = 10.5m$$

17. Question

A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to

empty the bowl?

Answer

Number of bottles = (volume of cylindrical shaped small bottles/ Internal volume of hemispherical bowl)

$$\Rightarrow n = \frac{\frac{2}{3}\pi r'^3}{\pi r''^2 h}$$

$$\Rightarrow n = \frac{2}{3} \times \frac{r'^3}{r''^2 h}$$

$$\Rightarrow n = \frac{2}{3} \times \frac{9^3}{\left(\frac{3}{2}\right)^2 \times 4} = 54$$

18. Question

The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder.

Answer

Internal radius of the hollow spherical shell = 6/2 = 3cm

And external radius of that shell = 10/2 = 5cm

Radius of the cylinder = 14/2 = 7

Let height of the cylinder be x cm

According to the question,

Volume of the cylinder = volume of the spherical shell

$$\pi(7)^2 \times x = \frac{4}{3} \times \pi \times (5^3 - 3^3)$$

$$\Rightarrow 49 \times x = \frac{4}{3}(125 - 27)$$

$$\Rightarrow 49 \times x = \frac{4}{3} \times 98$$

$$\Rightarrow x = \frac{4 \times 98}{3 \times 49} = \frac{8}{3} cm$$

$\therefore$ height of cylinder = $\frac{8}{3}$ cm

19. Question

A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Calculate the height of the cone.

Answer

Given, internal radius of hollow sphere, r = (4/2) = 2cm

External radius = 8/2 = 4cm

Volume of hollow sphere = $\frac{4}{3}\pi(R^2 - r^2)$

$= \frac{4}{3}\pi(8^2 - 4^2)$ --------(i)

Given, diameter of the cone = 8cm

∴ Radius of the cone = 4 cm

Let height of the cone be h

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi 4^2 h$ --------(ii)

Since hollow sphere is melted into a cone so their volumes are equal,

$$\Rightarrow \frac{4}{3}\pi(64 - 16) = \frac{1}{3}\pi 4^2 h$$

∴ h = 12cm

20. Question

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and the level of the water is raised by 6.75 cm. Find the radius of the ball.

Answer

Given, the radius of the cylindrical tube, r = 12cm

Level of water raised in tube, h = 6.75cm

Volume of cylinder = $\pi r^2 h$

$= \pi (12)^2 \times 6.75\ cm^3$

$= \pi \times 12^2 \times 6.75\ cm^3$ ----------(i)

let r be the radius of a spherical shell balls

volume of the sphere = $\frac{4}{3}\pi r^3$ -------(ii)

∵ volume of cylinder = volume of spherical ball

$$\therefore \pi \times 12^2 \times 6.75 = \frac{4}{3}\pi r^3$$

$$\Rightarrow r^3 = \frac{12^2 \times 6.75}{\frac{4}{3} \times \pi}$$

⇒ r = 9cm

∴ radius of spherical ball, r = 9cm

21. Question

500 persons have to dip in a rectangular tank which is 80 m long and 50 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3?

Answer

To find: the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3 and 500 persons have to dip in a rectangular tank which is 80 m long and 50 m broad.**Solution:** Length of rectangular tank, l = 80m

Breadth, b = 50m

displacement of water in rectangular tank by 1 person = 0.04 m^3

Total displacement of water in rectangular tank by 500 persons = 500 × 0.04 m^3 = 20m^3 -----(i)

Let depth of the rectangular tank be hThe amount of water displaced is equal to the volume of the tank.

Volume of rectangular tank = lbh

Volume of rectangular tank = 80 × 50 × h m^3 ----------(ii)

Equating (i) and (ii), we get

20 = 80 × 50 × h

⇒ h = 0.005 m As 1 m = 100 cmSo 0.005 m = 0.005 × 100 cm⇒ h = 0.5 cm

∴ rise in level of water due to angular displacement = 0.5cm

22. Question

A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimeters?

Answer

Given that, radius of cylindrical jar, r = 6cm

Depth/height of the jar, h = 2cm

Volume of the jar, V′ = π r^2 h

⇒ V′ = π $(6)^2$ (2) cm^3

Radius of the sphere = 1.5cm

So, volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.5)^3$

Volume of oil in cylindrical jar = volume of n spheres needed to raise the level by 2 cm

$$\therefore \pi (6)^2 (2) = n\times \frac{4}{3}\pi(1.5)^3$$

⇒ n = 16

∴ number of iron sphere needed = 16

23. Question

A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

Answer

Volume of hollow sphere before melting = volume of cone

$$\Rightarrow \frac{4}{3}\pi(R^3 - r'^3) = \frac{1}{3}\times\pi\times r''^2\times h''$$

Here, R = 4cm, r = 2cm, r'' = 4cm, therefore

$$\Rightarrow \frac{4}{3}\pi(4^3 - 2^3) = \frac{1}{3}\times\pi\times 4^2\times h''$$

⇒ h'' = 14cm = height of the cone

Now, slant height, l = $\sqrt{h''^2 + r''^2}$

$$= \sqrt{14^2 + 4^2}$$

= 14.56 cm

24. Question

The internal and external diameters of a hollow hemispherical vessel are 21 cm and 25.2 cm respectively. The cost of painting 1 cm^2 of the surface is 10 paise. Find the total cost to paint the vessel all over.

Answer

Given that, internal diameter of hollow hemisphere, r = 10.5cm

External radius, R = 12.6cm

Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$=2\pi(12.6)^2 + 2\pi(10.5)^2 + \pi((12.6)^2 - (10.5)^2)$

$=1843.38\ cm^2$

Given that, cost of painting 1 cm^2 of surface = 10paise

Therefore, total cost to paint 1843.38 cm^2 = $\frac{1843.38 \times 10}{100} = Rs184.34$

25. Question

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball of radius 9 cm is dropped into the tub and thus the level of water is raised by h cm. What is the value of h?

Answer

Given that, radius of cylindrical tube = 12cm

Let height of cylindrical tube be h

Volume of the cylinder = $\pi r_1^2 h = \pi (12)^2 h$ --------(i)

Also given that, spherical ball radius (r_2) = 9cm

Volume of sphere = $\frac{4}{3}\pi {r_1}^3$

$= \frac{4}{3}\pi 9^3$ ---------(ii)

Equating (i) and (ii), we get

$\Rightarrow \pi 12^2 h = \frac{4}{3}\pi 9^3$

⇒ h = 6.75cm

∴ level of water raised in tube = 6.75cm

26. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

Answer

Let radius of the third ball be r.

Volume of the larger ball = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi 3^3$

Volume of larger ball = sum of volume of smaller balls

$\Rightarrow \frac{4}{3}\pi 3^3 = \frac{4}{3}\pi (1.5)^3 + \frac{4}{3}\pi (2)^3 + \frac{4}{3}\pi r^3$

⇒ r^3 = 27 – 3.375 – 8

⇒ r = 2.5cm

27. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

Answer

Let the radius of third sphere be r_3

Volume of big spherical ball=volume of all three balls

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi {r_1}^3 + \frac{4}{3}\pi {r_2}^3 + \frac{4}{3}\pi {r_3}^3$$

$\Rightarrow R^3 = {r_1}^3 + {r_2}^3 + {r_3}^3$

$\Rightarrow 3 \times 3 \times 3 = (1.5 \times 1.5 \times 1.5) + (2 \times 2 \times 2) + {r_3}^3$

$\Rightarrow {r_3}^3 = 27 - (3.375 + 8)$

$\Rightarrow {r_3}^3 = 15.625$

$\Rightarrow r_3 = \sqrt[3]{15.625}$

$\Rightarrow r_3 = 2.5$ cm

27. Question

Prove that the surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

Answer

Curved surface area of sphere$= 4\pi r^2$

Let 'r' be the radius of sphere

Height of cylinder='2r'

Curved surface area of cylinder$= 2\pi r\, h$

$\Rightarrow 2\pi r(2r)$

$\Rightarrow 4\pi r^2$

∴ Surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

28. Question

The diameter of a metallic sphere is equal to 9 cm. It is melted and drawn into a long wire of diameter 2 mm having uniform cross-section. Find the length of the wire.

Answer

Volume of the metallic sphere=volume of the wire(cylinder in shape)

Radius of sphere $= \frac{9}{2}$ cm

=4.5 cm

Volume of sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi 4.5^3$

Radius of cylinder $= \frac{2}{2}$ = 1mm =0.1cm

Volume of cylinder $= \pi r^2 h$

⇒ $\pi r^2 h = \frac{4}{3}\pi R^3$

⇒ $r^2 h = \frac{4}{3}R^3$

⇒ 0.1 × 0.1 × h = $\frac{4}{3}$ × 4.5 × 4.5 × 4.5

⇒ 0.1 × 0.1 × h =4 × 1.5 × 4.5 × 4.5

⇒ h = 12150 cm

29. Question

An iron spherical ball has been melted and recast into smaller balls of equal size. If the radius of each of the smaller balls is 1/4 of the radius of the original ball, how many such balls are made? Compare the surface area, of all the smaller balls combined together with that of the original ball.

Answer

Let the radius of bigger spherical be R

Volume of bigger spherical ball = $\frac{4}{3}\pi R^3$

Radius of smaller spherical ball= $\frac{1}{4}$R

Volume of smaller ball $= \frac{4}{3}\pi (\frac{1}{4}R)^3$

Let number of equal size spherical balls be n

Volume of n equal spherical ball =Volume of bigger spherical ball

$\Rightarrow n \times \frac{4}{3}\pi \left(\frac{1}{4}R\right)^3 = \frac{4}{3}\pi R^3$

⇒ n × $(R/4)^3 = R^3$

⇒ $n = 4^3$

⇒ n = 64 balls

Surface area of bigger spherical ball =4π R^2

Surface area of smaller spherical ball $= 4\pi(\frac{1}{4}R)^2$

Ratio between the surface area of bigger and 64 smaller spherical ball

$$\Rightarrow \frac{64\times 4\pi(\frac{1}{4}R)^2}{4\pi R^2} = 4$$

30. Question

A tent of height 77 dm is in the form a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m^2. [Use $\pi = 22/7$]

Answer

Height of tent =77dm

=7.7m

Radius of the cylinder $= \frac{36}{2}$ =18 m

Height of cylinder =44dm =4.4m

Height of cone = (7.7-4.4) m

=3.3m

Curved Surface area of cylinder portion of the tent $= 2\pi \times r \times h$

= 2 × (22/7) × 18 × 4.4

= 497.83 m^2

Curved surface area of conical tent =π r l

Slant height of cone $= \sqrt{(3.3^2 + 18^2)}$

= 18.3m

Curved surface area of cone = (22/7) × 18 × 18.3

= 1035.26m^2

Total surface area of the tent = 497.83m^2 + 1035.26m^2

= 1533.09m^2

Total cost of canvas = 3.50 × 1533.09

= Rs 5365.82

31. Question

Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimension 16 cm × 8 cm × 8 cm when 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. [Use $\pi = 669/213$]

Answer

Volume of sphere $= \frac{4}{3}\pi R^3$

$$= \frac{4 \times 669 \times 8}{3 \times 213}$$

= 33.50 cm^3

Volume of 16 spheres = 16 × 33.50

= 536cm^3

Volume of rectangle = 16cm × 8cm × 8cm

= 1024cm^3

Volume of this liquid = 1024cm^3 – 536cm^3

= 488cm^3

32. Question

The largest sphere is to be curved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere.

Answer

Diameter of the sphere =Height of the cylinder

Diameter of the sphere =14 cm

Radius of the sphere $= \frac{14}{2}$ cm

=7 cm

Volume of the sphere $= \frac{4}{3} \pi R^3$

$$= \frac{4 \times 22 \times 7 \times 7 \times 7}{3 \times 7}$$

= 1437.3cm^3

33. Question

A copper sphere of radius 3 cm is melted and recast into a right circular cone of height 3 cm. Find the radius of the base of the cone.

Answer

Volume of the sphere $= \frac{4}{3} \pi R^3$

$$= \frac{4}{3} \pi 3^3$$

Volume of the cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \pi r^2 \times 3$$

Volume of the cone = volume of the sphere

$\frac{1}{3}\pi r^2 \times 3 = \frac{4}{3}\pi\ 3^3$

$\Rightarrow \frac{1}{3}\times r^2 \times 3 = \frac{4}{3}\times 3^3$

$\Rightarrow\ r^2\ = \frac{4\times3\times3\times3\times3}{3\times3}$

$\Rightarrow r^2 = 36$ cm

$\Rightarrow r = 6$ cm

34. Question

A vessel in the shape of a cuboid contains some water. If three identical spheres are immersed in the water, the level of water is increased by 2 cm. If the area of the base of the cuboid is 160 cm^2 and its height 12 cm, determine the radius of any of the spheres.

Answer

Given that area of cuboid =160 cm^2

Level of water is increased = 2 cm

Volume of vessel =160× 2 cm^3 ……. (1)

Volume of each sphere $= \frac{4}{3}\pi\ R^3$

Volume of 3 sphere $= 3\times\frac{4}{3}\pi\ R^3$ ……. (2)

From eq 1 and eq 2

$\Rightarrow 3\times(4/3)\ \pi\ R^3 = 160\times 2$

$\Rightarrow R^3 = \frac{160\times2}{3\times\frac{4}{3}\pi}$

$\Rightarrow R = 2.94$ cm

35. Question

A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Answer

Given: A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness.

To find: the thickness of the wire.

Solution: Diameter of copper wire =1 cm

Radius $= \frac{1}{2}$ cm

Height of copper rod =Length of copper rod =8 cm

Volume of rod $= \pi r^2 h$

$= \pi \times (\frac{1}{2})^2 \times 8$

Let the radius of wire be 'r' cm

As 1 m = 100 cm

length of the wire=Height of the wire =18 m =1800 cmAs rod is converted into wire,Therefore,

Volume of cylinder wire = Volume of cylindrical rod

$$\Rightarrow \pi r^2 h = \pi \times \left(\frac{1}{2}\right)^2 \times 8$$

$$\Rightarrow r^2 = \frac{1}{900}$$

$$\Rightarrow r^2 = \frac{8}{2 \times 2 \times 1800}$$

$$\Rightarrow r^2 = \frac{8^2}{\cancel{4}_1 \times 1800}$$

$$\Rightarrow r^2 = \frac{\cancel{8}^1}{\cancel{4}_1 \times \cancel{1800}_{900}}$$

$$\Rightarrow r = \sqrt{\frac{1}{900}}$$

$$\Rightarrow r = \pm \frac{1}{30}$$

As radius cannot be negative,

$$\Rightarrow r = \frac{1}{30}$$

⇒ r = 0.033 cm

Hence thickness of the wire is 0.033 cm.

$\Rightarrow \pi r^2 \times 0.18 = \pi \times (\frac{1}{2})^2 \times 8$

$\Rightarrow r^2 = \frac{1 \times 8}{0.18 \times 2 \times 2}$

$\Rightarrow r = \frac{1}{30}$ cm

$\Rightarrow$ r = 1/3 mm

$\therefore$ Diameter of cross section = (1/3) × 2 mm

= 0.67 mm

NOTE: When one figure is converted into another figure,the volume of two figures remain same.

36. Question

The diameters of internal and external surfaces of a hollow spherical shell are 10 cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

Answer

Diameter of internal surface =10 cm

$\therefore$ Radius of internal surface $= \frac{10}{2}$ cm =5 cm

Diameter of external surface =6 cm

$\therefore$ Radius of external surface $= \frac{6}{2}$ cm =3 cm

Volume of spherical shell hollow = $\left(\frac{4}{3}\right)\pi\ (R^3 - r^3)$

$= \left(\frac{4}{3}\right) \times \pi \times (5^3 - 3^3)$

Height of solid cylinder $= \frac{8}{3}$ cm

Let the radius of the solid cylinder be 'r' cm

Volume of the solid cylinder $= \pi r^2 h$

$= \frac{22}{7} \times r \times r \times \frac{8}{3}\ cm^3$

Volume of the solid cylinder = Volume of spherical shell hollow

$\Rightarrow \pi \times r \times r \times \frac{8}{3}\ cm^3 = \left(\frac{4}{3}\right) \times \pi \times (5^3 - 3^3)$

$\Rightarrow \pi \times r \times r \times \frac{8}{3} = \left(\frac{4}{3}\right) \times \pi \times (5^3 - 3^3)$

$\Rightarrow r^2 \times \frac{8}{3} = \left(\frac{4}{3}\right) \times (125 - 27)$

$\Rightarrow r^2 \times (8/3) = (4/3) \times 98$

$\Rightarrow r = 7\sqrt{3}$

Diameter of cylinder = $14\sqrt{3}$ cm

37. Question

A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two cones so formed. Also, find their curved surfaces.

Answer

Height of the cone (h_1)=3 cm

Radius of the cone (r_1) = 4 cm

Slant height of the cone (l_1) = 5 cm

Volume of cone (V_1)= $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 4^2 \times 3$ cm^3

$= 16\pi$ cm^3

Again after rotating

Height of the cone (h_2)=4 cm

Radius of the cone (r_2)=3 cm

Slant height of the cone (l_2)=5 cm

Volume of cone (V_2)= $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 3^2 \times 4$ cm^3

$= 12\pi$ cm^3

Difference between the volume two cones =16π – 12π

= 4π cm^3

Curved surface area of first cone =π × r_1× l_1

= π × 4 × 5

= 20 π cm^2

Curved surface area of second cone $= \pi \times r_2 \times l_2 = \pi \times 3 \times 5$ = 15 π cm^2

38. Question

How many coins 1.75 cm in diameter and 2 mm thick must be melted to form a cuboid 11 cm × 10 cm × 7 cm?

Answer

Volume of cuboid = l × b × h = 11 × 10 × 7

Diameter of coin = 1.75 cm

Radius$= \frac{1.75}{2}$ cm = 0.875 cm

Thickness of coin =2 mm = 0.2 cm

Volume of coin (cylinder) $= \pi r^2 h$

= (22/7) × 0.875 × 0.875 × 0.2 cm^3

Let number of coins be n

⇒ n × volume of coins = volume of cuboid

⇒ n × (22/7) × 0.875 × 0.875 × 0.2 = 11 × 10 × 7

$$\Rightarrow n = \frac{11 \times 10 \times 7 \times 7}{22 \times 0.875 \times 0.875 \times 0.2}$$

⇒ n = 1600

39. Question

A well with inner radius 4 m is dug 14 m deep. Earth taken out of it has been spread evenly all around a width of 3 m it to form an embankment. Find the height of the embankment.

Answer

Let the height of the embankment be 'h' m

Radius of well = 4 m

Height of the well = 14m

Volume of earth dug out (volume of cylinder)$= \pi r^2 h$

$$= \frac{22}{7} \times 4 \times 4 \times 14 \, m^3$$

= 704 m^3

Outer Radius of the embankment(R) = 4 m + 3 m = 7 m

Inner radius of the embankment(r) = 4 m

Volume of embankment =Volume of earth dug out

Volume of embankment $= \pi R^2 h - \pi r^2 h$

= πh (R^2 – r^2)

= πh (7^2 – 4^2)

= 33πh

Volume of embankment = Volume of earth dug out

⇒ 33πh = 704

$$\Rightarrow\ h\ = \frac{704 \times 7}{33 \times 22}$$

⇒ h = 6.78 m

40. Question

Water in a canal 1.5 m wide and 6 m deep is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

Answer

Speed of flowing water =10 km/hr

In 30 minutes length of flowing water =10× $\frac{30}{60}$ km

= 5 km = 5000 m

Volume of flowing water in 30 minutes =5000 × width × depth

Width of canal = 1.5 m

Depth of canal = 6 m

Volume of canal =5000 × 1.5 × 6 m^3

= 45000 m^3

Irrigated area in 30 minutes if 8 cm of flowing water is required $= \frac{45000}{0.08}$

= 562500 m^2

41. Question

A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km / h, in how much time will the tank be filled?

Answer

Let the time-taken by the pipe to fill the tank be x hrs.

Since the water is flowing at a rate of 3 kmph, therefore

Length of water column in x hrs is

3 × x = 3x km = 3000x m

Therefore the length of the pipe is $\frac{20}{2} = 10cm = 0.1m$

Therefore, the volume of water flowing through the pipe in x hours, V' = π r^2 h

= π $(0.1)^2$ (3000x) m^3

Also, volume of water that falls into the tank in x hours = π r^2 h

= π $(5)^2$ (2)

The volume of water flowing through the pipe in x hours = volume of water that falls into the tank in x hours

∴ π $(0.1)^2$ (3000x) = π $(5)^2$ (2)

⇒ 30x = 50

$$\Rightarrow x = \frac{50}{30} = \frac{100}{60} hrs$$

= 1hr 40min

42. Question

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.

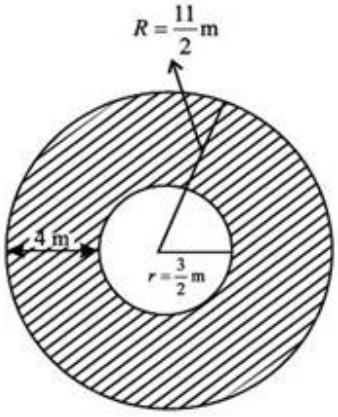
Answer

Let the height of the embankment be 'h' m

Radius of well $= \frac{3}{2} m$

=1.5 m

Height of the well =14m



Volume of earth dug out (volume of cylinder)$= \pi r^2 h$

$= \frac{22}{7} \times 1.5 \times 1.5 \times 14\, m^3$

=99 m^3

Outer Radius of the embankment(R) =1.5 m + 4 m

=5.5 m

Inner radius of the embankment(r) =1.5 m

Volume of embankment =Volume of earth dug out

Volume of embankment $= \pi R^2 h - \pi r^2 h$

$= \pi h(5.5^2 - 1.5^2)$

= 27.5πh

Volume of embankment =Volume of earth dug out

⇒ 27.5πh = 99

$\Rightarrow\ h\ = \frac{99 \times 7}{27.5 \times 22}\ m$h = 1.145

⇒ h = 9/8 = 1.125 m

43. Question

The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed $(Use\ \pi = 22/7)$.

Answer

Surface area of the sphere $= 4\pi r^2$

⇒ 4πr^2 = 616

$\Rightarrow\ 4 \times \frac{22}{7} \times r^2\ = \ 616$

$\Rightarrow\ r^2\ = \frac{616 \times 7}{4 \times 22}$

⇒ r^2 = 49

⇒ r = √49

⇒ r = 7 cm

Diameter of base = 7 × 2

= 14 cm

44. Question

The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14 cm long is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameters of the cylinder. $(Use\ \pi = 22/7)$

Answer

Height of hollow cylinder =14 cm

Let the radius of outer radius be 'R' cm

Let the radius of inner radius be 'r' cm

Difference between inner and outer Curved surface area= 88 cm

Curved surface area of hollow cylinder$= 2\pi(R-r)h$

⇒ 2π(R – r)h = 88

$$\Rightarrow R - r = \frac{88\times7}{2\times22\times14}$$

⇒ R – r = 1 …… (1)

Volume of metal = 176

Height of hollow cylinder = 14 cm

Volume of hollow cylinder (metal) = $\pi(R^2-r^2)h$

$$\Rightarrow \pi(R^2-r^2)h = 176$$

$$\Rightarrow (R^2-r^2) = \frac{176\times7}{22\times14}$$

$$\Rightarrow R^2-r^2 = 4$$

⇒ (R + r)(R – r) = 4

⇒ (R + r) = 4 …… (2)

Solving (1) and (2) we get,

R = 2.5cm

r = 1.5cm

∴ Outer diameter $= 2.5\times2$ cm

= 5 cm

Inner diameter $= 1.5\times2$ cm

= 3 cm

45. Question

The volume of a hemi-sphere is $2425\frac{1}{2}cm^3$. Find its curved surface area. $(Use\ \pi = 22/7)$

Answer

Volume of hemi-sphere $= \frac{2}{3}\pi r^3$

$\Rightarrow \frac{2}{3}\pi r^3 = 4851$

$\Rightarrow r^3 = \frac{4851 \times 3 \times 7}{22 \times 2 \times 2}$

⇒ r = 10.5 cm

Curved surface area$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 10.5^2$

= 693 cm^2

46. Question

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied out on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer

Height of the cylindrical bucket =32 cm

Radius of the cylindrical bucket = 18 cm

Volume of the bucket = $\pi r^2 h$

$= \pi \times 18^2 \times 32\ cm^3$

$= \pi \times 18 \times 18 \times 32$

Height of the conical heap =24 cm

Let Radius of the conical heap =r cm

Volume of conical heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times 24\ \text{cub.cm}$

Volume of conical heap=Volume of cylindrical bucket

$\Rightarrow \frac{1}{3}\pi \times r^2 \times 32\ cm^3 = \pi \times 18 \times 18 \times 32$

$\Rightarrow r^2 = \frac{18 \times 18 \times 32 \times 3}{24}$

⇒ r^2 = 1296 cm

$\Rightarrow r = \sqrt{1296}\, cm$

⇒ r = 36 cm

Slant height $= \sqrt{h^2 + r^2}$

∴ slant height $= \sqrt{24^2 + 36^2}$

$= 12\sqrt{13}\, cm$

47. Question

If the total surface area of a solid hemisphere is 462 cm^2, find its volume. $(Take\ \pi = 22/7)$

Answer

Total surface area of solid hemisphere $= 3\pi r^2$

⇒ $3\pi r^2 = 462$

⇒$3\times (22/7) \times r^2 = 462$

$\Rightarrow r^2 = \frac{462\times 7}{22\times 3}$

⇒ $r^2 = 49$

⇒ r=7 cm

Volume of solid hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi 7^3$ cm^3

$= \frac{2}{3}\times\frac{22}{7}\times 7\times 7\times 7$

= 718.67 cm^3

48. Question

150 spherical marbles, each of diameter 1.4 cm are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

Answer

Diameter of spherical marbles $= \frac{1.4}{2}$ cm

=0.7 cm

Diameter of cylinder vessel $= \frac{7}{2}\, cm$

=3.5 cm

Volume of 150 spherical balls =150× $\frac{4}{3}\pi R^3$

$= \frac{150\times4\times22\times0.7\times0.7\times0.7}{3\times7}$ cm^3

=215.6 cm^3

Let the rise in level of water be h cm

Volume of rise in level of water (Volume of cylinder) $= \pi r^2 h$

$= \frac{22}{7}\times3.5\times3.5\times h$

Volume of Rise in level of water in the vessel =volume of 150 spherical balls

⇒ (22/7) × 3.5 × 3.5 × h = 215.6

$\Rightarrow\ h\ = \frac{215.6\times7}{22\times3.5\times3.5}\ cm$

⇒ h = 5.6 cm

49. Question

A cylindrical tank full of water is emptied by a pipe at the rate of 225 liters per minute. How much time will it take to empty half the tank, if the diameter of its base is 3 m and its height is 3.5 m? $[Use\ \pi = 22/7]$

Answer

Diameter of the cylindrical base =3 m

∴ Radius of cylindrical tank $= \frac{3}{2}$m

= 1.5 m

Height of the tank =3.5 m

Volume of the tank $= \pi r^2 h$

$= \pi\times1.5^2\times3.5\ = \left(\frac{22}{7}\right)\times1.5\times1.5\times3.5$

= 24.75 m^3

Now, 1 m^3 = 1000 liters

24.75 m^3 =1000 × 24.75 liters

= 24750 liters

Full quantity of the water when it is full = 24750 m^3

Quantity of water when it is half filled$= \frac{24750}{2}$ liters

= 12375 liters

Time taken by it to empty 225 liters 0f water = 1 minute

∴ Time taken by it empty 12375 litersof water = $\frac{12375}{225}$ minutes

= 55 minutes

50. Question

A solid metallic sphere of radius 5.6 cm is melted and solid cones each of radius 2.8 cm and height 3.2 cm are made. Find the number of such cone formed.

Answer

Volume of the sphere = $\frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi\, 5.6^3$

Let number of cones be n

⇒n × volume of cone = volume of sphere

$\Rightarrow\ n\times\frac{1}{3}\pi 2.8^2\times 3.2\ =\frac{4}{3}\pi\, 5.6^3$

$\Rightarrow\ n\times\ \frac{1}{3}\times 2.8\times 2.8\times 3.2\ =\frac{4}{3}\times 5.6\times 5.6\times\ 5.6$

$\Rightarrow\ n\ =\frac{4\times 5.6\times 5.6\times 5.6\times 3}{3\times 2.8\times 2.8\times 3.2}$

⇒ n = 28

52. Question

Sushant has a vessel of the form of an inverted cone, open at the top of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\left(\frac{2}{5}\right)^{th}$ of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?

Answer

Height of the conical vessel, h’ = 11cm

It’s radius, r’ and r’’ resp.: r’ = 2.5cm r’’ = 0.25cm

Assume the number of spherical balls that were dropped in the vessel as ‘n’

Volume of water replaces by the spherical ball = volume of spherical balls dropped into the vessel

(2/5) × Volume of the cone = n × each spherical ball

$$\Rightarrow \frac{2}{5}\times\frac{1}{3}\pi r'^2 h = n\times\frac{4}{3}\pi r''^3$$

$\Rightarrow (2.5)^2 \times 11 = n \times 10 \times (0.25)^3$

$\Rightarrow n = 440$

53. Question

A solid cuboid of iron with dimensions 53 cm × 40 cm × 15 cm is melted and recast into a cylindrical pipe. The outer and inner diameters of pipe are 8 cm and 7 cm respectively. Find the length of pipe.

Answer

Outer and inner radius of the pipe: r′ = 8/2 = 4cm and r′′ = $\left(\frac{7}{2}\right)$ cm

Volume of cuboid = volume of pipe

L b h = π h[$R^2 - r^2$]

$$\Rightarrow 53 \times 40 \times 15 = \frac{22}{7} \times h\left[4^2 - \left(\frac{7}{2}\right)^2\right]$$

$$\Rightarrow 31800 = \frac{22}{7} \times h\left[\frac{15}{4}\right]$$

$\Rightarrow$ h = 2698.18cm ≈ 27m

54. Question

Water is flowing at the rate of 2.52 km / h through a cylindrical pipe into a cylindrical tank, the radius of the base is 40 cm. If the increase in the level of water in the tank, in half and hour is 3.15 m, find the internal diameter of the pipe.

Answer

Speed of water = 2.52kmph = 2520 m/hr

Length, h′ = 2520 m

Let the radius of the pipe be r′ and radius of the tank, r′′ = 40 cm = 0.4 m

Level of water in the tank in half and hour = 3.15 m

Level of water in the tank in an hour, h′′ = 6.30 m

Now, the volume of pipe = volume of the tank

$\Rightarrow$ π r'^2 h′ = π r''^2 h′′

$\Rightarrow r'^2 (2520) = (0.4)^2 (6.3)$

$$\Rightarrow r = \sqrt{\frac{(0.4)^2(6.3)}{2520}} = 0.02m = 2cm$$

∴ diameter = 2r′ = 2 × 2 = 4 cm

Exercise 16.2

1. Question

A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.

Answer

Diameter= 24 m

r =24/2= 12

Height of cylinder, H =1cm

Height of cone, h = 16-11= 5 m

Slant height (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{169}$ = 13 m

Surface area of tent = curved surface area of cylinder + curved surface area of cone

= 2πrH + πrl

= 2 × (22/7) × 12 × 11 + (22/7) × 12 × 13

= (22/7) × 12 (22 +13)

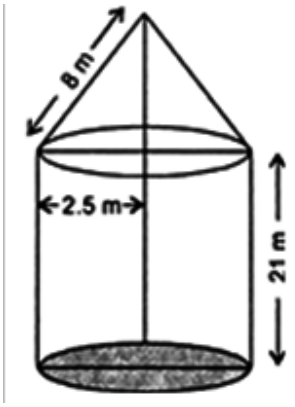
= (22/7) × 12 × 35

= 1320 m^2

2. Question

A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area and the volume of the rocket.

Answer



Radius of the cylinder portion (r) = 2.5 m

Height of the cylinder portion (h) = 21 m

∴ Surface area of the cylindrical portion = 2πrh

$= 2 \times \frac{22}{7} \times 2.5 \times 21$ m

= 330 m ……(1)

Radius of the conical portion (r) = 2.5 m

Slant height of the conical portion (l) = 8 m

∴ Curved surface area of the conical portion = πrl

= (22/7) × 2.5 × 8 m

= 62.86 m ……..(2)

Area of the circular top $= \pi r^2$

= (22/7) × 2.5 × 2.5 m^2

= 19.64 m^2 ……(3)

∴ Total surface area of the rocket

= (330 + 62.86 + 19.64) m^2 [From (1) , (2), (3)]

= 412.5 m^2

Volume of the cylindrical portion $= \pi r^2$h

$= \frac{22}{7} \times (2.5)^2 \times 21$

= 412.5 m^2

3. Question

A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m^2 $(Use\ \pi = 22/7)$.

Answer

Given: height of the tent, H = 77dm = 7.7 m

Height of the cylindrical portion, h = 44dm = 4.4m

∴ Height of the conical portion, h' = 7.7 – 4.4 = 3.3 m

Radius of the cylinder, r = 36/2 = 18m

Curved surface area of the cylindrical portion of the tent = 2 п r^2 h

= 2 × п × $(18)^2$ × 4.4

= 158.4 п m^2

Slant height of the conical portion of the tent = $\sqrt{3.3^2 + 18^2}$

= 18.3 m

∴ Curved surface area of the conical portion = п × 18 × 18.3

= 329.4 п m^2

∴ Total surface area of the tent = 158.4 п + 329.4 п

= 487.8 п

= 1533.09 m^2

∴ Canvas required to make the tent = 1533.09 m^2

Total cost of the canvas = 1533.09 × 3.50

= Rs5365.80

4. Question

A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm, respectively. Determine the surface area of the toy. $(Use\ \pi = 3.14)$

Answer

Height of the cone =4 cm

Radius of the cone = radius of hemisphere = 3 cm

Volume of toy = Volume of conical part + Volume of hemispherical part

Volume of cone $= \frac{\pi r^2}{3}$

∴ Volume of conical part = $\frac{3.14 \times 3^2 \times 4}{3}$

= 37.6 cm^2

Volume of hemisphere =$\frac{2\pi r^2}{3}$

Volume of toy = 37.6 +56.52 = 94.12 cm^2

And total surface area of toy = Curved surface area of conical part + Curved surface area of hemispherical part

Curved surface area of cone $= \pi rl$, Where l = $\sqrt{h^2 + r^2}$

= 3.14 × 3 × $\sqrt{4^2 + 3^2}$

= 47.1 c m^2

And curved surface area of hemisphere =2πr^2

= 2 × 3.14 × 3 × 3

= 56.52 c m^2

Then, total surface area of the toy = 47.1 + 56.52 = 103.62 c m^2

5. Question

A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the heights of the cylindrical and conical portions are 10 cm and 6 cm, respectively. Find the total surface area of the solid. (*Use* π = 22 / 7)

Answer

Height of cylinder (h) = 10 cm

Height of conical part = 6 cm

Slant height of cone (l) = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{3.5^2 + 6^2}$

⇒ l = 48.25 cm

Curved surface area of cone $= \pi$ rl

= π (3.5) (48.25)

= 76.408 cm^2 …(1)

Curved surface area of cylinder = 2π rh

=2π (3.5) (10)

=220 cm^2 …(2)

Curved surface area of hemisphere = $2\pi r^2$

= 2π $(3.5)^2$

= 77 cm^2 ….(3)

∴ Total curved surface area = Curved surface area of(cone + cylinder + hemisphere)

= 76.408 + 220 + 77

= 373.408

∴ Total surface area of solid = 373.408 cm^2

6. Question

A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.

Answer

Total surface area of toy = C.S.A. of cylinder + C.S.A. of hemisphere + C.S.A. of cone

= $(2\times \frac{22}{7}\times 5 \times 13) + (2\times\frac{22}{7} \times 5\times 5) + (\frac{22}{7} \times 5\times 13)$

= $\frac{22}{7}\times 5[(2\times 13) + (2\times 5)13]$

= $\frac{22}{7}\times 5[26+10+13]$

= $\frac{22}{7}\times 5\times 49$

=770 cm^2

7. Question

A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed in the tub. If the radius of the hemisphere is immersed in the tub. If the radius of the hemi-sphere is 3.5 cm and height of the cone outside the hemisphere is 5 cm, find the volume of the water left into hr tub. $(Take\ \pi = 22/7)$

Answer

Given radius of cylindrical tube(r) = 5 cm

Height of cylindrical tube (h) =9.8 cm

Volume of cylindrical =πr^2h

V1 =π 5^2(9.8)

= 770 cm

Given of hemisphere = 3.5 cm

Height of cone (h) = 5 cm

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi(3.5)r^3$

= 89.79 cm^3

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r(3.5)^2 5$

= 64.14 cm^3

Volume of cone + Volume of hemisphere (v2) = 89.79 + 64.14 = 154 cm^3

8. Question

A circus tent has cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 m. The heights of the cylindrical and conical portions are 4.2 m and 2.1 m respectively. Find the volume of the tent.

Answer

Given radius of Cylindrical base = 20 m

Height of Cylindrical part (h) = 4.2

Volume of Cylindrical = $\pi r^2 h$ ….(1)

V1 =п$(20)^2$ 4.2

= 5280 m^3

Volume 0f Cone =$\frac{1}{3}\pi r^2 h$

Height of conical part (h) = 2.1 m …..(2)

V2 $= \frac{\pi}{3}$ $(20)^2$ (2.1)

= 880 m^3

Volume of tent (v) = V1 + V2

V = 5280 + 880

= 6160 m^3

9. Question

A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of axis length 9 cm. Determine the capacity of the tank.

Answer

Diameter =21 cm

∴ Radius, r =10.5 cm

Length of the cylindrical part, l =18 cm

Height of the conical part, h = 9 cm

Volume of the tank = Volume of the cylindrical part + 2 × Volume of the conical part

$$= \pi r^2 L + 2 \times \frac{1}{3} \times \pi r^2 h$$

$$= \pi r^2 \left(L + \frac{2}{3} \times h\right)$$

= (22/7) × 10.5 × 10.5 × 24

= 8316 cm^3

10. Question

A conical hole is drilled in a circular cylinder of height 12 cm and base radius 5 cm. The height and the base radius of the cone are also the same. Find the whole surface and volume of the remaining cylinder.

Answer

Surface area = 210 π cm^2

Radius of cylinder(r) = 5 cm

Height of cylinder, (h) = 12 cm

Let, l be slant height of cone

$$l = \sqrt{r^2 + h^2}$$

Slant height of the cone = $\sqrt{5^2 + 12^2}$ =13 cm

Volume of cylinder = π × 5^2 × 12 = 300 cm^3

Volume of the conical hole = $\frac{1}{3}$ × π × 5 × 12 = 100 cm^3

Therefore, Volume of the remaining solid

= Volume of the cylinder – Volume of the removed conical part

= 300 π -100 π = 200 π cm^3

Curved surface of the cylinder

= 2 × π r h = 2 × π × 5 × 12 = 120π cm

Curved surface of cone = π r l = π × 5 × 13 = 65π cm

Base area of cylinder = π × 5^2 = 25cm^2

The whole surface area of the remaining solid includes the curved surface of the cylinder and cylinder and the cone and area of the base

Therefore, Whole surface area = 120 π + 65π + 25π =210π cm^2

11. Question

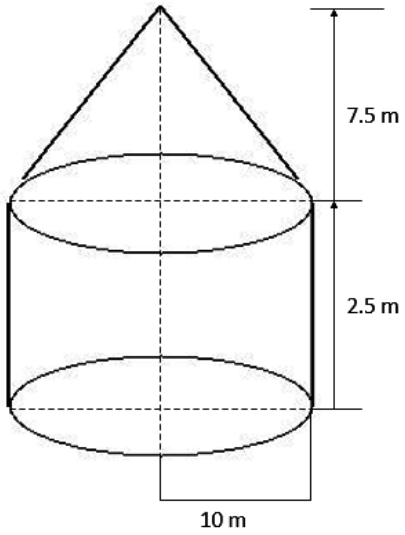
A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m. find the capacity of the tent and the cost of the canvas at Rs. 100 per square metre.

Answer

Given: A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m.

To find: the capacity of the tent and the cost of the canvas at Rs. 100 per square metre.

Solution:



height of Cylinder (h) = 2.5 m

Radius of cylinder = 20/2=10 m

height of cone(h_1)=7.5 m

Let "l" be the slant height of the cone,As we know $l^2 = \sqrt{r^2 + h^2}$

$\Rightarrow l^2 = \sqrt{10^2 + 7.5^2}$

$\Rightarrow l^2 = \sqrt{100 + 56.25}$

$\Rightarrow l^2 = \sqrt{156.25}$

$\Rightarrow l = 12.5$ m

Volume of a cylinder (v)= $\pi r^2 h$

$V_1 = \pi(10)^2(2.5)$ cm …..(1)

Volume of cone (V_1) = $\frac{1}{3}\Pi r^2 h_1$

Volume of cone (V_1) = $\frac{1}{3}\Pi(10)^2 \times 7.5$ ..... (2)

Volume of boiler = (1) + (2)

$V = V_1 + V_2$

$$V = \left(\Pi(10)^2(2.5)\right) + \left(\frac{1}{3}\Pi(10)^2 \times 7.5\right)$$

⇒ V = 250 Π + 250 Π⇒ V = 500 Π m^3

Now to find the cost find the curved surface of the tent.CSA of tent = CSA of cylinder + CSA of cone= 2Πrh + Πrl= Πr (2h+ l) $= \frac{22}{7} \times 10\left[2(2.5) + 12.5\right] = \frac{22}{7} \times 10[5 + 12.5]$

$$= \frac{22}{7} \times 10(17.5)$$

$$= \frac{3850}{7}$$

= 550 m^2

Cost of 1 m^2 canva = Rs 100

cost of 550 m^2 canva = 550× 100

= Rs 55000

12. Question

A boiler is in the form of a cylinder 2 m long with hemispherical ends each of 2 metre diameter. Find the volume of the boiler.

Answer

Given: radius of cylinder (r) = $\frac{20}{2}$ = 10 cm

Height of cylinder (h_1) = 2.5 m

Height of cone (h_2) = 7.5 m

Let l be slant height of cone

$$l = \sqrt{r^2 + h^2}$$

$\Rightarrow l = \sqrt{10^2 + 7.5^2}$

⇒ l = 12.5 m

Volume of cylinder (V_1) = $\pi r^2 h$

⇒ $V_1 = \pi\, r(10)^2(2.5)$ ……. (1)

Volume of cone (V_2) = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(10)^2\,(7.5)$ ……. (2)

Total capacity of tent = (1) + (2)

$V = V_1 + V_2$

⇒ $V = \pi(10)^2(2.5) + \frac{1}{3}\pi(10)^2\,(7.5)$

⇒ V = 250π + 250π

⇒ V = 500 π m^3

∴Total capacity of tent = 500 π m^3

13. Question

A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is $\frac{14}{3}m$ and the diameter of hemisphere is 3.5 m. Calculate the volume and the internal surface area of the solid.

Answer

Radius of the hemispherical part of the hollow cylinder = $\frac{3.5}{2} = \frac{7}{4}m$

Hollow cylinder and the hemisphere have the same base

∴ Radius of the cylinder = $\frac{7}{4}m$

Depth of the cylinder, h = $\frac{14}{3}m$

Volume of the vessel = Volume of cylindrical part + Volume of the hemispherical part

$= \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 \left(h + \frac{2}{3}r\right)$

$= \left(\frac{22}{7}\right)\left(\frac{7}{4}\right)^2 \left(\frac{14}{3} + \frac{2}{3}\left(\frac{7}{4}\right)\right)$

≈ 56.15 m^3

Internal surface area of the vessel = CSA of (cylinder + hemisphere)

= 2πrh + 2π r^2

= 2π r(h+r)

$$= 2\pi\left(\frac{7}{4}\right)\left(\frac{14}{3}+\frac{7}{4}\right)$$

= 70.58 $\approx 70\frac{7}{12}$ m^2

14. Question

A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Rs. 10 per dm^2.

Answer

Radius of hemispherical ends, r = 7 cm

Height of body (h + 2r) = 104 cm

Curved surface area of cylinder = 2πrh

= 2π (7)h …….(1)

Given: h + (2r) = 104 cm

⇒ h = 104 – 2 (7)

⇒ h =90 cm

Substitute the value of h in eq. (1),

Curved surface area of cylinder = 2π (7) (90)

= 3958.40 cm …..(2)

Curved area of 2 hemisphere = 2(2πr^2)

= 2(2 ×π ×7^2)

= 615.75 cm …….. (3)

Total curved surface area = 2+3

= 3958.40 + 615.75

= 4574.15 cm^2

Cost of polishing for 1 dm^2 = Rs.10

Cost of polishing for 54.75 dm^2 = 45.74×10

= Rs.457.60

15. Question

A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between the two vessels is filled with cork dust for heat insulation purposes. How many cubic centimeters of cork dust will be required?

Answer

Given height of cylindrical vessel (h) = 42 cm

Inner radius of vessel (r') = $\frac{14}{2}$cm =7 cm

Outer radius of vessel (r'') = $\frac{16}{2}$ =8 cm

Volume of cylinder = π ($r''^2 - r'^2$) h

= π (8^2 - 7^2) 42

= 1980 cm^3

16. Question

A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54 cm and the thickness of the iron sheet used in making the roller is 9 cm. find the mass of the roller, if 1 cm^3 of iron has 7.8 gm mass $(Use\ \pi = 3.14)$

Answer

Given that, internal radius of cylinder road roller (r1) = $\frac{54}{2}$ = 27cm

Thickness of road roller (t) =9 cm

Radius of cylinder road roller be R

t = R –r

t = R-27

R = 9 + 27 = 36 cm

Given height of cylindrical road roller (h) =1m

h = 100 cm

Volume of iron=πh(R^2-r^2)

= π(36^2 - 27^2) × 100

= 1780.38 cm

Mass of 1 cm of iron = 7.8 gm

Mass of 1780.38 cm of iron = 1780.38 × 7.8

=1388696.4 gm

∴ Mass of roller (m) = 1388.7 kg

17. Question

A vessel in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. find the inner surface area of the vessel.

Answer

Given radius of hemisphere & cylindrical (r)

$$= \frac{14}{2}$$

=7 cm

Given total height of vessel = 13 cm

∴ (h +r) = 13 cm

Inner surface area of vessel = 2π r (h +r)

= 2 ×π × 7 (13)

= 182π

=575 cm^2

18. Question

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer

Given radius of cone (r) = 3.5 cm

Total height of toy (h) = 15.5 cm

height of cone (l) = 15.5 – 3.5

= 12 cm

As we know,

$l^2 = r^2 + h^2$

$= 3.5^2 + 12^2$

$$= \left(\frac{7}{2}\right)^2 + 144$$

$$= \frac{49}{4} + 144$$

$$= \frac{49 + 576}{4}$$

$$= \frac{625}{4}$$

$$l = \sqrt{\frac{625}{4}}$$

$l = \frac{25}{2}$ l = 12.5

The curved surface area of a cone,S′ = πrl

= π (3.5) (12.5)

=137.5 cm^2…….. (1)

The curved surface area of the hemisphere, S′′ = $2\pi r^2$

= $2\pi(3.5)^2$

= 77 cm^2…… (2)

∴ Total surface area of the toy = 137.5 + 77 = 214.5 cm^2

19. Question

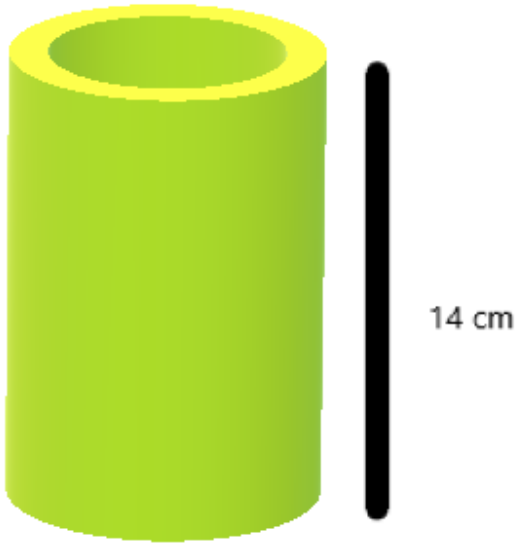
The difference between outside and inside surface areas of cylindrical metallic pipe 14 cm long is 44 m^2. If the pipe is made of 99 cm^3 of metal, find the outer and inner radii of the pipe.

Answer

Given: The difference between outside and inside surface areas of cylindrical metallic pipe 14 cm long is 44 m^2 and the pipe is made of 99 cm^3 of metal.

To find: the outer and inner radii of the pipe.

Solution:



Let the inner radius of pipe = r1

And the radius of outer cylinder = r2

Length of cylinder (h) = 14 cm

Surface area of hollow cylinder =2πh (r2 −r1)

Given surface area of cylinder = 44 m^2.

⇒ 2π h(r2 −r1) = 44

⇒ 2π (14) (r2 −r1) = 44

⇒ (r2 −r1) = $\frac{44}{28\pi}$

⇒ (r2 −r1) = $\frac{44 \times 7}{28 \times 22}$

⇒ (r2 −r1) = $\frac{\cancel{44}^{2} \times 7^{1}}{\cancel{28}^{4} \times \cancel{22}^{1}}$

⇒ (r2 −r1) = 1/2 ……. (1)

Given volume of a hollow cylinder = 99 cm^3

Volume of a hollow cylinder = π h($r2^2 - r1^2$)

⇒ π h ($r2^2$ - $r1^2$) = 99

⇒ 14π ($r2^2$ - $r1^2$) = 99Apply the formula a^2 - b^2 = (a - b) (a + b) in ($r2^2$ - $r1^2$).

⇒ 14π (r2 + r1) (r2 − r1) = 99Put the value of r2 - r1 from (1).

⇒ 14π (r2 + r1) $\frac{1}{2}$ = 99

$$\Rightarrow 14 \times \frac{22}{7} \times (r2 + r1) \times \frac{1}{2} = 99$$

$$\Rightarrow \cancel{14}^{2} \times \frac{\cancel{22}^{11}}{\cancel{7}_{1}} \times (r2 + r1) \times \frac{1}{\cancel{2}_{1}} = 99$$

⇒ 22 (r2+r1) = 99

$$\Rightarrow (r2 + r1) = \frac{99}{22}$$

⇒ (r2+r1) = $\frac{9}{2}$ ..... (2)

Equating (1) & (2) equations we get

r2 = 5/2 cm

Substituting r2 value in (1)

$$\frac{5}{2} - r1 = \frac{1}{2}$$

$$\Rightarrow \frac{5}{2} - \frac{1}{2} = r1$$

$$\Rightarrow r1 = \frac{5-1}{2}$$

$$\Rightarrow r1 = \frac{4}{2}$$

⇒ r1 = 2 cm

∴ Inner radius of pipe (r1) = 2 cm

Radius of outer cylinder (r2) = 5/2 cm

20. Question

A right circular cylinder having diameter 12 cm and height 15 cm is full ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Answer

Given radius of cylinder (r') = $\frac{12}{2}$ = 6 cm

Given radius of hemisphere (r'') = $\frac{6}{2}$ =3 cm

Given height of cylinder (h) = 15 cm

Height of cone (l) = 12 cm

Volume of cylinder = $\pi r^2 h$

= $\pi r(6)^2(15)$ cm ……. (1)

Volume of each cone = Volume of cone + Volume of hemisphere

V = $\pi(r)^2(l) + \frac{2}{3}\pi(r)^3$

V = $\pi(3)^2(12) + \frac{2}{3}\pi(3)^3$ …… (2)

Let number of cone be n

n (volume of each cone) = volume of cylinder

n = $[(\frac{1}{3}\pi(3)^2\ (12) + \frac{2}{3}\pi(3)^3] = \pi(16)^2\ 15$

⇒ n = 10

21. Question

A solid iron pole having cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that the mass of 1 cm^3 of iron is 8 gm.

Answer

Given radius of cylindrical part(r)= 12/2= 6 cm

Height of cylinder (h) = 110 cm

Length of cone (l)= 9 cm

Volume of cylinder= $\pi r^2 h$

V1= $\pi\ (6)^2\ 110\ cm^3$

Volume of cone=$\frac{1}{3}\pi r^2 l$

V2=$\frac{1}{3}\pi(6)^2 9$

=108 $\pi\ cm^3$

Volume of pole =v1 + v2

= $\pi(6)^2 110$+108 π

= 12785.14 108 π cm^3

Given mass of 1 cm^3 of iron = 8 gm

Mass of 1275.14 cm^3 of iron = 12785.14 × 8

= 102281.12

=102.2 kg

∴ Mass of pole for 12785.14 cm^3 of iron is 102.2 kg

22. Question

A solid toy is in the form of a hemisphere surmounted by a right circular come. Height of the cone is 2 cm and the diameter of the base is 4 cm. /if a right circular cylinder circumscribes the toy, find how much more space it will cover.

Answer

Given radius of cone, cylinder & hemisphere (r) = 4/2 = 2 cm

Height of cone (l) = 2 cm

Height of cylinder (h) = 4 cm

Volume of cylinder = πr^2h = $\pi(2)^2$(4) cm …….. (1)

Volume of cone = $\frac{1}{3}\pi r^2 l$

= $\frac{1}{3}\pi\,(2)^2 \times 2$

= $\frac{2}{3}$ × 4 × 2 cm ……. (2)

Volume of hemisphere = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(2)^3$

= $\frac{2}{3}\pi\,(8)^3$ …….. (3)

So remaining volume of cylinder when toy is instead to it

$$= \pi r^2 h - (\frac{1}{3}\pi r^2 l + \frac{2}{3}\pi r^3)$$

= eq.(1) – eq.(2) + eq.(3)

= п$(2)^2$(4) – ($\frac{\pi}{3} \times \frac{2}{3}$ × п × 8)

= 16п - $\frac{2}{3}$ п (4+8)

= 16п – 8п

= 8π cm^3

∴ So remaining volume of cylinder when toy is instead to it = 8π cm^3

23. Question

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottoms. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Answer

Given: radius of circular cone (r) = 60 cm

Height of a cone (l) = 120 cm

Volume of a cone = $\frac{1}{3}$ πr^2 l

= $\frac{1}{3}$ π$(60)^2$(120) cm …….. (1)

Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Given radius of hemisphere = 60 cm

$= \frac{2}{3}\pi(60)^3$ ……. (2)

Radius of cylinder (r) = 60 cm

Height of cylinder (h) = 180 cm

Volume of cylinder = πr^2h

= π$(60)^2$(180) cm^3 …… (3)

Volume of water left in cylinder = eq.(3) – eq.(1) + eq.(2)

$$= \frac{1}{3}\pi(60)^2(120) - \left[\frac{2}{3}\pi(60)^3 + \pi(60)^2(180)\right]$$

= 113.1 cm^3= 1.131m^3

24. Question

A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the value of water (i) displaced out of the cylinder. (ii) left in the cylinder. (*Take* $\pi = 22/7$)

Answer

Given internal radius (r1) = 10/2 = 5 cm

Height of cylindrical vessel (h) = 10.5 cm

Outer radius of cylindrical vessel (r2) = 7/2 = 3.5 cm

Length of cone (l) = 6 cm

(i)Volume of water displaced = Volume of cone

Volume of cone = $\frac{1}{3}\pi r^2 l$

= $\frac{1}{3}\pi(3.5)^2 \times 6$

= 76.9 cm^3

≈ 77 cm^3

Volume of water displayed = 77 cm^3

Volume of cylinder = $\pi r^2 h$

= $\pi(5)^2 10.5$

= 824.6

≈ 825 cm^3

(ii)Volume of water left in cylinder = Volume of cylinder – Volume of cone

= 825 -77 = 748 cm^3

25. Question

A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Determine the volume and total surface area of the remaining block.

Answer

Edge of wooden block (a) = 21 cm

Diameter of hemisphere = edge of cube

Radius = 21/2 = 10.5 cm

Volume of remaining block = Volume of box – Volume of hemisphere

$= a^3 - \frac{2}{3}\pi r^3$

$= (2)^3 - \frac{2}{3}\pi(10.5)^3$

= 6835.5 cm^3

Surface area of box = $6a^2$ …… (1)

Curved surface area of hemisphere = $2\pi r^3$ …….. (2)

Area of base of hemisphere $= \pi r^2$ …….. (3)

So remaining surface area of box = (1) – (2) + (3)

$= 6a^2 - \pi r^2\ 2\pi r^2$

$= 6(21)^2 - \pi(10.5)^2 + 2\pi(10.5)^2$

= 2992.5 cm^2

∴Remaining surface area of box=2992.5 cm^2

Volume of remaining block=6835.5 cm^3

26. Question

A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is 2/3 of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy. $(Use\ \pi = 22/7)$

Answer

Radius of base = 21cm and its volume = 2/3 × volume of the hemisphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\times\frac{2}{3}\pi r^3$$

$$\Rightarrow h = \frac{4}{3}r = \frac{4}{3}\times 21 = 28cm$$

Hence surface area of the toy = Surface area of the cone + Surface area of hemisphere

= π r ($\sqrt{21^2 + 28^2}$) + 2 π r^2

= π (21)[35 + 2 × 21]

= 5082 cm^2

27. Question

A solid is in the shape of a cone surmounted on a hemi-sphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. $(Use\ \pi = 22/7)$

Answer

Height of the cylinder = height of solid – radius of the cone

= 9.5 – 3.5 = 6cm

Volume of the solid = volume of the cone + volume of the cone

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi(3.5)^2(6) + \frac{2}{3}\pi(3.5)^3$$

= 166.83 cm^3

28. Question

An wooden toy is made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. $(Use\ \pi = 22/7)$

Answer

the height of the cylinder = 10 cm, and its base is of radius = 3.5 cm

Volume of wood in the toy = volume of cylinder – 2 × volume of hemisphere

$$= \pi r^2 h - 2\times\frac{2}{3}\pi r^3$$

$$= \pi(3.5)^2(10) - 2\times\frac{2}{3}\pi(3.5)^3$$

= 205.33 cm^3

29. Question

The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. $(Use\ \pi = 22/7)$

Answer

To be the largest, diameter of the sphere = edge of the cube

∴ its radius = 1/2 × edge

Volume of wood left = volume of cube – volume of sphere

$$= (7)^3 - \frac{4}{3}\pi\left(\frac{7}{2}\right)^3$$

= 343 – 179.67

= 163.33 cm^3

30. Question

From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. $(Take\ \pi = 22/7)$.

Answer

Slant height of the cone, l = $\sqrt{(2.8)^2 + (2.1)^2} = 3.5cm$

Remaining area of the cylinder = Curved surface area of the cylindrical part + Area of the conical part + Area of cylindrical base

= 2πrh + πrl + πr^2

= 2π × (2.1)× (2.8) + π × 2.1 × (3.5) + π $(2.1)^2$

= 36.96 + 23.1 + 13.86

= 73.92 cm^2

31. Question

The largest cone is curved out from one face of solid cube of side 21 cm. Find the volume of the remaining solid.

Answer

Largest cone is the right circular cone with base diameter and height = edge of the cube

∴ radius of the cone = (21/2) cm

and height of the cone = 21 cmwe know, volume of cube is $(side)^3$

and $volume\ of\ cone = \frac{1}{3}\pi r^2 h$

where 'r' is radius and 'h' is height of the cone.Volume of the remaining cube = volume of the cube – volume of cone cone)

$$= (21)^3 - \left(\frac{1}{3}\right)\left(\frac{22}{7}\right)\left(\frac{21}{2}\right)^2 (21)$$

$$= 9261 - \frac{4851}{2}$$

$$= 9261 - 2425.5$$

$$= 6835.5\ cm^3$$

32. Question

A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}\ cm^3$. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ` 10 per cm^2. $(Take\ \pi = 22/7)$

Answer

Let h be the height of the cone and r be radius of its base

Now, volume of the wooden toy = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$$= \frac{77}{6}(h + 7)$$

According to the question, $\frac{77}{6}(h + 6) = 166\frac{5}{6}$

⇒ h = 6cm

The height of the wooden toy = 6cm + 3.5 cm

= 9.5cm

Now, curved surface area of the hemispherical part = 2 π r^2

= 2π × $(3.5)^2$

=77cm^2

Hence the cost of painting the hemispherical part of the toy at the rate of ` 10 per cm^2 = 77 × 10 = Rs.770

33. Question

In Fig. 16.57, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. (*Take* $\pi = 22/7$)

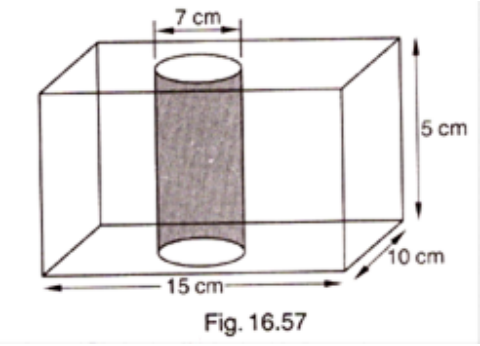


Fig. 16.57

Answer

Here, dimension of cuboid = 15 cm × 10 cm × 5 cm

Radius of cylindrical hole = $\frac{7}{2}$ cm

Surface area of the remaining block = surface area of cuboidal solid metallic block – 2 × base area of cylindrical hole

= $2(15 \times 10 + 10 \times 5 + 5 \times 15) - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

= 583 cm^2

Exercise 16.3

1. Question

A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of Rs. 1.20 per dm^2. (*Use* $\pi = 3.14$)

Answer

Radius of top of the bucket, r’ = 40/2 = 20 cm

Radius of bottom of the bucket, r'' = 20/2 = 10 cm

Depth, h = 12cm

Volume of the bucket = $\frac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$$= \frac{1}{3}\pi(20^2 + 10^2 + 20\times10)\times12$$

= 8800 cub. cm

Let l be slant height of the bucket

$$\Rightarrow l = \sqrt{(r'^2 - r''^2) + h^2}$$

$$\Rightarrow l = \sqrt{(20^2 - 10^2) + 12^2}$$

⇒ l = 15.62 cm

Total surface area of bucket = π (r' + r'') × l + π r''^{2}

= π (20 + 10) × 15.620 + π (10)^2

= 17.81 dm^2

Cost of tin sheet = 1.20 × 17.87 = Rs21.40

2. Question

A frustum of a right circular cone has a diameter of base 20 cm, of top 12 cm, and height 3 cm. Find the area of its whole surface and volume.

Answer

For frustum:

Base radius, r' = 20/2 = 10cm

Top radius, r'' = 12/2 = 6cm

Height, h = 3cm

Volume = $\frac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$$= \frac{1}{3}\pi(10^2 + 6^2 + 10\times6)\times3$$

= 616 cub. cm

Let l be the slant height of the cone, then

$$\Rightarrow l = \sqrt{(r' - r'')^2 + h^2}$$

$$\Rightarrow l = \sqrt{(10 - 6)^2 + 3^2}$$

⇒ l = 5cm

Total surface area of the frustum = π (r′ + r″) × l + π r'^2 + π r''^2

= π (20 + 10) × 15.620 + π $(10)^2$ + π $(6)^2$

= 678.85 cm^2

3. Question

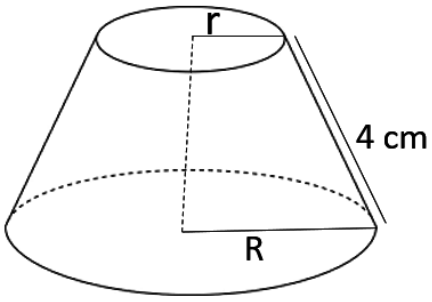
The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface of the frustum.

Answer

Given: The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm.

To find: the curved surface of the frustum.

Solution: For the given frustum:



Slant height, l = 4 cm

Perimeter of its circular ends = 18 cm and 6 cm

Let radius of larger and shorter ends be 'r' and 'R' resp., then

Perimeter of larger circular end = 2 π r = 18 cm

⇒ π r = 9

Similarly, 2π R = 6

⇒ π R = 3

Curved surface of the frustum = (π r +π R) × l

= (9 + 3) × 4= 12 × 4

= 48 cm^2

4. Question

The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.

Answer

For the frustum,

Perimeter of the larger end = 44cm

⇒ 2πr′ = 44

$$\Rightarrow\ r' = \frac{44}{2\times\frac{22}{7}} = 7cm$$

Perimeter of the smaller end = 33cm

⇒ 2πr″ = 33

$$\Rightarrow\ r'' = \frac{33}{2\times\frac{22}{7}} = \frac{21}{4}cm\ or\ 5.25cm$$

Height = 16cm

Its volume = $\frac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$$= \frac{1}{3}\pi\left(7^2 + \left(\frac{21}{4}\right)^2 + 7\times\frac{21}{4}\right)\times 16$$

$$= \frac{1}{3}\times\frac{22}{7}(784 + 441 + 588)$$

≈ 1900 cub. cm

Let l be the slant height of the cone, then

$$\Rightarrow\ l = \sqrt{(r' - r'')^2 + h^2}$$

$$\Rightarrow\ l = \sqrt{(7 - 5.25)^2 + 16^2}$$

⇒ l = 16.1cm

Curved surface area of frustum = π (r′ + r″) × l

= 3.14(7 + 5.25) × 16.1

= 619.65 cm^2

Total surface area of the frustum = π (r′ + r″) × l + π r'^2 + π r''^2

= 619.65 + 3.14 × (7^2 + 5.25^2)

= 860.275 cm^2

5. Question

If the radii of the circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm, find the capacity of the bucket. $(Use\ \pi = 22/7)$

Answer

For the bucket:

Height = 45cm

Radius of top = 28cm

Radius of bottom = 7cm

Volume of the bucket = $\frac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$$= \frac{1}{3}\pi(28^2 + 7^2 + 28\times7)\times45$$

$$= \frac{22}{7}\times15(784 + 49 + 196)$$

= 48510 cub. cm

6. Question

The height of a cone is 20 cm. A small cone is cut off from the top by a plane parallel to the base. If its volume be 1/125 of the volume of the original cone, determine at what height above the base the section is made.

Answer

Height of the cone = 20cm

Let the small cone was cut at the height x from the top then height of the small cone = x cm

From cones ABC and AEF,

$\frac{h'}{h''} = \frac{r'}{r''}$ (h', r' and h'', r'' are heights and radius of larger and smaller cones resp.)

$\Rightarrow \frac{20}{x} = \frac{r'}{r''}$ -------(i)

Given: Volume of the small cone = $\frac{1}{125}$ × volume of the large cone

And Volume of a cone = $\frac{1}{3}\pi(r^2)h$

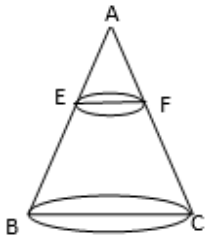
$$\therefore \frac{1}{3}\pi\left(r''^2\right)x = \frac{1}{125}\left(\frac{1}{3}\pi\left(r'^2\right)(20)\right)$$

$$\Rightarrow \frac{r''^2}{r'^2} = \frac{1}{125}\left(\frac{20}{x}\right)$$

$$\Rightarrow \frac{x^2}{20^2} = \frac{1}{125}\left(\frac{20}{x}\right)$$

$\Rightarrow x^3 = \left(\frac{20}{5}\right)^3$

⇒ x = 4cm



Height at which section is made = 20 – 4 = 16cm

7. Question

If the radii of the circular ends of a conical bucket 45 cm high are 5 cm and 15 cm respectively, find the surface area of the bucket.

Answer

Given: for the bucket,

Height = 45cm

r′ = 5cm

r′′ = 15cm

Curved surface area = π(r′ + r′′)l + π r''^2

= π(5 + 15)(26) + π $(15)^2$

= 745 π cm^2

Curved surface area of bucket = 745π cm^2

8. Question

The radii of the circular bases of a frustum of a right circular cone are 12 cm and 3 cm and the height is 12 cm. Find the total surface area and the volume of the frustum.

Answer

Given: radius of bases of frustum = 12cm and 3cm and its height = 12cm

Let l be the slant height of the cone, then

$\Rightarrow l = \sqrt{(r' - r'')^2 + h^2}$

$\Rightarrow l = \sqrt{(12 - 3)^2 + 12^2}$

⇒ l = 15cm

Total surface area of the frustum = π (r′ + r′′) × l + π r'^2 + π r''^2

= π (12 + 3) + π × (12^2 + 3^2)

= 378 cm^2

Volume = $\frac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$$= \frac{1}{3}\pi(12^2 + 3^2 + 12\times3)\times12$$

= 756 π cm^3

9. Question

A tent consists of a frustum of a cone capped by a cone. If the radii of the ends of the frustum be 13 m and 7 m, the height of the frustum be 8 m and the slant height of the conical cap be 12 m, find the canvas required for the tent.

(*Take* : π = 22 / 7)

Answer

Height of the frustum, h = 8m

Radii of frustum are: r′ = 13m and r′′ = 7m

Let l be the slant height of the frustum

$$\Rightarrow l = \sqrt{(r' - r'')^2 + h^2}$$

$$\Rightarrow l = \sqrt{(13 - 7)^2 + 8^2}$$

⇒ l = 10cm

Curved surface area of frustum, A′ = π (r′ + r′′) × l

= π (13 + 7) × 10

= 628.57 cm^2

Slant height of the conical cap = 12m

Base radius of the cap = 7m

Curved surface of the cap, A′′ = πrl

= π × 7 × 12

= 264m^2

Total canvas required = A′ + A′′

= 628.57 + 264

= 892.57 m^2

∴ Total canvas = 892.57 m^2

10. Question

A reservoir in the form of the frustum of a right circular cone contains 44×10^7 litres of water which fills it completely. The radii of the bottom and top of the reservoir are 50 metres and 100 metres respectively. Find the depth of water and the lateral surface area of the reservoir. $(Take: \pi = 22/7)$

Answer

Let height and slant height of the frustum be h and l.

Radius, r′ = 100m, r′′ = 50m

Volume of frustum = 44×10^7 litres = 44×10^4 m^3

$$\frac{1}{3} \times \frac{22}{7}(100^2 + 50^2 + 100 \times 50)h = 44 \times 10^7$$

$$h = \frac{44 \times 10^4 \times 7 \times 3}{22 \times 17500} = 24m$$

Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

$$\Rightarrow l = \sqrt{(100 - 50)^2 + 24^2}$$

⇒ l = 55.46m

Lateral surface area of the frustum = π l (r′ + r′′)

$$= \frac{22}{7} \times 55.46 \times 150$$

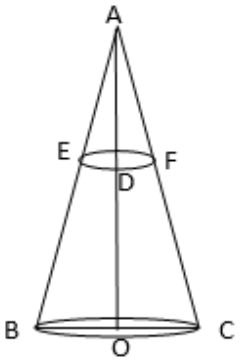
= 26145.4 m^2

11. Question

A metallic right circular cone 20 cm high and whose vertical angle is 90° is cut into two parts at the middle point of its axis by a plane parallel to the base. If the frustum so obtained be drawn into a wire of diameter (1/16) cm, find the length of the wire.

Answer

let ABC be the given cone.



Here, the height of the metallic cone AO = 20cm

7. Statistics

Exercise 7.1

1. Question

Calculate the mean for the following distribution:

x :	5	6	7	8	9
y :	4	8	14	11	3

Answer

X	y	yx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	N = 40	∑yx = 281

Mean = $\frac{\sum yx}{N}$

= $\frac{281}{40}$ = 7.025

2. Question

Find the mean of the following data:

x:	19	21	23	25	27	29	31
y:	13	15	16	18	16	15	13

Answer

X	y	yx
19	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	N = 106	∑yx = 2650

Mean = $\frac{\sum yx}{N}$

= $\frac{2650}{106}$ = 25

3. Question

If the mean of the following data is 20.6. Find the value of p.

x:	10	15	p	25	35
y:	3	10	25	7	5

Answer

X	y	yx
10	3	30
15	10	150
P	25	25p
25	7	175
35	5	175
	N = 50	∑yx = 530 + 25p

Given,

Mean = 20.6

$\frac{\sum yx}{N}$ = 20.6

$\frac{530+25p}{50}$ = 20.6

530 + 25p = 20.6 (50)

25p = 20.6 (50) – 530

p = $\frac{500}{25}$ = 20

4. Question

If the mean of the following data is 15, find p.

x :	5	10	15	20	25
y :	6	p	6	10	5

Answer

X	y	yx
5	6	30
10	p	10p
15	6	90
20	10	200
25	5	125
	N = p + 27	∑yx = 10p + 445

Given,

Mean = 15

$\frac{\sum yx}{N}$ = 15

$\frac{10p+445}{p+27}$ = 15

10p + 445 = 15 (p + 27)

10p + 445 = 15p + 405

15p – 10p = 445 – 405

5p = 40

p = $\frac{40}{5}$ = 8

5. Question

Find the value of p for the following distribution whose mean is 16.6

x :	8	12	15	p	20	25	30
y :	12	16	20	24	16	8	4

Answer

X	y	yx
8	12	96
12	16	192
15	20	300
P	24	24p
20	16	320
25	8	200
30	4	120
	N = 100	∑yx = 24p + 1228

Given,

Mean = 16.6

$\frac{\sum yx}{N}$ = 16.6

$\frac{24p+1228}{100}$ = 16.6

24p + 1228 = 1660

24p = 1660 – 1228

24p = 432

$p = \frac{432}{24} = 18$

6. Question

Find the missing value of p for the following distribution whose mean is 12.58

x:	5	8	10	12	p	20	25
y:	2	5	8	22	7	4	2

Answer

X	y	yx
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7p
20	4	80
25	2	50
	N = 50	∑yx = 524 + 7p

Given,

Mean = 12.58

$\frac{\sum yx}{N}$ = 12.58

$\frac{524+7p}{50}$ = 12.58

524 + 7p = 12.58 (50)

7p = 629 – 524

$p = \frac{105}{7} = 15$

7. Question

Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
y:	6	8	15	p	8	4

Answer

X	y	yx
3	6	18
5	8	40
7	15	105
9	P	9p
11	8	88
13	4	52
	N = p + 41	∑yx = 9p + 303

Given,

Mean = 7.68

$\frac{\sum yx}{N}$ = 7.68

$\frac{9p+303}{p+41}$ = 7.68

9p + 303 = 7.68 (p + 41)

9p – 7.68p = 314.88 – 303

1.32p = 11.88

p = $\frac{11.88}{1.32}$ = 9

8. Question

Find the value of p, if the mean of the following distribution is 20.

x:	15	17	19	20+p	23
y:	2	3	4	5p	6

Answer

X	y	yx
15	2	30
17	3	51
19	4	76
20 + p	5p	$100p + 5p^2$
23	6	138
	N = 5p + 15	$\sum yx = 295 + 100p + 5p^2$

Given,

Mean = 20

$\frac{\sum yx}{N}$ = 20

$\frac{295+100p+5\,(p*p)}{5p+15}$ = 20

$295 + 100p + 5p^2 = 100p + 300$

$295 + 5p^2 = 300$

$5p^2 = 300 - 295$

$5p^2 - 5 = 0$

$5\,(p^2 - 1) = 0$

$p^2 - 1 = 0$

$(p + 1)\,(p - 1) = 0$

p = ± 1

If p + 1 = 0, p = - 1 (Reject)

Or p – 1 = 0, p = 1

9. Question

The following table gives the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students.

Age (in year):	15	16	17	18	19	20
No. of students:	3	8	10	10	5	4

Answer

Age (x)	No. of students (y)	yx
15	3	45
16	8	128
17	10	170
18	10	180
19	5	95
20	4	80
	N = 40	∑yx = 698

Mean = $\frac{\sum yx}{N}$

= $\frac{698}{40}$ = 17.45

Therefore, mean age is 17.45 years.

10. Question

Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of Candidates	Average Score
I	60	75
II	48	80
III	Not available	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Answer

Let the number of candidates from school III be P.

Schools	No. of candidates (N ;)	Average score (x ;)
I	60	75
II	48	80
III	P	55
IV	40	50

Given,

Average score of all schools = 66

$\frac{N1x1+N2x2+N3x3+N4x4}{N1+N2+N3+N4}$ = 66

$\frac{60*75+48*80+P*55+40*50}{60+48+P+40}$ = 66

$\frac{4500+3840+55P+2000}{148+P}$ = 66

10340 + 55P = 66P + 9768

10340 – 9768 = 66P – 55P

P = $\frac{572}{11}$ = 52

11. Question

Five coins were simultaneously tossed 1000 times and at each toss the numbers of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

Answer

No. of heads per toss (x)	No. of tosses (y)	yx
0	38	0
1	144	144
2	342	684
3	287	861
4	164	656
5	25	125

Mean number of heads per toss = $\frac{\sum yx}{N}$

= $\frac{2470}{1000}$ = 2.47

Therefore, mean = 2.47

12. Question

Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution 50.

x: 10	30	50	70	90
y: 17	f1	32	f2	19

Answer

X	y	yx
10	17	170
30	f1	30f1
50	32	1600
70	f2	70f2
90	19	1710
	N = 120	∑yx = 30f1 + 70f2 + 3480

Given, mean = 50

$\frac{\sum yx}{N}$ = 50

$\frac{30f1+70f2+3480}{120}$ = 50

30f1 + 70f2 + 3480 = 50 * 120

30f1 + 70f2 + 3480 = 6000 (i)

Also, ∑y = 120

17 + f1 + 32 + f2 + 19 = 120

f1 + f2 = 52

f1 = 52 – f2

Substituting value of f1 in (i), we get

30 (52 – f2) + 70f2 + 3480 = 6000

40f2 = 960

f2 = 24

Hence, f1 = 52 – 24 = 28

Therefore, f1 = 28 and f2 = 24

13. Question

The arithmetic mean of the following data is 14. Find the value of k.

x_i:	5	10	15	20	25
y_i:	7	k	8	4	5

Answer

X	y	yx
5	7	35
10	K	10k
15	8	120
20	4	80
25	5	125
	N = 24 + k	∑yx = 360 + 10k

Given,

Mean = 14

$\frac{\sum yx}{N}$ = 14

$\frac{360+10k}{24+k}$ = 14

360 + 10k = 336 + 14k

24 = 4k

k = 6

Hence, the value of k is 6

14. Question

The arithmetic mean of the following data is 25, find the value of k.

x_i:	5	15	25	35	45
y_i:	3	k	3	6	2

[CBSE 2001]

Answer

X	y	yx
5	3	15
15	K	15k
25	3	75
35	6	210
45	2	90
	N = 14 + k	∑yx = 15k + 390

Given,

Mean = 25

$\frac{\sum yx}{N}$ = 25

$\frac{15k+390}{14+k}$ = 25

15k + 390 = 25 (14 + k)

15k + 390 = 350 + 25k

40 = 10k

k = 4

15. Question

If the mean of the following data is 18.75. Find the value of p.

x_i:	10	15	p	25	30
y_i:	5	10	7	8	2

Answer

X	y	yx
10	5	50
15	10	150
P	7	7p
25	8	200
30	2	60
	N = 32	∑yx = 7p + 460

Given,

Mean = 18.75

$\frac{\sum yx}{N}$ = 18.75

$\frac{7p+460}{32}$ = 18.75

7p + 460 = 18.75 (32)

7p + 460 = 600

7p = 140

p = 20

Hence, the value of p is 20

Exercise 7.2

1. Question

The number of telephone calls received at an exchange per interval for 250 successive one-minute intervals are given in the following frequency table.

No. of calls (x):	0	1	2	3	4	5	6
No. of intervals (f):	15	24	29	46	54	43	39

Compute the mean number of calls per interval.

Answer

Let the assumed mean (A) = 3

Number of calls (x_i)	Number of intervals (f_i)	$u_i = x_i - A = x_i - 3$	f_iu_i
0	15	-3	-45
1	24	-2	-48
2	29	-1	-29
3	46	0	0
4	54	1	54
5	43	2	86
6	39	3	117
	N = 250		$\sum f_iu_i = 135$

Mean number of calls = A + $\frac{\sum fiui}{N}$

= 3 + $\frac{135}{250} = \frac{885}{250}$

= 3.54

2. Question

Five coins were simultaneously tossed 1000 times, and at each toss the number of heads was observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss

No. of heads pre toss(x):	0	1	2	3	4	5
No. of tosses (f):	38	144	342	287	164	25

Answer

Let the assumed mean (A) = 2

No. of heads per toss (x_i)	No. of intervals (f_i)	$U_i = x_i - A = x_i - 2$	f_iu_i
0	38	-2	-76
1	144	-1	-144
2	342	0	0
3	287	1	287
4	164	2	328
5	25	3	75
	N = 1000		$\sum f_iu_i = 470$

Mean number of heads per toss = A + $\frac{\sum fiui}{N}$

$= 2 + \frac{470}{1000} = 2 + 0.47$

$= 2.47$

3. Question

The following table gives the number of branches and number of plants in the garden of a school.

No. of branches (x):	2	3	4	5	6
No. of plants (f):	49	43	57	38	13

Answer

Let the assumed mean (A) = 4

No. of branches (x_i)	No. of plants (f_i)	$u_i = x_i - A = x_i - 4$	f_iu_i
2	49	-2	-98
3	43	-1	-43
4	57	0	0
5	38	1	38
6	13	2	26
	N = 200		$\sum f_iu_i = -77$

Average number of branches per plant = A + $\frac{\sum fiui}{N}$

= 4 + $\frac{-77}{200} = \frac{800-77}{200}$

= 3.62 (approx.)

4. Question

The following table gives the number of children of 150 families in a village

No. of children (x):	0	1	2	3	4	5
No. of families (f):	10	21	55	42	15	7

Find the average number of children per family.

Answer

Let the assumed mean (A) = 2

No. of children (x_i)	No. of families (f_i)	$u_i = x_i - A = x_i - 2$	f_iu_i
0	10	-2	-20
1	21	-1	-21
2	55	0	0
3	42	1	42
4	15	2	30
5	7	3	21
	N = 150		$\sum f_iu_i = 52$

Average number of children per family = $A + \frac{\sum fiui}{N}$

$= 2 + \frac{52}{150} = \frac{300+52}{150}$

= 2.35 (approx)

5. Question

The marks obtained out of 50, by 102 students in a Physics test are given in the frequency table below:

Marks (x):	15	20	22	24	25	30	33	38	45
Frequency (f):	5	8	11	20	23	18	13	3	1

Find the average number of marks.

Answer

Let the assumed mean (A) = 25

Marks (x_i)	Frequency (f_i)	$u_i = x_i - A = x_i - 25$	f_iu_i
15	5	-10	-50
20	8	-5	-40
22	11	-3	-33
24	20	-1	-20
25	23	0	0
30	18	5	90
33	13	8	104
38	3	13	39
45	1	20	20
	N = 102		$\sum f_iu_i = 110$

Average number of marks = A + $\frac{\sum fiui}{N}$

$= 25 + \frac{110}{102} = \frac{2550+110}{102}$

$= \frac{2660}{102} = 26.08$ (approx)

6. Question

The number of students absent in a class were recorded every day for 120 days and the information is given in the following frequency table:

No. of students absent (x):	0	1	2	3	4	5	6	7
No. of days (f):	1	4	10	50	34	15	4	2

Find the mean number of students absent per day.

Answer

Let the assumed mean (A) = 3

No. of students absent (x_i)	No. of days (f_i)	$u_i = x_i - A = x_i - 3$	f_iu_i
0	1	-3	-3
1	4	-2	-8
2	10	-1	-10
3	50	0	0
4	34	1	34
5	15	2	30
6	4	3	12
7	2	4	8
	N = 120		$\sum f_iu_i = 63$

Mean number of students absent per day = A + $\frac{\sum fiui}{N}$

= 3 + $\frac{63}{120}$ = $\frac{360+63}{120}$

= 3.53 (approx)

7. Question

In the first proof reading of a book containing 300 pages the following distribution of misprints was obtained:

No. of misprints per pages (x):	0	1	2	3	4	5
No. of pages (f):	154	95	36	9	5	1

Find the average number of misprints per page.

Answer

To find : the average number of misprints per page.

Solution : Use the shortcut method to find the mean of given data.For that,Let the assumed mean be (A) = 2,The deviation of values x_i from assumed mean be $d_i = x_i - A$.

Now to find the mean:First multiply the frequencies in column (ii) with the value of deviations in column (iii) as f_id_i.

Construct the table using above information.The table is as follows:

No. of misprints per pages (x_i)	No. of pages (f_i)	$_{di} = x_i - A = x_i - 2$	f_id_i
0	154	-2	-308
1	95	-1	-95
2	36	0	0
3	9	1	9
4	5	2	10
5	1	3	3
	N = 300		$\sum f_id_i$= -381

Now add the sum of all entries in column (iii) to obtain $\sum_{i=1}^{n} f_i d_i$

and the sum of all frequencies in the column (ii) to obtain $\sum_{i=1}^{n} f_i = N$

So,Average number of misprints per day = A + $\frac{\sum f_i d_i}{N}$

where, N = total number of observations

⇒ Mean = 2 + $\frac{-381}{300}$

Mean = $\frac{600-381}{300}$

Mean = $\frac{219}{300}$

Mean = 0.73

8. Question

The following distribution gives the number of accidents met by 160 workers in a factory during a month.

No. of accidents (x):	0	1	2	3	4
No. of workers (f):	70	52	34	3	1

Find the average number of accidents per worker.

Answer

Let the assumed mean (A) = 2

No. of accidents (x_i)	No. of workers (f_i)	$u_i = x_i - A = x_i - 2$	f_iu_i
0	70	-2	-140
1	52	-1	-52
2	34	0	0
3	3	1	3
4	1	2	2
	N = 160		$\sum f_iu_i = -187$

Average number of accidents per day workers = $A + \frac{\sum fiui}{N}$

$= 2 + \left(-\frac{187}{160}\right) = \frac{320-187}{160}$

= 0.83

9. Question

Find the mean from the following frequency distribution of marks at a test in statistics:

Marks (x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

Answer

Let the assumed mean (A) = 25

Marks (x_i)	No. of students (f_i)	$u_i = x_i - A = x_i - 25$	f_iu_i
5	15	-20	-300
10	50	-15	-750
15	80	-10	-800
20	76	-5	-380
25	72	0	0
30	45	5	225
35	39	10	390
40	9	15	135
45	8	20	160
50	6	25	150

	N = 400		$\sum f_i u_i$ = -1170

Mean = A + $\frac{\sum fiui}{N}$

= 25 + $\frac{-1170}{400}$ = $\frac{10000-1170}{400}$

= 22.075

Exercise 7.3

1. Question

The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees) (in rupees)	Frequency (f)	Expenditure (in rupees) (x)	Frequency (f)
100-150	24	300-350	30
150-200	40	350-400	22
200-250	33	400-450	16
250-300	28	450-500	7

Find the average expenditure (in rupees) per household.

Answer

Let the assumed mean (A) = 275

Class interval	Mid value (x_i)	$d_i = x_i - 275$	$u_i = \frac{xi-275}{50}$	Frequency (f_i)	f_iu_i
100 – 150	125	-150	-3	24	-72
150 – 200	175	-100	-2	40	-80
200 – 250	225	-50	-1	33	-33
250 – 300	275	0	0	28	0
300 – 350	325	50	1	30	30
350 – 400	375	100	2	22	44
400 – 450	425	150	3	16	48
450 – 500	475	200	4	7	28
				N = 200	$\sum f_iu_i = -35$

We have, A = 275

h = 50

Mean = A + h * $\frac{\sum fiui}{N}$

$= 275 + 50 * \frac{-35}{200}$

$= 275 - 8.75$

$= 266.25$

2. Question

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Numbers of plants:	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Numbers of houses:	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Answer

Let us find class marks (x_i) for each interval by using the relation.

Class mark (x_i) = $\frac{Upper class\ limit + Lower class\ limit}{2}$

Now we may compute x_i and $f_i x_i$ as follows:

Number of plants	Number of house (f_i)	x_i	f_ix_i
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	5	7	35
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
	N = 20		$\sum f_ix_i = 162$

Fro, the table we may observe that,

$\Sigma f_i = 20$

$\Sigma f_ix_i = 162$

Mean, $\bar{X} = \frac{\sum fixi}{\sum fi}$

$= \frac{162}{20} = 8.1$

So mean number of plants per house is 8.1

We have used for the direct method values x_i and f_i are very small.

3. Question

Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in Rs):	100-120	120-140	140-160	160-180	180-200
Numbers of workers:	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Answer

Let the assumed mean (A) = 150

Class interval	Mid value (x_i)	$d_i = x_i - 150$	$u_i = \frac{xi-150}{20}$	Frequency (f_i)	f_iu_i
100 – 120	110	-40	-2	12	-24
120 – 140	130	-20	-1	14	-14
140 – 160	150	0	0	8	0
160 – 180	170	20	1	6	6
180 - 200	190	40	2	10	20
				N = 50	$\sum f_iu_i$ = - 12

We have, A = 150

h = 20

Mean = A + h * $\frac{\sum fiui}{N}$

= 150 + 20 * $\frac{-12}{50}$

= 150 - $\frac{24}{5}$ = 150 – 4.8

= 145.2

4. Question

Thirty women were examined in a hospital by a doctor and the number of heart beats per minute recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute:	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women:	2	4	3	8	7	4	2

Answer

We may find class mark of each interval (x_i) by using the relation.

x_i = $\frac{Upper\ class\ limit+Lower\ class\ limit}{2}$

Class size of this data = 3

Now taking 75.5 as assumed mean (A), we may calculate d_i, u_i, f_iu_i as following:

No. of heart beat per minute	Number of women (f_i)	x_i	$d_i = x_i - 75.5$	$u_i = \frac{xi-75.5}{h}$	f_iu_i
65 – 68	2	66.5	-9	-3	-6
68 – 71	4	69.5	-6	-2	-8
71 – 74	3	72.5	-3	-1	-3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
	N = 30				$\sum f_iu_i$ = 4

Now we may observe from table that $\sum f_i = 30$, $\sum f_iu_i = 4$

Mean ($\bar{X}$) = $d_i + \frac{\sum iui}{\sum fi} * h$

= $75.5 + \frac{4}{30} * 3$

= 75.5 + 0.4 = 75.9

So mean of heart beat per minute of women are 75.9 beats per minute.

5. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-6	6-12	12-18	18-24	24-30
Frequency:	6	8	10	9	7

Answer

Let A assumed mean be 15.

Class interval	Mid value (x_i)	$d_i = x_i - 15$	$u_i = \frac{xi-15}{6}$	f_i	f_iu_i
0 – 6	3	-12	-2	6	-12
6 – 12	9	-6	-1	8	-8
12 – 18	15	0	0	10	0
18 – 24	21	6	1	9	9
24 – 30	27	12	2	7	14
				N = 40	$\sum f_iu_i = 3$

A = 15, h = 6

Mean = A + h * $\frac{\sum fixi}{N}$

= 15 + 6 * $\frac{3}{40}$

= 15 + 0.45

= 15.45

6. Question

Find the mean of each of the following frequency distributions:

Class interval:	50-70	70-90	90-110	110-130	130-150	150-170
Frequency:	18	12	13	27	8	22

Answer

Let the assumed mean (A) be 100.

Class interval	Mid value (x_i)	$d_i = x_i - 100$	$u_i = \frac{xi-100}{20}$	f_i	f_iu_i
50 – 70	60	-40	-2	18	-36
70 – 90	80	-20	-1	12	-12
90 – 110	100	0	0	13	0
110 – 130	120	20	1	27	27
130 – 150	140	40	2	8	16
150 – 170	160	60	3	22	66
				N = 100	$\sum f_iu_i = 61$

A = 100, h = 20

Mean = A + h * $\frac{\sum fiui}{N}$

= 100 + 20 * $\frac{61}{100}$

= 100 + 12.2

= 112.2

7. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-8	8-16	16-24	24-32	32-40
Frequency:	6	7	10	8	9

Answer

Let the assumed mean (A) = 20

Class interval	Mid – value (x_i)	$d_i = x_i - 20$	$u_i = \frac{xi-20}{8}$	Frequency (f_i)	f_iu_i
0 – 8	4	-16	-2	6	-12
8 – 16	12	-8	-1	7	-17
16 – 24	20	0	0	10	0
24 – 32	28	8	1	8	8
32 - 40	36	16	2	9	18
				N = 40	$\sum f_iu_i = 7$

We have, A = 20

h = 8

Mean = A + h * $\frac{\sum fiui}{N}$

= 20 + 8 * $\frac{7}{40}$

= 20 + 1.4 = 21.4

8. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-6	6-12	12-18	18-24	24-30
Frequency:	7	5	10	12	6

Answer

Let the assumed mean (A) = 15

Class interval	Mid value (x_i)	$d_i = x_i - 15$	$u_i = \frac{xi-15}{6}$	Frequency (f_i)	f_iu_i
3 – 6	3	-12	-2	7	-14
6 – 12	9	-6	-1	5	-5
12 – 18	15	0	0	10	0
18 – 24	21	6	1	12	12
24 - 30	27	12	2	6	12
				N = 40	$\sum f_iu_i = 5$

We have, A = 15

h = 6

Mean = A + h * $\frac{\sum fiui}{N}$

$= 15 + 6 * \frac{5}{40}$

$= 15 + 0.75 = 15.75$

9. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-10	10-12	20-30	30-40	40-50
Frequency:	9	12	15	10	14

Answer

Let the assumed mean (A) = 25

Class interval	Mid value (x_i)	$d_i = x_i - 25$	$u_i = \frac{xi-25}{10}$	Frequency (f_i)	f_iu_i
0 – 10	5	-20	-2	9	-18
10 – 20	15	-10	-1	12	-12
20 – 30	25	0	0	15	0
30 – 40	35	10	1	10	10
40 – 50	45	20	2	14	28
				N = 60	$\sum f_iu_i = 8$

We have, A = 25

h = 10

Mean = A + h * $\frac{\sum fiui}{N}$

= 25 + 10 * $\frac{8}{60}$

= 25 + $\frac{8}{6}$

= 25 + $\frac{4}{3}$ = 26.333

10. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-8	8-16	16-24	24-32	32-40
Frequency:	5	9	10	8	8

Answer

Let the assumed mean (A) = 20

Class interval	Mid value (x_i)	$d_i = x_i - 20$	$u_i = \frac{xi-20}{8}$	Frequency (f_i)	f_iu_i
0 – 8	4	-16	-2	5	-10
8 – 16	12	-8	-1	9	-9
16 – 24	20	0	0	10	0
24 – 32	28	8	1	8	8
32 – 40	36	16	2	8	16
				N = 40	$\sum f_iu_i = 5$

We have, A = 20

h = 8

Mean = A + h * $\frac{\sum fiui}{N}$

= 20 + 8 * $\frac{5}{40}$

= 20 + 1 = 21

11. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-8	8-16	16-24	24-32	32-40
Frequency:	5	6	4	3	2

Answer

Let the assumed (A) = 20

Class interval	Mid value (x_i)	$d_i = x_i - 20$	Frequency (f_i)	$u_i = \frac{xi-20}{8}$	f_1u_1
0 – 8	4	-16	5	-2	-10
8 -16	12	-8	6	-1	-6
16-24	20	0	4	0	0
24-32	28	8	3	1	3
32-40	36	16	2	2	4
			N = 20		$\sum f_iu_i = - 9$

We have, A = 20

h = 8

Mean = A + h * $\frac{\sum fiui}{N}$

= 20 + 8 * $\frac{-9}{20}$

= 20 - $\frac{72}{20}$ = 20 – 3.6

= 16.4

12. Question

Find the mean of each of the following frequency distributions:

Class interval:	10-30	30-50	50-70	70-90	90-110	110-130
Frequency:	5	8	12	20	3	2

Answer

Let the assumed mean (A) = 60

Class interval	Mid value (x_i)	$d_i = x_i - 60$	$u_i = \frac{xi-60}{20}$	Frequency (f_i)	f_iu_i
10-30	20	-40	-2	5	-10
30-50	40	-20	-1	8	-8
50-70	60	0	0	12	0
70-90	80	20	1	20	20
90-110	100	40	2	3	6
110-130	120	60	3	2	6
				N = 50	$\sum f_iu_i = 14$

We have, A = 60

h = 20

Mean = A + h * $\frac{\sum fiui}{N}$

= 60 + 20 * $\frac{14}{50}$

= 60 + 5.6

= 65.6

13. Question

Find the mean of each of the following frequency distributions:

Class interval:	25-35	35-45	45-55	55-65	65-75
Frequency:	6	10	8	12	4

Answer

Let the assumed mean (A) = 50

Class interval	Mid value	$d_i = x_i - 50$	$u_i = \frac{xi-50}{10}$	Frequency (f_i)	f_iu_i
25-35	30	-20	-2	6	-12
35-45	40	-10	-1	10	-10
45-55	50	0	0	8	0
55-65	60	10	1	12	12
65-75	70	20	2	4	8
				N = 40	$\sum f_iu_i = -2$

We have, A = 50

h = 10

Mean = A + h * $\frac{\sum fiui}{N}$

= 50 + 10 * $\frac{-2}{40}$

= 50 – 0.5

= 49.5

14. Question

Find the mean of each of the following frequency distributions:

Classes:	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Frequency:	14	22	16	6	5	3	4

Answer

Let the assumed mean (A) = 42

Class interval	Mid value (x_i)	$d_i = x_i - 42$	$U_i = \frac{xi-42}{5}$	Frequency (f_i)	f_iu_i
25-29	27	-15	-3	14	-42
30-34	32	-10	-2	22	-44
35-39	37	-5	-1	16	-16
40-44	42	0	0	6	0
45-49	47	5	1	5	5
50-54	52	10	2	3	6
55-59	57	15	3	4	12
				N = 10	$\sum f_iu_i = -79$

We have, A = 42

h = 5

Mean = A + h * $\frac{\sum fiui}{N}$

= 42 + 5 * $\frac{-79}{10}$

= 42 - $\frac{79}{14}$

$= \frac{588-79}{14}$

= 36.357

15. Question

For the following distribution, calculate mean using all suitable methods:

Size of items:	1-4	4-9	9-16	16-27
Frequency:	6	12	26	20

Answer

By direct method

Class interval	Mid value (x_i)	Frequency (f_i)	$f_i x_i$
1-4	2.5	6	15
4-9	6.5	12	78
9-16	12.5	26	325
16-27	21.5	20	430
		N = 64	$\sum f_i u_i$ = 848

Mean = $\frac{\sum fixi}{N} + A$

13. Probability

Exercise 13.1

1. Question

The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?

Answer

P(happening) + P'(not happening) = 1

Given: P = 0.85

∴ P' = 1 – 0.85 = 0.15

2. Question

A die is thrown. Find the probability of getting:

(i) a prime number

(ii) 2 or 4

(iii) a multiple of 2 or 3

(iv) an even prime number

(v) a number greater than 5

(vi) a number lying between 2 and 6

Answer

Sample space, S = 6

(i) Number of event of getting a prime number, n(E) = 3

$$\therefore \ \mathrm{P} = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(iii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

(iv) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{1}{6}$

(v) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{1}{6}$

(vi) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

3. Question

In a simultaneous throw of a pair of dice, find the probability of getting:

(i) 8 as the sum

(ii) a doublet

(iii) a doublet of prime numbers

(iv) a doublet of odd numbers

(v) a sum greater than 9

(vi) an even number on first

(vii) an even number on one and a multiple of 3 on the other

(viii) neither 9 nor 11 as the sum of the numbers on the faces

(ix) a sum less than 6

(x) a sum less than 7

(xi) a sum more than 7

(xii) at least once

(xiii) a number other than 5 n any dice.

(xiv) even number on each die

(xv) 5 as the sum

(xvi) 2 will come up at least once

(xvii) 2 will not come either time

Answer

Sample space = 36

(i) n(E) = 5

$$\therefore \mathrm{P} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

(ii) n(E) = 6

$$\therefore \mathrm{P} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(iii) n(E) = 3

$$\therefore \mathrm{P} = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(iv) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(v) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(vi) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(vii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{11}{36}$

(viii) Number of event with sum 9 or 11 = 6

∴ Number of events of not getting a sum of either 9 or 11 = 36 – 6 = 30

$$\mathrm{P} = \frac{n(E)}{n(S)} = \frac{30}{36} = \frac{5}{6}$$

(ix) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

(x) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

(xi) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

(xii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{11}{36}$

(xiii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{25}{36}$

4. Question

Three coins are tossed together. Find the probability of getting:

(i) exactly two heads

(ii) at most two heads

(iii) at least one head and one tail.

(iv) no tails

Answer

Sample space, n(S) = 8

(i) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{3}{8}$

(ii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iii) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{6}{8} = \frac{3}{4}$

(iv) $\mathrm{P} = \frac{n(E)}{n(S)} = \frac{1}{8}$

5. Question

What is the probability that an ordinary year has 53 Sundays?

Answer

Ordinary year has 365 days (52 weeks + 1 day)

That 1 day may be any day of the 7 days of a week

Therefore, total number of possible outcomes, n(S) = 7

And event of getting 53 Sunday is that 1 day is Sunday. Thus, n(E) = 1

$$\therefore \mathrm{P} = \frac{n(E)}{n(S)} = \frac{1}{7}$$

6. Question

What is the probability that a leap year has 53 Tuesdays and 53 Mondays?

Answer

A leap year has 366 days (52 weeks + 2 days)

That two days may be (sun, mon) (mon, tue) (tue, wed) (wed, thurs) (thurs, fri) (fri, sat) (sat, sun)

∴ n(S) = 7

Number of events for the occurrence of 53 Tuesdays and 53 Mondays, n(E) = 1

$$\therefore \mathrm{P} = \frac{n(E)}{n(S)} = \frac{1}{7}$$

7. Question

A and B throw a pair of dice. If A throws 9, find B's chance of throwing a higher number.

Answer

Given: A and B throw a pair of dice. A throws 9.

To find: B's chance of throwing a higher number.

Solution: Sample space of throwing a pair of dice is:

$$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$

$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$

$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$

$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$

$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)$

Sample space, n(S) = 36let E be the number of events of getting sum higher than 9 i.e getting 10,11 and 12.

The pairs are: $(4,6),(5,5),(5,6),(6,4),(6,5),(6,6)$

then n(E) = 6

We know:

$$Probability\ of\ an\ event = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

So,

$\mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

8. Question

Two unbiased dice are thrown. Find the probability that the total of the numbers on the dice is greater than 10.

Answer

Possible outcomes, n(S) = 36

Number of events of getting sum greater than 10, n(E) = 3

$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

9. Question

A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is

(i) a black king

(ii) either a black card or a king

(iii) black and a king

(iv) a jack, queen or a king

(v) neither a heart nor a king

(vi) spade or an ace

(vii) neither an ace nor a king

(viii) neither a red card nor a queen.

(ix) other than an ace

(x) a ten

(xi) a spade

(xii) a black card

(xiii) the seven of clubs

(xiv) jack

(xv) the ace of spades

(xvi) a queen

(xvii) a heart

(xviii) a red card

(xix) neither a king nor a queen

Answer

Total number of outcomes, n(S) = 52

(i) n(E) = 2

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(ii) n(E) = 26 + 2 = 28

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$$

(iii) n(E) = 2

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iv) n(E) = 12

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(v) n(E) = 36

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

(vi) n(E) = 16

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

(vii) n(E) = 44

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{44}{52}$$

(viii) n(E) = 24

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{24}{52} = \frac{6}{13}$$

(ix) n(E) = 48

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{48}{52} = 12/13$$

(x) n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(xi) n(E) = 13

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(xii) n(E) = 26

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(xiii) n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{52}$$

(xiv) n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(xv) n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{52}$$

(xvi) n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(xvii) n(E) = 13

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(xviii) n(E) = 26

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(xix) n(E) = 44

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{44}{52} = \frac{11}{13}$

10. Question

In a lottery of 50 ticket bears a prime numbered 1 to 50, one ticket is drawn. Find the probability that the drawn ticket bears a prime number.

Answer

Total number of possible outcomes, n(S) = 50

Number of events of getting a prime number, N(E) = 15

Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{15}{50} = \frac{3}{10}$

11. Question

An urn contains 10 red and 8 white balls. One ball is drawn at random. Find the probability that the ball drawn is white.

Answer

Total number of possible outcomes, n(S) = 10 + 8 = 18

Number of events of getting white ball, N(E) = 8

Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{8}{18} = \frac{4}{9}$

12. Question

A bag contains 3 red balls, 5 black balls and 4 white balls. A balls is drawn at random from the bag. What is the probability that the ball drawn is:

(i) white?

(ii) red?

(iii) black?

(iv) not red?

Answer

Total number of possible outcomes, n(S) = 12 {3 red balls, 5 black balls and 4 white balls)

(i) n(E) = 4

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{12} = \frac{1}{3}$$

(ii) n(E) = 3

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

(iii) n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{12}$$

(iv) n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{12} = \frac{3}{4}$$

13. Question

What is the probability that a number selected from the numbers 1, 2, 3,…, 15 is a multiple of 4?

Answer

Sample space, n(S) = 15

Number of events of getting numbers multiple of 4, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{15} = \frac{1}{5}$$

14. Question

A bag contains 6 red, 8 black and 4 white balls. A ball is drawn at random. What is the probability that ball drawn is not black?

Answer

Sample space, n(S) = 18

Number of events of getting balls not black, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{18} = \frac{5}{9}$$

15. Question

A bag contains 5 white and 7 red balls. One ball is drawn at random. What is the probability that ball drawn is white?

Answer

Sample space, n(S) = 12

Number of events of getting white ball, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{12}$$

16. Question

Tickets numbered from 1 to 20 are mixed up and a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 7?

Answer

Sample space, n(S) = 20

Number of events of getting a multiples of 3 or 7 on the drawn ticket, n(E) = 8 {3,6,9,12,15,18,7,14}

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

17. Question

In a lottery there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Answer

Sample space, n(S) = 35

Number of events of getting a prize, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{35} = \frac{2}{7}$$

18. Question

If the probability of winning a game is 0.3, what is the probability of loosing it?

Answer

We not that, P(E) + P'(E) = 1

Here, P(E) = 0.3

P'(E) = 1 – 0.3 = 0.7

19. Question

A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is:

(i) red

(ii) black or white

(iii) not black

Answer

Sample space, n(S) = 15

(i) n(E) = 7

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{7}{15}$$

(ii) n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{15}$$

(iii) n(E) = 5

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{15} = \frac{1}{3}$$

20. Question

A bag contains 4 red, 5 black and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is:

(i) white

(i) red

(iii) not black

(iv) red or white

Answer

Here, n(S) = 15

(i) n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{15} = \frac{2}{5}$$

(ii) n(E) = 4

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{15}$$

(iii) number of events of getting black balls, n(E) = 5

$\therefore$ probability of getting black balls, P(E) = $\frac{n(E)}{n(S)} = \frac{5}{15} = \frac{1}{3}$

Hence, probability of getting black balls = 1 – P(E) = 1 – $\frac{1}{3} = \frac{2}{3}$

(iv) number of events of getting a red or white ball, n(E) = 10

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{15} = \frac{2}{3}$$

21. Question

A black die and a white die are thrown at the same time. Write all the possible outcomes. What is the probability?

(i) that the sum of the two numbers that turn up is 8?

(ii) of obtaining a total of 6?

(iii) of obtaining a total of 10?

(iv) of obtaining the same number on both dice?

(v) of obtaining a total more than 9?

(vi) that the sum of the two numbers appearing on the top of the dice is 13?

(vii) that the sum of the numbers appearing on the top of the dice is less that or equal to 12?

(viii) that the product of numbers appearing on the top of the two dice is 2.

(ix) that the difference of the numbers appearing on the top of the two dice is 2.

Answer

Sample space, n(S) = 36

(i) N(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

(ii) n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

(iii) n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(iv) n(E) = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(v) n(E) = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(vi) The maximum sum is 12. So, getting a sum of number appearing on the top of the two dice as 13 is an impossible event.

∴ Probability, P(E) = 0

(vii) n(E) = 36

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{36}{36} = 1$$

(viii) n(E) = 2

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

(ix) n(E) = 8

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$$

22. Question

One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting:

(i) a king of red suit

(ii) a face card

(iii) a red face card

(iv) a queen of black suit

(v) a jack of hearts

(vi) a spade

Answer

Number of possible outcomes, n(S) = 52

(i) n(E) = 2

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(ii) n(E) = 12

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(iii) n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

(iv) n(E) = 2

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(v) n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$$

(vi) n(E) = 13

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

23. Question

Five cards – ten, jack, queen, king and an ace of diamonds are shuffled face downwards. One card is picked at random.

(i) What is the probability that the card is a queen?

(ii) If a king is drawn first and put aside, what is the probability that the second card picked up is the (a) ace? (b) king?

Answer

Total number of possible outcomes, n(S) = 5

(i) Number of events of drawing a queen, n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{5}$$

(ii) If a king is drawn first and put aside, total number of remaining cards, n(S) = 4

a) Probability that second card is ace, $\mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{4}$

b) Since there was only a single king and when it is put aside, then n(E) = 0

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = 0$$

24. Question

A bag contains 3 red balls and 5 black balls. A ball is drawn at random form the bag. What is the probability that the ball drawn is:

(i) red

(ii) black

Answer

Total number of possible outcomes, n(S) = 8

(i) Number of possible outcomes of drawing a red ball, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{8}$$

(ii) Number of possible outcomes of drawing a black ball, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{8}$$

25. Question

A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears

(i) a two digit number

(ii) a number which is a perfect square

Answer

Total number of possible outcomes, n(S) = 89

(i) Number of favorable outcomes of getting a two digit number = 81

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{81}{89}$$

(ii) Number of favorable outcomes of getting a perfect square = 8

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{8}{89}$$

26. Question

A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the number 1, 2, 3,…, 12 as shown in Fig. 13.3. What is the probability that it will point to:

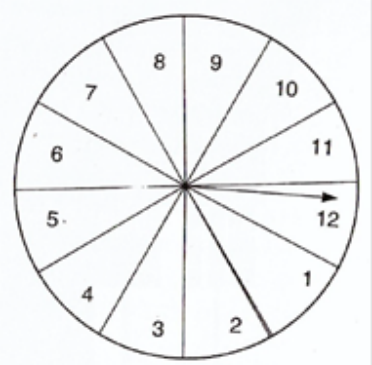


Fig. 13.3

(i) 10?

(ii) an odd number?

(iii) a number which is multiple of 3?

(iv) an even number?

Answer

The possible number of outcomes, n(S) = 12

(i) Number of favorable outcomes, n(E) = 1

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$$

(ii) Number of favorable outcomes, n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

(iii) Number of favorable outcomes, n(E) = 4

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{12} = \frac{1}{3}$$

(iv) Number of favorable outcomes, n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

27. Question

Two customers are visiting a particular shop in the same wee (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another. What is the probability that both will visit the shop on:

(i) the same day?

(ii) different days?

(iii) consecutive days?

Answer

Total number of days to visit the shop, n(S) = 36

(i) Number of favorable outcomes, n(E) = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) n(E') = 1 – n(E) = $1 - \frac{1}{6} = \frac{5}{6}$

(iii) Number of favorable outcomes, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

28. Question

In a class, there are 18 girls and 16 boys. The class teacher wants to choose one pupil for class monitor. What she does, she writes the name of each pupil on a card and puts then into a basket and mixes thoroughly. A child is asked to pick one card from the basket. What is the probability that the name written on the card is:

(i) the name of a girl

(ii) the name of a boy

Answer

Total number of possible outcomes, n(S) = 34

(i) Number of favorable outcomes, n(E) = 18

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{18}{34} = \frac{9}{17}$$

(ii) Number of favorable outcomes, n(E) = 16

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{16}{34} = \frac{8}{17}$$

29. Question

Why is tossing a coin considered to be a fair way of deciding which team should choose ends in a game of cricket?

Answer

Number of possible outcomes, n(S) = 2

Probability of getting a head, P(E') = $\frac{1}{2}$ = probability of getting a tail

Since both probabilities are equal for both of the events, therefore both of the players have equal chances to win the game and thus this is a fair way to make this decision that who will play first.

30. Question

What is the probability that a number selected at random from the number 1, 2, 2, 3, 3, 3, 4, 4, 4, 4 will be their average?

Answer

Total number of possible outcomes = 10

Average of the numbers = $\frac{sum\ of\ the\ given\ n\ numbers}{n} = \frac{30}{10} = 3$

∴ Number of favorable outcomes, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{10}$$

31. Question

The faces of a red cube and a yellow cube are numbered from 1 to 6. Both cubes are rolled. What is the probability that the top face of each cube will have the same number?

Answer

Total number of possible outcomes, n(S) = 36

Number of favorable outcomes, n(E) = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

32. Question

The probability of selecting a green marble at random form a jar that contains only green, white and yellow marbles is 1/4. The probability of selecting a white marble at random form the same jar is 1/3. If this jar contains 10 yellow marbles. What is the total number of marbles in the jar?

Answer

Let the number of green marbles be x and the number of white marbles be y

Total number of possible outcomes, n(S) = x + y + 10

$$\mathrm{P(E)} = \frac{n(E)}{n(S)}$$

Probability of green marbles = 1/4

$$\Rightarrow \frac{x}{x+y+10} = \frac{1}{4}$$

⇒ x + y + 10 = 4x

⇒ 3x – y – 10 = 0 ----------(i)

Probability of white marbles = $\frac{1}{3}$

$$\Rightarrow \frac{y}{x+y+10} = \frac{1}{3}$$

⇒ x – 2y + 10= 0 ----------(ii)

Solving eq. (i) and (ii), we get

x = 6 and y = 8

Thus, total number of marbles in the jar = x + y + 10 = 6 + 8 + 10 = 24

33. Question

There are 30 cards, of same size, in a bag on which numbers 1 to 30 are written. One card is taken out of the bag at random. Find the probability that the number on the selected card is not divisible by 3.

Answer

Total number of possible outcomes, n(S) = 30

Number of favorable outcomes of selecting a card divisible by 3, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{30} = \frac{1}{3}$$

Number of favorable outcomes of not selecting a card divisible by 3 = 1 – P(E)

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

34. Question

A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is (i) red or white (ii) not black (iii) neither white nor black.

Answer

Total number of possible outcomes, n(S) = 20

(i) Number of favorable outcomes, n(E) = 13

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{13}{20}$$

(ii) Number of events of getting a black ball, n(E) = 7

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{7}{20}$$

Probability of not getting a black ball = 1 – P(E)

$$= 1 - \frac{7}{20} = \frac{13}{20}$$

(iii) Number of favorable outcomes, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{20} = \frac{1}{4}$$

35. Question

Find the probability that a number selected from the number 1 to 25 is not a prime number when each of the given numbers is equally likely to be selected.

Answer

Total number of possible outcomes, n(S) = 25

Number of events of getting a prime number, n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{25}$$

Number of events of not getting a prime number = 1 – P(E)

$$= 1 - \frac{9}{25} = \frac{16}{25}$$

36. Question

A bag contains 8 red, 6 white and 4 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is

(i) red or white

(ii) not black

(iii) neither white or black

Answer

Total number of possible outcomes, n(S) = 18

(i) Number of favorable outcomes, n(E) = 14

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{14}{18} = \frac{7}{9}$$

(ii) Number of events of getting a black ball, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{18} = \frac{2}{9}$$

Probability of not getting a black ball = 1 – P(E)

$$= 1 - \frac{2}{9} = \frac{7}{9}$$

(iii) Number of favorable outcomes, n(E) = 8

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{18} = \frac{4}{9}$$

37. Question

Find the probability that a number selected at random from the numbers 1, 2, 3,…, 35 is a

(i) prime number

(ii) multiple of 7

(iii) a multiple of 3 or 5

Answer

Total number of possible outcomes, n(S) = 35

(i) Number of favorable outcomes, n(E) = 11

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{11}{35}$$

(ii) Number of favorable outcomes, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{35} = \frac{1}{7}$$

(iii) Number of favorable outcomes, n(E) = 16

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{16}{35}$$

38. Question

From a pack of 52 playing cards Jacks, queens, kings and aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is

(i) a black queen

(ii) a red card

(iii) a black jack

(iv) a picture card (Jacks, queens and kings are picture cards).

Answer

Total number of possible outcomes, n(S) = 52 – 2 – 2 – 2 – 2 = 44

(i) Number of favorable outcomes, n(E) = 2

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{44} = \frac{1}{22}$$

(ii) Number of favorable outcomes, n(E) = 26 – 8 = 18

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{18}{44} = \frac{9}{22}$$

(iii) Number of favorable outcomes, n(E) = 2

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{44} = \frac{1}{22}$$

(iv) Number of favorable outcomes, n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{44} = \frac{3}{22}$$

39. Question

A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out.

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Answer

(i) Since the bag contains lemon flavored candies only thus there is no chance of getting orange flavored candy.

∴ P(E) = 0

(ii) The bag contains lemon flavored candies only thus this is a sure event of getting lemon flavored candy

∴ P(E) = 1

40. Question

It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Answer

Let E be the event of 2 students having the same birthday and _E' be the event of not getting the same birthday

Given: P(E) = 0.992

We know that, P(E) + P(E') = 1

∴ P(E') = 1 – P(E)

= 1 – 0.992

= 0.008

41. Question

A bag contains 3 red balls and 5 black balls is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?

Answer

Total number of possible outcomes, n(S) = 8

(i) Number of favorable outcomes, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{8}$$

(ii) Probability of not drawing a red ball = 1 – P(E)

$$= 1 - \frac{3}{8} = \frac{5}{8}$$

42. Question

A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red (ii) white (iii) not green

Answer

Total number of possible outcomes, n(S) = 5 + 8 + 4 = 17

(i) Number of favorable outcomes, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{17}$$

(ii) Number of favorable outcomes, n(E) = 8

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{17}$$

(iii) Number of events of drawing a green marble = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{17}$$

Probability of not drawing green marble = 1 – P(E)

$$= 1 - \frac{4}{17} = \frac{13}{17}$$

43. Question

(i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and not replaced. Now bulb is drawn at random from the box, find the probability that it bears (i) a two digit number (ii) a perfect square number (ii) a number divisible by 5.

Answer

(i) Total number of possible outcomes, n(S) = 20

Number of favorable outcomes, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

(ii) Total number of possible outcomes, n(S) = 19

Number of favorable outcomes, n(E) = 15

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{15}{19}$$

44. Question

A box contains 90 discs which are numbered from 1 to 90. If one discs is drawn at random from the box, find the probability that it bears (i) a two digit number (ii) a perfect square number (iii) a number divisible by 5.

Answer

Total number of possible outcomes, n(S) = 90

(i) Number of favorable outcomes, n(E) = 81

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{81}{90}$$

(ii) Number of favorable outcomes, n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{90} = \frac{1}{10}$$

(iii) Number of favorable outcomes, n(E) = 18

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{18}{90} = \frac{1}{5}$$

45. Question

A lot consists of 144 ball pens of which 20 are defective and others good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it?

(ii) She will not buy it?

Answer

Total number of possible outcomes, n(S) = 144

(i) Number of favorable outcomes, n(E) = 124

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{124}{144} = \frac{31}{36}$$

(ii) Probability of not buying a defective pen = 1 – P(E)

$$= 1 - \frac{31}{36} = \frac{5}{36}$$

46. Question

12 defective pens are accidently mixed with 132 good ones. It is not possible to just look at pen and tell whether or not it is defective. One pen is taken out at random from this lit. Determine the probability that the pen taken out is good one.

Answer

Total number of possible outcomes, n(S) = 144

Number of favorable outcomes, n(E) = 132

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{132}{144} = \frac{11}{12}$$

47. Question

Five cards – the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put a side, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Answer

Total number of possible outcomes, n(S) = 5

(i) Number of favorable outcomes, n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{5}$$

(ii) If queen is drawn and put aside, total number of remaining cards, n(S) = 4

a) Number of favorable outcomes, n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{4}$$

b) Total number of events of getting a queen = 0 as there is no queen

∴ P(E) = 0

48. Question

Harpreet tosses two different coins simultaneously (say, one is of Re 1 and other of Rs 2). What is the probability that he gets at least one head?

Answer

Total number of possible outcomes, n(S) = 4

Number of favorable outcomes, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{4}$$

49. Question

Two dice, one blue and one grey, are thrown at the same time.

Complete the following table:

Events: Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability											

From the above table a student argues that there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument?

Answer

Total number of possible outcomes, n(S) = 36

Number of favorable outcomes = n(E)

We know that, $\mathrm{P(E)} = \frac{n(E)}{n(S)}$

Sum of numbers on the two dice	2	3	4	5	6	7	8	9	10	11	12
n(E)	1	2	3	4	5	6	5	4	3	2	1
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

50. Question

Cards marked with numbers 13, 14, 15, …, 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the card drawn is

(i) divisible by 5

(ii) a number is a perfect square

Answer

Total number of possible outcomes, n(S) = 48

(i) Number of favorable outcomes, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{48} = \frac{5}{24}$$

(ii) Number of favorable outcomes, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{48} = \frac{1}{12}$$

51. Question

A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.

Answer

Given: number of red balls = 6

Let number of blue balls be x

Total number of possible outcomes, n(S) = 6 + x

Number of favorable outcomes = n(E)

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)}$$

P(blue ball) = 2P(red ball)

$$\Rightarrow \frac{x}{x+6} = \frac{2\times6}{x+6}$$

⇒ x = 12

∴ number of blue balls = 12

52. Question

A bag contains tickets numbered 11, 12, 13, …, 30. A tickedt is taken out from the bag at random. Find the probability that the number on the drawn ticket (i) is a multiple of 7

(ii) is greater than 15 and a multiple of 5.

Answer

Total number of possible outcomes, n(S) = 20

(i) Number of favorable outcomes, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{20}$$

(ii) Number of favorable outcomes, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{20}$$

53. Question

The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) heart (ii) queen

(iii) clubs.

Answer

Total number of possible outcomes, n(S) = 52 – 3 = 49

(i) Number of favorable outcomes, n(E) = 13

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{13}{49}$$

(ii) Number of favorable outcomes, n(E) = 4 – 1 = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{49}$$

(iii) Number of favorable outcomes, n(E) = 13 – 3 = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{49}$$

54. Question

Two dice are thrown simultaneously. What is the probability that:

(i) 5 will not come up on either of them?

(ii) 5 will come up on at least one?

(iii) 5 will come up at both dice?

Answer

Total number of possible outcomes, n(S) = 36

(i) Number of favorable outcomes, n(E) = 25

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{25}{36}$$

(ii) Number of favorable outcomes, n(E) = 11

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{11}{36}$$

(iii) Number of favorable outcomes, n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{36}$$

55. Question

Fill in the blanks:

(i) Probability of a sure event is ……. .

(ii) Probability of an impossible event is …….. .

(iii) The probability of an event (other than sure and impossible event) lies between …….. .

(iv) Every elementary event associated to a random experiment has ……. Probability.

(v) Probability of an event A+ Probability of event 'not A' = …….. .

(vi) Sum of the probabilities of each outcome in an experiment is ……. .

Answer

(i) 1

(ii) 0

(iii) 0 and 1

(iv) Equal

(v) 1

(vi) 1

56. Question

Examine each of the following statements and comment:

(i) If two coins are tossed at the same time, there are 3 possible outcomes – two heads, two tails, or one of each. Therefore, for each outcome, the probability of occurrence is 1/3.

(ii) If a die is thrown once, there are two possible outcomes – an odd number or an even number. Therefore, the probability of obtaining an odd number is and the probability of obtaining an even number is 1/2.

Answer

(i) Incorrect statement

If 2 coins are tossed at the same time = 4 possible outcomes (HH, HT, TH, TT}

Therefore probability of each outcome = 1/4

(ii) Correct statement

When a dice is thrown once, event of occurrence of 3 odd numbers and 3 even numbers are possible out of 6 outcomes. Therefore, P(even number) = P(odd number) = $\frac{3}{6} = \frac{1}{2}$

57. Question

A box contains 100 red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box, then find the probability that it will be (i) a blue card (ii) not a yellow card (iii) neither yellow nor a blue card

Answer

Total number of possible outcomes, n(S) = 100 + 200 + 50 = 350

(i) Number of favorable outcomes, n(E) = 50

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{50}{350} = \frac{1}{7}$$

(ii) Number of favorable outcomes, n(E) = 100 + 50 = 150

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{150}{350} = \frac{3}{7}$$

(iii) Number of favorable outcomes, n(E) = 100

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{100}{350} = \frac{2}{7}$$

58. Question

A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

Answer

Total number of possible outcomes, n(S) = 50

Number of favorable outcomes, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{50} = \frac{2}{25}$$

59. Question

A box contains cards numbered 3, 5, 7, 9, …, 35, 37. A card is drawn at random from the box. Find the probability that the number on the drawn card is a prime number.

Answer

Total number of possible outcomes, n(S) = 18

Number of favorable outcomes, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{18} = \frac{5}{9}$$

60. Question

A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest. Which of the above you prefer more.

Answer

Total number of possible outcomes, n(S) = 12

(i) Number of favorable outcomes, n(E) = 3

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{12}$$

(ii) Number of favorable outcomes, n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{12} = \frac{3}{4}$$

61. Question

Cards numbered 1 to 30 are put in a bag. A card is drawn at random from this bag. Find the probability that the number on the drawn card is

(i) not divisible by 3

(ii) a prime number greater than 7

(iii) not a perfect square number.

Answer

Total number of possible outcomes, n(S) = 30

(i) Number of favorable outcomes of numbers divisible by 3, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{30} = \frac{1}{3}$$

Probability of getting a number not divisible by 3 = 1 – P(E)

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) Number of favorable outcomes, n(E) = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{30} = \frac{1}{5}$$

(iii) Number of favorable outcomes of getting a perfect square, n(E) = 5

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{5}{30} = \frac{1}{6}$$

Probability of not getting a perfect square = 1 – P(E)

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

62. Question

A dice is rolled twice. Find the probability that

(i) 5 will not come up either time.

(ii) 5 will come up exactly one time

Answer

Total number of possible outcomes, n(S) = 36

(i) Number of favorable outcomes, n(E) = 11 {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5) (4,5), (6,5)}

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{11}{36}$$

(ii) Number of favorable outcomes, n(E) = 11 – 1 = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

63. Question

A piggy bank contains hundred 50 paise coins, fifty ` 1 coins, twenty ` 2 coins and ten ` 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, find the probability that the coin which fell

(i) will be a 50 paise win

(ii) will be of value more than ` 1

(iii) will be of value less than ` 5

(iv) will be a ` 1 or ` 2 coin

Answer

Total number of possible outcomes, n(S) = 100 + 50 + 20 + 10 = 180

(i) Number of favorable outcomes, n(E) = 100

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{100}{180} = \frac{5}{9}$$

(ii) Number of favorable outcomes, n(E) = 20 + 10 = 30

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{30}{180} = \frac{1}{6}$$

(iii) Number of favorable outcomes, n(E) = 100 + 50 + 20 = 170

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{170}{180} = \frac{17}{18}$$

(iv) Number of favorable outcomes, n(E) = 50 + 20 = 70

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{70}{180} = \frac{7}{18}$$

64. Question

All the black face cards are removed from a pack of 52 cards. The remaining cards are well shuffled and then a card is drawn at random. Find the probability of getting a

(i) face card (ii) red card

(iii) black card (iv) king.

Answer

Total number of possible outcomes, n(S) = 52 – 6 = 46

(i) Number of favorable outcomes, n(E) = 12 – 6 = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{26} = \frac{3}{23}$$

(ii) Number of favorable outcomes, n(E) = 26

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{26}{46} = \frac{13}{23}$$

(iii) Number of favorable outcomes, n(E) = 26 – 6 = 20

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{20}{46} = \frac{10}{23}$$

(iv) Number of favorable outcomes, n(E) = 4 – 2 = 2

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{2}{46} = \frac{1}{23}$$

65. Question

Cards numbered from 11 to 60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn cards is

(i) an odd number

(ii) a perfect square number

(iii) divisible by 5

(iv) a prime number less than 20

Answer

Total number of possible outcomes, n(S) = 50

(i) Number of favorable outcomes, n(E) = 25

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{25}{50} = \frac{1}{2}$$

(ii) Number of favorable outcomes, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{50} = \frac{2}{25}$$

(iii) Number of favorable outcomes, n(E) = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{50} = \frac{1}{5}$$

(iv) Number of favorable outcomes, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{50} = \frac{2}{25}$$

66. Question

A bag contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the card thoroughly. Find the probability that the number on the drawn card is

(i) an odd number

(ii) a multiple of 5

(iii) a perfect square

(iv) an even prime number.

Answer

Total number of possible outcomes, n(S) = 49

(i) Number of favorable outcomes, n(E) = 25

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{25}{49}$$

(ii) Number of favorable outcomes, n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{49}$$

(iii) Number of favorable outcomes, n(E) = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{7}{49} = \frac{1}{7}$$

(iv) Number of favorable outcomes, n(E) = 1

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{1}{49}$$

67. Question

All kings and queens are removed from a pack of 52 cards. The remaining cards are well – shuffled and then a card is randomly drawn from it. Find the probability that this card is (i) a red face card (ii) a black card.

Answer

Total number of possible outcomes, n(S) = 52 – 8 = 44

(i) Number of favorable outcomes, n(E) = 2

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{2}{44} = \frac{1}{22}$$

(ii) Number of favorable outcomes, n(E) = 26 – 4 = 22

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{22}{44} = \frac{1}{2}$$

68. Question

All jacks, queens and kings are removed from a pack of 52 playing cards. The remaining cards are well – shuffled and then a card is randomly drawn from it. Find the probability that this card is

(i) a black face card (ii) a red card

Answer

Total number of possible outcomes, n(S) = 52 – 12 = 40

(i) Since all jacks,

Number of favorable outcomes, n(E) =

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} =$$

69. Question

Red queens and black jacks are removed from a pack of 52 playing cards. A cards is drawn at random from the remaining cards, after reshuffling them. Find the probability that the card drawn is

(i) a king (ii) of red colour

(iii) a face card (iv) a queen

Answer

Total number of possible outcomes, n(S) = 52 – 2 – 2 = 48

(i) Number of favorable outcomes, n(E) = 4

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{4}{48} = \frac{1}{12}$$

(ii) Number of favorable outcomes, n(E) = 24

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{24}{48} = \frac{1}{2}$$

(iii) Number of favorable outcomes, n(E) = 12 – 4 = 8

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{48} = \frac{1}{6}$$

(iv) Number of favorable outcomes, n(E) = 2

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{2}{48} = \frac{1}{24}$$

70. Question

A box contains 20 cards numbered from a 1 to 20. A card is drawn at random form the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Answer

Total number of possible outcomes, n(S) = 20

(i) Number of favorable outcomes, n(E) = 10 + 3 = 13 {(2, 4, 6, 8, 10, 12, 14, 16, 18, 20) (3, 9, 15)}

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{13}{20}$$

(ii) Number of favorable outcomes, n(E) = 8

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

71. Question

In a bag there are 44 identical cards with figure of circles, of which 9 are blue and rest are green and 20 squares of which 11 are blue and rest are green. One card is drawn from the bag at random. Find the probability that it has the figure of (i) square (ii) green colour (iii) blue circle and (iv) green square.

Answer

Total number of possible outcomes, n(S) = 44

(i) Number of favorable outcomes, n(E) = 20

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{20}{44} = \frac{5}{11}$$

(ii) Number of favorable outcomes, n(E) = (24 – 9) + (20 – 11) = 24

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{24}{44} = \frac{6}{11}$$

(iii) Number of favorable outcomes, n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{44}$$

(iv) Number of favorable outcomes, n(E) = 9

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{9}{44}$$

72. Question

All red face cards are removed from a pack of playing cards. The remaining cards are well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is (i) a red card (ii) a face card and (iii) a card of clubs.

Answer

Total number of possible outcomes, n(S) = 52 – 6 = 46

(i) Number of favorable outcomes, n(E) = 26 – 6 = 20

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{20}{46} = \frac{10}{23}$$

(ii) Number of favorable outcomes, n(E) = 12 – 6 = 6

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{6}{46} = \frac{3}{23}$$

(iii) Number of favorable outcomes, n(E) = 13 – 3 = 10

$$\therefore \mathrm{P(E)} = \frac{n(E)}{n(S)} = \frac{10}{46} = \frac{5}{23}$$

CCE - Formative Assessment

1. Question

Cards each marked with one of the numbers 4, 5, 6, ..., 20 are placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting an even number?

Answer

The total number of elementary events are: 4,5,6 …..20 = 20 – 3= 17

Let E be the event of getting an even number on the random draw

Then total favorable outcomes are: 4, 6, 8, 10, 12, 14, 16, 18, 20

Total number of favorable outcomes = 9

P (even number) = P (E) = 9/17

2. Question

One card is drawn from a well shuffled deck of 52 playing cards. What is the probability of getting a non-face card?

Answer

Total numbers of elementary events are: 52

Let E be the event of getting a non –face card

The total number of favorable outcomes are ; 52 – 12 = 40: being four cards each of king , queen, jack being a face card , deleted from the favorable outcomes number

P (non-face card) = P (E) = 40/52 = 20/ 26 = 10/ 13

3. Question

A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag. What is the probability of getting a white ball or a green ball?

Answer

Total numbers of elementary events are: 5+8 + 7 = 20

Let E be the event of getting a green ball or a white ball

Total numbers of favorable outcomes are: 8+ 7 = 15

P (white or green ball) = P (E) = 15/20 = 3/4

4. Question

A die is thrown once. What is the probability of getting a prime number?

Answer

Total numbers of elementary events are: 6 being a dice having six faces

Let E be the event of getting a prime number

Total favorable outcomes are: 1, 2, 3, 5

Total number of favorable events = 4

P (prime number) = P (E) = 4/6 = 2/3

5. Question

A die is thrown once. What is the probability of getting a number lying between 2 and 6?

Answer

Total numbers of elementary events are: 6 being a dice having six faces

Let E be the event of getting a prime number

The favorable outcomes are: 3, 4, 5,

Total number of favorable outcomes are = 3

P (number between 2 and 6) = P (E) = 3/6 =1/2

6. Question

A die is thrown once. What is the probability of getting an odd number?

Answer

Total numbers of elementary events are: 6 being a dice having six faces

Let E be the event of getting a prime number

The favorable events are: 1, 3, 5

Number of favorable events = 3

P (odd number) = P (E) = 3/6 =1/2

7. Question

If E denote the complement or negation of an even E, what is the value of $P(E) + P(\bar{E})$?

Answer

E denotes the complement of an event E then $\overline{E}$ denotes the affirmation aspect and the probability of any event`s occurring or non-occurring , together is always 1

So P(E) + P($\overline{E}$) = 1

8. Question

One card is drawn at random from a well shuffled deck of 52 cards. What is the probability of getting an ace?

Answer

Total numbers of elementary events are 52

Let E be the event of getting an ace

The numbers of favorable events are: 4

P (an ace) = P (E) = 4/52 = 1/13

9. Question

Two coins are tossed simultaneously. What is the probability of getting at least one head?

Answer

Total numbers of elementary events are 2+ 2 = 4

Let E be the event of getting at least two heads

The favorable outcomes are: HH, HT, TH

Total numbers of favorable outcomes are 3

P (ATLEAST one head) = P (E) = 3/4

Note: at least one head means the outcome with one head or more will be considered favorable and the outcome with no head at all will be ruled out.

10. Question

Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3?

Answer

Total numbers of elementary events are = 20

Let E be the event of drawing a multiple of 3

The favorable outcomes are: 3, 6, 9, 12, 15, 18

Total numbers of favorable outcomes are = 6

P (multiple of 3) = P (E) = 6/20 =3/10

11. Question

From a well shuffled pack of cards, a card is drawn at random. Find the probability of getting a black queen.

Answer

Total numbers of elementary events are 52

Let E be the event of drawing a black queen

Total numbers of favorable outcomes are 2

P (black queen) = P (E) = 2/52 = 1/26

12. Question

A die is thrown once. Find the probability of getting a number less than 3.

Answer

Total number of elementary events are : 6

Let E be the event of getting a number less than 3

Favorable events are: 1, 2

Total numbers of favorable events are =2

P (number less than 3) = P (E) = 2/6 =1/3

13. Question

Two coins are tossed simultaneously. Find the probability of getting exactly one head.

Answer

Total numbers of elementary events are 2+ 2 = 4

Let E be the event of getting one head

Favorable outcome are: TH , HT

Numbers of favorable outcomes are= 2

P (exactly one head) = P (E) = 2/4= 1/2

14. Question

A die is thrown once. What is the probability of getting a number greater than 4?

Answer

Total numbers of elementary events are 6

Let E be the event of getting number greater than 4

The favorable outcomes are: 5, 6

Number of favorable outcomes are = 2

P (number greater than 4) = P (E) = 2/6 = 1/3

15. Question

What is the probability that a number selected at random from the numbers 3, 4, 5, ..., 9 is a multiple of 4?

Answer

Total number of elementary events are = 7

Let E be the event of getting a multiple of 4

Favorable outcome are: 4, 8

Numbers of favorable outcome are= 2

P (multiple of 4) = P (E) = 2/7

16. Question

A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Answer

Total number of elementary events are= 26

Let E be the event of choosing a consonant

The favorable outcomes are: B, C, D, F, G, H, J, K, L, M, N, P, Q, R, S, T, V, W, X, Y, Z

Numbers of favorable outcomes are = 21

P (getting a consonant) = P (E) = 21/26

1. Question

If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, then the probability that it is odd, is

A. $\frac{4}{9}$

B. $\frac{5}{9}$

C. $\frac{1}{9}$

D. $\frac{2}{3}$

Answer

Total numbers of elementary events are = 9

Let E be the event of getting an odd number

The favorable outcomes are: 1, 3, 5, 7, 9

Numbers of favorable events= 5

P (odd number) = P (E) = 5/9

2. Question

In Q. No. 1, the probability that the digit is even, is

A. $\frac{4}{9}$

B. $\frac{5}{9}$

C. $\frac{1}{9}$

D. $\frac{2}{3}$

Answer

Total numbers of elementary events are = 9

Let E be the event of getting an even number

Favorable outcomes are: 2, 4, 6, 8

Numbers of favorable outcomes are: 4

P (even number) = P (E) = 4/9

3. Question

In Q. No. 1, the probability that the digit is a multiple of 3 is

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{9}$

D. $\frac{2}{9}$

Answer

Total numbers of elementary events are = 9

Let E be the event of getting a multiple of 3

Favorable outcomes are: 3, 6, 9

Numbers of favorable outcomes are: 3

P (multiple of 3) = P (E) = 3/9 = 1/3

4. Question

If three coins are tossed simultaneously, then the probability of getting at least two heads, is

A. $\frac{1}{4}$

B. $\frac{3}{8}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer

Total numbers of elementary events are: 2 x 2 x 2 = 8

Let E be the event of getting at least two heads

Favorable outcomes are: THH, HHT, HHH, HTH

Numbers of favorable outcomes are: 4

P (at least two heads) = P (E) = 4/8 = 1/2

5. Question

In a single throw of a die, the probability of getting a multiple of 3 is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$

Answer

Total numbers of elementary events are: 6

Let E be the event of getting multiple of 3

Favorable events are: 3, 6

Numbers of favorable events are: 2

P (getting multiple of 3) = P (E) = 2/6 = 1/3

6. Question

A number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3 the probability that |x| < 2 is

A. $\frac{5}{7}$

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. $\frac{1}{7}$

Answer

Total numbers of elementary events are: 7

Let E be the event of getting $|x| < 2$

This means -2 < x < 2

Favorable outcomes are: -1, 0, 1

Numbers of favorable outcomes are: 3

We know,

$$P(E) = \frac{Number\ of\ favorable\ outcomes}{Total\ outcomes}$$

P ($|x| < 2$) = P (E) = 3/7

7. Question

The probability of guessing the correct answer to a certain test questions is x/12 . If the probability of not guessing the correct answer to this question is 2/3, then x =

A. 2

B. 3

C. 4

D. 6

Answer

Probability of guessing the correct answer is = P (E) = x/12

Probability of guessing the incorrect answer is = P (not E) = 2/3

P (E) + P (not E) = 1

x/12 + 2/3 = 1

x+8 = 12

x = 4

8. Question

A bag contains three green marbles, four blue marbles, and two orange marbles. If a marble is picked at random, then the probability that it is not an orange marble is

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{4}{9}$

D. $\frac{7}{9}$

Answer

Total numbers of elementary events are: 3+ 4 + 2= 9

Let E be the event of getting not an orange marble

Numbers of Favorable outcomes are: 3 + 4 = 7

P (not an orange marble) = P (E) = 7/9

9. Question

A number is selected at random from the numbers 3, 5, 5, 7, 7, 7, 9, 9, 9, 9 The probability that the selected number is their average is

A. $\frac{1}{10}$

B. $\frac{3}{10}$

C. $\frac{7}{10}$

D. $\frac{9}{10}$

Answer

Total numbers of elementary events are: 10

Let E be the event of getting selected number being the average

Favorable outcome is = 70 /10 = 7

Average = total of observations/ numbers of observation

Total of observations = 3+ 5+ 5+ 7+ 7+ 7+ 9+ 9 + 9+ 9 = 70

Numbers of observation = 7

Numbers of favorable outcomes are: 3

P (selecting average number) = P (E) = 3/10

10. Question

If a number x is chosen from the numbers 1, 2, 3, and a number y is selected from the numbers 1, 4, 9. Then, P (xy < 9)

A. $\frac{7}{9}$

B. $\frac{5}{9}$

C. $\frac{2}{3}$

D. $\frac{1}{9}$

Answer

Total numbers of elementary events are: 3 x 3 = 9

Let E be the event of (xy < 9)

Favorable outcomes are: 1× 1 =1, 1×4 = 4, 2×1 =2, 2×4= 8, 3×1 =3

Numbers of favorable outcomes are: 5

P (xy < 9) = 5/9

11. Question

The probability of throwing a number greater than 2 with a fair dice is

A. $\frac{3}{5}$

B. $\frac{2}{5}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer

Total numbers of elementary events are: 6

Let E be the event of getting a number greater than 2

Favorable outcomes are: 3, 4, 5, 6

Numbers of favorable outcomes are: 4

P (number>2) = P (E) = 4/6 =2/3

12. Question

A card is accidently dropped from a pack of 52 playing cards. The probability that it is an ace is

A. $\frac{1}{4}$

B. $\frac{1}{13}$

C. $\frac{1}{52}$

D. $\frac{12}{13}$

Answer

Total numbers of elementary events are: 52

Let E be the event of getting the dropped card as an ace

Favorable outcomes are: 4

P (ace) = P(E) = 4/52 = 1/13

13. Question

The probability that a non-leap year has 53 sundays, is

A. $\frac{2}{7}$

B. $\frac{5}{7}$

C. $\frac{6}{7}$

D. $\frac{1}{7}$

Answer

Total numbers of elementary events are: 7

Let E be the event of having 53 Sundays

Note- a non-leap year has 365 days or 52 weeks and 1 day. This one day could be any day amongst Sunday, Monday, Tuesday, Wednesday, Thursday, Friday or Saturday. Out of these favorable event is one; Sunday.

Number of favorable outcome is = 1

P (Sunday) = P (E) = 1/7

14. Question

A number is selected from numbers 1 to 25. The probability that it is prime is

A. $\frac{2}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{9}{25}$

Answer

Total numbers of elementary events are: 25

Let E be the event of getting a prime number

Favorable outcomes are: 2, 3, 5, 7, 11, 13, 17, 19, 23

Numbers of favorable outcomes are = 9

P (prime number) = P (E) = 9/25

15. Question

In a single throw of a pair of dice, the probability of getting the sum a perfect square is

A. $\frac{1}{18}$

B. $\frac{7}{36}$

C. $\frac{1}{6}$

D. $\frac{2}{9}$

Answer

Total numbers of elementary events are: 6 x 6 = 36

Let E be the event of getting a perfect square

Favorable outcomes are: 4, 9

4 and 9 can be rolled in combinations of: (2,2), (1,3), (3,1) , (3,6), (4,5), (5,4), (6,3)

Numbers of favorable outcomes are = 7

P (perfect square) = P (E) = 7/36

16. Question

Which of the following cannot be the probability of an event?

A. $\frac{2}{3}$

B. -1.5

C. 15%

D. 0.7

Answer

Probability of an event cannot be negative or more than one so -1.5 can never be probability of any event.

17. Question

If P(E) = 0.05, then P(not E) =

A. - 0.05

B. 0.5

C. 0.9

D. 0.95

Answer

P (E) = 0.05 given

P (E) + P (not E) = 1

P (not E) = 1 – 0.05 = 0.95

18. Question

Which of the following cannot be the probability of occurrence of an event?

A. 0.2

B. 0.4

C. 0.8

D. 1.6

Answer

Probability of the occurrence of an event cannot be greater than 1.

1.6 Which is greater than 1 is the correct answer.

19. Question

The probability of a certain event is

A. 0

B. 1

C. 1/2

D. no existent

Answer

Probability of a certain event is always 1.

20. Question

The probability of an impossible event is

A. 0

B. 1

C. 1/2

D. non-existent

Answer

Probability of an impossible event is always zero (0)

21. Question

What is the probability that a non-leap year has 53 Sundays?

A. $\frac{6}{7}$

B. $\frac{1}{7}$

C. $\frac{5}{7}$

D. None of these

Answer

Total numbers of elementary events are: 7

Let E be the event of having 53 Sundays

Note- a non-leap year has 365 days or 52 weeks and 1 day. This one day could be any day amongst Sunday, Monday, Tuesday, Wednesday, Thursday, Friday or Saturday. Out of these favorable event is one; Sunday.

Number of favorable outcome is = 1

P (Sunday) = P (E) = 1/7

22. Question

Aarushi sold 100 lottery tickets in which 5 tickets carry prizes. If Priya purchased a ticket, what is the probability of Priya winning a prize?

A. $\frac{19}{20}$

B. $\frac{1}{25}$

C. $\frac{1}{20}$

D. $\frac{17}{20}$

Answer

Total numbers of elementary events are: 100

Let E be the event of winning a prize

Numbers of favorable outcomes are = 5

P (prize winning) = P (E) = 5/100 = 1/20

23. Question

A number is selected from first 50 natural numbers. What is the probability that it is a multiple of 3 or 5?

A. $\frac{13}{25}$

B. $\frac{21}{50}$

C. $\frac{12}{25}$

D. $\frac{23}{50}$

Answer

Total numbers of elementary event are: 50

Let E be the event of getting a multiple of 3 or 5

Favorable outcomes are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 5, 10, 20, 25, 35, 40, 50

Numbers of favorable outcomes are: 23

P (multiple of 3 or 5) = P (E) = 23/50

24. Question

Two numbers 'a' and 'b' are selected successively without replacement in that order from the integers 1 to 10. The probability that a/b is an integer, is

A. $\frac{17}{45}$

B. $\frac{1}{5}$

C. $\frac{17}{90}$

D. $\frac{8}{45}$

Answer

Total numbers of elementary events are: 10 x 9= 90

Being numbers are of non-replacing nature and order matters

Let E be the event of a/b as an integer

Favorable events are:

2/1, 3/1 , 4/1, 4/2, 5/1 , 6/1, 6/2 , 6/3 , 7/1 , 8/4 , 8/2 , 8/1, 9/3 , 9/1 , 10/5 , 10/2 , 10/1

Numbers of favorable outcomes are: 17/90

P (a/b is an integer) = P (E) = 17/90

25. Question

Two dice are rolled simultaneously. The probability that they show different faces is

A. $\frac{2}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{5}{6}$

Answer

Total numbers of elementary events are: 6 x 6 = 36

Let E be the event of getting of showing different faces on both the dice

Numbers of favorable events on first dice are: 6

Numbers of favorable events on second dice are: 5

Total numbers of favorable events are = 6 × 5 = 30

P (each dice showing different face) = 30/36 = 10/12 = 5/6

26. Question

What is the probability that a leap year has 52 Mondays?

A. $\frac{2}{7}$

B. $\frac{4}{7}$

C. $\frac{5}{7}$

D. $\frac{6}{7}$

Answer

Total numbers of elementary events are: 7

Let E be the event of having exactly 52 Mondays

Note- a leap year has 366 days or 52 weeks and 2 day. Two days could be any days amongst Sunday, Monday, Tuesday, Wednesday, Thursday, Friday or Saturday.

Favorable outcomes are: Tuesday, Wednesday, Thursday, Friday, Saturday,

Number of favorable outcome is = 5

P (52 Mondays) = P (E) = 5/7

27. Question

A month is selected at random in a year. The probability that it is March or October, is

A. $\frac{1}{12}$

B. $\frac{1}{6}$

C. $\frac{3}{4}$

D. None of these

Answer

Total numbers of elementary events are: 12

Let E be the event of having month as March or October

Numbers of favorable events are = 2

P (March or October) = P(E) = 2/12 = 1/6

28. Question

From the letters of the word "MOBILE", a letter is selected. The probability that the letter is a vowel, is

A. $\frac{1}{3}$

B. $\frac{3}{7}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

Answer

Total numbers of elementary events are: 6

Let E be the event of selecting a vowel

Favorable outcomes are: O, I, E

Numbers of favorable events are = 3

P (vowel) = P (E) = 3/6 = 1/2

29. Question

If a two digit number is chosen at random, then the probability that the number chosen is a multiple of 3, is

A. $\frac{3}{10}$

B. $\frac{29}{100}$

C. $\frac{1}{3}$

D. $\frac{7}{25}$

Answer

Total numbers of elementary event are: 90

Let E be the event of choosing a multiple of 3

Favorable outcomes are: 3× 4 =12 , 3×5= 15, 3×6= 18, 3×7= 21,….. 3×33= 99

Numbers of favorable outcomes are= 30

P(multiple of 3) = P (E) = 30/90 = 1/3

30. Question

Two dice are thrown together. The probability of getting the same number on both dice is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{12}$

Answer

Total numbers of elementary events are: 6x 6= 36

Let E be the event of getting same number on both dice together

Favorable outcomes are: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

Numbers of favorable outcomes are: 6

P(both same numbers) = P (E) = 6/36 = 1/6

31. Question

A die is thrown once. The probebility of getting a prime number is

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{6}$

Answer

Total numbers of elementary events are: 6

Let E be the event of getting a prime number

Favorable events are: 2, 3, 5

Numbers of favorable events are= 3

P (prime number) = P (E) = 3/6 = 1/2

32. Question

The probability of getting an even number, when a die is thrown once is